Research Article

A Two-Stage Chance Constrained Approach with Application to Stochastic Intermodal Service Network Design Problems

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1. Introduction

As a vital component of logistics and economy, intermodal freight transportation (IFT) facilitates international trade among most countries in the world. With the growth of international shipping, IFT is playing a more and more significant role in global transportation. Generally, the IFT is defined as the transportation of a load from its origin to its destination by a sequence of at least two different modes of transportation [1], where the load is transported in one and the same transportation unit [2]. Based on previous work [3, 4], the research issues on IFT can be summarised as five categories: intermodal transportation policy [3], intermodal network design [5], intermodal service network design [6], intermodal routing, and empty container reposition [7].

This paper focuses on the intermodal service network design and, more specifically, the stochastic intermodal service network design (SISND) in a sea-rail network, which is a type of service network design (SND). The SISND is defined as to determine the intermodal services, the specification of terminal operations, and the routing of container demands [6], which is associated with the tactical planning level [8]. Table 1 lists the key literature on service network design problems which consider stochastic characteristics.

According to the transportation mode considered, the literature on SND can be sorted into two categories, i.e., unimodal SND and intermodal SND. In the early development of unimodal SND [9], the SND model was probably first constructed for railway transportation, in which freight routing, block policy, makeup policy, and classification workload allocation were addressed simultaneously. Zhu et al. [8] proposed a two-layer space-time network to depict the operation and decision in a railway transportation system. The railway SND was formulated as a mixed integer programming model and then solved by a tabu search heuristic. Zhu [10] and Zhu et al. [11] extended this work and developed a
Table 1: Overview of service network design problems.

<table>
<thead>
<tr>
<th>Transportation mode</th>
<th>Stochastic travel time</th>
<th>Stochastic transfer time</th>
<th>Stochastic demand</th>
<th>Key references</th>
</tr>
</thead>
<tbody>
<tr>
<td>Railway</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Zhu et al. [8, 11]; Crainic et al. [9]; Zhu [10]; Pedersen et al. [12]; Andersen et al. [13]; Teypaz et al. [14]; Andersen et al. [15]; Chouman and Crainic [16]; Lai and Lo [17]; Meng and Wang [18]; Shintani et al. [19]; Huang et al. [20]</td>
</tr>
<tr>
<td>Maritime</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Lai and Lo [17]; Meng and Wang [18]; Shintani et al. [19]; Huang et al. [20]</td>
</tr>
<tr>
<td>Others</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Armacost et al. [21]</td>
</tr>
<tr>
<td>Railway</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Lium et al. [32, 33]</td>
</tr>
<tr>
<td>Maritime</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>An and Lo [34]</td>
</tr>
<tr>
<td>Others</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Bai et al. [38]</td>
</tr>
<tr>
<td>Intermodal</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Crainic and Rousseau [22]; Meng et al. [23]; Riessen et al. [24]; Andersen et al. [25]</td>
</tr>
<tr>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Hrušovský et al. [36]; Lanza [37]</td>
</tr>
<tr>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Demir et al. [35]</td>
</tr>
<tr>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>This work</td>
</tr>
</tbody>
</table>

three-layer space-time network to deal with scheduled SND for railway freight transportation. A metaheuristic integrating ellipsoidal search and slope scaling was introduced to solve the problem. Later on, some literature is concerned with SND taking into consideration the asset management [12–16] such as locomotives, railcars, cranes, and crews.

In addition to railway transportation, there are also articles concentrating on maritime and other SND. For example, Lai and Lo [17] studied ferry SND involving fleet size optimisation, routing, and scheduling. The model was formulated as a network flow problem with multiple origin-destination pairs, with the aim of balancing the operator cost and passenger cost. Meng and Wang [18] and Shintani et al. [19] both considered the liner shipping SND incorporating empty container reposition, while Huang et al. [20] presented a mixed integer linear programming model considering liner SND, fleet deployment, and empty container reposition. Additionally, Armacost et al. [21] focused on the express shipment SND in an overnight air network, which considered aircraft route design, aircraft assignment, and package routing. The model was tested on the UPS Next Day Air delivery network to demonstrate its performance.

Compared with the unimodal transportation, intermodal transportation has a large number of advantages such as faster transhipment, lower cost, increased flexibility, higher productivity, and improved safety [1]. As a result, intermodal SND has recently received more and more attention. The pioneering research of Crainic and Rousseau [22] established a general modelling framework for multimodal freight SND based on a network optimisation model. The problem was solved by decomposition and column generation principles. Meng et al. [23] presented a linear programming model to formulate the intermodal liner shipping SND in an inland and maritime network. The model considered the laden container and the empty container separately and captured several important issues including liner service design, laden container routing, and empty container reposition. Riessen et al. [24] proposed a model based on a path-based formulation and a minimum flow network formulation to combine the self-operated service and subcontracted service to address the intermodal SND within European gateway services network. Moreover, Andersen et al. [25] analysed the consequence of collaborating service synchronisation removing border operations and investigated a more comprehensive model which integrated SND, vehicle management, and fleet coordination.

All the literature mentioned above considers the deterministic SND. However, in practice, IFT is subject to a variety of uncertain factors. For example, Meng et al. [26] reviewed the research on containership routing and scheduling problems and indicated that there are too many uncertainties in containerised maritime transportation, such as container demand [27], port time [28], and travel time [29]. Yang et al. [30] constructed a weighted min-max chance constrained model to solve the train routing problem for achieving a minimal transportation cost, in which the demand, transportation cost, and transportation capacity were treated as fuzzy variables. Furthermore, Milenković and Bojić [31] investigated rail freight car fleet sizing problem by considering the fuzziness and randomness of freight demand. In the railway system, travel time of freight trains is frequently affected by passenger trains due to the relatively lower priority of freight trains, which contributes to uncertainty; meanwhile, the freight demand also fluctuates over space and time. Similarly, in the maritime system, stochasticity at sea and port poses a big challenge for liner shipping companies because of unexpected weather and variable operation efficiency.
Obviously, modelling these components by their expected values cannot capture the characteristics of real-life problems. In some cases, the optimal solution acquired under deterministic conditions may lead to a poor or even infeasible design, due to various stochastic factors. Therefore, it is essential to incorporate the stochasticity of freight demand, travel time, and terminals transfer time in the IFT SND. As a consequence, how to tackle the stochastic demand and time parameters (such as travel time and transfer time) has become one of the most significant challenges faced by freight companies. So far, research on stochastic SND is limited to Lium et al. [32, 33] for railway transportation, An and Lo [34] for maritime transportation, Demir et al. [35], Hrušovský et al. [36] and Lanza [37] for intermodal transportation, and Bai et al. [38] for other types of transportation. Specifically, Lium et al. [33] introduced the stochastic freight demand into SND formulation and investigated the difference between solutions under deterministic and stochastic conditions. An and Lo [34] established a model for ferry SND with uncertain demand under user equilibrium flows, in which regular and ad hoc services were taken into account. Demir et al. [35] developed a stochastic intermodal mixed integer programming model for the green intermodal SND with uncertain travel time and uncertain demand. The objective was to minimise the weighted sum of transportation cost, late delivery cost, and CO₂ emissions cost. Sample average approximation (SAA) method was used to solve this problem. For stochastic SND of other transportation types, Bai et al. [38] described a two-stage stochastic model for stochastic freight delivery SND with vehicle rerouting, in which the stochasticity of demand was captured.

SND is a typical NP-hard problem. Thus, highly efficient algorithms are needed for solving SISND and generating a practical transportation plan. There has been extensive research on various algorithms to solve stochastic SND. Hoff et al. [39] developed a metaheuristic based on neighbourhood search for stochastic SND by integrating exact and heuristic methods, while Crainic et al. [40] introduced a metaheuristic with the progressive hedging algorithm to divide their stochastic problem into several deterministic problems. Hrušovský et al. [36] proposed a hybrid methodology framework combining simulation and optimisation approaches. The methodology was implemented on real-life instances to illustrate its advantages. Although more time-consuming, the stochastic programming model can provide more flexibility and robustness for planners to deal with uncertain and fuzzy information. However, to the best of our knowledge, no research has considered stochastic travel time, stochastic transfer time, and stochastic container demand simultaneously in IFT.

The rest of this paper is structured as follows. Section 2 describes the SISND problem with stochastic travel time, transfer time, and container demand. Section 3 formulates this problem as a two-stage chance constrained programming problem. Section 4 presents the proposed solution algorithm involving SAA method and ant colony optimisation, while Section 5 implements the methodology on a real-life intermodal network and discusses the computational results. The conclusions are drawn in Section 6.

2. Problem Description

Based on the characteristics of sea-rail IFT system, the SISND problem in a sea-rail network is complicated in three aspects. First, compared with traditional freight transportation, the goods transported by containers is more time-sensitive and perishable. Hence, besides transportation cost, the SISND problem is also required to consider transportation time (e.g., delivery time at destinations), which may contribute to late delivery penalty cost. Second, both train and ship services have their service paths, capacities, operation costs, and travel times. Therefore, the coordination of individual rail and ship services has to be considered, which makes the SISND problem more complex. Third, stochastic times and demands may decrease the performance of a transportation plan and sometimes may even make it infeasible, which further increases the difficulties in achieving a robust transportation plan. In response to the complexities mentioned above, we introduce the SISND problem in this paper to minimise the expected total cost, by designing the optimal intermodal service and specifying services for each container demand from its origin to its destination, where stochastic time parameters and demands are considered.

To illustrate the problem, we first consider a simple sea-rail intermodal network with three railway stations A, B, and C, two intermodal hubs D and E where containers are transshipped from trains to ships, and one destination F, as shown in Figure 1.

![Figure 1: A simple sea-rail intermodal network.](image-url)
transportation is a type of consolidated based transportation, different train service combinations should be considered. According to the intermodal network in Figure 1, there are four potential train service designs and 108 possible intermodal routes for transporting these container demands, as shown in Table 2.

For freight companies, the estimated container demand is usually used to generate the transportation plan. However, it cannot reflect the variability of the real world. The fluctuation of container demand has a significant impact on routing container shipment and can even lead to an infeasible routing plan in some cases. In this case, the capacity chance constraints regarding such stochastic container demands are required. For instance, assuming that the container demands $p_1$ and $p_2$ both select Ship 1 and service BD, (1) imposes that the total container volume cannot exceed the capacity of service Ship 1 with the probability of at least $\varepsilon_1$,

$$\Pr \left[ q^{p_1} + q^{p_2} \leq z^{\text{Ship 1}} \cdot \text{cap}^{\text{Ship 1}} \right] \geq \varepsilon_1, \quad (1)$$

where $q^{p_1}$ and $q^{p_2}$ denote the volume of $p_1$ and $p_2$, respectively, $\text{cap}^{\text{Ship 1}}$ denotes the capacity of service Ship 1, and $z^{\text{Ship 1}}$ denotes the service frequency. Furthermore, the chance constraints with respect to arc capacity and node transfer capacity are also essential and shown in (2) and (3), respectively,

$$\Pr \left[ q^{p_1} + q^{p_2} \leq \text{cap}_{BD} \right] \geq \varepsilon_1, \quad (2)$$

$$\Pr \left[ q^{p_1} + q^{p_2} \leq \text{cap}_{BD} \right] \geq \varepsilon_1, \quad (3)$$

where $\text{cap}_{BD}$ and $\text{cap}_D$ denote the capacity of arc BD and the transfer capacity of node D, respectively. For each container demand, there is a due time at destination ports. Late delivery is allowed but will incur penalty. For example, we assume that the optimal intermodal route of $p_1$ is

$$A \rightarrow B \rightarrow D \rightarrow F. \quad (4)$$

Let $\tau^p_{\text{dtime}}$ denote the due time of $p_1$, $t^{AB}$ and $t^{BD}$ the travel time of train services AB and BD, respectively, $t^{\text{Ship 1}}$ the travel time of ship service Ship 1, and $t_B$ and $t_D$ the transfer time at nodes B and D, respectively. For container demand $p_1$, it needs to transfer from service AB to service BD, and then from service BD to service Ship 1. Thus, when the travel time and transfer time are both stochastic, the on-time delivery chance constraints have to be considered as well in (5), which requires $p_1$ arriving at the destination port with a probability of no less than $\varepsilon_2$,

$$\Pr \left[ t^{AB} + t_B + t^{BD} + t_D + t^{\text{Ship 1}} \leq \tau^p_{\text{dtime}} \right] \geq \varepsilon_2. \quad (5)$$

Based on the constraints introduced in (1), (2), (3), and (5), a two-stage chance constrained programming model for the SISND problem with random variables is constructed in Section 3. This SISND formulation is then solved in Section 4.

<table>
<thead>
<tr>
<th>Service design</th>
<th>Train services operated</th>
<th>Ship services operated</th>
<th>Intermodal routes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>AB, BD, AD AC, CE</td>
<td>Ship 1/2/3/4/5/6</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>AB, BD AC, CE</td>
<td>Ship 1/2/3/4/5/6</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>AC, CE, AE AB, BD</td>
<td>Ship 1/2/3/4/5/6</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>AC, CE AB, BD</td>
<td>Ship 1/2/3/4/5/6</td>
<td></td>
</tr>
</tbody>
</table>
3. A Two-Stage Chance Constrained SISND Problem with Stochastic Time and Demand Variables

In this section, we depict the two-stage chance constrained optimisation model for the SISND problem, which is used for the selection of intermodal services and route plans for container demands. Specifically, Section 3.1 defines the notations to be used in the remainder of the article, based on which Section 3.2 provides the formulation for the SISND problem.

3.1. Notations. This section lists notations used in Table 3, including indices, sets, input parameters, auxiliary parameters, and decision variables.

3.2. Mathematical Formulation. In this paper, we formulate the SISND problem in a sea-rail intermodal network as a two-stage chance constrained programming model, which makes service design decisions and a series of resource decisions to allocate container demands. The sea-rail intermodal network is represented by a directed graph \( G = (V, A) \), where \( V \) stands for the set of nodes and \( A \) the set of arcs.

Our problem is formulated based on the following assumptions.

Assumption 1. Each container demand can be transported by only one service path.

Assumption 2. All container demands can arrive later than the due times but will incur penalty cost which is proportional to the delay time and the demand volume.

Assumption 3. The railway transportation cost and travel time on arcs are proportional to the arc distance.

Assumption 4. Only direct train services are considered. Thus, container demands can be transported directly to their destinations by one direct service without reclassification at intermediate stations. Alternatively, container demands can also be sent by a sequence of direct services.

The objective function is the expected total cost which includes fixed cost, variable cost, transfer cost, and late delivery penalty cost.

(i) Fixed cost consists of crew cost, locomotives cost [8], administration cost [27], and other resources cost. It is formulated by (6), where the first term represents the fixed cost for operating train services while the second term for operating ship services:

\[
\sum_{s \in S_{train}} c_{train}^s x^s z^s + \sum_{s \in S_{ship}} c_{ship}^s x^s z^s
\]

(ii) Variable cost is relevant to the fuel consumption, infrastructure fees, etc. and is formulated in

\[
\sum_{a \in A} c_a y_a^p, \quad p \in P
\]

(iii) The transfer cost is made up of the unloading, transportation, and loading cost during the transfer process as given in

\[
\sum_{i \in V \setminus (V_d \cup V_d')} c_i y_i^p, \quad p \in P
\]

(iv) Late delivery penalty cost is incurred when the container demand does not arrive at the destination on time due to the stochasticity of travel time and transfer time. The penalty cost is proportional to the delay time as given in

\[
y^p \gamma \left( \sum_{r \in R} \sum_{a \in A} y_{train}^p \delta_{t a}^r + \sum_{i \in V \setminus (V_d \cup V_d')} y_i^p t_i \right)
\]

\[
+ \sum_{s \in S_{ship}} y_{ship}^p t_s - \sum_{s \in S_{train}} y_{train}^p t_s \right), \quad p \in P
\]

The total cost \( C^p \) for each unit of container demand is then the sum of variable cost, transfer cost, and late delivery penalty cost.

\[
C^p = \sum_{a \in A} c_a y_a^p + \sum_{i \in V \setminus (V_d \cup V_d')} c_i y_i^p
\]

\[
+ y^p \gamma \left( \sum_{r \in R} \sum_{a \in A} y_{train}^p \delta_{t a}^r + \sum_{i \in V \setminus (V_d \cup V_d')} y_i^p t_i \right)
\]

\[
+ \sum_{s \in S_{ship}} y_{ship}^p t_s - \sum_{s \in S_{train}} y_{train}^p t_s \right)
\]

The two-stage SISND problem is to minimise the expected total cost, where the first stage minimises the fixed cost to operate services and the second stage minimises the transportation cost for all container demands.

\[
P0
\]

\[
f^*(X, Y) = \min \sum_{s \in S_{train}} c_{train}^s x^s z^s + \sum_{s \in S_{ship}} c_{ship}^s x^s z^s + E [Q(X, Y, \xi)]
\]

where

\[
Q(X, Y, \xi) = \min \sum_{p \in P} q^p C^p
\]

subject to the following constraints.
### Table 3: Notations.

#### Indices and sets

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>Set of nodes</td>
</tr>
<tr>
<td>$V_o$</td>
<td>Set of origin railway stations, $V_o \subseteq V$</td>
</tr>
<tr>
<td>$V_h$</td>
<td>Set of intermodal transfer hubs, where containers are transferred from train services to ship services, $V_h \subseteq V$</td>
</tr>
<tr>
<td>$V_d$</td>
<td>Set of destinations including railway stations and destination ports, $V_d \subseteq V$</td>
</tr>
<tr>
<td>$A$</td>
<td>Set of arcs</td>
</tr>
<tr>
<td>$A_i^+$</td>
<td>Set of outward arcs of node $i$, $A_i^+ \subseteq A$</td>
</tr>
<tr>
<td>$A_i^-$</td>
<td>Set of inward arcs of node $i$, $A_i^- \subseteq A$</td>
</tr>
<tr>
<td>$P$</td>
<td>Set of container demands</td>
</tr>
<tr>
<td>$S$</td>
<td>Set of transportation services including train services and ship services</td>
</tr>
<tr>
<td>$S_i^+$</td>
<td>Set of services with origin node $i$, $S_i^+ \subseteq S$</td>
</tr>
<tr>
<td>$S_i^-$</td>
<td>Set of services with destination node $i$, $S_i^- \subseteq S$</td>
</tr>
<tr>
<td>$S_{\text{train}}$</td>
<td>Set of train services, $S_{\text{train}} \subseteq S$</td>
</tr>
<tr>
<td>$S_{\text{ship}}$</td>
<td>Set of ship services, $S_{\text{ship}} \subseteq S$</td>
</tr>
<tr>
<td>$o^p$</td>
<td>Origin of container demand $p$, $o^p \in V_o$</td>
</tr>
<tr>
<td>$d^p$</td>
<td>Destination of container demand $p$, $d^p \in V_d$</td>
</tr>
<tr>
<td>$i$</td>
<td>A node in sea-rail intermodal network, $i \in V$</td>
</tr>
<tr>
<td>$a$</td>
<td>A transportation link, $a \in A$</td>
</tr>
<tr>
<td>$p$</td>
<td>A container demand, $p \in P$</td>
</tr>
<tr>
<td>$r,s$</td>
<td>Two transportation services, $r,s \in S$</td>
</tr>
</tbody>
</table>

#### Input parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q^p$</td>
<td>The volume of container demand $p$ measured in Twenty-foot Equivalent Units, which is a random variable (TEUs)</td>
</tr>
<tr>
<td>$T_{\text{duetime}}^p$</td>
<td>Due time when container demand $p$ must arrive at its destination, i.e. the latest delivery time (hours)</td>
</tr>
<tr>
<td>$t^s$</td>
<td>Travel time of service $s$, which is a random variable (hours)</td>
</tr>
<tr>
<td>$t_s$</td>
<td>Travel time of arc $a$, which is a random variable (hours)</td>
</tr>
<tr>
<td>$t_i$</td>
<td>Transfer time at node $i$, which is a random variable (hours)</td>
</tr>
<tr>
<td>$c_{\text{train}}^s$</td>
<td>Fixed cost (locomotives, crew, etc.) for operating a train service $s \in S_{\text{train}}$ (if the service is provided) (US$/train service)</td>
</tr>
<tr>
<td>$c_{\text{ship}}^s$</td>
<td>Fixed cost for operating a ship service $s \in S_{\text{ship}}$ (US$/ship service)</td>
</tr>
<tr>
<td>$c_a$</td>
<td>Unit cost for transporting one container on arc $a$ (US$/TEU)</td>
</tr>
<tr>
<td>$c_i$</td>
<td>Unit transfer cost for loading, unloading and transporting operations between two consecutive services at node $i$ (US$/TEUs)</td>
</tr>
<tr>
<td>$c_{\text{penalty}}$</td>
<td>Unit penalty cost for late delivery (US$/TEU/day)</td>
</tr>
<tr>
<td>$c_{\text{cap}}$</td>
<td>Capacity of service $s$ (TEUs)</td>
</tr>
<tr>
<td>$c_{\text{cap}}p$</td>
<td>Capacity of arc $a$ (TEUs)</td>
</tr>
<tr>
<td>$c_{\text{cap}}p_i$</td>
<td>Transfer capacity of node $i$ (TEUs)</td>
</tr>
</tbody>
</table>

#### Auxiliary variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q^s$</td>
<td>The number of containers transported via service $s$ (TEUs)</td>
</tr>
<tr>
<td>$q_a$</td>
<td>The number of containers on arc $a$ (TEUs)</td>
</tr>
<tr>
<td>$\delta_a^s$</td>
<td>A binary variable, equal to 1 if arc $a$ is on the route of service $s$, and 0 otherwise</td>
</tr>
<tr>
<td>$\gamma^p$</td>
<td>A binary variable, equal to 1 if container demand $p$ is delayed, and 0 otherwise</td>
</tr>
<tr>
<td>$L$</td>
<td>A large enough number</td>
</tr>
<tr>
<td>$\xi$</td>
<td>A random vector including all random variables, $\xi = (q^p, t^s, t_p, p \in P, s \in S, i \in V)$</td>
</tr>
</tbody>
</table>

#### Decision variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^s$</td>
<td>A binary variable, equal to 1 if the service $s$ is operated; 0 otherwise</td>
</tr>
<tr>
<td>$y_{p,s}^{\text{train}}$</td>
<td>A binary variable, equal to 1 if the trains service $s \in S_{\text{train}}$ is used for container demand $p$; 0 otherwise</td>
</tr>
<tr>
<td>$y_{p,s}^{\text{ship}}$</td>
<td>A binary variable, equal to 1 if the ship service $s \in S_{\text{ship}}$ is used for container demand $p$; 0 otherwise</td>
</tr>
<tr>
<td>$y_p^{\text{train}}$</td>
<td>A binary variable, equal to 1 if the container demand $p$ is transported on arc $a$; 0 otherwise</td>
</tr>
<tr>
<td>$y_p^{\text{ship}}$</td>
<td>A binary variable, equal to 1 if the container demand $p$ is transferred at node $i$; 0 otherwise</td>
</tr>
<tr>
<td>$z^s$</td>
<td>Service frequency of service $s$, i.e. the number of service $s$ operated within the planning horizon</td>
</tr>
<tr>
<td>$X$</td>
<td>A vector that consists of all first stage decision variables, $X = (x^s, s \in S)$</td>
</tr>
<tr>
<td>$Y$</td>
<td>A vector that consists of all second stage decision variables, $Y = (y_{p,s}^{\text{train}}, y_{p,s}^{\text{ship}}, y_p^{\text{train}}, y_p^{\text{ship}}, z^s, i \in V, a \in A, p \in P, s \in S)$</td>
</tr>
</tbody>
</table>
(a) Flow Conservation Constraints

\[
\sum_{a \in A_i^p} y_a^p = \sum_{a \in A_i^p} y_a^s = 1, \quad \forall i = o^p, \ p \in P
\]  
(13)

\[
\sum_{a \in A_i^p} y_a^p = \sum_{a \in A_i^p} y_a^s = 0, \quad \forall i \in V \setminus (V_0 \cup V_d), \ p \in P
\]  
(14)

\[
\sum_{a \in A_i^p} y_a^p = \sum_{a \in A_i^p} y_a^s = -1, \quad \forall i = d^p, \ p \in P
\]  
(15)

\[
\sum_{p \in P} q_p y_{p}^{\text{train}} = q^t, \quad \forall s \in S_{\text{train}}
\]  
(16)

Equations (13)-(15) enforce flow conservation at origin, intermediate, and destination nodes, respectively. Equations (16) and (17) enforce flow conservation for train and ship services, respectively. Equation (18) enforces flow conservation for arcs.

(b) Capacity Chance Constraints

\[
\Pr \left\{ \sum_{p \in P} q_p y_{p}^{\text{train}} \leq z^t \cdot c^p \right\} \geq \varepsilon_1, \quad \forall s \in S_{\text{train}}
\]  
(19)

\[
\Pr \left\{ \sum_{p \in P} q_p y_{p}^{\text{ship}} \leq z^s \cdot c^p \right\} \geq \varepsilon_1, \quad \forall s \in S_{\text{ship}}
\]  
(20)

\[
\Pr \left\{ \sum_{p \in P} q_p y_a^p \leq c^a \right\} \geq \varepsilon_1, \quad \forall a \in A
\]  
(21)

\[
\Pr \left\{ \sum_{p \in P} q_p y_i^p \leq c^p \right\} \geq \varepsilon_1, \quad \forall i \in V \setminus (V_0 \cup V_d)
\]  
(22)

Equations (19) and (20) ensure that the flows via train and ship are within their capacities with a possibility of at least \(\varepsilon_1\). Similarly, (21) limits the flow on arcs, while (22) restricts transfer workload at intermediate nodes.

(c) On-Time Delivery Chance Constraints

\[
\Pr \left\{ \sum_{p \in P} y_{p}^{\text{train}} \cdot \delta_{t_i}^p + \sum_{i \in V \setminus (V_0 \cup V_d)} y_i^p \cdot t_i + \sum_{s \in S_{\text{ship}}} y_{p}^{\text{ship}} \cdot t_s \leq T_{\text{due time}}^p \right\} \geq \varepsilon_2, \quad \forall p \in P
\]  
(23)

Equation (23) ensures that each container demand can arrive at the destination port before its due time with a possibility of at least \(\varepsilon_2\).

(d) Decision Variables Constraints

\[
\sum_{s \in S_{\text{ship}}} y_{p}^{\text{ship}} = 1, \quad \forall p \in P
\]  
(24)

\[
\sum_{p \in P} y_{p}^{\text{train}} = 1, \quad \forall p \in P
\]  
(25)

\[
\sum_{p \in P} y_p^s = 1, \quad \forall p \in P
\]  
(26)

\[
y_{p}^{\text{train}} \leq x^t, \quad \forall p \in P, \ s \in S_{\text{train}}
\]  
(27)

\[
y_{p}^{\text{ship}} \leq x^s, \quad \forall p \in P, \ s \in S_{\text{ship}}
\]  
(28)

\[
y^t \leq x^t \cdot L, \quad \forall s \in S
\]  
(29)

\[
\sum_{s \in S_{\text{ship}}} y_{p}^{\text{ship}} \geq y_p^s, \quad \forall i \in V_h, \ p \in P
\]  
(30)

Equations (24)-(26) ensure that only one intermodal container route comprising several train services, one intermodal transfer hub, and one ship service can be selected to transport each container demand. Equations (27) and (28) specify that service \(s\) cannot be used to transport containers if it is not operated, while (29) represents that service \(s\) must be selected before allowing for its service frequency. Equation (30) enforces that the container demand cannot transfer at an intermodal transfer hub if the ship service departing from this hub is not operated.

4. The Solution Algorithm for the Two-Stage Chance Constrained SISND Problem

This section is dedicated to explaining the hybrid heuristic algorithm we propose for solving the aforementioned two-stage chance constrained SISND problem. The algorithm consists of two parts: (1) the SAA method for converting the stochastic problem to deterministic sample average approximation problems, by replacing the original distribution of random variables with an empirical distribution obtained from a random sample, and (2) the ant colony optimisation (ACO) algorithm for solving the converted problems.

4.1. Sample Average Approximation Method. Although chance constrained problems have been studied for almost 60 years, they are still difficult to solve numerically, even for simple problems. One reason is that the feasibility of a solution is hard to check because of the difficulty of computing chance constraints. The other reason is that the feasible region defined by chance constraints is not convex generally [41].

In the chance constrained problem (11)-(30), the expectation in the objective function and the chance constraints are very difficult to calculate, even for simple function forms. In this paper, we apply the SAA method to solve our SISND
problem with chance constraints, which is a mature approach to solve stochastic optimisation problems [42]. The SAA scheme approximates the expected objective function and chance constraints by the corresponding sample average function based on Monte Carlo simulation [43]. In detail, let \(\xi = (\xi^1, \xi^2, \ldots, \xi^N)\) be an independent sample which comprises \(N\) realisations of the random vector \(\xi\) according to the probability distributions of random variables, i.e.,

\[
\xi^n = [q^\text{p}(n), t^\text{p}(n), t_a(n), t_i(n)] \quad |p \in P, s \in S, a \in A, i \in V_n|, \quad n = 1, 2, \ldots, N
\]  

where \(q^\text{p}(n), t^\text{p}(n), t_a(n), t_i(n)\) are the values of all random variables; then \(E[Q(X, Y, \xi)]\) is approximated by \((1/N) \sum_{n=1}^{N} Q(X, Y, \xi^n)\). The chance constraints are also approximated in a similar way as follows. Denote function \(II(t)\) as

\[
II(t) = \begin{cases} 
1, & \text{if } t \geq 0 \\
0, & \text{if } t < 0 
\end{cases}
\]

Then the probability in (19) is approximated as

\[
\Pr \left\{ \sum_{p \in P} q^\text{p} y^\text{ps}_{\text{train}} \leq z^s c^\text{p} \right\} \approx 1 - \mu
\]

\[
\approx \frac{1}{N} \sum_{n=1}^{N} II \left( z^s c^\text{p} - \sum_{p \in P} q^\text{p}(n) y^\text{ps}_{\text{train}} \right)
\]

Thus, the two-stage chance constrained programming model \(\textbf{P0}\) can be converted to the following SAA problem \(\textbf{PI}\):

\[
f_{N^0}(X, Y) = \min \sum_{s \in S_{\text{train}}} c^\text{train}_s x^s + \sum_{s \in S_{\text{ship}}} c^\text{ship}_s x^s
\]

\[
+ \frac{1}{N} \sum_{n=1}^{N} Q(X, Y, \xi^n)
\]

subject to (13)-(18), (24)-(30), and (35)-(39):

\[
\frac{1}{N} \sum_{n=1}^{N} II \left( z^s c^\text{p} - \sum_{p \in P} q^\text{p}(n) y^\text{ps}_{\text{train}} \right) \geq \epsilon_1,
\]

\(\forall s \in S_{\text{train}}\) \quad (35)

\[
\frac{1}{N} \sum_{n=1}^{N} II \left( q^\text{p} c^\text{p} - \sum_{p \in P} q^\text{p}(n) y^\text{ps}_{\text{ship}} \right) \geq \epsilon_1, \quad \forall s \in S_{\text{ship}}
\]  

\[
\frac{1}{N} \sum_{n=1}^{N} II \left( c a_p - \sum_{p \in P} q^\text{p}(n) y^p_{a} \right) \geq \epsilon_1, \quad \forall a \in A
\]

By generating \(M\) independent samples, each containing \(N\) realisations of \(\xi\), we can formulate an associated SAA problem. By solving the SAA problem for each sample, we get their optimal solutions, denoted by \((X^m, Y^m), m = 1, 2, \ldots, M\), and treat them as candidate solutions for \(\textbf{P0}\). Without loss of generality, we assume that the corresponding optimal values of the objective function, denoted by \(f^1_{N^0}, f^2_{N^0}, \ldots, f^M_{N^0}\), are rearranged as \(f^1_{N^0} \leq f^2_{N^0} \leq \cdots \leq f^M_{N^0}\). Thus, \(f^1_{N^0}\) yields the lower bound of the objective function of \(\textbf{P0}\) suggested by Luedtke and Ahmed [44], which, when \(M \geq \log_2 (1/\mu), 0 < \mu < 1\), is valid with a confidence level \(1 - \mu\) in

\[
\Pr \left\{ f^*_{N^0} \leq f^* \right\} \geq 1 - \mu
\]

In addition, each candidate solution is checked by a posteriori analysis to see whether the constraints are satisfied [44]. Here we generate an independent test sample containing \(N'\) realisations of the random vector \(\xi\), i.e., \(\xi^1, \xi^2, \ldots, \xi^{N'}\). For all candidate solutions, the possibilities of chance constraints are recalculated by using the test sample, based on which feasible solutions to \(\textbf{P0}\) are derived. For any feasible solution \((\tilde{X}, \tilde{Y})\), the upper bound stated by Verweij et al. [42] for the optimal value \(f^*\) of \(\textbf{P0}\) can be estimated by

\[
\frac{1}{N'} \sum_{n=1}^{N'} Q(\tilde{X}, \tilde{Y}, \xi^n) + \frac{1}{N'} \sum_{n=1}^{N'} Q(\tilde{X}, \tilde{Y}, \xi^n)
\]

From the above \(M\) candidate solutions, we choose the one which is feasible for \(\textbf{P0}\) and has the smallest estimated objective value of \(\textbf{P0}\) as the optimal solution, denoted by \((X^*, Y^*)\). The quality of the optimal solution can be evaluated by the optimality gap (i.e., the difference between optimal value and lower bound) calculated in (42) as follows:

\[
f_{N^0}(X^*, Y^*) - f^1_{N^0}
\]

where \(f_{N^0}(X^*, Y^*)\) is recomputed by using the test sample with size \(N'\) and \(f^1_{N^0}\) provides a lower bound as mentioned above.

4.2. **Ant Colony Optimisation for SAA Problem.** The deterministic SAA problem \(\textbf{PI}\) converted from the SISND problem is still NP-hard. In this subsection, we employ ACO
algorithm to solve the SAA problems. ACO is a heuristic algorithm for solving combinatorial optimisation problems [45], which is first proposed by Dorigo et al. [46] and applied to the travelling salesman problems (TSP). Recently, ACO has been widely applied to different research fields such as vehicle routing [47], traffic signal plan [48], reactive power management [49], and economic dispatch [50]. The details about this algorithm are described as follows.

We put initial pheromone trails on each service. A probability function in (43) is defined to select the service to be operated:

\[ P_s^k = \frac{\tau_s^\alpha}{\sum_{r \in L(k)} \tau_r^\alpha} \]  

(43)

where \( P_s^k \) is the probability of operating service \( s \) by ant \( k \), \( L(k) \) the set of services not selected by ant \( k \), \( \alpha \) the parameter to regulate the influence of pheromone trail \( \tau_s \), and \( \tau_s \) the intensity of pheromone trail on service \( s \).

Similarly, intermodal container route of each demand is also constructed by ACO algorithm. After determining the services to operate, a probability function in (44) is defined to select the service used to transport container demand \( p \):

\[ p_{ps} = \frac{\tau_{ps}^\alpha \eta_{ps}^\beta}{\sum_{r \in L(k)} \tau_r^\alpha \eta_r^\beta} \]  

(44)

where \( p_{ps} \) is the probability of choosing service \( s \) to transport container demand \( p \) by ant \( k \), \( L(k) \) is the set of services not selected to transport container demand \( p \) by ant \( k \), \( \alpha \) is the parameter to regulate the influence of pheromone trail \( \tau_{ps} \), \( \beta \) is the parameter to regulate the influence of heuristic information \( \eta_{ps} \), \( \tau_{ps} \) is the intensity of pheromone trail on container demand \( p \) transported by service \( s \), and \( \eta_{ps} = 1/d_s \) is the heuristic information of container demand \( p \) transported by service \( s \), where \( d_s \) is the route cost of service \( s \).

In the process of searching the optimal solution, pheromone trails on services change dynamically iteration by iteration. Pheromone trails are updated based on evaporation rate and increase of pheromone trail as follows:

\[ \tau_s(T) = (1 - \rho) \tau_s(T - 1) + \Delta \tau_s(T - 1) \]  

(45)

\[ \tau_{ps}(T) = (1 - \rho) \tau_{ps}(T - 1) + \Delta \tau_{ps}(T - 1) \]  

(46)

where

\( \rho \in (0, 1) \) is pheromone trail evaporation rate

\( \tau_s(T) \) is pheromone trail on service \( s \) at iteration \( T \)

\( \Delta \tau_s(T - 1) \) is increase of pheromone trail on service \( s \) at iteration \( T - 1 \)

\( \tau_{ps}(T) \) is pheromone trail on container demand \( p \) transported by service \( s \) at iteration \( T - 1 \)

\( \Delta \tau_{ps}(T - 1) \) is increase of pheromone trail on container demand \( p \) transported by service \( s \) at iteration \( T - 1 \)

As given in (45), if service \( s \) is operated, the pheromone trail on this service is increased by \( \Delta \tau_s(T - 1) = \phi / f_k \), where \( \phi \) is a predefined coefficient to adjust the effect of increasing pheromone trail [51] and \( f_k \) is the total cost calculated by ant \( k \). Otherwise, if service \( s \) is not operated, the increased pheromone trail is zero. The way of updating pheromone trails in (46) is similar to that of (45).

4.3. A Hybrid Heuristic Algorithm. As a metaheuristic search method, ACO has a high efficiency in solving combinatorial optimisation problems. Hence, in this paper, the SAA method and ACO algorithm are integrated to develop a hybrid heuristic algorithm for solving the two-stage chance constrained programming model, where SAA is used to simulated stochastic travel time, transfer time, and container demand, and ACO is employed to yield the optimal service design and intermodal container routes. The procedure of the hybrid heuristic algorithm is illustrated in Figure 2.

5. Numerical Example

In this section, we use a practical sea-rail intermodal network to demonstrate the two-stage chance constrained programming model, and to assess the proposed hybrid heuristic algorithm for solving the SISND problem with stochastic time parameters and container demands. We also compare the results under deterministic and stochastic conditions in Section 5.1 and investigate the effect of stochastic factors on optimal solutions and the performance of the solutions in Section 5.2.

5.1. Case Study. The case study is on a realistic sea-rail intermodal network from China to Singapore. As depicted in Figure 3, this intermodal network comprises 17 railway stations, 1 destination port, and 2 intermodal transfer hubs where containers can be transhipped from train services to ship services.

It is assumed that 12 container demands need to be transported, including inland demands and container demands, and their details are given in Table 4. To transport these container demands, 42 train services and 6 ship services are available, which are listed in Tables S1-S2 in the Supplementary Materials. Each service is characterised by its origin, destination, service path, service distance, service time, fixed cost, and variable cost. Unit transfer cost \( c_i \) and unit penalty cost \( c_{penalty} \) are assumed to be 25 (US$/TEU) and 50 (US$/TEU/day), respectively. In addition, confidence levels \( \epsilon_1 \) and \( \epsilon_2 \) are both set as 0.9.

The heuristic algorithm incorporating SAA method and ACO is coded in MATLAB R2012a. The programme is carried out on a desktop PC with a core i5 2.50GHz processor and 4GB RAM.

We first test the case with deterministic parameters. The optimal operated services and intermodal container routes in this deterministic case are shown in Table 5, leading to a total system cost of $184,275.

We then test the case with stochastic parameters, where a multiplier \( \lambda \) is introduced to describe the variability of the
stochastic travel times of trains and ships. In this problem, the travel time $t_a$ on a railway arc $a$ follows a normal distribution, i.e., $t_a \sim N(\mu_a, \sigma_a)$ with $\mu_a$ the mean travel time and $\sigma_a = \lambda \sigma_a^0$ the standard deviation, where $\sigma_a^0 = 1 \text{ hour}$. Therefore, the total travel time of a train service also follows a normal distribution. Similarly, the travel time $t_s$ by ship service $s$ is also assumed to follow a normal distribution, i.e., $t_s \sim N(\mu_s, \sigma_s)$ with $\mu_s$ the mean value and $\sigma_s = \lambda \sigma_s^0$ the standard deviation, where $\sigma_s^0 = 1 \text{ day}$. In addition, the transfer time and container demand volume both follow uniform distributions, i.e., $t_i \sim U(m_i, n_i)$ and $q_p \sim U(m_p, n_p)$, where $n_i - m_i = \theta \Delta t$, $\Delta t = 1 \text{ hour}$, and $n_p - m_p = \omega \Delta q$, $\Delta q = 1 \text{ TEU}$. For algorithm parameters, we set $M = 20$, $N = 20$ and $N' = 1000$ [52]. Letting $\lambda = 6$, $\theta = 1$ and $\omega = 1$. In addition, the confidence levels are both assumed to be 0.9. The optimal operated services and intermodal container routes are solved and displayed in Table 6.

Comparing the results of the deterministic and stochastic cases in Tables 5 and 6, the stochastic travel time, transfer time, and container demand lead to not only a different service design, but also a different intermodal container route. The number of operated services in the stochastic case is 15, including 11 train services and 4 ship services, while in the deterministic case 8 train services and 4 ship services are operated. Besides the service design, the routes of some container demands also change: demands 6, 7, 10, and 12 change ships service, while demands 1, 2, 4, 6, and 7 change train services.
Table 4: The details of container demands.

<table>
<thead>
<tr>
<th>Container demand</th>
<th>Origin</th>
<th>Destination</th>
<th>Freight volume (TEUs)</th>
<th>Due time (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Xian</td>
<td>Singapore</td>
<td>19</td>
<td>354</td>
</tr>
<tr>
<td>2</td>
<td>Xian</td>
<td>Shijiazhuang</td>
<td>14</td>
<td>24</td>
</tr>
<tr>
<td>3</td>
<td>Xian</td>
<td>Taiyuan</td>
<td>11</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>Taiyuan</td>
<td>Singapore</td>
<td>16</td>
<td>334</td>
</tr>
<tr>
<td>5</td>
<td>Taiyuan</td>
<td>Shijiazhuang</td>
<td>13</td>
<td>24</td>
</tr>
<tr>
<td>6</td>
<td>Shijiazhuang</td>
<td>Singapore</td>
<td>18</td>
<td>338</td>
</tr>
<tr>
<td>7</td>
<td>Shijiazhuang</td>
<td>Hong Kong</td>
<td>22</td>
<td>346</td>
</tr>
<tr>
<td>8</td>
<td>Shijiazhuang</td>
<td>Shijiazhuang</td>
<td>24</td>
<td>346</td>
</tr>
<tr>
<td>9</td>
<td>Zhengzhou</td>
<td>Xuzhou</td>
<td>15</td>
<td>24</td>
</tr>
<tr>
<td>10</td>
<td>Xuzhou</td>
<td>Singapore</td>
<td>12</td>
<td>328</td>
</tr>
<tr>
<td>11</td>
<td>Xuzhou</td>
<td>Jinan</td>
<td>13</td>
<td>24</td>
</tr>
<tr>
<td>12</td>
<td>Jinan</td>
<td>Singapore</td>
<td>12</td>
<td>326</td>
</tr>
</tbody>
</table>

Table 5: The service and route choice of the deterministic case.

<table>
<thead>
<tr>
<th>Container demand</th>
<th>Services selected</th>
<th>Intermodal container route</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2,12,5,Ship 1</td>
<td>Xian → Houma → Taiyuan → Shijiazhuang → Bazhou → Tianjin → Ship 1</td>
</tr>
<tr>
<td>2</td>
<td>2,12</td>
<td>Xian → Houma → Taiyuan → Shijiazhuang</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>Xian → Houma → Taiyuan</td>
</tr>
<tr>
<td>4</td>
<td>3,Ship 3</td>
<td>Taiyuan → Shijiazhuang → Bazhou → Tianjin → Ship 3</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td>Taiyuan → Shijiazhuang</td>
</tr>
<tr>
<td>6</td>
<td>5,Ship 1</td>
<td>Shijiazhuang → Bazhou → Tianjin → Ship 1</td>
</tr>
<tr>
<td>7</td>
<td>30,11,Ship 4</td>
<td>Zhengzhou → Xian → Heze → Shijiazhuang → Bazhou → Tianjin → Ship 4</td>
</tr>
<tr>
<td>8</td>
<td>30</td>
<td>Zhengzhou → Xian → Heze → Yanzhou → Jinan</td>
</tr>
<tr>
<td>9</td>
<td>32</td>
<td>Zhengzhou → Shangqiu → Xuzhou</td>
</tr>
<tr>
<td>10</td>
<td>35,11,Ship 6</td>
<td>Xuzhou → Yanzhou → Jinan → Jiaozhou → Qingdao → Ship 6</td>
</tr>
<tr>
<td>11</td>
<td>35</td>
<td>Xuzhou → Yanzhou → Jinan</td>
</tr>
<tr>
<td>12</td>
<td>11,Ship 4</td>
<td>Jinan → Jiaozhou → Qingdao → Ship 4</td>
</tr>
</tbody>
</table>

Table 6: The service and route choice of the stochastic case.

<table>
<thead>
<tr>
<th>Container demand</th>
<th>Services selected</th>
<th>Intermodal container route</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7,5,Ship 1</td>
<td>Xian → Houma → Xian → Heze → Yanzhou → Jinan</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>Xian → Houma → Xian → Heze</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>Xian → Houma → Taiyuan</td>
</tr>
<tr>
<td>4</td>
<td>12,15,Ship 3</td>
<td>Taiyuan → Shijiazhuang → Hengdou → Bazhou → Tianjin → Ship 3</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td>Taiyuan → Shijiazhuang</td>
</tr>
<tr>
<td>6</td>
<td>23,11,Ship 5</td>
<td>Shijiazhuang → Hengdou → Dezhou → Jinan → Jiaozhou → Qingdao → Ship 5</td>
</tr>
<tr>
<td>7</td>
<td>30,11,Ship 5</td>
<td>Zhengzhou → Xian → Heze → Yanzhou → Jinan → Jiaozhou → Qingdao → Ship 5</td>
</tr>
<tr>
<td>8</td>
<td>30</td>
<td>Zhengzhou → Xian → Heze → Yanzhou → Jinan</td>
</tr>
<tr>
<td>9</td>
<td>32</td>
<td>Zhengzhou → Shangqiu → Xuzhou</td>
</tr>
<tr>
<td>10</td>
<td>35,36,Ship 1</td>
<td>Xuzhou → Yanzhou → Jinan → Dezhou → Tianjin → Ship 1</td>
</tr>
<tr>
<td>11</td>
<td>35</td>
<td>Xuzhou → Yanzhou → Jinan</td>
</tr>
<tr>
<td>12</td>
<td>36,Ship 2</td>
<td>Jinan → Dezhou → Tianjin → Ship 2</td>
</tr>
</tbody>
</table>
Table 7: Costs of the deterministic case and stochastic case.

<table>
<thead>
<tr>
<th>Case</th>
<th>Fixed cost (US$)</th>
<th>Variable cost (US$)</th>
<th>Transfer cost (US$)</th>
<th>Late delivery cost (US$)</th>
<th>Total cost (US$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic case</td>
<td>126,025</td>
<td>50,850</td>
<td>7,400</td>
<td>0</td>
<td>184,275</td>
</tr>
<tr>
<td>Stochastic case</td>
<td>147,175</td>
<td>56,582</td>
<td>4,787</td>
<td>8,258</td>
<td>216,802</td>
</tr>
</tbody>
</table>

Table 8: The optimal service design under different multiplier $\lambda$.

<table>
<thead>
<tr>
<th>Container demand</th>
<th>Service selected</th>
<th>Case 1 ($\lambda=0$)</th>
<th>Case 2 ($\lambda=1$)</th>
<th>Case 3 ($\lambda=2$)</th>
<th>Case 4 ($\lambda=3$)</th>
<th>Case 5 ($\lambda=4$)</th>
<th>Case 6 ($\lambda=5$)</th>
<th>Case 7 ($\lambda=6$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2,12,5,Ship 1</td>
<td>2,12,5,Ship 1</td>
<td>4,5,Ship 2</td>
<td>2,3,Ship 3</td>
<td>2,12,5,Ship 3</td>
<td>2,3,Ship 3</td>
<td>2,3,Ship 3</td>
<td>7,5,Ship 1</td>
</tr>
<tr>
<td>2</td>
<td>2,12</td>
<td>2,12</td>
<td>4</td>
<td>2,12</td>
<td>2,12</td>
<td>2,12</td>
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<td>3</td>
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<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
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<td>36,Ship 2</td>
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5.2. Sensitivity Analysis

5.2.1. Sensitivity to Stochastic Travel Time. In this subsection, we investigate the impact of travel time on the optimal solution of the two-stage chance constrained programming model. The services selected by each container demand under different multiplier $\lambda$ (i.e., different standard deviations $\sigma_a$ and $\sigma_e$) are calculated and displayed in Table 8. The confidence levels $\epsilon_1$ and $\epsilon_2$ are all the same. As shown in Table 8, the operated services and intermodal container routes are strongly affected by the stochastic travel time for each container demand. With the increase of multiplier $\lambda$, high travel time variability forces the demands to choose the routes with short delivery time. For example, when $\lambda = 0$ (i.e., no variability), container demand 1 chooses train services 2, 12, 5, and Ship 1 with a delivery time of 297.9 hours. In contrast, when $\lambda = 2$, it selects services 2, 3, and Ship 2 with a shorter delivery time of 268.9 hours. With respect to the service design, the train and ship services used also change with multiplier $\lambda$. In case 1, the operated services consist of 8 train services and 4 ship services, while 11 train services and 4 ship services are used in case 7.

As illustrated in Figure 4, the late delivery cost and total cost both rise considerably with $\lambda$, while other costs fluctuate within a certain range. This implies that stochastic travel time not only influences the service design and intermodal container routes, but also increases the operation cost such as late delivery cost and total cost. Meanwhile, the total delay time (i.e., the sum of delay time for each container demand) also grows obviously with $\lambda$ in Figure 4(f).

In addition, we explore the impact of travel time variability on punctuality, i.e., the percentage of on-time delivery. As illustrated in Figure 5, the punctuality drops with the increase of multiplier $\lambda$ for these seven solutions corresponding to different cases, which indicates that higher travel time variability can result in lower punctuality.

5.2.2. Sensitivity to Stochastic Transfer Time. In addition to stochastic travel time, the impact of stochastic transfer time on the design of intermodal service network is also examined under different variabilities of stochastic transfer time reflected by the value of $\theta$. As depicted in Figure 6, more operation cost is incurred by stochastic transfer time compared with the deterministic case. Specifically, late delivery cost, total cost, and total delay time ascend significantly with the increase of $\theta$, while fixed cost and variable cost also grow at a relatively slow rate with fluctuations. On the other hand, the transfer cost of stochastic case is less than that of deterministic case. That can be explained by the increasing number of operated services in the stochastic case, which results in less transferred containers and transfer cost.

5.2.3. Sensitivity to Stochastic Demand. This subsection conducts the sensitivity analysis of stochastic demand. In order to examine the impact of stochastic demand, we test the stochastic case with the same confidence levels under different multiplier $\omega$, i.e., different level of stochastic demand. The
Figure 4: The costs under different multiplier $\lambda$. (a) Fixed cost with multiplier $\lambda$. (b) Variable cost with multiplier $\lambda$. (c) Transfer cost with multiplier $\lambda$. (d) Late delivery cost with multiplier $\lambda$. (e) Total cost with multiplier $\lambda$. (f) Total delay time with multiplier $\lambda$.

Figure 5: The punctualities under different multiplier $\lambda$. 
total costs with the settings \( \omega = 1, 2, 3, 4, 5, 6 \) are investigated. As shown in Figure 7, the total cost obtained by the stochastic case is much more than that of the deterministic case and increases with the value of multiplier \( \omega \). This indicates that the accuracy of estimated container demand is extremely important for freight companies and more cost is required to maintain the same service level.

5.2.4. Sensitivity to Confidence Levels. In this subsection, we analysed the impact of two confidence levels on total
cost, i.e., the least possibility $\varepsilon_1$ for capacity restriction and the least possibility $\varepsilon_2$ for on-time delivery. We test the stochastic case by varying $\varepsilon_1$ and $\varepsilon_2$ from 0.5 to 0.9. It can be seen from Figure 8 that, with the increase of confidence level $\varepsilon_1$, the total cost fluctuates but presents an increasing tendency. Similarly, the total cost goes up with confidence level $\varepsilon_2$. Mathematically, the feasible region shrinks gradually with the rise of these two confidence levels. As a result, the solution with the minimal total cost cannot satisfy the chance constraints with high confidence levels and becomes infeasible. Therefore, one of suboptimal solutions with more total cost is selected as the optimal solution.

5.2.5. Sensitivity to Sample Size $N$. The sample size is a key factor which has an effect on the computational time and the quality of optimal solutions. Accordingly, we discuss the performance of SAA method with sample size $N = 20, 30, 40, 50, 60, 70, 80, 90, 100$. As shown in Table 9, the average optimal SAA value of $M$ candidate solutions decreases with sample size. Meanwhile, the computational
Table 9: The results of SAA with $M = 20$ and $N' = 1000$.

<table>
<thead>
<tr>
<th>Sample size $N$</th>
<th>Average value (US$)</th>
<th>Minimal value (US$)</th>
<th>Maximal value (US$)</th>
<th>Lower bound (US$)</th>
<th>Optimality gap (US$)</th>
<th>CPU time (s)</th>
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<tbody>
<tr>
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<td>229,281</td>
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time is increasing. In addition, we found that even if the sample size is relatively small, a good solution can be obtained.

However, the optimality gap does not present an obviously increasing or decreasing tendency but fluctuates with the sample size. Because the stopping criterion of the hybrid heuristic algorithm is based on the maximum iteration rather than the optimality gap, the algorithm stops the searching process when reaching the maximum iteration, even if the optimality gap is sometimes not small enough. As a result, the optimality gap is relatively high for some sample size $N$.

6. Conclusions

In this paper, the stochastic intermodal service network design (SISND) problem with stochastic travel time, transfer time, and container demand is formulated as a two-stage chance constrained programming model to minimise total cost in an intermodal sea-rail network. A hybrid heuristic algorithm incorporating SAA method and ACO algorithm is proposed to solve the SISND problem under capacity and on-time delivery chance constraints with predetermined confidence levels. A numerical example is conducted on an intermodal sea-rail network from China to Singapore to demonstrate the validity of the proposed model and the effectiveness of the proposed algorithm in solving the SISND problem. Sensitivity analysis is conducted to examine the impact of stochastic travel time on the route assigned for each intermodal container demand, operated services, late delivery cost, total cost, and punctuality.

The results reveal that stochasticity can result in different optimal operated services and intermodal container routes compared to the deterministic case. In addition, the stochasticity in travel time, transfer time, and container demand not only influences operation cost, but incurs late delivery cost and the change of service design. As the travel time variability increases, the late delivery cost and total cost grow, while the punctuality decreases. This implies that, due to higher travel time variability, higher operation cost is required to satisfy the chance constraints under the same confidence levels and to maintain a certain service level.

Future research can be developed in two directions. First, the formulation can be extended to integrate container routing, service design, and empty container reposition. Second, the proposed solution algorithm should be compared with other algorithms to further verify its performance. In addition, more efficient algorithms including exact algorithms, heuristic algorithms, and the combination of them need to be developed for large scale SISND problems.

Data Availability

The data used to support the findings of this study are included within the article and the supplementary information file.

Disclosure

The funder had no role in study design, data collection and analysis, decision to publish, or preparation of the manuscript.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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Supplementary Materials

S1: the lists of candidate services.docx: candidate train and ship services for the numerical example. (Supplementary Materials)

References


