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Research Article

Robust Train Scheduling Problem with Optimized Maintenance Planning on High-Speed Railway Corridors: The China Case

Chuntian Zhang,[1] Yuan Gao ,[1] Wenjie Li,[2] Lixing Yang ,[1] and Ziyou Gao[1]

[1]*State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University, Beijing 100044, China*
[2]*Sid and Reva Department of Civil, Environmental and Infrastructure Engineering, George Mason University, Fairfax, VA 22030, USA*

Correspondence should be addressed to Yuan Gao; gaoyuan@bjtu.edu.cn

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Simultaneously considering train scheduling problem and maintenance planning problem with uncertain travel time, we propose a two-stage integrated optimization model for the sunset-departure and sunrise-arrival trains (SDSA-trains). Specifically, in the first stage, we obtain an optimal solution of the SDSA-trains under each scenario, which leads to the minimum total travel time. In the second stage, a robust SDSA-train schedule is generated based on the optimal solutions of the first stage. The key is that we consider two operation modes to solve the conflict between the SDSA-trains and the maintenances. Some state variables are used to deal with train operation mode selection. Furthermore, some linearization techniques are used to formulate a mixed-integer linear programming (MILP) model. Finally, numerical experiments are implemented to prove the effectiveness of the proposed model and optimization method.

1. Introduction

Desirable train schedules are important to ensure the efficient operation of the high-speed railway system, but some uncertain events (e.g., bad weather conditions, equipment failure) often occur due to the complexity of the real-world conditions and affect the nominal train schedules. Therefore, finding a robust train schedule is very important for the high-speed railway system. Its robustness is not only related to the trains' anti-interference ability during its operation but also the efficiency of train operation. In general, the uncertain events will directly affect the travel time of each train, which makes the travel time uncertain on each section. Thus, an effective method to obtain a robust train schedule with the minimum deviation from the expected travel time to reduce the delay propagation is very crucial for train operatives and managers.

In addition to finding a robust train schedule, the railway infrastructure also should be in good condition to ensure that tracks are in appropriate state for trains. In order to ensure this condition, regular maintenance works (i.e., maintenance windows) on high-speed railway are executed at night in China. As is known, when regular maintenance is executed, the whole railway system does not work. Especially, high-speed railway in China is the longest and busiest high-speed railway system in the world. The regular maintenance works from 0 : 00 to 6 : 00 am for high-speed railway system become a hindrance of efficient operation. Moreover, at this situation, some long-distance passengers have to spend most of daytime in high-speed trains, which reduces their attractiveness to long-distance passengers.

In recent years, sunset-departure and sunrise-arrival trains (SDSA-trains) on high-speed railway have been introduced to attract more long-distance passengers. A SDSA-train departs from the origin station in the evening and arrives at the destination station in the next morning. The travel distance of the SDSA-train is usually between 1500 km and 2500 km. It should be mentioned that when traveling at night, a SDSA-train is not only a means of transport but also a hotel for passengers, which is more in accord with passenger travel behavior.

Obviously, there is a conflict between the SDSA-train schedules and maintenance window plans on high-speed railway. Therefore, maintenance window must be considered when generating a robust train schedule. By combining these aforementioned aspects, we intend to solve the problem in the following three aspects. First, in order to generate a robust train schedule, we propose a two-stage integrated optimization model. In the first stage, we generate an integrated optimization model of train scheduling and maintenance planning on high-speed railway corridors under each scenario. In the second stage, we formulate a biobjective optimization model to minimize total travel time of all SDSA-trains in the premise of meeting the robustness requirement. Note that the second stage also should satisfy the constraints established in the first stage. Second, we consider two operation modes to solve the conflict between the SDSA-train scheduling and maintenance planning, namely, (1) dwelling and waiting at high-speed railway station (Mode 1) and (2) switching from high-speed railway to *normal-speed* railway (Mode 2). Note that paralleling along each main high-speed railway corridor, there is an old railway corridor, which is referred to as *normal-speed* railway. In order to solve SDSA-train operation modes selection problems, we introduce some state variables and analyze the relationship among these variables. Third, in order to make the model be handled easily, some linearization techniques are used and nonlinear constraints and objective function are equivalently replaced by linear constraints and objective function. To verify the proposed integrated optimization model, we carry out a series of numerical examples. The computational result shows that the proposed model can be solved to optimality.

The rest of this paper is organized as follows. Section 2 reviews relevant literature on robust train scheduling and maintenance planning problems. Section 3 describes in detail the SDSA-trains' operation modes, when there is a conflict with maintenance window. In Section 4, a two-stage integrated optimization model is formulated to generate robust train scheduling. In Section 5, the proposed model is applied to the SDSA-trains on Beijing-Guangzhou high-speed railway corridor. The numerical experiments are implemented to verify the effectiveness and efficiency of the model. Finally, some conclusions and further works are discussed in Section 6.

2. Literature Review

2.1. Nominal and Robust Train Scheduling. Due to the importance of train schedule in the operation of railway systems, lots of researches have been conducted on train scheduling problem in the past decades. In general, there are mainly two study versions dealing with the train timetabling problem, *i.e.*, *nominal* and *robust* versions. Roughly speaking, the nominal version aims to optimize an objective function (*e.g.*, minimizing the passenger travel time, maximizing the passenger satisfaction) in the premise of satisfying the track capacity constraints, whereas the robust version mainly aims to reduce the delay propagation when there is a disturbance or disruption on the railway network.

In the past years, there have been many works which have treated the nominal version of the problem. Cacchiani et al. [1] pointed out that the nominal timetable problem can be studied mainly from two aspects of cyclic (periodic) timetable and noncyclic (nonperiodic) timetable. Nachtigall and Voget [2] generated a timetable with minimum passenger waiting times for periodically served railway networks. They proposed a method based on a genetic algorithm combined with a greedy heuristic and an optimization procedure to solve the problem. In order to achieve more potential for optimization, Liebchen and Möhring [3] integrated the decisions of network planning, line planning and vehicle scheduling into the task of periodic timetabling and showed how to extend the periodic event scheduling problem (PESP) model to consider this integration. Caimi et al. [4] extended the well-known periodic event scheduling problem (PESP) to the flexible periodic event scheduling problem (FPESP), allowing the departure and arrival times to be flexible time slots rather than the exact time. The computational results on real-world instance proved that the model was effective. Besides, Kroon and Peeters [5], Lindner [6] and Peeters and Kroon [7] formulated an integer linear programming (ILP) model to solve the periodic timetable problem. Some researchers, however, studied noncyclic timetabling problems. Generally speaking, noncyclic version had a greater advantage in a competitive market, which aimed to generate the optimal timetables by maximizing the overall profit. Brännlund et al. [8] proposed an ILP model to obtain a profit maximization timetable for a set of trains. They used a Lagrangian relaxation solution approach to solve the problem, where the track capacity constraints were relaxed. Caprara et al. [9] proposed a graph theoretical formulation to formulate an ILP model and aimed to maximize the sum of the profits of the scheduled trains. This formulation was relaxed in a Lagrangian way. The relaxed constraints were associated only with nodes of the aforementioned graph. Cacchiani et al. [10] considered the customary formulation of noncyclic train timetabling and aimed to maximize profits for compatible paths in a suitable graph. They analyzed the existing ILP models and proposed some new models to solve the problem. Cacchiani et al. [11] solved the nonperiodic version and the train platforming problem (TPP) from a tactical perspective. Cacchiani et al. [11] showed some standard but successful solution approaches based on integer programming. Through considering the dynamic characteristics of passenger flow, Shi et al. [12] proposed an effective method to simultaneously optimize the train timetable and accurate passenger flow control strategies on an oversaturated metro line.

Furthermore, finding robust version of the problem is a major practical issue that has received a lot of attention in recent years. Delays occurring at an operational level make nominal timetable infeasible, but the train schedules with better robustness can reduce the deviation degree between the train adjustment plan and the basic plan. There are many aspects of robust train timetabling problems to be considered. Kroon et al. [13] stressed that real-time railway operations were subject to stochastic disturbances. They proposed a stochastic optimization model to improve the robustness of a given cyclic railway timetable by allocating time supplements and buffer times in a given timetable. The experimental results showed that the average delays of trains can be significantly reduced by making relatively minor modifications to a given timetable. Khan and Zhou [14] considered various stochastic disturbances and developed a two-stage stochastic optimization model. They aimed to minimize the total planned trip time and reduce the expected schedule delay. Furthermore, they tested the effectiveness of the model on the real data of Beijing-Shanghai high-speed rail corridor in China. Cacchiani et al. [15] pointed out that Lagrangian heuristic was a very effective way to solve robust schedule issues. Besides, they showed how to modify the existing Lagrange heuristic to generate robust solutions. Shafia et al. [16] proposed a robust train timetabling model, which could deal with the disturbances existed among traveling times. They used a branch-and-bound (B&B) algorithm and a new heuristic beam search (BS) algorithm to solve the large-scale problems in a rational amount of time. Besides, Yang et al. [17, 18] studied robust train timetabling problems by considering the uncertainty of passenger. Yang et al. [17] studied the timetabling problem with fuzzy passenger demand. They considered the two objectives of fuzzy total passengers' time and total delay time. Yang et al. [18] regarded the number of passengers boarding/leaving each train at each station as a random variable. They formulated a 0-1 mixed-integer programming model and designed a branch-and-bound algorithm to solve the model.

2.2. Consideration of Train Scheduling and Maintenance Planning at the Same Time. In addition to the study of the railway timetable problem, railway infrastructure maintenance problem has also attracted a lot of attentions from researchers in the past twenty years. In the early research of railway maintenance plan, Higgins [19] aimed at determining the optimal maintenance activities and crews allocation on a single track line so as to minimize the round-trip scheduled trains and reduce completion time. The proposed nonlinear model was solved by a tabu search heuristic. Budai et al. [20, 21] also studied the railway infrastructure maintenance problem. Budai et al. [20] presented an optimization model to improve railway maintenance decisions. Some greedy heuristics were proposed to solve the model aiming to minimize possession costs. Later, genetic algorithms, memetic algorithms and the two-phase opportunity-based heuristic were proposed by Budai [21] to solve the preventive maintenance scheduling problem. Lidén and Joborn [22] established maintenance cost model, freight traffic cost model and passenger traffic cost model to evaluate the effects of maintenance windows on the maintenance costs and the train traffic and transportation costs. In addition, they also explored the effects of different maintenance window widths on maintenance costs and freight traffic costs. Zhou et al. [23] proposed a train-set circulation optimization model to minimize the total connection time and maintenance costs, and this model was solved by an efficient multiple-population genetic algorithm (MPGA). A realistic high-speed railway case was given to show the effectiveness of the proposed model and algorithm.

In recent five years, some researchers have begun to consider the train scheduling problem and maintenance planning problem at the same time. Aken et al. [24] studied the train timetable adjustment problem (TTAP) under the condition of maintenances. They proposed a mixed-integer linear programming model and used a row generation approach to generate an alternative timetable. The alternative timetable has the minimum deviation from the original timetable. Lidén and Joborn [25] presented a mixed-integer programming model for long term tactical plan, which simultaneously considered train services and railway network maintenances. The experimental results showed that the model could handle large-scale instances. Luan et al. [26] solved the problem of integrated optimization of train scheduling and preventive maintenance planning from a micro perspective. They proposed a novel integrated mixed-integer linear programming model by regarding a maintenance task as a virtual train, and the model was solved by a heuristic based on Lagrangian relaxation. Arenas et al. [27] presented a mixed-integer linear programming formulation that rearranged a timetable to cope with the capacity consumption produced by maintenance activities. They also considered a microscopic approach to solve the problem.

Note that Vansteenwegen et al. [28] took the influences of the planned infrastructure works into account and obtained a more robust timetable. They aimed to adjust the published timetable as small as possible in case of a planned infrastructure unavailable to keep the level of passenger service as high as possible, which was implemented by an iterative approach. The developed algorithm allowed small modifications to the routing and the published timetable; besides they improved the robustness of solutions. In this paper, we also consider the impact of maintenance planning on train scheduling and generate a robust train schedule by establishing a two-stage integrated optimization model.

3. Problem Description and Assumptions

3.1. Problem Description

3.1.1. Railway Corridors for SDSA-Trains. When solving the problem of SDSA-trains timetable optimization, the impact of high-speed railway maintenance window must be considered. The SDSA-train cannot pass directly when there is a maintenance window, so it has to wait at the high-speed railway station or switch to the normal-speed railway, as shown in Figure 1. *Case*1 represents SDSA-trains select wait at the high-speed railway station to avoid the conflict with the maintenance window, and *Case*2 represents SDSA-trains switch from high-speed railway to normal-speed railway when the next segment high-speed railway undergoes maintenance window. Note that the normal-speed railway is parallel to the corresponding high-speed railway. There is a physical connection between them at major stations. Besides, the maintenance window is not considered on normal-speed railway.

Figure 2 clearly describes the connecting relationship between high-speed railway and its parallel normal-speed railway. There are 3 major stations (*S*1, *S*2, and *S*3) belonging to both high-speed railway and its parallel normal-speed railway. It provide the possibility for SDSA-trains to switch from high-speed railway to normal-speed railway. Besides, only major stations can serve as origin stations and destination stations. Figure 2 shows that the 3 major stations divide the

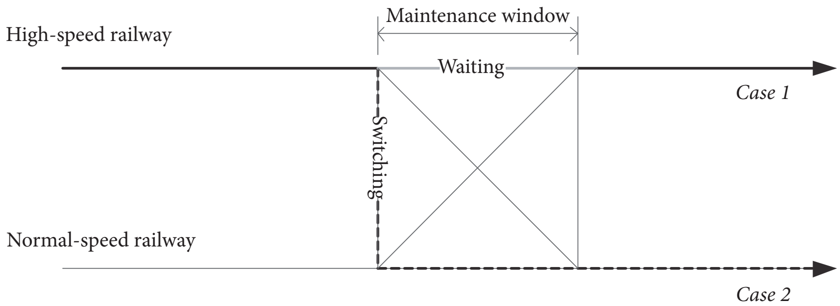


FIGURE 1: Waiting and switching of SDSA-trains to avoid the conflict with the maintenance window.

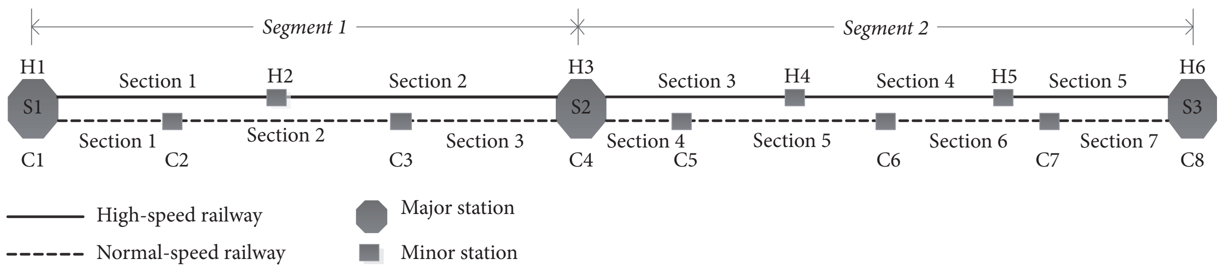
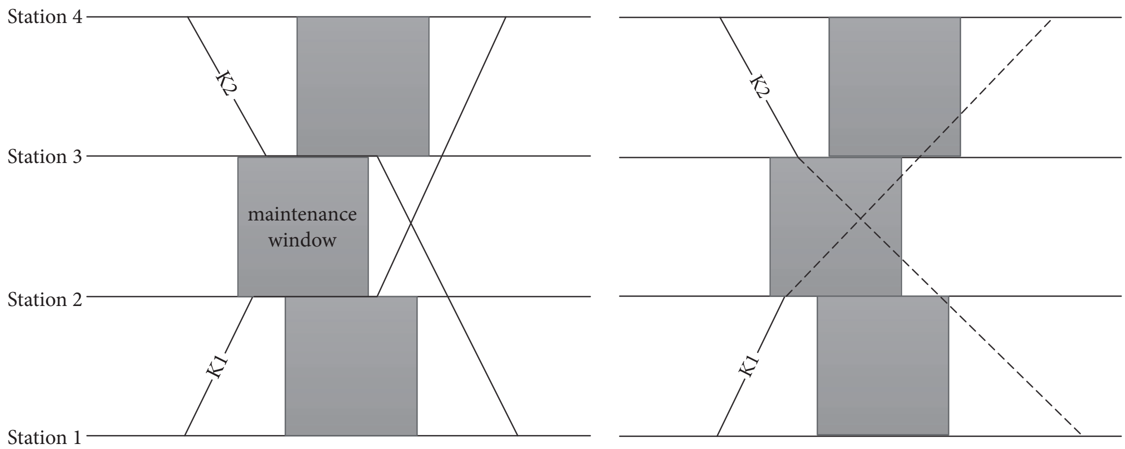


FIGURE 2: A high-speed railway corridor and its parallel normal-speed railway corridor.



(a) *Waiting*

(b) *Switching*

FIGURE 3: Illustration of SDSA-trains operation modes.

high-speed railway corridor into 2 segments, while all high-speed railway stations divide the high-speed railway corridor into 5 sections. Similarly, there are 2 segments and 7 sections on the normal-speed railway.

In this paper, we consider that the maintenance window on the whole line is asynchronous. It is synchronous on each segment. For example, Section 1 and Section 2 have the same maintenance window on high-speed railway, because they all belong to Segment 1. However, the maintenance windows on Segment 1 and Segment 2 are asynchronous.

3.1.2. Scheduling SDSA-Trains by Considering Conflicts with Maintenance Windows. Based on above analysis, there are two operation modes to be selected for SDSA-train when there is a conflict with maintenance window. Figure 3 shows these two operation modes (only major stations are considered) are simply named Mode 1 and Mode 2. Figure 3(a) shows Mode 1, which corresponds to *Case*1 in Figure 1. It means that the SDSA-train operates only on the high-speed railway; namely, when the next segment high-speed railway undergoes maintenance, the SDSA-train just waits at a major station until the maintenance is finished. For example, in Figure 3(a), train $K1$ waits for the second segment maintenance window at station 2 until the maintenance is finished. Figure 3(b) shows Mode 2, which corresponds to *Case*2 in Figure 1. A SDSA-train switches from high-speed railway to normal-speed railway at a proper major station if the next segment maintenance is executed. Then the SDSA-train runs on the normal-speed railway until it reaches its destination, seeing train $K1$ in Figure 3(b). The same analysis

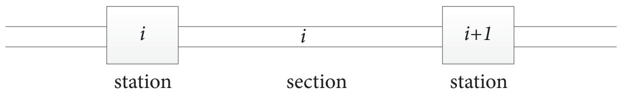


FIGURE 4: The relationship between section indexes and station indexes.

is also reasonable for the opposite train $K2$ in Figures 3(a) and 3(b).

3.2. Assumptions. To better illustrate the model, the following basic assumptions are made in this study:

Assumption 1. In order to ensure safety, maintenance on a segment cannot be separated. The widths of maintenance window are the same and predetermined on all segments.

Assumption 2. We do not consider the extra acceleration/deceleration times of SDSA-trains, because the consideration of acceleration/deceleration times does not make the problem be solved difficultly, but it makes the formula tedious.

Assumption 3. The detailed number of passengers boarding/alighting of each SDSA-train at each station are ignored, while the maximum capacity of each SDSA-train and the number of passenger demands at each major station are considered.

Assumption 4. The time cost of switching from high-speed railway to normal-speed at major stations is tiny compared to the total travel time. For simplicity, we assume that the time of switching is 0.

Assumption 5. From a safety perspective, when a train runs from origin station to destination station, the operation of switching from one railway line to another is allowed to occur at most once.

4. Mathematical Formulations

4.1. Notations and Parameters. For convenience, the notations used in this paper are listed in Table 1.

Remark 6. In Figure 2, we have $N_H = \{H1, H2, \cdots, H6\}$ and $N_C = \{C1, C2, \cdots, C8\}$. Since there are 3 major stations ($S1$, $S2$, and $S3$), the set of major stations is denoted by $N_m = \{S1, S2, S3\}$. Accordingly, mappings $H(i)$ and $C(i)$, $i \in N_m$, are defined as follows:

$$\begin{aligned} H(S1) &= H1, \\ H(S2) &= H3, \\ \text{and } H(S3) &= H6, \\ C(S1) &= C1, \\ C(S2) &= C4, \\ \text{and } C(S3) &= C8. \end{aligned} \quad (1)$$

Remark 7. Figure 4 shows the relationship between section indexes and station indexes.

4.2. Decision Variables. Note that this paper aims to formulate a two-stage integrated optimization model to schedule SDSA-trains and plan maintenance windows. Clearly, the set of decision variables include the start and end times of the maintenance windows and the arrival and departure times of the SDSA-trains, all of which are continuous variables. Moreover, some binary variables are used to describe stop pattern, departure orders of trains, and the operation modes selection of SDSA-trains. Table 2 lists all the decision variables and the corresponding definitions.

4.3. First Stage: Generation of Train Scheduling and Maintenance Planning under Each Scenario

4.3.1. Objective Function. Typically, the railway company aims to provide passengers with more convenient and quick train. Thus, in the first stage, we use objective function (2) to minimize the total travel time of SDSA-trains in order to achieve greater social benefits under different scenarios.

$$\min F_1 = \sum_{k=1}^{T_n} \left(a_{s,k,D(k)}^H - d_{s,k,O(k)}^H \right). \quad (2)$$

4.3.2. Constraints

(a) Basic Constraints of SDSA-Train

$$\sum_{k=1}^{T_n} x_{s,k,i}^H \cdot Cap_k \ge Q_i, \quad i \in N_m. \quad (3)$$

$$d_{k,O(k)}^- \le d_{s,k,O(k)}^H \le d_{k,O(k)}^+, \quad k \in T_n. \quad (4)$$

$$a_{k,D(k)}^- \le a_{s,k,D(k)}^H \le a_{k,D(k)}^+, \quad k \in T_n. \quad (5)$$

$$a_{s,k,H(i)}^H = a_{s,k,C(i)}^C, \quad k \in T_n,\ i \in N_m,\ H(i) \in N_H,\ C(i) \in N_C. \quad (6)$$

$$d_{s,k,H(i)}^H = d_{s,k,C(i)}^C, \quad k \in T_n,\ i \in N_m,\ H(i) \in N_H,\ C(i) \in N_C. \quad (7)$$

Table 1: Notations.

Notations	Definition
N	set of all stations.
N_H	set of high-speed railway stations.
N_C	set of normal-speed railway stations.
N_m	set of major stations.
i, j	index of stations.
$H(i)$	mapping from set N_m to set N_H, *i.e.*, $i \in N_m, H(i) \in N_H$.
$C(i)$	mapping from set N_m to set N_C, *i.e.*, $i \in N_m, C(i) \in N_C$.
T	set of all trains.
T_n	set of SDSA-trains.
T_h	set of existing high-speed trains.
T_c	set of existing normal-speed trains.
k, l	index of trains.
S	set of all scenarios.
s	index of scenarios.
$O(k)/D(k)$	origin/destination station of SDSA-train k.
$r^H_{i,min}/r^C_{i,min}$	minimum running time on section i on high-speed railway/normal-speed railway.
$r^H_{i,max}/r^C_{i,max}$	maximum running time on section i on high-speed railway/normal-speed railway.
$t^H_{s,k,i}/t^C_{s,k,i}$	running time of train k on section i on high-speed railway/normal-speed railway under scenario s.
h^{aH}_i	minimal headway between consecutive arrival at station i on high-speed railway.
h^{dH}_i	minimal headway between consecutive departure from station i on high-speed railway.
h^{aC}_i	minimal headway between consecutive arrival at station i on normal-speed railway.
h^{dC}_i	minimal headway between consecutive departure from station i on normal-speed railway.
τ^H_i	the minimum dwelling time of each train at station i on high-speed railway.
τ^C_i	the minimum dwelling time of each train at station i on normal-speed railway.
$[d^-_{k,O(k)}, d^+_{k,O(k)}]$	time window of SDSA-train k departure from origin station.
$[a^-_{k,D(k)}, a^+_{k,D(k)}]$	time window of SDSA-train k arrival at destination station.
$[w^-_i, w^+_i]$	time window of maintenance at high-speed railway station i.
Q_i	the total passenger demand at major station i on high-speed railway.
Cap_k	the maximum capacity of SDSA-train k.
D	the width of maintenance window on high-speed railway.
M	a large positive number.

Table 2: Decision variables.

Variables	Definition
$s^w_{s,i}/e^w_{s,i}$	actual start/end time of maintenance window at major station i on high-speed railway under scenario s, $s \in S, i \in N_m$.
$d^H_{s,k,i}/d^C_{s,k,i}$	departure time of train k on high-speed railway/normal-speed railway from station i under scenario s, $s \in S$.
$a^H_{s,k,i}/a^C_{s,k,i}$	arrival time of train k on high-speed railway/normal-speed railway at station i under scenario s, $s \in S$.
$x^H_{s,k,i}$	= 1 if train k stops at station i under scenario s; =0, otherwise. $s \in S, k \in T_n, i \in N_H$.
$x^C_{s,k,i}$	= 1 if train k stops at station i under scenario s; =0, otherwise. $s \in S, k \in T_n, i \in N_C$.
$O_{s,k,l,i}$	= 1 if train k departs from station i earlier than train l under scenario s; =0, otherwise. $s \in S$.
$\rho_{s,k,i}$	= 1 if train k runs on high-speed railway from major station i to major station $i+1$ under scenario s; =0, otherwise. $s \in S, k \in T_n, i \in N_m$.
$Z^{(0)}_{s,k,i}$	= 1 if train k waits at major station i when there is a conflict with maintenance under scenario s; =0, otherwise. $s \in S, k \in T_n, i \in N_m$.
$Z^{(1)}_{s,k,i}$	= 1 if train k switches from high-speed railway to normal-speed railway at major station i under scenario s; =0, otherwise. $s \in S, k \in T_n, i \in N_m$.

$$d_{s,k,i}^{H} - a_{s,k,i}^{H} \geq \tau_i^{H} \cdot x_{s,k,i}^{H}, \quad k \in T_n, \ i \in N_H. \tag{8}$$

$$x_{s,k,i}^{H} \leq \rho_{s,k,j}, \quad k \in T_n, \ i \in N_H, \ j \in N_m, \ i \in \{H(j), H(j)+1, \ldots, H(j+1)-1\}. \tag{9}$$

$$d_{s,k,i}^{C} - a_{s,k,i}^{C} \geq \tau_i^{C} \cdot x_{s,k,i}^{C}, \quad k \in T_n, \ i \in N_C. \tag{10}$$

$$x_{s,k,i}^{C} \leq 1 - \rho_{s,k,j}, \quad k \in T_n, \ i \in N_C, \ j \in N_m, \ i \in \{C(j), C(j)+1, \ldots, C(j+1)-1\}. \tag{11}$$

$$a_{s,k,i+1}^{H} - d_{s,k,i}^{H} = t_{s,k,i}^{H} \cdot \rho_{s,k,j} + \left(a_{s,k,i+1}^{C} - d_{s,k,i}^{C}\right) \cdot \left(1 - \rho_{s,k,j}\right), \quad i \in N_H, \ j \in N_m. \tag{12}$$

$$a_{s,k,i+1}^{C} - d_{s,k,i}^{C} = t_{s,k,i}^{C} \cdot \left(1 - \rho_{s,k,j}\right) + \left(a_{s,k,i+1}^{H} - d_{s,k,i}^{H}\right) \cdot \rho_{s,k,j}, \quad i \in N_C, \ j \in N_m. \tag{13}$$

$$d_{s,l,i}^{H} - d_{s,k,i}^{H} \geq h_i^{dH} - M \cdot \left(3 - O_{s,k,l,i} - \rho_{s,l,j} - \rho_{s,k,j}\right), \quad k, l \in T_n \cup T_h, \ i \in N_H, \ j \in N_m. \tag{14}$$

$$d_{s,k,i}^{H} - d_{s,l,i}^{H} \geq h_i^{dH} - M \cdot \left(3 - O_{s,l,k,i} - \rho_{s,l,j} - \rho_{s,k,j}\right), \quad k, l \in T_n \cup T_h, \ i \in N_H, \ j \in N_m. \tag{15}$$

$$a_{s,l,i}^{H} - a_{s,k,i}^{H} \geq h_i^{aH} - M \cdot \left(3 - O_{s,k,l,i-1} - \rho_{s,l,j} - \rho_{s,k,j}\right), \quad k, l \in T_n \cup T_h, \ i \in N_H, \ j \in N_m. \tag{16}$$

$$a_{s,k,i}^{H} - a_{s,l,i}^{H} \geq h_i^{aH} - M \cdot \left(3 - O_{s,l,k,i-1} - \rho_{s,l,j} - \rho_{s,k,j}\right), \quad k, l \in T_n \cup T_h, \ i \in N_H, \ j \in N_m. \tag{17}$$

$$d_{s,l,i}^{C} - d_{s,k,i}^{C} \geq h_i^{dC} - M \cdot \left(1 - O_{s,k,l,i} + \rho_{s,l,j} + \rho_{s,k,j}\right), \quad k, l \in T_n \cup T_c, \ i \in N_C, \ j \in N_m. \tag{18}$$

$$d_{s,k,i}^{C} - d_{s,l,i}^{C} \geq h_i^{dC} - M \cdot \left(1 - O_{s,l,k,i} + \rho_{s,l,j} + \rho_{s,k,j}\right), \quad k, l \in T_n \cup T_c, \ i \in N_C, \ j \in N_m. \tag{19}$$

$$a_{s,l,i}^{aC} - a_{s,k,i}^{C} \geq h_i^{aC} - M \cdot \left(1 - O_{s,k,l,i-1} + \rho_{s,l,j} + \rho_{s,k,j}\right), \quad k, l \in T_n \cup T_c, \ i \in N_C, \ j \in N_m. \tag{20}$$

$$a_{s,k,i}^{C} - a_{s,l,i}^{C} \geq h_i^{aC} - M \cdot \left(1 - O_{s,l,k,i-1} + \rho_{s,l,j} + \rho_{s,k,j}\right), \quad k, l \in T_n \cup T_c, \ i \in N_C, \ j \in N_m. \tag{21}$$

Constraint (3) is established to satisfy the passenger demands, which is the most important constraint for SDSA-trains stop plan. Constraint (4) is used to guarantee the departure time from origin station in a suitable time range to make SDSA-trains more attractive. The same applies to constraint (5). Constraints (6) and (7) are high-speed railway and normal-speed railway connection constraints at major stations. According to Assumption 4 and mappings $H(i)$ and $C(i)$, $i \in N_m$, it is equivalent to say that SDSA-train k "arrives at major station i," "arrives at high-speed railway station $H(i)$," or "arrives at normal-speed railway station $C(i)$." In other words, at major station i, we have $a_{s,k,H(i)}^{H} = a_{s,k,C(i)}^{C}$; that is, constraint (6) holds. Similarly, constraint (7) holds.

Constraints (8)-(11) are established to ensure that the dwelling time is long enough to complete passengers' boarding and alighting if the SDSA-train k stops at the station i. Constraints (8) and (10) are the basic dwelling time constraints. Constraints (9) and (11) are established for the operation of SDSA-trains switch from high-speed railway to normal-speed railway at major station. Obviously, for constraint (9), if the SDSA-train k runs on normal-speed railway (i.e., $\rho_{s,k,j} = 0$), then the SDSA-train k cannot dwell at high-speed railway station (i.e., $x_{s,k,i}^{H} = 0$); if the SDSA-train k runs on high-speed railway (i.e., $\rho_{s,k,j} = 1$), then the SDSA-train k may dwell at high-speed railway station (i.e., $x_{s,k,i}^{H} \leq 1$). Similarly, constraint (11) holds.

Constraint (12) expresses the running time between two consecutive high-speed railway stations. If $\rho_{k,j} = 1$, the running time of SDSA-train k from station i to station $i+1$ on high-speed railway under scenario s is equivalent to $t_{s,k,i}^{H}$; if $\rho_{k,j} = 0$, $a_{s,k,i+1}^{H} - d_{s,k,i}^{H} = a_{s,k,i+1}^{C} - d_{s,k,i}^{C}$ always holds. The same applies to constraint (13). Besides, constraints (14)-(21) are the necessary headway constraints (including the arrival headway constraints and departure headway constraints) to guarantee the safe operations on the high-speed railway corridor and normal-speed railway corridor.

(b) Additional Constraints of SDSA-Train When There Is a Conflict with Maintenance

$$w_i^{-} \leq s_{s,i}^{w} \leq e_{s,i}^{w} \leq w_i^{+}, \quad i \in N_m. \tag{22}$$

$$e_{s,i}^{w} = s_{s,i}^{w} + D, \quad i \in N_m. \tag{23}$$

$$Z_{s,k,i}^{(0)} + Z_{s,k,i}^{(1)} \geq 1, \quad \text{if } \rho_{s,k,i-1} = 1 \text{ and } s_{s,i}^{w} \leq a_{s,k,H(i)}^{H} < e_{s,i}^{w}, \ k \in T_n, \ i \in N_m. \tag{24}$$

$$Z_{s,k,i}^{(0)} + Z_{s,k,i}^{(1)} \geq 1, \quad \text{if } \rho_{s,k,i-1} = 1, \ a_{s,k,H(i)}^{H} < s_{s,i}^{w} \text{ and } a_{s,k,H(i+1)}^{H} > s_{s,i}^{w}, \ k \in T_n, \ i \in N_m. \tag{25}$$

$$d_{s,k,H(i)}^{H} \geq e_{s,i}^{w} \text{ and } \rho_{s,k,i} = 1, \quad \text{if } Z_{s,k,i}^{(0)} = 1, \ k \in T_n, \ i \in N_m. \tag{26}$$

$$\rho_{s,k,i} \leq \rho_{s,k,i-1}, \quad k \in T_n, \ i \in N_m. \tag{27}$$

$$Z_{s,k,i}^{(0)} + Z_{s,k,i}^{(1)} \leq 1, \quad k \in T_n, \ i \in N_m. \tag{28}$$

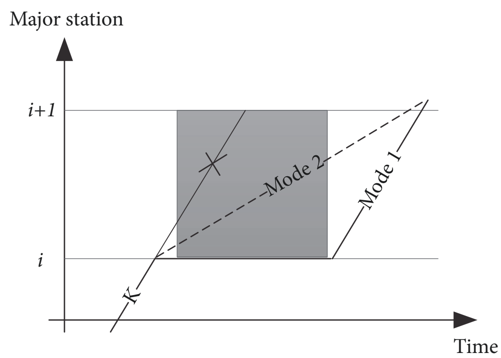


Figure 5: An illustration of constraint (25).

$$Z_{s,k,i}^{(0)} \leq \rho_{s,k,i-1} \text{ and } Z_{s,k,i}^{(0)} \leq \rho_{s,k,i}, \quad k \in T_n, \ i \in N_m \tag{29}$$

$$Z_{s,k,i}^{(1)} \geq \rho_{s,k,i-1} - \rho_{s,k,i}, \quad k \in T_n, \ i \in N_m. \tag{30}$$

$$\rho_{s,k,i} \leq 1 - Z_{s,k,i}^{(1)}, \quad k \in T_n, \ i \in N_m. \tag{31}$$

Constraint (22) ensures that maintenance starting time $s_{s,i}^{w}$ and ending time $e_{s,i}^{w}$ are in a reasonable time interval $[w_i^-, w_i^+]$. Constraint (23) means that the widths of maintenance window (*i.e.*, D) are the same and predetermined on each segment. Constraints (24)-(26) are the operation modes selection constraints. Constraint (24) indicates the fact that when SDSA-train k arrives at major station i, if the maintenance on segment from major station i to major station $i+1$ is being executed, then SDSA-train k must select an operation mode (i.e., Mode 1 or Mode 2) as shown in Figure 3. Constraint (25) indicates the fact that when SDSA-train k arrives at major station i, there is no conflict with the maintenance window, but it will conflict with the maintenance window at major station $i+1$ if SDSA-train k continues to run, which is illustrated in Figure 5. Then SDSA-train k must select Mode 1 or Mode 2, namely, waiting at major station i or switching to normal-speed railway corridor. Furthermore, if Mode 1 is selected for SDSA-train k to avoid the conflict with the maintenance window at major station i, then its waiting end time should satisfy constraint (26).

Constraints (27)-(31) are state-mode constraints. Obviously, when a SDSA-train conflicts with maintenance window at a major station, the railway line which it will run on may change. Binary variables $\rho_{s,k,i}$, $Z_{s,k,i}^{(0)}$, and $Z_{s,k,i}^{(1)}$ record the selection of SDSA-trains together. From a security perspective, we do not allow the SDSA-trains to switch back to high-speed railway when Mode 2 is selected, so constraint (27) is given. Constraint (28) indicates that at most one operation mode can be selected for each SDSA-train at major stations. Constraint (29) ensures that the SDSA-trains can only run on high-speed railway if Mode 1 is selected. Constraints (30)-(31) ensure that Mode 2 can be successfully selected. Obviously, only when $\rho_{s,k,i-1} = 1$ and $\rho_{s,k,i} = 0$ are satisfied at the same time will the operation of switching from high-speed railway to normal-speed railway happen. Moreover, when $\rho_{s,k,i-1}$ and $\rho_{s,k,i}$ take other values, constraint (30) always holds. Similarly, constraint (31) always holds.

4.4. Second Stage: Generation of Robust Train Scheduling and Maintenance Planning. With the train scheduling and maintenance planning generated in the first stage under different scenarios, the second stage aims to generate a robust train scheduling and maintenance planning. We aim to achieve the following two objectives in this stage, namely, (1) the total travel time of all SDSA-trains is minimized and (2) the generated train scheduling is more robust (i.e., the new train scheduling has the minimum deviation from the train scheduling under different scenarios). Thus, the objective function of the second stage is formulated as follows:

$$\begin{aligned} \min F_2 = {} & \omega_1 \cdot \sum_{k=1}^{T_n} \left(a_{k,D(k)}^{H} - d_{k,O(k)}^{H} \right) + \omega_2 \\ & \cdot \left(\sum_{s=1}^{S} \sum_{k=1}^{T_n} \left| \left(a_{k,D(k)}^{H} - d_{k,O(k)}^{H} \right) - \left(a_{s,k,D(k)}^{H} - d_{s,k,O(k)}^{H} \right) \right| \right). \end{aligned} \tag{32}$$

$d_{k,O(k)}^{H}$ is the departure time of train k on high-speed railway from origin station $O(k)$. $a_{k,D(k)}^{H}$ is the arrival time of train k on high-speed railway at destination station $D(k)$. These two variables are applicable to the optimization of the second stage.

The new train scheduling and maintenance planning also should satisfy the same constraints (3)-(31) set in the first stage to ensure safe operations of trains and meet the travel demands of passengers. Note that we consider the travel time as a decision variable in the second stage to formulate a biobjective optimization model, which is different from the first stage. The corresponding running time constraints of the second stage are shown as follows:

$$\begin{aligned} r_{i,min}^{H} - M \cdot \left(1 - \rho_{s,k,j}\right) & \leq a_{s,k,i+1}^{H} - d_{s,k,i}^{H} \\ & \leq r_{i,max}^{H} + M \cdot \left(1 - \rho_{s,k,j}\right), \quad i \in N_H, \ j \in N_m. \end{aligned} \tag{33}$$

$$\begin{aligned} r_{i,min}^{C} - M \cdot \rho_{s,k,j} \leq a_{s,k,i+1}^{C} - d_{s,k,i}^{C} \leq r_{i,max}^{C} + M \cdot \rho_{s,k,j}, \\ i \in N_C, \ j \in N_m. \end{aligned} \tag{34}$$

Based on the above analysis, the second-stage optimization model can be formulated as follows:

$$\begin{aligned} \min \quad & F_2 \\ \text{s.t.} \quad & \text{Constraints } (3)-(11), \text{constraints } (14) \\ & -(31), \text{and constraints } (33)-(34). \end{aligned} \tag{35}$$

4.5. Linearization. Note that constraints (12)-(13) and (24)-(26) and the second-stage objective function F_2 are nonlinear, so we will linearize them with some linearization techniques.

4.5.1. Linearize the Nonlinear Constraints. We find that constraints (12) and (13) are not easy to linearize, but we can replace them with the following two identical linearization formulas:

$$\begin{aligned} & t^H_{s,k,i} - M\cdot\left(1-\rho_{s,k,j}\right) \le a^H_{s,k,i+1} - d^H_{s,k,i} \\ & \quad \le t^H_{s,k,i} + M\cdot\left(1-\rho_{s,k,j}\right), \quad i\in N_H,\ j\in N_m. \\ & t^C_{s,k,i} - M\cdot\rho_{s,k,j} \le a^C_{s,k,i+1} - d^C_{s,k,i} \le t^C_{s,k,i} + M\cdot\rho_{s,k,j}, \\ & \qquad i\in N_C,\ j\in N_m. \end{aligned} \tag{36}$$

Some extra binary variables (i.e., α_1, α_2, β_1, and β_2) are used to linearize constraints (24)-(26) in the integrated optimization model. The following lemmas are easy to verify, so we do not give a detailed proof in this paper.

Lemma 8. *For $i \in N_m$, $H(i) \in N_H$, $i \ge 2$, and $k \in T_n$, constraint (24) can be equivalently replaced by the following linear constraint:*

$$Z^{(0)}_{s,k,i} + Z^{(1)}_{s,k,i} \ge -2 + \alpha_1 + \alpha_2 + \rho_{s,k,i-1}, \tag{37}$$

where α_1 is a binary variable, indicating the order relation between $a^H_{s,k,H(i)}$ and $s^w_{s,i}$. Similarly, α_2 is a binary variable, indicating the order relation between $a^H_{s,k,H(i)}$ and $e^w_{s,i}$, i.e.,

$$\begin{aligned} \alpha_1 &= \begin{cases} 1, & if\ a^H_{s,k,H(i)} \ge s^w_{s,i} \\ 0, & if\ a^H_{s,k,H(i)} < s^w_{s,i} \end{cases} \\ \alpha_2 &= \begin{cases} 1, & if\ a^H_{s,k,H(i)} < e^w_{s,i} \\ 0, & if\ a^H_{s,k,H(i)} \ge e^w_{s,i} \end{cases} \end{aligned} \tag{38}$$

It is easy to verify that definitions of α_1 and α_2 can be equivalently transformed to the following linear and binary constraints:

$$\begin{aligned} \alpha_1 &> \frac{a^H_{s,k,H(i)} - s^w_{s,i}}{M}, \\ \alpha_1 &\le \frac{a^H_{s,k,H(i)} - s^w_{s,i}}{M} + 1; \\ \alpha_2 &\ge \frac{e^w_{s,i} - a^H_{s,k,H(i)}}{M}, \\ \alpha_2 &< \frac{e^w_{s,i} - a^H_{s,k,H(i)}}{M} + 1; \\ & \qquad \alpha_1, \alpha_2 \in \{0,1\}, \end{aligned} \tag{39}$$

where M is a sufficiently large positive value.

Similarly, constraint (25) can be linearized according to the following lemma.

Lemma 9. *For $i \in N_m$, $H(i) \in N_H$, $i \ge 2$, and $k \in T_n$, constraint (25) can be equivalently replaced by the following linear constraint:*

$$Z^{(0)}_{s,k,i} + Z^{(1)}_{s,k,i} \ge -2 + \beta_1 + \beta_2 + \rho_{s,k,i-1}, \tag{40}$$

where β_1 is a binary variable, indicating the order relation between $a^H_{s,k,H(i)}$ and $s^w_{s,i}$. Similarly, β_2 is a binary variable, indicating the order relation between $a^H_{s,k,H(i+1)}$ and $s^w_{s,i}$.

Further, constraint (26) can be linearized directly without introducing extra additional variables, and it can be linearized according to the following lemma.

Lemma 10. *For $i \in N_m$, $H(i) \in N_H$, and $k \in T_n$, constraint (26) can be equivalently replaced by the following linear constraints:*

$$d^H_{s,k,H(i)} \ge e^w_{s,i} - M\cdot\left(1 - Z^{(0)}_{s,k,i}\right) - M\cdot\left(1-\rho_{s,k,i}\right). \tag{41}$$

4.5.2. Linearize the Nonlinear Objective Function. Different from the nonlinear constraints, the nonlinear objective function F_2 is caused by the absolute value. In order to linearize it, we use the following lemma.

Lemma 11. *Assume that $\mathbf{X}$ is the feasible set and $v_0(x)$ and $v_1(x)$ are linear functions of $x \in \mathbf{X}$.*

Model

$$\begin{aligned} \max_x \quad & v_0(x) + \left|v_1(x)\right| \\ \text{s.t.} \quad & \mathbf{x}\in\mathbf{X}, \end{aligned} \tag{42}$$

is equivalent to the following model with linear objective function:

$$\begin{aligned} \max_{x,C_0,C_1} \quad & C_0 + C_1 \\ \text{s.t.} \quad & C_0 = v_0(x), \\ & C_1 \ge v_1(x), \\ & C_1 \ge -v_1(x), \\ & \mathbf{x}\in\mathbf{X}. \end{aligned} \tag{43}$$

The proof of Lemma 11 is simple. According to Lemma 11, the second-stage objective function F_2 of model (32) is reformulated as

$$\min \widetilde{F_2} = \sum_{k=1}^{T_n}\left(\omega_1\cdot C^{tra}_k + \omega_2\cdot\sum_{s=1}^{S} C^{tra}_{s,k}\right). \tag{44}$$

where C^{tra}_k and $C^{tra}_{s,k}$ satisfy the following constraints:

$$C^{tra}_k = a^H_{k,D(k)} - d^H_{k,O(k)}, \quad k\in T_n \tag{45}$$

$$\begin{aligned} C^{tra}_{s,k} &\ge \left(a^H_{k,D(k)} - d^H_{k,O(k)}\right) - \left(a^H_{s,k,D(k)} - d^H_{s,k,O(k)}\right), \\ & \qquad k\in T_n,\ s\in S \end{aligned} \tag{46}$$

$$\begin{aligned} C^{tra}_{s,k} &\ge -\left(\left(a^H_{k,D(k)} - d^H_{k,O(k)}\right) - \left(a^H_{s,k,D(k)} - d^H_{s,k,O(k)}\right)\right), \\ & \qquad k\in T_n,\ s\in S. \end{aligned} \tag{47}$$

Obviously, the second-stage objective function F_2 can be equivalently replaced by $\widetilde{F}_2$. This stage model also should satisfy constraints (45)-(47) in addition to the constraints contained in model (35).

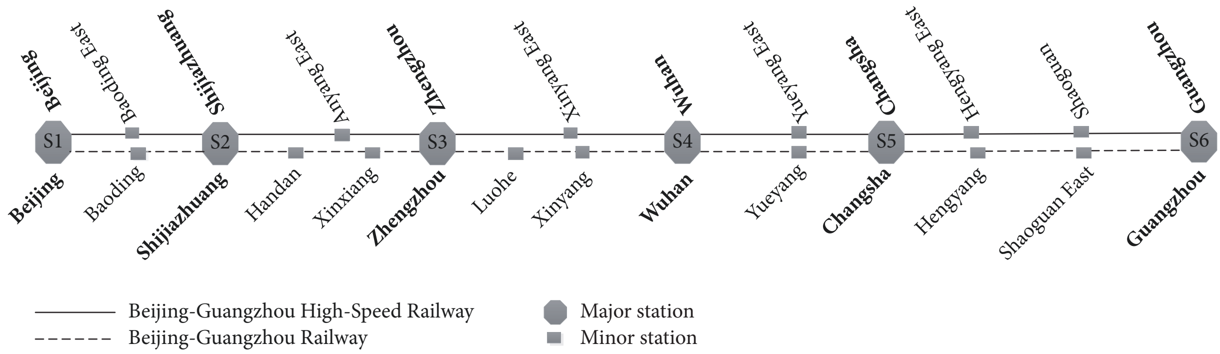


Figure 6: Topological relations of Beijing-Guangzhou High-Speed Railway and Beijing-Guangzhou Railway.

5. Numerical Experiments

In this section, we carry out computational experiment based on the Beijing-Guangzhou High-Speed Railway and the Beijing-Guangzhou Railway in China to show the effectiveness and efficiency of our proposed model. All the experiments are carried out on a computer with Intel(R) Core(TM) i5-3337U@1.80GHz CPU and 8.00GB RAM using Microsoft Windows 7(64bit) OS. The programming is coded in MATLAB 2014b. All MILP models are solved by CPLEX 12.5 to optimality with a gap of 0.01%.

5.1. Setup. In this experiment, we plan to insert 10 SDSA-trains (5 downstream and 5 upstream) into the existing timetable under each scenario. Besides, we will consider the double track Beijing-Guangzhou High-Speed Railway and the Beijing-Guangzhou Railway. Beijing-Guangzhou High-Speed Railway is the longest high-speed railway line in the world. It consists of 36 stations with a total length 2, 298 km. However, some of the stations are very small. In order to make the calculation simpler and without loss of generality, we only select 12 typical stations to carry out numerical experiments. Similarly, we also select 14 typical stations on Beijing-Guangzhou Railway to carry out numerical experiments. The topological relations between the two railway lines are shown in Figure 6.

Obviously, there are 6 major stations in the selected corridor, which divide Beijing-Guangzhou Railway corridor into 5 segments. According to Section 3, we know that there are 5 different maintenance works on the high-speed railway corridor. Besides, the Beijing-Guangzhou High-Speed Railway is divided into 11 sections and the Beijing-Guangzhou Railway is divided into 13 sections, depending on all the selected stations. For more clarity, Tables 3 and 4 are given to show the distances and running times of each section. Note that the minimum and maximum running times of each section are listed. For example, in the first row of Table 3, (37, 49) means that the minimum running time of section 1 is 37 minutes and the maximum running time of section 1 is 49 minutes.

In order to keep a safe distance of consecutive trains, the minimum arrival headway and departure headway should be set in advance. On the high-speed railway, at minor stations, the minimum arrival headway is set to 2 minutes and the minimum departure headway is set to 3 minutes, i.e., h_i^{aH}=2 and h_i^{dH}=3; at major stations, h_i^{aH}=3 and h_i^{dH}=5. Similarly, on the normal-speed railway, at minor stations, we set h_i^{aC}=3 and h_i^{dC}=4; at major stations, we set h_i^{aC}=4 and h_i^{dC}=5. The minimum dwelling time should be set, and we have $\tau_i^H = 2$ and $\tau_i^C = 3$. Besides, we are interested in giving the passenger demand at each major station, which determines the SDSA-train stop plan to some extent. The downstream passenger demands at major stations are 1000, 550, 600, 600, 650, and 800; the upstream passenger demands at major stations are 800, 650, 600, 600, 550, and 1000. Though, this may have a little difference from the actual situation, and the capacity of the SDSA-train at each station is still set to 630, *i.e.,*Cap_k = 630.

The original timetable we use is the actual timetable on April 15st, 2017. Note that we only consider the existing trains that may have conflict with SDSA-trains. After processing, 73 downstream existing trains and 58 upstream existing trains on Beijing-Guangzhou High-Speed Railway remained; 32 downstream existing trains and 23 upstream existing trains on Beijing-Guangzhou Railway remained.

The numerical experiments are divided into two stages. In the first stage, we generate 10 different train schedules and maintenance plans under different scenarios. In the second stage, a robust train scheduling is designed based on the first stage.

5.2. The Numerical Experiments of the First Stage. In the first stage, we generate 10 different scenarios. Table 5 clearly shows some related parameters, "Dep." represents departure time window, and "Arr." represents arrival time window. Because a SDSA-train departs from its origin station on the first night and arrives at its destination station on the next morning, we simply set the start of the second day as 1440. In addition, we assume that the width of maintenance window on each segment is 240 minutes, *i.e.,* $D = 240$; the time window of maintenance at each station is from 0: 00 to 6: 00 am, i.e., $[w_i^-, w_i^+] = [1440, 1800]$. Note that the above parameters are the same under each scenario.

Further, we carry out numerical experiments based on the different disturbance times obtained in 10 different scenarios.

Table 3: Information of distance and running time on the high-speed railway.

Downstream (Beijing ⟶ Guangzhou)			Upstream (Guangzhou ⟶ Beijing)		
Section	Distance (km)	Running time (min)	Section	Distance (km)	Running time (min)
1	139	(35, 49)	**1**	227	(38, 50)
2	142	(33, 41)	**2**	303	(61, 69)
3	235	(50, 68)	**3**	177	(40, 46)
4	177	(39, 50)	**4**	147	(31, 39)
5	337	(70, 86)	**5**	215	(46, 56)
6	199	(39, 50)	**6**	199	(39, 46)
7	215	(45, 60)	**7**	337	(66, 86)
8	147	(30, 42)	**8**	177	(34, 48)
9	177	(34, 42)	**9**	235	(45, 63)
10	303	(56, 67)	**10**	142	(28, 39)
11	227	(49, 63)	**11**	139	(28, 46)

Table 4: Information of distance and running time on the normal-speed railway.

Downstream (Beijing ⟶ Guangzhou)			Upstream (Guangzhou ⟶ Beijing)		
Section	Distance (km)	Running time (min)	Section	Distance (km)	Running time (min)
1	150	(72, 97)	**1**	221	(133, 143)
2	131	(64, 78)	**2**	300	(186, 204)
3	165	(79, 98)	**3**	186	(114, 137)
4	157	(77, 122)	**4**	147	(76, 84)
5	90	(43, 52)	**5**	215	(117, 136)
6	140	(64, 85)	**6**	234	(119, 154)
7	162	(78, 97)	**7**	162	(80, 101)
8	234	(116, 139)	**8**	140	(69, 115)
9	215	(115, 142)	**9**	90	(42, 56)
10	147	(78, 88)	**10**	157	(75, 98)
11	186	(113, 128)	**11**	165	(79, 95)
12	300	(186, 207)	**12**	131	(69, 76)
13	221	(138, 180)	**13**	150	(82, 105)

Each scenario is an assumption of a set of running time of the trains on the sections. In order to consider all possible disturbances more comprehensively, the value of the running time of different scenarios should include all cases between the minimum running time and the maximum running time as much as possible. We first get the minimum running time and the maximum running time of the section by processing the obtained actual timetable as shown in Tables 3 and 4. Secondly, the running time of each section is divided into 10 statistical intervals according to the increasing order of running time. Finally, different scenarios are generated based on the 10 statistical intervals. To more clearly illustrate the method of generating the scenarios, we give an example of how to make the parameter value of different scenarios in the downstream of the high-speed railway, as shown in Table 6.

The value of the parameter R_s is the same under each scenario s, as shown in Table 6. That is to say, the scenarios parameters are generated randomly only once for each section of the railway under each scenario. Note that the generated scenarios parameters not only apply to the high-speed railway downstream but also the high-speed railway upstream, the normal-speed railway downstream, and the normal-speed railway upstream.

Table 7 shows the objectives and computation times of the experiments of each scenario. Each scenario involves 12636 decision variables and 52392 constraints.

To further analyze the influence of scenarios parameters on the objectives, Figure 7 is given.

Figure 7 presents the change of the objectives under different scenarios parameters. It is reasonable to see that the objective function value increases with the increase of the scenarios parameter values. For example, the parameter value generated in scenario 1 is the smallest in all 10 scenarios, corresponding to the shortest running time in all 10 scenarios (i.e., the maximum running speed). Generally, the total travel time increases with the increases of running time of section.

5.3. The Numerical Experiments of the Second Stage. Obviously, there are two objectives in the second stage. The weights of the two objectives are represented by ω_1 and ω_2, respectively. In this paper, we consider that the two objectives

Table 5: The parameters of SDSA-trains.

No.	Downstream (Beijing ⟶ Guangzhou)			Upstream (Guangzhou ⟶ Beijing)		
	Trip	Dep.	Arr.	Trip	Dep.	Arr.
1	Beijing⟶Guangzhou	[1020, 1140]	[1740, 2040]	Guangzhou⟶Beijing	[1020, 1140]	[1740, 2040]
2	Beijing⟶Guangzhou	[1160, 1200]	[1740, 2040]	Guangzhou⟶Beijing	[1160, 1200]	[1740, 2040]
3	Beijing⟶Changsha	[1220, 1260]	[1740, 2040]	Guangzhou⟶Shijiazhuang	[1220, 1260]	[1740, 2040]
4	Beijing⟶Guangzhou	[1280, 1320]	[1740, 2040]	Guangzhou⟶Beijing	[1280, 1320]	[1740, 2040]
5	Shijiazhuang⟶Guangzhou	[1260, 1320]	[1740, 2040]	Changsha⟶Beijing	[1260, 1320]	[1740, 2040]

Table 6: Scenarios setting example: downstream of high-speed railway.

Section	Running time(min)	Scenario 1	Scenario 2	......	Scenario 9	Scenario 10
1	(35, 49)	$35+R_1\cdot(49\text{-}35)$	$35+R_2\cdot(49\text{-}35)$	......	$35+R_9\cdot(49\text{-}35)$	$35+R_{10}\cdot(49\text{-}35)$
2	(33, 41)	$33+R_1\cdot(41\text{-}33)$	$33+R_2\cdot(41\text{-}33)$	......	$33+R_9\cdot(41\text{-}33)$	$33+R_{10}\cdot(41\text{-}33)$
3	(50, 68)	$50+R_1\cdot(68\text{-}50)$	$50+R_2\cdot(68\text{-}50)$	......	$50+R_9\cdot(68\text{-}50)$	$50+R_{10}\cdot(68\text{-}50)$
4	(39, 50)	$39+R_1\cdot(50\text{-}39)$	$39+R_2\cdot(50\text{-}39)$	......	$39+R_9\cdot(50\text{-}39)$	$39+R_{10}\cdot(50\text{-}39)$
5	(70, 86)	$70+R_1\cdot(86\text{-}70)$	$70+R_2\cdot(86\text{-}70)$	......	$70+R_9\cdot(86\text{-}70)$	$70+R_{10}\cdot(86\text{-}70)$
6	(39, 50)	$39+R_1\cdot(50\text{-}39)$	$39+R_2\cdot(50\text{-}39)$	......	$39+R_9\cdot(50\text{-}39)$	$39+R_{10}\cdot(50\text{-}39)$
7	(45, 60)	$45+R_1\cdot(60\text{-}45)$	$45+R_2\cdot(60\text{-}45)$	......	$45+R_9\cdot(60\text{-}45)$	$45+R_{10}\cdot(60\text{-}45)$
8	(30, 42)	$30+R_1\cdot(42\text{-}30)$	$30+R_2\cdot(42\text{-}30)$	......	$30+R_9\cdot(42\text{-}30)$	$30+R_{10}\cdot(42\text{-}30)$
9	(34, 42)	$34+R_1\cdot(42\text{-}34)$	$34+R_2\cdot(42\text{-}34)$	......	$34+R_9\cdot(42\text{-}34)$	$34+R_{10}\cdot(42\text{-}34)$
10	(56, 67)	$56+R_1\cdot(67\text{-}56)$	$56+R_2\cdot(67\text{-}56)$	......	$56+R_9\cdot(67\text{-}56)$	$56+R_{10}\cdot(67\text{-}56)$
11	(49, 63)	$49+R_1\cdot(63\text{-}49)$	$49+R_2\cdot(63\text{-}49)$	......	$49+R_9\cdot(63\text{-}49)$	$49+R_{10}\cdot(63\text{-}49)$

R_s: scenarios parameters.
$R_1 \in [0, 0.1]$; $R_2 \in [0.1, 0.2]$; $R_3 \in [0.2, 0.3]$; $R_6 \in [0.3, 0.4]$; $R_5 \in [0.4, 0.5]$;
$R_6 \in [0.5, 0.6]$; $R_7 \in [0.6, 0.7]$; $R_8 \in [0.7, 0.8]$; $R_9 \in [0.8, 0.9]$; $R_{10} \in [0.9, 1]$.

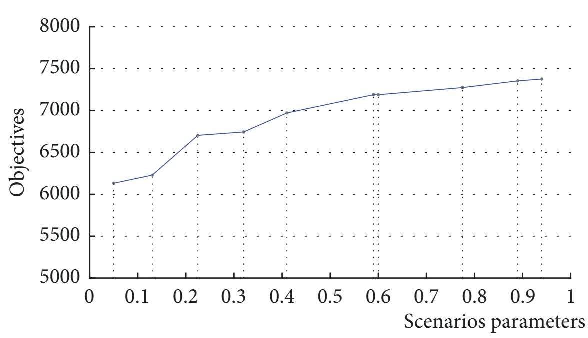


Figure 7: The change of the objectives under different scenarios parameters.

have the same importance; namely, the weight of the two objectives in the second stage are all set as 0.5, i.e., $\omega_1 = \omega_2 = 0.5$. This is the robust optimization model proposed in this paper, which is abbreviated with RO-II. In order to show the difference between the solutions of robust optimization and other cases, we have done the following two additional numerical experiments. One is to optimize the model without considering the above 10 scenarios. That means only the first item in the objective function (32) of the second stage is considered. This case can be called nonrobust optimization, which is abbreviated with NRO-II. The other is to consider the average running time of the 10 scenarios in the first stage. It is solved by the first stage model, which is abbreviated with AS-I. Table 8 shows the objectives, computation times, and total deviation from 10 scenarios of these experiments.

In order to show the deviation between these contrast experiments and each scenario, Figure 8 is given.

In Figure 8, the deviation from each curve is the absolute value of the difference between the contrast experiment and each scenario. Obviously, nonrobust optimization has the least total travel time, but the biggest total deviation. Trains tend to choose the maximum operating speed in this case. Therefore, the solution of nonrobust optimization has a small deviation from scenario 1 and scenario 2 but a large deviation from other scenarios. The timetable generated by nonrobust optimization may require a large adjustment when encountering some uncertain events. However, the robust optimization timetable generated in this paper can better adapt to different running times. It needs less adjustment when running in a specific scenario. In addition, the deviation of robust optimization is very close to the average running time, and the total travel time of robust optimization is shorter than the average running time. Furthermore, we also analyzed the deviations of some trains, as shown in Figure 9.

Obviously, in Figure 9, for each SDSA-train, the solution of robust optimization can also better adapt to different running times.

Figures 10, 11(a), and 11(b) depict the time-space diagrams of SDSA-trains and existing trains in the case of robust optimization with $\omega_1 = \omega_2 = 0.5$. Figure 10 mainly depicts the

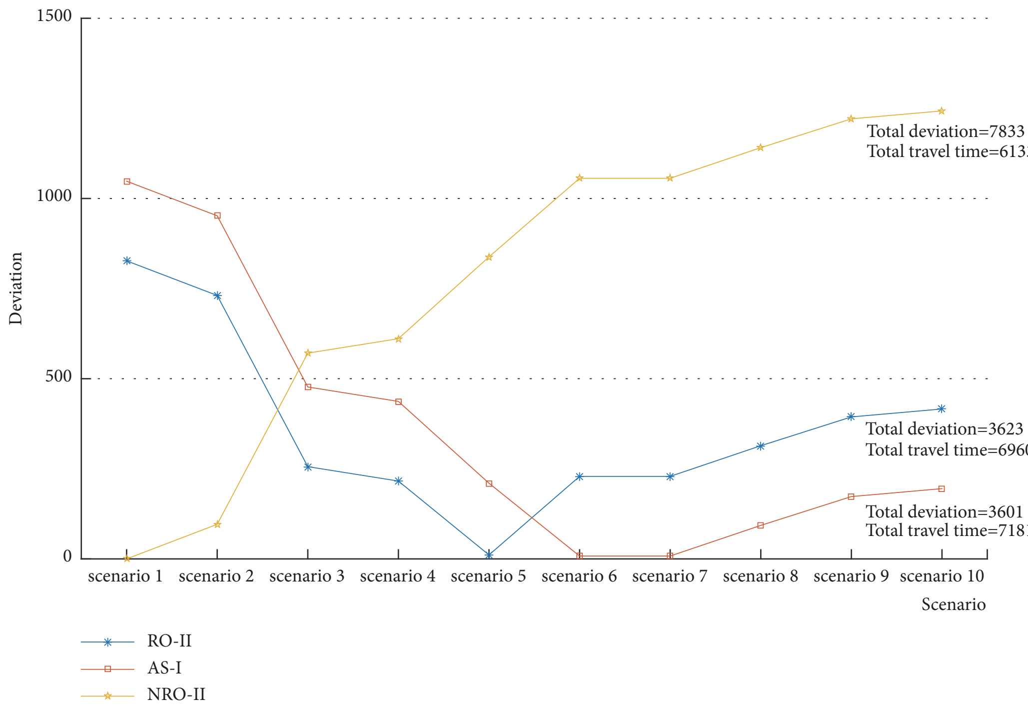


FIGURE 8: The deviation between the three sets of contrast experiments and each scenario.

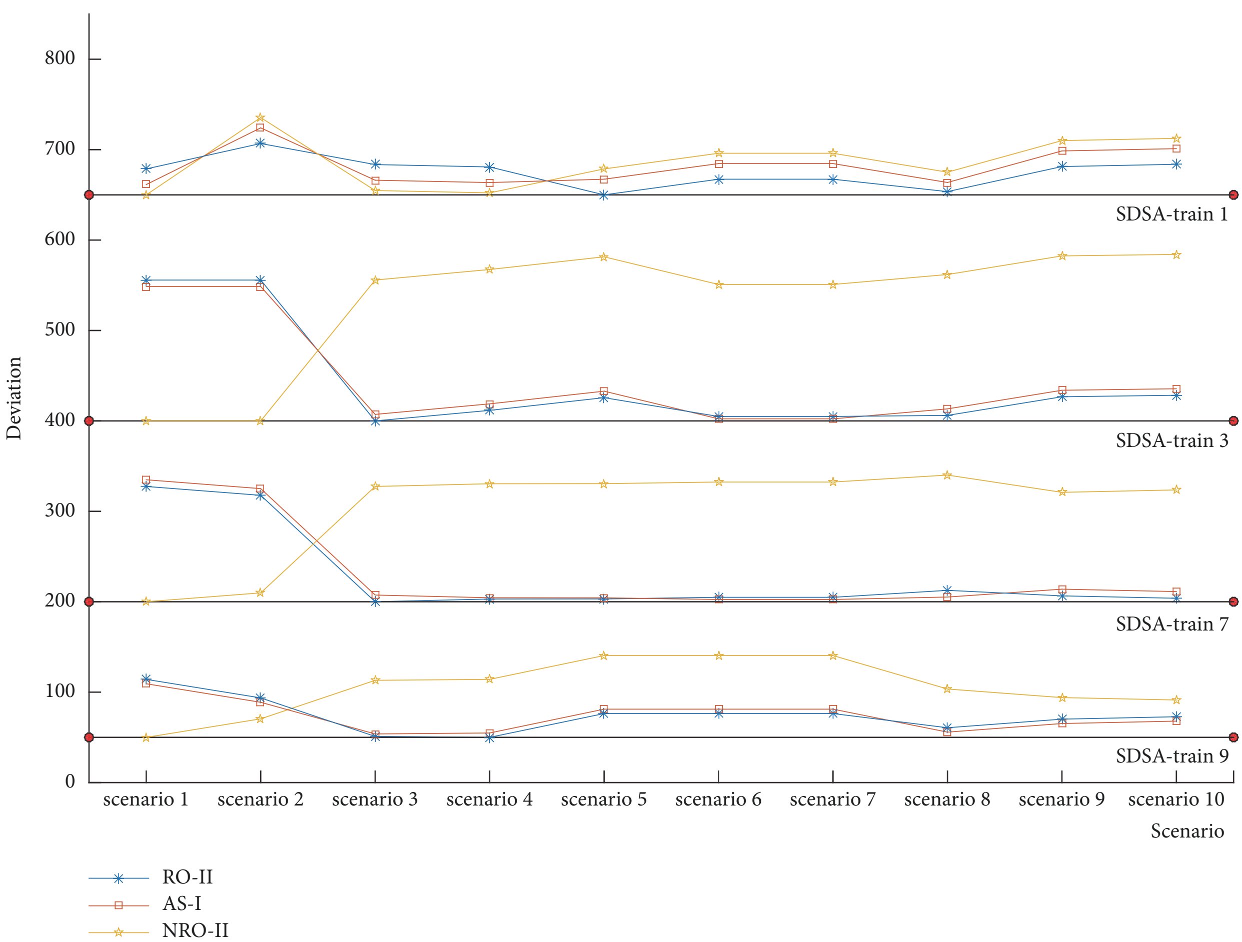


FIGURE 9: The deviations of some trains between the three sets of contrast experiments and each scenario.

Table 7: Computational results under different scenarios.

	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
Objectives (unit: min)	6133	6229	6704	6744	6971
Computation time (unit: s)	22	25	17	20	23
	Scenario 6	Scenario 7	Scenario 8	Scenario 9	Scenario 10
Objectives (unit: min)	7189	7189	7274	7354	7376
Computation time (unit: s)	9	13	16	5	4

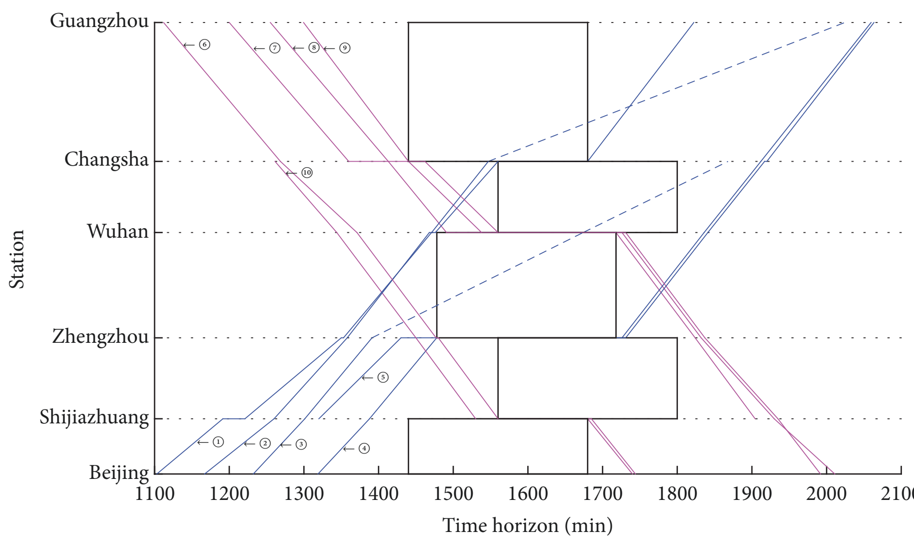


Figure 10: Time-space diagram of SDSA-trains at major stations.

Table 8: Computational results of different contrast experiments.

	RO-II	NRO-II	AS-I
Total travel time (unit: min)	6960	6133	7181
Total deviation from 10 scenarios	3623	7833	3601
Computation time (unit: s)	6	41	17

SDSA-trains operation at major stations and the maintenance windows on high-speed railway segments, in which solid lines represent running on high-speed railway, while dashed lines represent running on normal-speed railway. Note that maintenance windows are asynchronous on each segment. In Figure 11(a), the green lines represent the time-space diagrams of existing high-speed trains. The blue dashed lines represent running on normal-speed railway. Obviously, the high-speed railway corridor is unavailable from 0: 00 to 6: 00 am for existing high-speed trains. In Figure 11(b), the green lines represent the time-space diagrams of existing normal-speed trains and the blue dashed lines represent running on high-speed railway. Note that only SDSA-train 2 and SDSA-train 3 are involved in the operation of running on the normal-speed railway, so we only depict these two SDSA-trains in Figure 11(b).

We can see that only SDSA-train 2 and SDSA-train 3 select Mode 2 when there is a conflict with maintenance windows. Specifically, SDSA-train 2 switches from high-speed railway to normal-speed railway at major station Changsha; SDSA-train 3 switches from high-speed railway to normal-speed railway at major station Zhengzhou. Note that if SDSA-train 2 selects Mode 1 at major station Changsha, there will be a shorter travel time. However, it is not conducive to the generation of robust train scheduling. Other SDSA-trains all select Mode 1 when there is a conflict with the maintenance windows on the next high-speed railway segment.

Furthermore, we analyze the SDSA-train stop plans based on Figures 11(a) and 11(b). There are three cases that may make trains stop at station. Specifically, the first case is to ensure the safe operations of all trains. The second case is to meet passengers' demands. The third case is to avoid conflicts with the maintenance windows. Note that, in the first case, it is necessary to ensure the safe operations between the SDSA-train and the SDSA-train and the SDSA-train and the existing train. It should be emphasized that we assume that the existing train schedules cannot be affected, so the SDSA-trains have to stop at stations when there is a conflict with the existing train schedules. It is clear and reasonable to see that an existing high-speed train overtakes SDSA-train 1 at high-speed railway station Baoding East and SDSA-train 2 overtakes SDSA-train 1 at high-speed railway station Xinyang East. In the second case, the reason that SDSA-train 6 stops at high-speed railway station Zhengzhou is to meet passengers' demands. In the third case, the reason that SDSA-trains select Mode 1 (i.e., waiting at high-speed railway station) is to avoid conflicts with the maintenance windows.

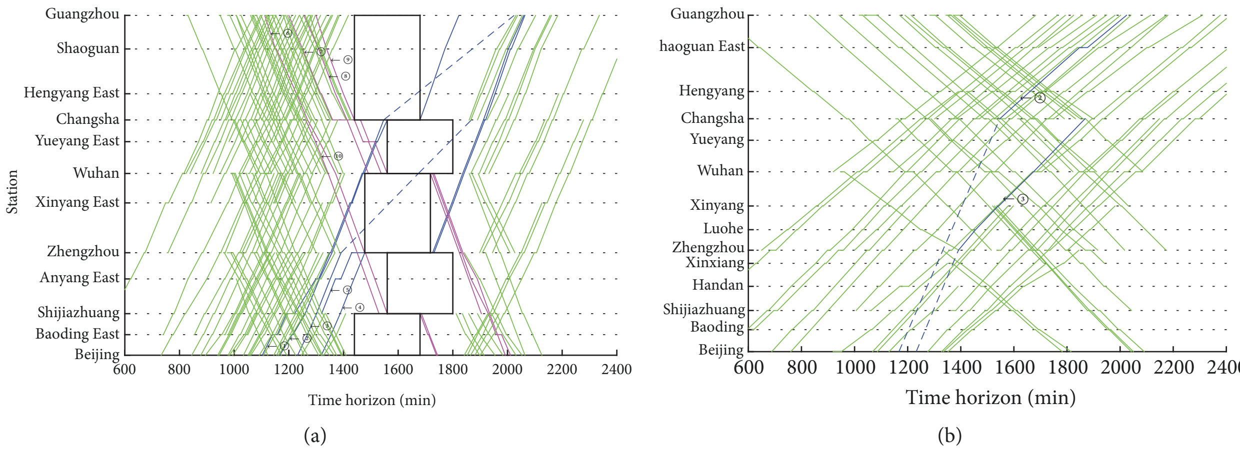


Figure 11: Time-space diagrams of SDSA-trains and existing trains on the high-speed railway (a) and the normal-speed railway (b).

6. Conclusion and Future Work

Taking into account the disturbances in the actual operations, we propose a two-stage method to generate a robust train scheduling and maintenance planning. In the first stage, we generate an integrated optimization model of train scheduling and maintenance planning on high-speed railway corridors under each scenario. Note that we consider two operation modes of SDSA-trains under each scenario. The selection of operation modes is realized by introducing binary state variables. In the second stage, a robust SDSA-train schedule is generated based on the optimal solutions of the considered scenarios. Besides, in order to make the model be handled easily, some linearization techniques are used. The nonlinear constraints and objective function are equivalently replaced by linear constraints and objective function. Finally, numerical experiments show the effectiveness of the proposed model and optimization method.

The following aspects should also be considered in future research. First, we do not consider the uncertainty of passengers' demands and the number of passengers boarding/alighting of each train at each station. These are two very important issues for future research. Second, line planning can be integrated into the issue. Due to the line planning, the origin/destination stations and stop plans can be determined in advance. Third, we can use other modeling methods and propose some efficient algorithms to solve this problem. For example, we can build a space-time network model and use Lagrangian-relaxation-based algorithms to solve it. These aspects can be studied in depth in the future.

Data Availability

Data used in this manuscript are available on the China railway official website at http://www.12306.cn/mormhweb/.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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