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Research Article

Managing Recurrent Congestion of Subway Network in Peak Hours with Station Inflow Control

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Station inflow control (SIC) is an important and effective method for reducing recurrent congestion during peak hours in the Beijing, Shanghai, and Guangzhou subway systems. This work proposes a practical and efficient method for establishing a static SIC scheme in normal weekdays for large-scale subway networks. First, a traffic assignment model without capacity constraint is utilized to determine passenger flow distributions on the network. An internal relationship between station inflows and section flows is then constructed. Second, capacity bottlenecks are identified by considering the transport capacity of each section. Then, a feedback-based bottleneck elimination strategy is established to search target control stations and determine their control time and control strength. To validate the effectiveness of the proposed approach, a decision support system coded in the C# programming language was developed, and the Beijing subway was used as a case study. The results indicate that the proposed method and tool are capable of practical applications, and the generated SIC plan has better performance over the existing SIC plan. This study provides a practical and useful method for operation agencies to construct SIC schemes in the subway system.

1. Introduction

1.1. Motivation. Station inflow control (SIC) is to relieve congestion and ensure operational safety by controlling the number of passengers entering a station [1]. It is usually applied during excessive congestion such as peak hours and other expected high passenger flows. Though SIC causes travel delays and inconvenience to passengers, it plays an important role in maintaining safety when travel demand greatly exceeds capacity. In April 2011, the Beijing subway first employed SIC at 17 stations during morning peak hours [2]. Since then, SIC has been gradually implemented in other subway systems in China, such as the Guangzhou and Shanghai subways. With the continuous growth of travel demand, the number of controlled stations and the length of control time have increased greatly. At the end of 2016, the number of controlled stations in the Beijing subway reached 74, covering almost 25% of all stations [3].

Up to now, a practical and efficient method to generate SIC schemes for large-scale transit networks is still lacking. Current SIC scheme is established almost depending on operator's experience. According to the regulations on operation safety management for urban rail transit, which was released by the Beijing municipal commission of transport, a station should be controlled when passenger inflows reach the warning state of 70% of the load capacity of the station [1]. This regulation is the only official guidance for carrying out SIC actions. However, it does not provide a clear and system-wide optimization approach. For instance, congestion of a certain station may not be caused by large inflows but by limited train capacity. Hence, a system-wide optimization approach that considers not only large inflows but also train capacities should be constructed.

To address the SIC issue in subway systems, previous works have proposed mathematical optimization models to establish a cooperative SIC plan within multiple stations [4, 5]. Nevertheless, these methods have low computational

efficiency and lack capability to address the problem in large-scale networks, because of complex solution and oversimplification of the practical problem in formulating model. It is well known that capacity bottlenecks are the critical reason for congestion. Hence, we start from this basic point and construct a new feedback-based bottleneck elimination strategy to determine the SIC plan. Compared to the existed mathematical models, the proposed method has direct meaning and high computational efficiency and can be easily expanded by considering practical factors in different situations. The major characteristics of the model include the following. (a) A feedback-based bottleneck elimination algorithm is used to generate the control scheme, which can provide quantitative support for operation managers, including control stations, control times, and control strength. (b) The method has high computational efficiency and capability for applications in large-scale rail transit networks. (c) The proposed approach has good extensibility with regard to practical factors, such as platform load capacity and traffic environment outside the station.

1.2. SIC Applications in Beijing Subway. The Beijing subway developed rapidly in recent years, consisted of 18 lines with a total length of 574 km by the end of 2016. The ridership reached 3.66 billion trips in 2016 (based on the count of unlinked trips, as every transfer in the system is counted as an additional trip), which means more than 10 million trips per day [6]. To relieve congestion, train headways in some lines have been reduced to the minimum time (less than two minutes). However, travel demand is much greater than the transport capacity, and the congestion is becoming serious.

The Beijing Subway firstly employed SIC measures in April of 2011, which controlled seventeen stations in Line 1 and Line BT [2]. The control time starts from 7:00 am to 9:00 am on weekdays. Then, the number of controlled stations continues to grow as travel demand increases. Figure 1 presents the number of controlled stations and annual ridership in the last five years.

In practice, three control levels are generally employed based on the crowding extent in a station. The first level limits passengers entering platform when platform capacity is insufficient. The second level limits passengers going to purchasing area, such as the area close to ticket gates. If the congestion cannot be relieved by these two control actions, then a third level is adopted, which limits passengers entering station by setting fences to slow down flow speed or closing station gates. Control level increases with the extent of congestion growth. Representative SIC measures applied in the Beijing subway are shown in Figure 2.

According to a survey of the Beijing subway, SIC schemes will be updated every three months in accordance with the variation of travel demand. If a station meets with heavy congestion, station masters will apply for a SIC action to the Beijing Subway Operation Company. Then, operation agencies decide whether to authorize the application depending on flow status and previous experience. However, there exist drawbacks in the redesign process for establishing a new

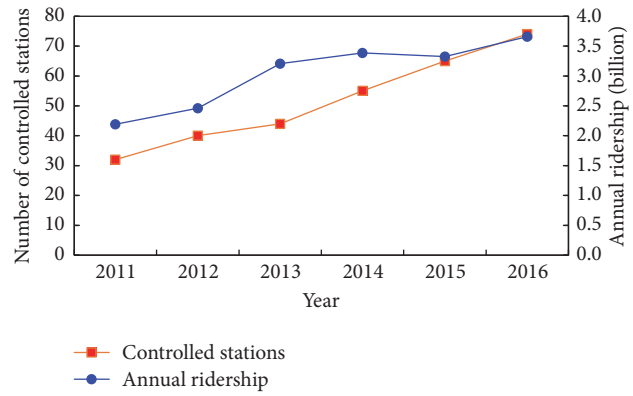


Figure 1: The number of controlled stations and annual ridership of the Beijing subway.

SIC plan. First, station masters only make decisions based on a single station's crowdedness, not from the system wide. Second, there is no scientific method to support operation managers to decide which station should be controlled. Moreover, a SIC scheme only determines which station and what time should be controlled, not including the control strength.

1.3. Objective and Organization. This work aims to provide a practical method for generating static SIC schemes for large-scale subway networks only in peak hours and guides SIC actions through a quantization reference of control strength. A proper SIC scheme could ensure operational safety and improve the service level for passengers.

The remainder of this paper is organized as follows. Section 2 provides related works on SIC actions and congestion remission measures for rail transit system. The methodology to determine a control plan based on a feedback-based bottleneck elimination approach is given in Section 3. Afterwards, a discussion of its implementation in the Beijing subway is provided in Section 4. Finally, conclusions and future research are discussed.

2. Literature Review

Congestion is an intractable problem for worldwide traffic and transit systems. Two important strategies are usually used to manage congestion, which are capacity enhancement and travel demand management. For subway systems, it is difficult to improve capacity because physical facilities have upper limits on capacity, and a long time is required to build new lines. Measures of demand management become effective and positive tools for relieving congestion. In practice, two types of congestion-relief measures are widely used. The first one is time-varying price approach which has been used in the London [7], Melbourne subway systems [8, 9], and so on. The second is station inflow control which is broadly implemented in the subway systems of China. Next, we will summarize the related works on these two aspects.

(a) Limit to enter platform

(b) Close ticket gates

(c) Speed reduction by fences

(d) Close station gates

Figure 2: Representative SIC measures in the Beijing subway.

2.1. Time-Dependent Pricing. The differential price approach to shift demand from peak periods is not new to either research or practice, which has been implemented in many large cities in the US from the 1970s [10]. There are two basic strategies of differential pricing measure, peak surcharges and off-peak discounts. In the field of subway system, Santiago underground system firstly employed time-varying pricing strategy to reduce peak congestion in 1986 [11]. Thereafter, many subway systems have implemented the varying price strategy to manage peak demand. A recent review by LEK finds that around 40% of urban rail networks worldwide provide some form of peak surcharge and/or off-peak discount [12]. The most famous differential pricing strategy is "early bird tickets" in Melbourne metro, which provides free tickets if trips are completed before 7:00 am. After a half year of the implementation of the policy, it is found that 23% of commuters transfer from peak hours to off-peak periods, and the departure time of travelers moves forward about 42 minutes in average. The pricing strategy reduces peak demand and relieves congestion significantly [9]. More applications of differential price policy in subway systems can refer to the reviews of Hale and Charles [13] and Liu and Charles [14].

The mechanism of differential pricing for congestion relief is to influence passenger's travel behaviors and even the demand in temporal-spatial domains. Pricing strategy can intervene with the choices of passengers on departure time, destination, travel mode, and travel route. Usually the effect of a new price policy needs a long time to appear. Compared with the elastic pricing strategy, inflow control manages peak demand in a direct and forced way, by removing the mountain peak of the demand. For a mature subway system, pricing strategy is more preferable than control actions, while in developing subway systems where travel demand grows and fluctuates greatly, inflow control is an efficient measure for reliving congestion and maintaining operational safety.

2.2. Station Inflow Control. Station inflow control has been widely implemented in the subway systems of China, including Beijing, Tianjin, Shanghai, Guangzhou, and Chengdu subway. With the peak congestion being serious, how to perform SIC actions becomes an absorbing topic for researchers and practitioners. Previous works can be divided into three categories: macroscopic principles for SIC strategy, mesoscopic methods for constructing a SIC plan, and microscopic measures for carrying out SIC actions in a station. This work falls into the second category on how to construct SIC schemes.

Macroscopic principles provide overall guidance for SIC. Liu and Jiang pointed out earlier that SIC actions should be executed at three levels of "station-line-network" [15]. In practice, congestion of a station may not be caused by large inflows but for limited train capacity, because the stations ahead of the congestion station occupy most of the train capacity. Hence, collaborative control within multiple stations should be considered. Based on a balance principle, Liu et al. proposed a cooperative control strategy between two stations [16]. Huang et al. studied propagation pattern of congestion and proposed a collaborative control strategy in

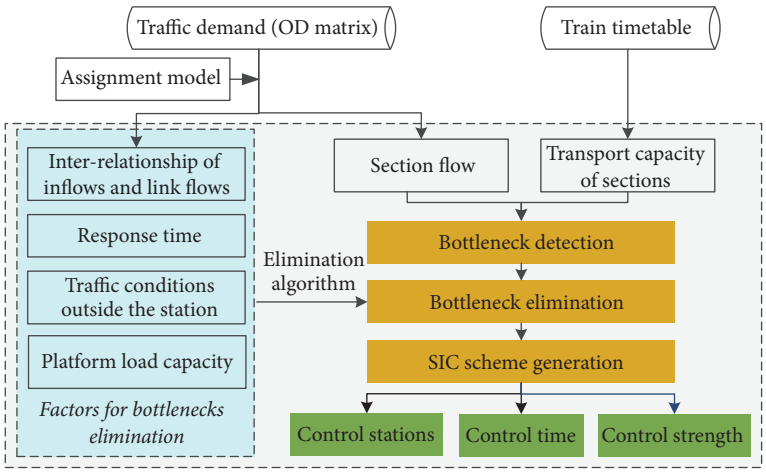


Figure 3: Framework for generating a SIC scheme.

a single line [17]. Cooperative inflow control within multiple stations has been a consensus agreement for carrying out SIC actions.

On the mesoscopic level of method construction for SIC plans, Zhao et al. firstly proposed a mathematical optimization model to establish SIC schemes for a single line, with the objective of minimizing travel delay and maximizing the turnover of passenger flows [4]. Lu et al. also proposed a linear integer programming model on the basis of network topology and definition of passenger demand; however, the model is only for a line [18]. Then, Yao et al. established a coordinated passenger inflow control model for networks. In order to improve the efficiency and capability of the model for a network, a static assignment model is used to establish the internal flow relationship [5]. With the view of reducing travel delay, Guo et al. built a cooperative model for a network with constraints on the capacity of stations [19], and Wang et al. established an integer programming model based on an analysis of passenger delay and alight and board processes [20]. In conclusion, these models try to use optimization method to solve the SIC problem. Though cooperative control strategy can be considered, large numbers of parameters and complex solution limit their capability in real large-scale networks. The critical point of these models is establishing internal flow relationships, whose complexity increases greatly with network scale grows. In practice, congestion usually takes place in a regional wide. When we construct a SIC plan, there is no need to consider the stations far away from the target congestion station. Different from optimization methods, we establish a heuristic inflow control algorithm to establish SIC schemes.

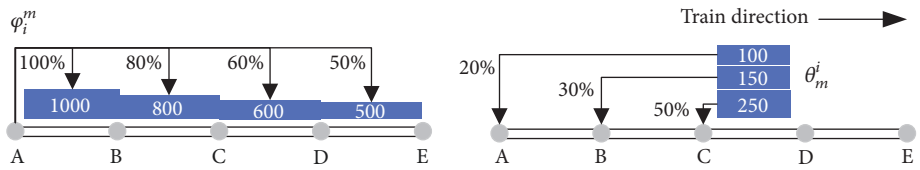
Moreover, some works focus on SIC actions implemented in stations, such as ticket gates optimization and physical fences setting for inflow control [21]. Dou et al. provide a cloud model to determine what time to start control actions for a station [22]. Xie studied the detail control actions for transfer stations [23]. In addition to normal control actions, Coulson et al. propose a new strategy of monetary incentives to reduce congestion by redirecting of passengers to less crowded exits [24]. However, it is difficult to use a normalized method to determine control actions, because each station has specific physical structure and organization rules. How to establish detail SIC actions is out of range of this work.

3. Methodology

This section discusses a feedback-based bottleneck elimination algorithm for generating SIC schemes. Firstly, the framework to generate SIC scheme is provided. Then, the internal relationship between station inflows and section flows is established using a capacity-unconstrained assignment model, which provides important parameters for constructing SIC plans. Thirdly, capacity bottlenecks are identified by considering section flows and corresponding transport capacities. Fourthly, the elimination processes for a single bottleneck and multiple bottlenecks are presented separately. Lastly, the flowchart for generating a time-dependent SIC plan for a network is given.

3.1. Framework for Generating an SIC Scheme. It is well known that bottleneck caused by the imbalance between supply and demand is the origin of congestion. We proceed from this point to establish a feedback-based elimination algorithm to remove bottlenecks and then search the corresponding target control stations. Usually, a SIC scheme contains three parts, namely, control stations, time, and strength. The framework for SIC scheme generation is shown in Figure 3.

3.2. Flow Relationship Construction. In a rail transit system, transport capacity has an upper limit, and passengers should wait for the next train if the capacity of the incoming train is insufficient. In order to obtain the original status of flow distributions on a network, the capacity limitation of section is not considered in our traffic assignment model but will be taken into consideration later. A Logit-based stochastic user equilibrium (SUE) assignment model is applied to establish the relationship between station inflows and section (link)



(a) The station-section travel rate (b) The section-station capacity occupation rate

Figure 4: Schematic of relationship between station inflows and section flows.

flows [25]. In the model, there are mainly two types of relationships: (1) the inflows of a station that travel through given sections and their corresponding travel through rates and (2) the link flows of a section that comes from particular stations and their corresponding capacity occupation rates.

A subway network is represented by a directed graph $G = (N, E)$, where $N = \{1, 2, \ldots, i, j\}$ is the set of stations (nodes) and $E = \{1, 2, \ldots, m, n\}$ is the set of sections (links). The analysis time (such as 7:00 am to 9:00 am) is divided into short time spans with the same length, which are denoted by $T = \{1, 2, \ldots, t\}$. Let $d_{ij}(t)$ be the travel demand from station i to station j in time interval t, where $q_m(t)$ and $C_m(t)$ are the section flow and transport capacity of section m at time t, respectively. Define R_{ij} as the set of feasible routes for the OD (Origin-Destination) pair (i, j), and r_{ijk} as the kth route, $r_{ijk} \in R_{ij}$. Let $W_{r_{ijk}}$ be the generalized travel cost of the kth route.

The route cost is represented by a generalized cost function which is composed of four parts: (1) access and egress walk time; (2) on-board time; (3) waiting time; and (4) transfer time. The walk time contains access and egress walk time, which can be calculated by the distance corresponding to the walk link and the average walking speed of travelers. Normally, the on-board time for passengers travelling through sections is constant, which can be fixed from train schedules (timetable). The waiting time includes two components: (a) waiting time at the origin station; (b) waiting time at the transfer station if a transfer is required. The average waiting time at an origin/transfer station is equal to half of the train headway time. The transfer time is similar to the access or egress walk time. Then the generalized cost of a feasible route can be formulated as follows:

$$\begin{aligned} W_{r_{ijk}} &= \mu^a \left(W_{r_{ijk}}^{\text{access}} + W_{r_{ijk}}^{\text{egress}}\right) + \mu^b W_{r_{ijk}}^{\text{board}} \\ &\quad + \mu^c \left(W_{r_{ijk}}^{o\text{-wait}} + \delta W_{r_{ijk}}^{t\text{-wait}}\right) + \delta \mu^d W_{r_{ijk}}^{\text{transfer}}, \end{aligned} \tag{1}$$

where $W_{r_{ijk}}$ is the generalized cost of the route r_{ijk}. μ^a, μ^b, μ^c, μ^d are weight factors for the cost corresponding to different travel processes; $\delta = 1$ if there is a transfer; otherwise, $\delta = 0$; $W_{r_{ijk}}^{o\text{-wait}}$ and $W_{r_{ijk}}^{t\text{-wait}}$ are waiting time at origin and transfer station, respectively. The weight factors for different travel processes can be estimated by the maximum likelihood method through travel surveys. Referring to the previous work on traffic assignment for the Beijing subway [26], $\mu^a = 0.21$, $\mu^b = 0.14$, $\mu^c = 0.28$, and $\mu^d = 0.37$. The value of factors may be variant for different subway systems.

Suppose the OD matrix at time t is given, which can be accurately determined from automatic fare collection (AFC) records. Take an OD pair (i, j) as an example to show the process of establishing the internal flow relationship. According to the Logit-based SUE model, the probability for passengers in OD pair (i, j) choosing the kth route is given by (2) and the flow volume of the kth route can be computed using (3).

$$p_{r_{ijk}} = \frac{\exp\left(W_{r_{ijk}}/\overline{W}\right)}{\sum_{r_{ijk}}^{R_{ij}} \left(W_{r_{ijk}}/\overline{W}\right)}, \quad \forall i \in N,\ j \in N,\ r_{ijk} \in R_{ij}, \tag{2}$$

$$q_{r_{ijk}}(t) = d_{ij}(t) \cdot p_{r_{ijk}}, \quad \forall i \in N,\ j \in N,\ r_{ijk} \in R_{ij}, \tag{3}$$

where $p_{r_{ijk}}$ is the choice probability and $\overline{W}$ is the average cost of all feasible routes in the OD pair (i, j).

Assign the whole demand of the OD pair (i, j) to all feasible routes; then passenger flows of section m that comes from this OD can be determined, as shown in the following:

$$q_m^{ij}(t) = \sum_{r_{ijk}}^{R_{ij}} q_{r_{ijk}}(t) \cdot \delta_{r_{ijk}}^m, \quad \forall i \in N,\ j \in N,\ m \in E, \tag{4}$$

where $q_m^{ij}(t)$ refers to the passenger flow of section m, which comes from OD pair (i, j), and $\delta_{r_{ijk}}^m$ is a binary variable: if $\delta_{r_{ijk}}^m = 1$, section m belongs to the route r_{ijk}; otherwise $\delta_{r_{ijk}}^m = 0$.

Next, assign all demands of all OD pairs to the network; the total section flow of section m can be calculated by

$$q_m(t) = \sum_{i=1}^{N} \sum_{j=1}^{N} q_m^{ij}(t), \quad \forall m \in N,\ t \in T. \tag{5}$$

In this work, two important parameters are defined to describe the internal relationship between station inflows and section flows. These are the station-section travel through rate φ_i^m and the section-station capacity occupation rate θ_m^i. The two parameters describe the correlation from opposite perspectives, where φ_i^m starts from station view to explain that the inflow of a station will go through specific sections, and θ_m^i aims to explain that section flows come from particular stations. An example of these two parameters is shown in Figure 4.

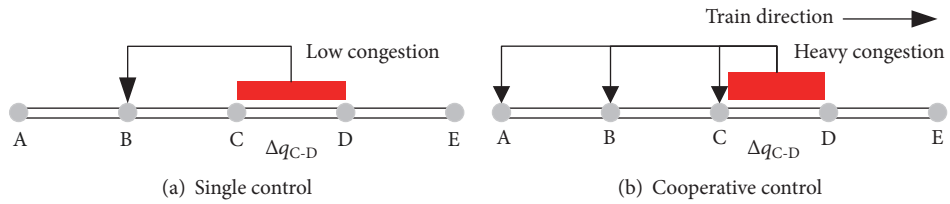


Figure 5: Schematic of the single bottleneck elimination process.

In Figure 4(a), assume the inflow volume of station A is 1000 passengers and the flows traveling through the subsequent sections are 1000, 800, 600, and 500. Therefore, the station-section travel rates φ_i^m are 100%, 80%, 60%, and 50% respectively. The station-section travel rate can be computed by

$$\varphi_i^m(t) = \frac{\sum_{j=1}^{N} q_m^{ij}(t)}{\sum_{j=1}^{N} d_{ij}(t)}, \quad \forall i \in N,\ t \in T,\ m \in E, \tag{6}$$

where $\varphi_i^m(t)$ represents the percentage of passengers who depart from station i traveling through section m at time t.

Suppose the section flows between stations C and D are 500 passengers, as shown in Figure 4(b), in which the numbers of passengers from stations A, B, and C are 100, 150, and 250, respectively. Then, the section-station capacity occupation rates are 20%, 30%, and 50%. The formulation of the section-station capacity occupation rate is given by

$$\theta_m^i(t) = \sum_{j=1}^{N} \frac{q_m^{ij}(t)}{q_m(t)}, \quad \forall i \in N,\ m \in E,\ t \in T, \tag{7}$$

where $\theta_m^i(t)$ represents the percentage of passenger flows from station i in all flows traveling through section m at time t.

The section-station capacity occupation rate θ_i^m should satisfy

$$\sum_{i=1}^{N} \theta_i^m(t) = 1, \quad \forall m \in N,\ t \in T. \tag{8}$$

3.3. Capacity Bottlenecks Identification. A bottleneck emerges when section flows exceed its transport capacity. In practice, passengers should queen on the platform for unable boarding. Note that there is no capacity limitation in our traffic assignment model; then the bottleneck is exhibited as the surplus flow rather than section capacity. Define Δq_m as the surplus flow of section m, as shown in the following:

$$\Delta q_m(t) = q_m(t) - C_m(t), \quad \forall m \in E,\ t \in T. \tag{9}$$

The transport capacity can be calculated from the train timetable directly or from train headways, as shown in (10). Because of the randomness of passengers' arrival and imbalance in the distribution of passengers on different vehicles, the available capacity of a train is usually less than the theoretical capacity. Here, the peak hour factor (PHF) is used to describe the reduction of transport capacity [27] and is determined using (11).

$$C_m(t) = n_m \cdot D \cdot \gamma_{\text{PHF}} \cdot \varsigma, \tag{10}$$

where n_m is the number of trains running through section m during time t, D means the nominal load capacity for a train (usually 1440 passengers per train), γ_{PHF} is the PHF ($0.25 \le \gamma_{\text{PHF}} \le 1.0$), and ς is the permissible overload rate ($\varsigma \le 130\%$).

$$\gamma_{\text{PHF}} = \frac{F_{60}}{4 \cdot F_{15}}, \tag{11}$$

where F_{60} is the inflows of a certain line in an hour and F_{15} is the maximum peak flow during 15 minutes in this hour.

3.4. Single Bottleneck Elimination Algorithm. The elimination algorithm for a single bottleneck is the basis for removing multiple bottlenecks. We will describe how to remove a single bottleneck first. There are two types of control strategies for eliminating a bottleneck: single-station control and cooperative multistations control, as shown in Figure 5. Consider the bottleneck in Figure 5 as an example to show the elimination process. Suppose section C-D is a capacity bottleneck and $\Delta q_{\text{C-D}}$ is the overload flow. While $\Delta q_{\text{C-D}}$ is low, the bottleneck can be removed by controlling station B (assume the flows of section C-D mainly come from station B). When the congestion is heavy, multiple stations, such as stations A, B, and C, should be controlled. Two important questions should be answered when we remove the bottleneck, that is, (1) how to select the target control station/stations and (2) how to determine the control strength of each station that ensures the bottleneck can be removed.

3.4.1. Target Control Stations Selection. The purpose of target control station selection is to determine which station/stations should be controlled. There is no doubt that it is more effective to control the stations with larger flows traveling through the bottleneck section. We can choose the target control stations according to the section-station capacity occupation rate θ_m^i. Define overload rate δ as the congestion pressure of the bottleneck, as shown in

$$\delta = \frac{q_m}{C_m}. \tag{12}$$

The higher the congestion pressure is, the more the stations should be controlled. Moreover, the more stations controlled,

Table 1: The proposed values for determining the number of target control stations.

Overload rate (δ)	Number of target control stations (k)
$\delta < 110\%$	1
$110\% \le \delta < 125\%$	2
$125\% \le \delta < 140\%$	3
$140\% \le \delta$	4

the lower control strengths that are executed. Therefore, there is not just one control strategy for removing a certain bottleneck. In this work, a suggested value determined by referring to management experience and congestion pressure is used to identify the number of target control stations. By mining and analyzing the distribution characteristics of the Beijing subway, proposed values were determined and are provided in Table 1. Once the number of control stations is determined, the top k stations according to the section-station capacity occupation rate can be selected as the target control stations.

3.4.2. Station Weight Calculation. If we only consider the internal relationship between station inflows and section flows, the control strength can be determined directly. However, in practice, other factors should be taken into consideration. Define a station weight γ_i^m to describe the contribution of the control station i to eliminate the bottleneck m. Four typical factors are considered in this work.

(1) Flow relationship: the section-station capacity occupation rate represents the capacity utilization of sections. It is the most important parameter used to determine the control station weight. The larger the rate, the more important the station in removing the bottleneck.

(2) Response time: response time describes the temporal connection between control stations and bottlenecks, which is expressed by the running time of the train between the control station and the bottleneck. When the control station is near the bottleneck, the effect of the control action is more obvious.

(3) Traffic conditions outside the station: traffic condition is assistant factor when we carry out SIC actions. Two representative factors are considered in this work, which are the area of the square outside the station, and the bus operation condition. First, there should be sufficient space for passengers queuing outside the station. The smaller the station square, the lower the control strength. Second, if there are enough buses around the station, travelers have high possibility to transfer to a bus instead of metro. In fact, the frequency, service area, and level of buses influent passengers' transfer behavior. However, it is a hard work to consider these influences in depth, because each bus line has its specific service area and characteristics. For available data, a simple index of the number of bus lines outside the station is used to describe the bus operation condition in this work. Usually, the more the bus lines, the higher the control strength.

(4) Platform load capacity: platform load capacity is another major factor that determines the control strength, which can be measured based on the platform area and the designed flow density. It is safer and robust for a platform with a larger capacity. Hence, if the platform capacity of a station is insufficient, greater control strength should be used to maintain safety.

The station weight can be expressed as a function of the above factors, as shown in

$$\gamma_i^m = \mu_1 \frac{\theta_m^i}{\sum_{i\in N_m'} \theta_m^i} + \mu_2 \frac{1/\Delta t_i^m}{\sum_{i\in N_m'} 1/\Delta t_i^m} + \mu_3 \frac{S_i^o}{\sum_{i\in N_m'} S_i^o} + \mu_4 \frac{V_i}{\sum_{i\in N_m'} V_i} + \mu_5 \frac{S_i^p}{\sum_{i\in N_m'} S_i^p}, \tag{13}$$

where N_m' is the set of target control stations for bottleneck section m; Δt_i^m is the train running time between station i and bottleneck m; S_i^o is the area of the station square; S_i^p is the area of platform; V_i is the number of bus lines around the station; μ indicates the importance of these factors. In this work, $\mu_1 = 0.4, \mu_2 = 0.1, \mu_3 = 0.1, \mu_4 = 0.1, \mu_5 = 0.3$.

For a given bottleneck, the weights of target control stations should satisfy the constraint of (14). The station weights can be normalized by (15).

$$\sum_{i\in N_m'} \gamma_i^m(t) = 1, \tag{14}$$

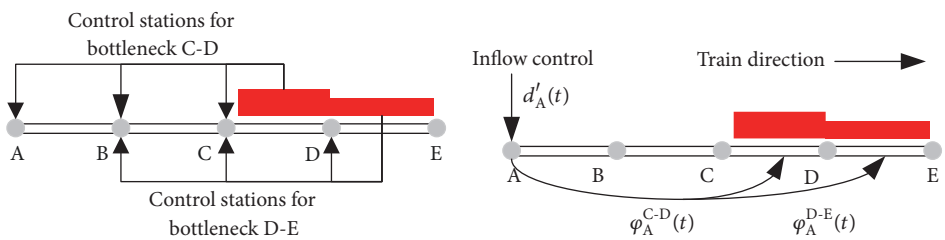
$$\gamma_i^m(t) = \frac{\widehat{\gamma}_i^m(t)}{\sum_{i\in N_m'} \widehat{\gamma}_i^m(t)}, \tag{15}$$

where $\gamma_i^m(t)$ is the final station weight and $\widehat{\gamma}_i^m(t)$ is the initial control weight from (13).

3.4.3. Control Strength Calculation. After the target control stations and the corresponding control weights are obtained, the control strength needs to be determined. The inflow control rate is defined to quantify the control strength, which is the percentage of passengers who are limited to entering the station relative to the travel demand, as shown in the following:

$$\beta_i(t) = \frac{d_i'(t)}{d_i(t)} \cdot 100\%, \quad \forall i \in N,\ t \in T, \tag{16}$$

where $\beta_i(t)$ is the control rate, $d_i(t)$ is the whole travel demand of station i at time t, and $d_i'(t)$ is the number of passengers who are limited to entering the station.



(a) Schematic for eliminating multiple bottlenecks (b) Flow relationship among multiple bottlenecks

Figure 6: Schematic for the internal relationship between multiple bottlenecks.

To remove bottleneck m with overload flow $\Delta q_m(t)$, the effective inflows that need to be controlled can be calculated by $\Delta q_m(t)\gamma_i^m(t)$. Note that this is the effective flow, not the real flow, because not all passengers who enter station i will travel through bottleneck m. Here, we can use the station-section travel through rate to revise the control inflows, as shown in

$$d_i'(t) = \Delta q_m(t) \cdot \frac{\gamma_i^m(t)}{\varphi_i^m(t)}. \tag{17}$$

The control rate of the stations is given by

$$\beta_i(t) = \frac{\Delta q_m(t) \cdot \gamma_i^m(t) / \varphi_i^m(t)}{d_i(t)} \cdot 100\%, \quad \forall i \in N_m'. \tag{18}$$

3.5. Multibottlenecks Elimination Algorithm. There are always many bottlenecks on the network, and these bottlenecks are interconnected through passenger flows. For example, if we remove a bottleneck with heavy congestion, another light bottleneck close to this bottleneck may disappear. Consider the example in Figure 6, which shows the elimination process for multiple bottlenecks. Suppose both section C-D and section D-E are bottlenecks, and their target control station sets are {A, B, C} and {B, C, D}, respectively. When we control the inflows of stations A, B, and C to remove bottleneck C-D, the link flows through section D-E may reduce, or the bottleneck may even disappear. Hence, the connections among bottlenecks should be considered when eliminating multiple bottlenecks. The internal flow relationship among these bottlenecks is illustrated in Figure 6(b). The reduced flows of the related links can be computed by the station-section travel through rate, as shown in

$$\overline{q}_m(t) = q_m(t) - \sum_{i \in N_m'} d_i'(t) \cdot \varphi_i^m(t), \quad \forall m \in E,\ t \in T, \tag{19}$$

where $\overline{q}_m(t)$ is the updated section flow of section m after station i is under control.

To rationally remove all bottlenecks, an iterative elimination algorithm is employed in this work:

(1) Sort all bottlenecks according to their overload rate in descending order and select the first one as the target bottleneck.

(2) Establish the inflow control plan for the target bottleneck by using the single bottleneck elimination algorithm; see Section 3.4.

(3) Update the section flows of the related sections using (19).

(4) Identify the bottlenecks on the network after updating the section flows.

(5) If all the bottlenecks are removed, the SIC plan is finished; otherwise, go to step (1).

3.6. SIC Scheme Generation for a Network. Based on the above approach for generating a SIC plan for a single time interval, the algorithm for construction of a SIC scheme throughout the peak hours (7:00 am to 9:00 am) will be established in this section. A SIC scheme usually contains three elements, namely, control stations, control time, and strength. Discretize the continuous time into short time periods of equal length (such as 30 minutes). For each time span, the proposed algorithm can be used to generate a portion of the SIC plan.

It should be noted that the passengers who are limited to entering the station in time interval t will influence the travel demand in next time period $t + 1$. Moreover, travel behaviors of passengers can be influenced by SIC actions, such as travel mode change and departure time reschedule. Then the temporal-distribution of demand can be changed. However, it is very hard to consider the interactive influence when we construct the SIC plan. In this work, it is supposed that travel demand is constant, and passengers will wait outside the station and not transfer to other travel modes.

Then, the travel demand at time $t + 1$ should be updated by considering the passengers delayed at time t, as shown in (20). Moreover, the section flows related to the control station should also be updated, as shown in (21).

$$\overline{d}_i(t+1) = d_i(t+1) + d_i'(t), \quad \forall t \in T,\ t \geq 1, \tag{20}$$

where $\overline{d}_i(t)$ is the updated inflow of station i in time $t + 1$, if the station is controlled in time t.

$$\overline{q}_m(t+1) = q_m(t+1) + d_i'(t) \cdot \varphi_i^m(t+1), \quad \forall t \in T,\ t \geq 1, \tag{21}$$

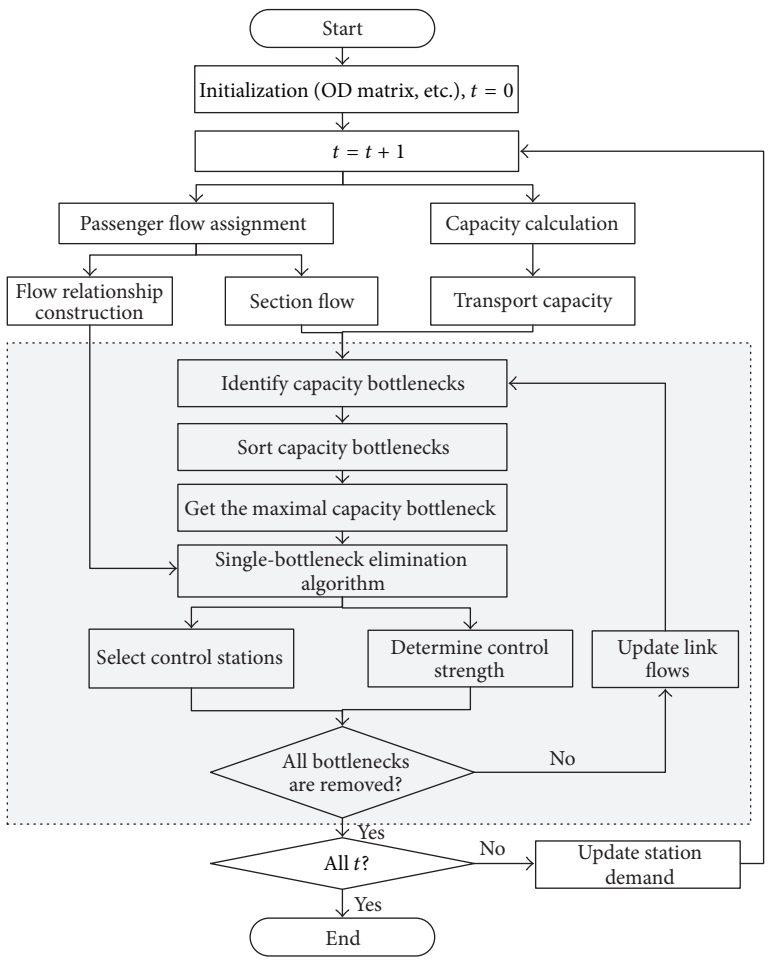


Figure 7: Algorithm for constructing SIC plan in whole peak hours.

where $\overline{q}_m(t)$ is the updated section flow of section m in time $t + 1$.

The iterative algorithm based on the bottleneck elimination approach to generate the whole SIC scheme during peak hours is presented in Figure 7.

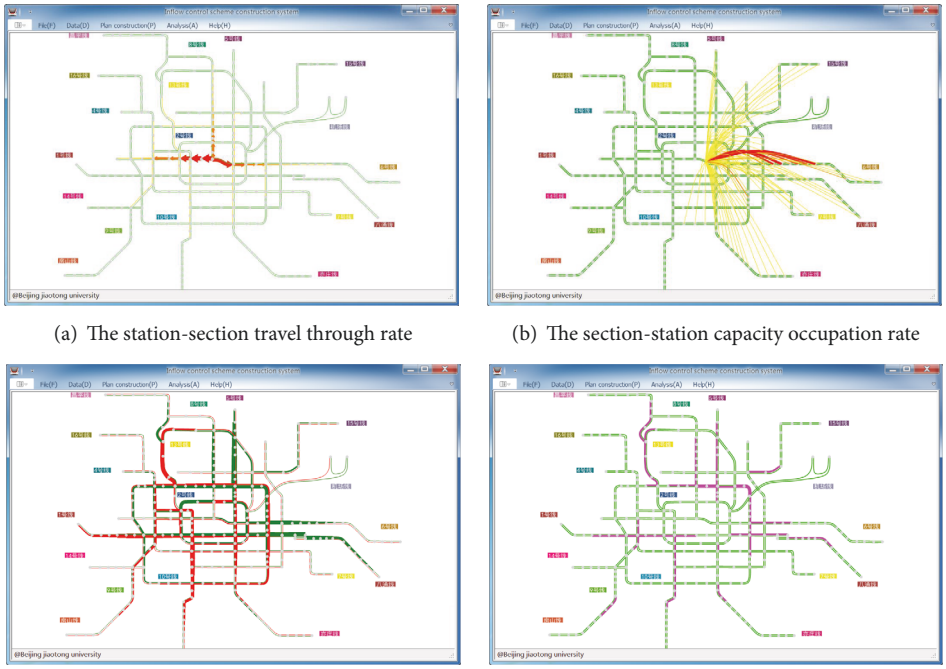
4. Method Implementation

4.1. Tool Description. A decision support tool based on the proposed method was developed in the C# programming language and is used to validate the efficiency and performance of the approach. The tool was executed on an Intel PC under Windows 7 with a 3.8 GHz CPU and 4 G RAM. Figure 8 shows the main interfaces of the tool. Thanks to the Beijing Subway Operation Co. Ltd. for providing data, the tool has been tested in the Beijing subway system. The results demonstrate that it has high efficiency for applications in large transit networks, and reasonable SIC plans can be obtained.

The main capabilities of the tool include the following: (a) constructing time-dependent internal-relationship between station inflows and section flows and representing them in graphical visualization; (b) generating SIC schemes based on the iterative bottleneck elimination algorithm and automatically exporting the plan in the form of a report; (c) evaluating the performance of the generated SIC plan; (d) tracking the detailed elimination process for a particular bottleneck.

4.2. Timeframe for Updating SIC Plan. Though the proposed method has no limitation on the iteration step length for updating SIC plan, other factors from applicability and robustness should be considered in practice. Firstly, we will analyze the stability of the model inputs, which are time-dependent OD tables. The relative deviation shown in (22) is used to measure the stability of OD flows. Figure 9 shows the deviations of OD flows between two related weekdays (the day and the same weekday of last week) in different time intervals. It can be clearly found that the deviations increase greatly with the time interval shortening. If the time step is five minutes, the average deviation is over 50%. There is no doubt that SIC plans are not credible if a short time span is chosen to update the plan.

$$\mathrm{RE}(t) = \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \frac{\left| d_{ij}(t) - \overline{d}_{ij}(t) \right| / d_{ij}(t)}{M} \times 100\%, \qquad (22)$$



(a) The station-section travel through rate

(b) The section-station capacity occupation rate

(c) Flow distribution on network

(d) Bottleneck distribution on network

Figure 8: Main interfaces of the decision support tool.

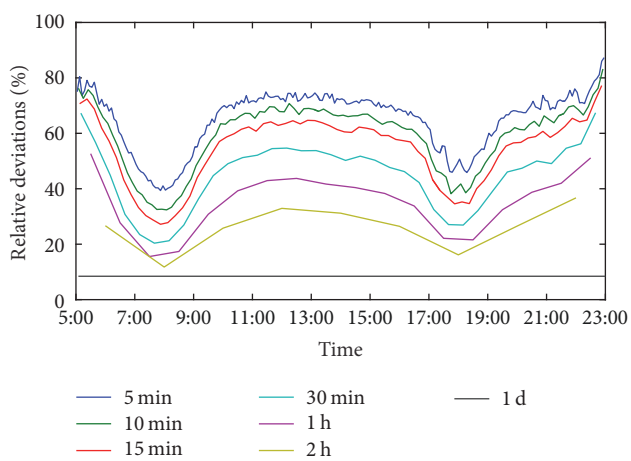


Figure 9: Relative deviation of OD flows in different time interval.

where RE(t) is the average relative deviation of OD flows in time interval t; M is the total number of OD pairs; and $d_{ij}(t)$ and $\overline{d}_{ij}(t)$ mean the volume of flows in a certain day and the contrastive day of last week, respectively.

Secondly, the feasibility of implementation of control actions should also be considered when we make the plan. Because rail transit stations lack flexible control equipment, it is difficult to change control actions frequently, such as fences. Consider the variation of travel demand and the feasibility of implementation in practice; we suggest that the timeframe (time length) for updating SIC plan should not be less than 30 minutes.

4.3. Case Study of Beijing Subway

4.3.1. Data. The Beijing subway, one of the largest and most congested transit systems in China, was analyzed as a case study. To avoid the fluctuating influence of travel demand, the average travel demand over five weekdays from Monday April 10th to Friday April 14th in 2017 is used in the model. The demand was obtained from the automatic fare collection (AFC) system. Time-dependent inflows of each line are provided in Figure 10. The SIC action is usually carried out during the morning and evening peak hours. However, only morning peak hours (7:00 am to 9:00 am) are discussed in the study.

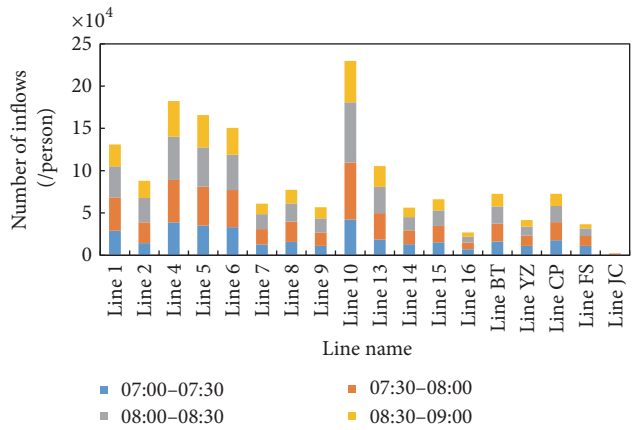


Figure 10: Time-dependent inflows of each line during morning peak hours.

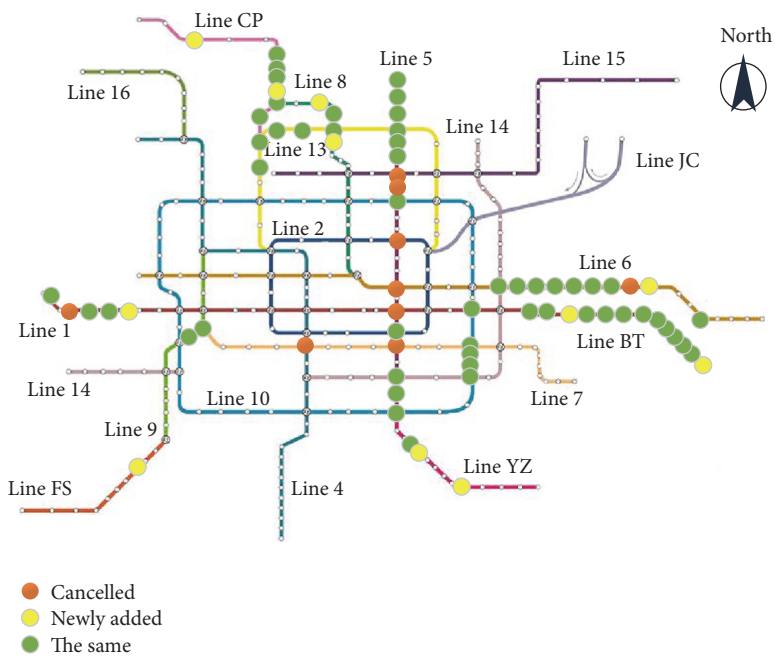


Figure 11: Distribution of the generated and actual SIC plans for the network (morning peak hours).

The time span for updating the SIC scheme is set to be 30 minutes. Hence, there are four small time intervals during the morning peak hours. Transport capacity is an important parameter for identifying bottlenecks. In this work, the train timetable is employed to calculate the transport capacity for each line or section, which is presented in Table 2.

Moreover, traffic conditions around the station (such as the bus and square area) are obtained from a traffic survey, and platform load capacity is determined from the design map of stations. These parameters are crucial to determining the station weight when removing a particular bottleneck. Considering the copious amount of information in these parameters, they are not represented here in detail.

Note that Line 4, Line 14, and Line 16, which belong to Beijing Mass Transit Rail (MTR) Corporation, are excluded from the SIC scheme. The main reason is the different operational concepts between the Beijing subway and MTR Corporation. Regular SIC actions are never applied to these lines.

4.3.2. Results. In April of 2017, the Beijing subway controlled 61 stations during morning peak hours. The generated SIC plan from the proposed method includes 63 stations. Comparing with the actual SIC plan, 52 stations are the same as the actual controlled stations, 9 stations should be cancelled, and 11 new stations should be controlled. Figure 11 shows the distribution of the actual and generated SIC plans for the network, in which the "canceled" means the station is controlled in practice but not included in the generated plan, and "newly added" has an opposite meaning. Table 3 presents the detailed plan of each line.

Table 2: Transport capacity for each line during peak hours.

Line	Vehicle capacity (person)	Train formation (vehicles)	Train capacity (person/train)	PHF	Frequency (trains/h)		Transport capacity (person/h)	
					7:00–8:00	8:00–9:00	7:00–8:00	8:00–9:00
Line 1	240	6	1440	0.83	30	30	35850	35850
Line 2	240	6	1440	0.77	27	30	29782	33091
Line 4	240	6	1440	0.89	24	24	30789	30789
Line 5	240	6	1440	0.89	27	27	34638	34638
Line 6	240	8	1920	0.86	18	18	29609	29609
Line 7	240	8	1920	0.84	16	16	25943	25943
Line 8	240	6	1440	0.80	16	19	18450	21910
Line 9	240	6	1440	0.85	18	18	22027	22027
Line 10	240	6	1440	0.81	20	15	23235	17427
Line 13	240	6	1440	0.85	22	25	26853	30514
Line 14	310	6	1860	0.86	15	15	23984	23984
Line 15	240	6	1440	0.86	13	13	16092	16092
Line 16	240	6	1440	0.85	12	12	14709	14709
Line BT	240	6	1440	0.86	20	20	24681	24681
Line YZ	240	6	1440	0.85	10	10	12258	12258
Line CP	240	6	1440	0.85	10	10	12240	12240
Line FS	240	6	1440	0.79	10	10	11340	11340
Line JC	224	4	896	0.97	6	6	5230	5230

Table 3: The number of controlled stations in each line.

Line name	Actual plan	Generated plan	Canceled stations	Newly added stations
Line 1	6	6	−1	1
Line 2	0	0	0	0
Line 4	0	0	0	0
Line 5	16	11	−5	0
Line 6	9	9	−1	1
Line 7	2	0	−2	0
Line 8	1	3	0	2
Line 9	2	2	0	0
Line 10	5	5	0	0
Line 13	4	4	0	0
Line 14	0	0	0	0
Line 15	0	0	0	0
Line 16	0	0	0	0
Line BT	9	11	0	2
Line YZ	1	3	0	2
Line CP	6	8	0	2
Line FS	0	1	0	1
Line JC	0	0	0	0
SUM	61	63	−9	11

Consider too many items in the SIC scheme, only an example of the detailed plan for Line 1 and BT is shown in Table 4. The control rate represents the control strength, which equals the percentage of limited passengers and all inflows. It can also be regarded as the reduction of speed of passengers entering the station. In Table 4, the station names are represented by their acronyms.

Table 5 provides the detailed elimination process for key bottlenecks (the heaviest congestion) in Line 1 and BT between 7:30 am and 8:00 am. For a certain bottleneck, the target control stations, station weights, and control volumes can be tracked. Through tracking the bottleneck elimination process, we can determine the function of each control station for a certain bottleneck.

4.3.3. Performance Evaluation. Measuring the performance of SIC schemes is important for carrying out a new plan in practice. However, it is difficult because the actual plan of the Beijing subway is not clear. We can only know the control stations and a rough range of control times (such as 7:00 am to 9:00 am), and we do not know pivotal information about the control strength of each controlled station. Therefore, the number of delayed passengers cannot be determined. In this work, the section flows are used to evaluate the performance in an indirect way. First, time-varying link flows of each section under the actual SIC plan can be obtained from the Revenue Clearance Center (RCC) of the Beijing subway, which represents the performance of the practical plan. Then, section flows under the generated SIC scheme can be gained from the flow assignment model. The relative deviation of section flows between the practical and generated plan is used to measure the SIC performance, as shown in

$$\delta(t) = \sum_{m \in E'} \frac{\left(q'_m(t) - q''_m(t)\right) / q''_m(t)}{N} * 100\%, \tag{23}$$

where $\delta(t)$ is the average relative deviation of all section flows, $q'_m(t)$ is the section flow of section m at time t using the generated SIC scheme, $q''_m(t)$ is the actual flow from the RCC, and E' is the set of sections with N elements.

Table 6 shows the average relative deviation of section flows in each line. Note that the travel demand used to generate SIC plan is the average OD tables from five consecutive weekdays in April of 2017. Therefore, the actual section flows are the corresponding mean value. The following can be seen:

(1) In examining the results over all the peak hours, the change of relative deviations is very small. It is easy to understand that SIC action only influences the departure time of travelers rather than the travel demand. Hence, the total flows through the section will not change greatly.

(2) In view of the small time span, the section flows in the controlled lines generally increase during 7:00 am~8:00 am and decrease during 8:00 am~9:00 am to a certain degree. This indicates that passengers enter the station earlier and that the new plan has a positive impact on reducing passengers' travel delays.

(3) The larger the section flow values change, the more controlled stations there are in these lines, such as Line 1 and Line 5. These lines are the most congested lines on the network. Put another way, it is necessary to adjust the actual SIC plan in these lines to improve the flow management.

4.3.4. Guidance for SIC Actions. After the SIC scheme for a network is obtained, there is another important work that should be considered, which is how to determine the detail control actions for a specific station. In practice, station masters decide how to control flows according to their experience and the specific operational environment

Table 4: An example of the generated SIC scheme.

ID	Station name	Line name	Control rate (%)				
			7:00–7:30	7:30–8:00	8:00–8:30	8:30–9:00	9:00–9:30
(1)	PGY	Line 1	4.56	34.53	41.92	/	/
(2)	GCL	Line 1	1.74	15.93	15.51	/	/
(3)	BJYLY	Line 1	/	25.00	18.72	/	/
(4)	BBS	Line 1	4.33	33.47	36.66	/	/
(5)	YQL	Line 1	/	24.99	/	/	/
(6)	WKS	Line 1	/	24.97	/	/	/
(8)	SH	Line 1	/	/	49.72	/	/
(9)	SHD	Line 1	/	/	9.73	/	/
(10)	CMDX	Line BT	22.85	59.63	40.26	/	41.60
(11)	SQ	Line BT	41.11	52.05	52.75	38.80	47.90
(12)	GZ	Line BT	44.26	69.17	47.25	16.14	/
(14)	TZBY	Line BT	24.98	50.00	49.95	/	/
(15)	GY	Line BT	50.00	70.00	70.00	28.13	/
(17)	LY	Line BT	54.17	70.00	70.00	27.70	/

Note. The "/" represents no control.

Table 5: Elimination processes of certain bottlenecks (7:30 am~8:00 am).

Line name	Bottleneck	Target control stations	Control weight	Control volume (Person)	Inflows (Person)	Control rate (%)
Line 1	WSL->GZF	PGY	0.314	2109	6107	34.53
		BBS	0.160	588	3693	15.93
		GGL	0.077	961	3842	25.00
		BBYLY	0.127	992	2964	33.47
		YQL	0.217	814	3257	24.99
		WKS	0.105	537	2151	24.97
Line BT	CMDX->GBD	LY	0.240	1971	2815	70.00
		GY	0.264	1798	2569	70.00
		CMDX	0.093	1412	2368	59.63
		GZ	0.125	1593	2303	69.17
		SQ	0.096	1039	1996	52.05

of the station. This is a difficult work because each station may have different physical structure, organization rules for passenger flows, flow characteristics of different entrances and exits, and so on. This work cannot provide a normalized method to determine the detail station control actions.

However, the control rate/strength of the scheme could provide a guidance for setting control actions. The control rate represents the percentage of passengers who are limited to entering the station relative to the travel demand, which can also be regarded as the flow speed reduction rate. Then, what we need to do is to use some proper measures to slow down the inflow speed. We think the microsimulation tool, such as LEGION and VISSIM, which is widely used for evacuation evaluation for stations, is a useful method to determine the detail actions.

5. Conclusion

In this study, an algorithm based on a bottleneck elimination strategy for generating SIC schemes was proposed, which has high computational efficiency for application to large-scale subway networks. The approach has a clear meaning and can be expanded easily by considering other factors. A tool based on the proposed method has been developed in the C# programming language. A case study of the Beijing subway validates that there are good performance and practical value of the model. The study can support managers in constructing scientifically SIC plans and help station masters adopt proper actions in reference to the control rate.

A static SIC plan is suitable for recurrent congestion during peak hours because commute travel is very stable in the subway system. However, there are also sudden changes

Table 6: Deviations of section flows between the new scheme and the actual scheme.

ID	Line	Relative deviation (%)					
		7:00–7:30	7:30–8:00	8:00–8:30	8:30–9:00	9:00–9:30	Average
(1)	Line 1*	**4.66**	**5.66**	**4.78**	−7.75	−12.19	−0.97
(2)	Line 2	**0.54**	**3.58**	−0.12	0.49	−1.07	0.68
(3)	Line 4	**1.83**	**2.77**	**1.49**	**1.03**	−0.81	1.26
(4)	Line 5*	**10.33**	**8.95**	−6.24	−7.21	−8.97	−0.63
(5)	Line 6*	**3.88**	**7.11**	−7.40	−6.50	−3.93	−1.37
(6)	Line 7*	**0.32**	**0.73**	**0.67**	**0.29**	0.33	0.47
(7)	Line 8*	**6.16**	**7.75**	−7.31	−3.78	−1.61	0.24
(8)	Line 9*	**0.22**	−1.79	−2.14	0.83	−1.58	−0.89
(9)	Line 10*	−1.17	**0.84**	−0.84	**1.38**	0.51	0.14
(10)	Line 13*	−1.15	−0.89	**1.35**	−1.06	0.28	−0.29
(11)	Line 14	**0.10**	**0.10**	−1.07	−0.47	0.03	−0.26
(12)	Line 15	**0.72**	−0.54	**0.96**	−0.27	−1.33	−0.09
(13)	Line 16	**0.34**	−0.21	**1.63**	−1.15	−1.71	−0.22
(14)	Line BT*	−0.29	**1.02**	**0.31**	−0.13	1.26	0.43
(15)	Line YZ*	−1.93	**1.71**	−1.21	−0.35	−0.26	−0.41
(16)	Line CP*	−1.90	**3.12**	**4.31**	−5.58	−1.07	−0.22
(17)	Line FS	**0.65**	**6.37**	−3.42	−4.64	−2.96	−0.80
(18)	Line JC	−1.11	**0.03**	−0.01	0.55	1.10	0.11

Note. ∗ means there is control stations on these lines.

in congestion pattern caused by mass flows, unexpected accidents, severe weather, and so on. Online self-adaptive control approaches can be developed in the future by considering real-time passenger flow status, such as passenger flow density on the platform and train.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this work.

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