Hindawi
Journal of Advanced Transportation
Volume 2019, Article ID 1524178, 12 pages
https://doi.org/10.1155/2019/1524178

Research Article

Day-to-Day Evolution of Traffic Flow with Dynamic Rerouting in Degradable Transport Network

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Received 28 April 2019; Revised 2 September 2019; Accepted 25 September 2019; Published 23 December 2019

Academic Editor: Oded Cats

Random events like accidents and vehicle breakdown, degrade link capacities and lead to uncertain travel environment. And whether travelers adjust route or not depends on the utility difference (dynamic rerouting behavior) rather than a constant. Considering travelers' risk-taking behavior in uncertain environment and dynamic rerouting behavior, a new day-to-day traffic assignment model is established. In the proposed model, an exponential-smoothing filter is adopted to describe travelers' learning for uncertain travel time. The cumulative prospect theory is used to reflect route utility and its reference point is adaptive and set to be the minimal travel time under a certain on-time arrival probability. Rerouting probability is determined by the difference between expected utility and perceived utility of previously chosen route. Rerouting travelers choose new routes in a logit model while travelers who do not choose to reroute travel on their previous routes again. The proposed model's several mathematical properties, including fixed point existence, uniqueness, and stability condition, are investigated through theoretical analyses. Numerical experiments are also conducted to validate the proposed heuristic stability condition, show the effects of four main parameters on dynamic natures of the system, and investigate the differences of the system based on expected utility theory and cumulative prospect theory and with static rerouting behavior and dynamic rerouting behavior.

1. Introduction

Transportation network modeling mainly examines the final equilibrium state for the past few decades on an explicit or implicit assumption that if equilibrium exists, then it will also occur. The assumption is very ideal; in fact, it was quite contrary to happen [1]. To explain whether final equilibrium state can be achieved and its attainment, a substantial stream of research has been developed to look into "day-to-day" flow dynamics. They can be divided into two categories based on types of approaches, i.e., stochastic ones [2–7] or the deterministic [8–22]. The everyday traffic flow is stochastic in stochastic one while it is deterministic in deterministic one. Under the deterministic framework, traffic flow adjustment has been formulated in either continuous time [8–17] or discrete time [18–26]. In reality, traffic flows are not continuously adjustable over days because of activity constraints [26]. Hence, it is more appropriate to model traffic flow dynamics as a discrete-time system with each discrete epoch representing a lag over which travels may be repeated, e.g., days or weekdays [26].

Horowitz [18] established a day-to-day (DTD) deterministic discrete time traffic assignment model in which perceived travel time is a weighted average of experienced travel times on previous days. Jha et al. [19] further considered the confidence of the perception and established a perception update model based on Bayesian model. Huang et al. [20] modelled the adaptation of advanced traveler information system and its effect on route choice. Under uncertain traffic network, Xu et al. [25] suggested adopting the route choice model based on cumulative prospect theory. Guo et al. [26] presented a link-based model to avoid the numeration of a large number of routes.

In these models, traffic flows are the result of travelers' perceived travel costs by learning experienced travel costs. The experienced travel costs depend on travelers' route choice and

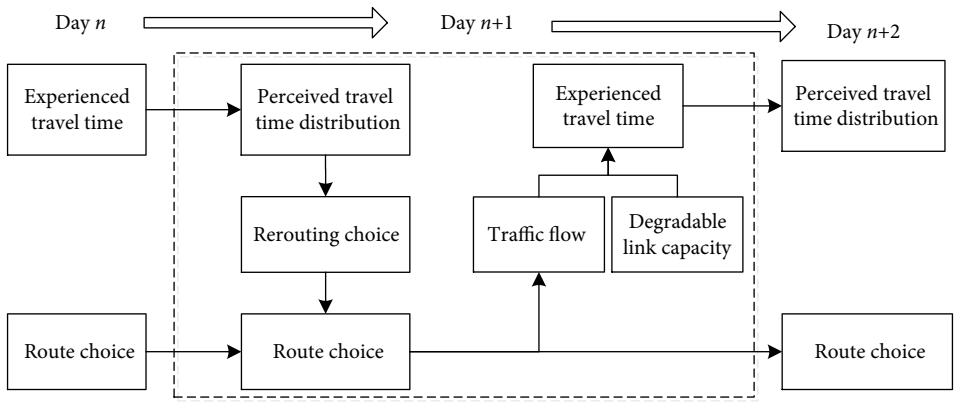

Figure 1: Framework of day-to-day traffic assignment model.

the traffic network. In reality, traffic networks frequently suffer from some minor events including vehicle breakdown, accidents and so on. Such minor incidents occur randomly in our daily commutes and degrade the link capacity, leading to travel time uncertainty [27]. However, most of the DTD models previously introduced do not address travel time uncertainty led by randomly degradable link capacity.

Meantime, they commonly assume that travelers always adjust their routes to pursue the optimal utility or that a fixed proportion of travelers adjust their route considering their inertia. However, both of them do not coincide with reality. For one thing, travelers do not always seek the optimal solution because they accept a satisfactory solution [28]. For another, travelers' inertia changes with the situation, which is validated through a set of laboratory experiments by Mahmassani and Liu [29]. For example, when the travel cost on a certain route significantly change, it is likely for travelers to give up their inertial behaviors and to choose another route.

This paper simultaneously considers the influence of travel time uncertainty on route choice and travelers' dynamic rerouting behavior (DR) to construct a more realistic DTD traffic assignment model. In our model, travel time mean and variation of a route is updated by an exponential smooth filter. Perceived route utility is reference-dependent and follows cumulative prospect theory (CPT). Travelers' rerouting choice is dynamically determined by the difference between expected travel utility and utility of previously chosen route. Based on the model, the natures of the dynamic system are further investigated, which provides the valuable information for network design and management.

The remainder of this paper is organized as follows. The new day-to-day learning model will be elaborated in Section 2. In Section 3, we further discuss the existence, the uniqueness and the stability of the fixed point of the evolutionary model. These theoretical analyses are verified by numerical experiments in Section 4. Section 5 gives conclusions and some suggestions on the future direction.

2. Model Description

Figure 1 gives the framework of our day-to-day traffic assignment model. The model includes three main parts: perceived route travel time distribution, rerouting choice and route choice. A traveler will be able to obtain the information on travel time of his chosen route after finishing a trip. Assuming that all travelers communicate the information with each other, each traveler will be able to know travel time distribution of all routes. Based on it, travelers will predict the route travel time distribution next day, which is described by the perceived route travel time distribution model. Travelers will further get know of attractiveness of all routes and decide whether to reroute and which route to choose if rerouting. The rerouting probability positively depends on the difference between traveler's expected utility and the perceived utility of the previously chosen route. Based on it, the rerouting choice model is established. Then, rerouting travelers will seek the maximal utility and choose the optimal route while travelers not to reroute will travel on their previous routes. The chosen probability of a route consists of them, which is exactly our route choice model. We specifically illustrate these models in the next section.

2.1. Perceived Route Travel Time Distribution. Assumed that all travelers communicate experienced travel time with each other after finishing trips, travelers are able to get the route travel time distribution. Because route travel time distribution is normal regardless of link travel time distribution as long as Lindenberg's condition is satisfied [24, 27], travelers fit the route travel time with a normal distribution. Its mean and variance should be consistent with:

$$T_{r,w}^{n} \sim N\left(E\left(T_{r,w}^{n}\right), \operatorname{var}\left(T_{r,w}^{n}\right)\right),\ T_{r,w}^{n} \in \left[\underline{T}_{r,w}, \overline{T}_{r,w}\right],\ \forall r \in R_w, w \in W, \tag{1}$$

$$E\left(T_{r,w}^{n}\right) = \sum_{l \in L} \Lambda_{r,l} E\left(t_l^n\right),\ \forall r \in R_w,\ w \in W, \tag{2}$$

$$\operatorname{var}\left(T_{r,w}^{n}\right) = \sum_{l \in L} \Lambda_{r,l} \operatorname{var}\left(t_l^n\right),\ \forall r \in R_w,\ w \in W, \tag{3}$$

where $T_{r,w}^{n}$ is the experienced travel time distribution on route r between origin and destination (O–D) pair w on day n. $\underline{T}_{r,w}$ and $\overline{T}_{r,w}$ are, respectively, infimum and supremum of route travel time. The former is the free-flow travel time of route r

while the latter is set as the 99.74 percentile route travel time on day n. $\Lambda_{r,l}$ is the route-link index whose value is 1 if link l belongs to route r, zero otherwise. And t_l^n is the travel time distribution of link l on day n.

On the condition that Bureau of Public Road (BPR) link performance function is used to describe link travel time, that link capacity distributions are independent of each other, that the link capacity distribution is independent of traffic flow volume on it and that the link capacity is uniformly distributed with the design capacity as its upper bound and the worst degraded capacity as its lower bound, the mean and variance of the link travel time can be derived as follows [27]:

$$E(t_l^n) = t_l^{free} + \alpha t_l^{free}(x_l^n)^\beta \frac{\left(1-(\varepsilon_l)^{1-\beta}\right)}{(\overline{u}_l)^\beta(1-\varepsilon_l)(1-\beta)}, \ \forall l \in L, \quad (4)$$

$$\begin{aligned}\operatorname{var}(t_l^n) = \alpha^2\left(t_l^{free}\right)^2(x_l^n)^{2\beta}\Bigg\{&\frac{\left(1-(\varepsilon_l)^{1-2\beta}\right)}{(\overline{u}_l)^{2\beta}(1-\varepsilon_l)(1-2\beta)} \\ &-\left[\frac{\left(1-(\varepsilon_l)^{1-\beta}\right)}{(\overline{u}_l)^\beta(1-\varepsilon_l)(1-\beta)}\right]^2\Bigg\}, \ \forall l \in L,\end{aligned} \quad (5)$$

where subscript l refers to a particular link; t_l^{free}, $\overline{u}_l$ and t_l^n, respectively, are link $l's$ free-flow travel time, design capacity and travel time with flow x_l^n; α and β are deterministic parameters; and ε_l is the worst capacity degradable rate of link l.

Based on the newly obtained travel time distribution, travelers will update their perception. We assume that the update process is consistence with an exponential smoothing filter. Hence, the perceived travel time distribution can be expressed as:

$$\begin{aligned}\widetilde{T}_{r,w}^{n+1} =&(1-\lambda)\widetilde{T}_{r,w}^n + \lambda T_{r,w}^n, \forall \widetilde{T}_{r,w}^{n+1} \in \left[\underline{T}_{r,w}, \overline{T}_{r,w}\right], \\ &r \in R_w, w \in W,\end{aligned} \quad (6)$$

where $\widetilde{T}_{r,w}^{n+1}$ is the perceived route travel time distribution on day $n+1$; $\lambda(0 \le \lambda \le 1)$ is a constant that reflects travelers' learning rate.

According to the work of Lou and Cheng [24], the mean and variance of perceived travel time distribution can be calculated as follows:

$$\widetilde{T}_{r,w}^{n+1} \sim N\left(E\left(\widetilde{T}_{r,w}^{n+1}\right), \operatorname{var}\left(\widetilde{T}_{r,w}^{n+1}\right)\right), \quad \forall r \in R_w, w \in W, \quad (7)$$

$$E\left(\widetilde{T}_{r,w}^{n+1}\right) = (1-\lambda)E\left(\widetilde{T}_{r,w}^n\right) + \lambda E(T_{r,w}^n), \quad \forall r \in R_w, w \in W, \quad (8)$$

$$\begin{aligned}\operatorname{var}\left(\widetilde{T}_{r,w}^{n+1}\right) =&(1-\lambda)\operatorname{var}\left(\widetilde{T}_{r,w}^n\right) + \lambda\operatorname{var}(T_{r,w}^n) \\ &+ \lambda(1-\lambda)\left(E(T_{r,w}^n) - E\left(\widetilde{T}_{r,w}^n\right)\right)^2, \\ &\forall r \in R_w, w \in W.\end{aligned} \quad (9)$$

2.2. Perceived Utility. With different behavioral assumptions on travelers' risk-taking behaviors, various models have been proposed to describe the route utility in uncertain environment including the expected-utility-theory-based models [30, 31], the travel time budget models [27], the late arrival penalty model [32], α-reliable mean-excess travel time model [33] and the cumulative-prospect-theory-based models [34, 35]. Substantial evidence indicates that cumulative prospect theory (CPT) provides a well-supported descriptive paradigm for decision making under uncertainty because it captures risk-attitude conversion and high estimation of low probability [36–38]. Therefore, we use it to describe perceived utilities for alternative routes.

In the CPT, the reference point is a key parameter which directly determines the perceived gain and loss. Individual adjusts it according to the change [39]. Hence, we use an adaptive adjustment reference point, like Xu et al. [35]. The reference point is the time that travelers budget to ensure a desired on-time arrival probability. If the desired on-time arrival probability is set as ρ, the reference point on the day $n+1$, $RTT_w^{n+1}(\rho)$, is the minimum of the ρth percentile travel times of all alternative routes. It adaptively changes with travelers' perception. Specifically, it is expressed as:

$$RTT_w^{n+1}(\rho) = \min_{r \in R_w}\left(F_{r,w}^{n+1}\right)^{-1}(\rho), \quad \forall w \in W, \quad (10)$$

$$\begin{aligned}F_{r,w}^{n+1}(\tau) &= \Pr\left(\widetilde{T}_{r,w}^{n+1} < \tau\right) \\ &= \int_{\underline{T}_{r,w}}^{\tau} \frac{1}{\sqrt{2\pi\operatorname{var}\left(\widetilde{T}_{r,w}^{n+1}\right)}} e^{-\left(\tau - E\left(\widetilde{T}_{r,w}^{n+1}\right)\right)^2/2\operatorname{var}\left(\widetilde{T}_{r,w}^{n+1}\right)} d_\tau, \quad \forall r \in R_w, w \in W,\end{aligned} \quad (11)$$

where $F_{r,w}^{n+1}$ is the cumulative distribution function of route r travel time between O–D pair w on day $n+1$.

The perceived prospect value for route r between O–D pair w on day $n+1$, denoted as $V_{r,w}^{n+1}(\tau)$, can be formulated as:

$$\begin{aligned}V_{r,w}^{n+1}(\tau) =&\int_{\underline{T}_{r,w}}^{RTT_w^{n+1}(\rho)} \frac{d_{h\left(F_{r,w}^{n+1}(\tau)\right)}}{d_\tau} g_{r,w}^{n+1}(\tau) d_\tau \\ &+ \int_{RTT_w^{n+1}(\rho)}^{\overline{T}_{r,w}} -\frac{d_{h\left(1-F_{r,w}^{n+1}(\tau)\right)}}{d_\tau} g_{r,w}^{n+1}(\tau) d_\tau, \\ &\forall r \in R_w, w \in W,\end{aligned} \quad (12)$$

where τ represents the travel time of route r between O–D pair w on day n.

$h(\cdot)$ and $g_{r,w}^{n+1}(\cdot)$ in the Formula (12) are, respectively, the weighting function and the value function. In the context of route choice, they can be specifically written as [24]:

$$h(\rho) = \frac{\rho^\gamma}{\left(\rho^\gamma + (1-\rho)^\gamma\right)^{1/\gamma}}, \quad (13)$$

$$g_{r,w}^{n+1}(\tau) = \begin{cases}\left(RTT_w^{n+1} - \tau\right)^\mu, \tau \le RTT_w^{n+1} \\ -\eta\left(\tau - RTT_w^{n+1}\right)^\upsilon, \tau < RTT_w^{n+1}\end{cases}, \quad \forall r \in R_w, w \in W, \quad (14)$$

where ρ is the probability; parameter γ refers to the adjustment level of the probability in the travelers' decision-making process; parameters μ and υ represent the degree of diminishing sensitivity of the value function. Typically, $0 < \mu, \upsilon < 1$ and

thus the value function exhibits risk-aversion over gains and risk-seeking over losses. And parameter $\eta(\eta \geq 1)$ is loss-aversion coefficient that indicates that travelers are more sensitive to losses than gains.

2.3. Rerouting Choice. A traveler has an expected utility for a trip and judges whether to reroute based on it. If the perceived utility of the route that he previously chose is higher than the expectation, he will not change route because satisfactory solution is good enough for travelers; otherwise, he will possibly choose to reroute. And the larger the difference between his expected prospect value and the perceived prospect value of the route that he previously chose, it is more likely for the traveler to change its routes. Based on these, rerouting probability of travelers choosing route r between O–D pair w on n on $n+1$, $\chi_{r,w}^{n+1}\left(\nabla V_{r,w}^{n+1}\right)$, is given by Equations (15) and (16).

$$\chi_{r,w}^{n+1}\left(\nabla V_{r,w}^{n+1}\right) = \begin{cases} \frac{\chi_0\left(\nabla V_{r,w}^{n+1}\right)^3}{\left(\nabla V_{r,w}^{n+1}\right)^3+\varpi} & \nabla V_{r,w}^{n+1} \geq 0 \\ 0 & \nabla V_{r,w}^{n+1} < 0 \end{cases}, \quad \forall r \in R_w, w \in W, \tag{15}$$

$$\nabla V_{r,w}^{n+1} = AV_w^{n+1} - V_{r,w}^{n+1}, \quad \forall r \in R_w, w \in W, \tag{16}$$

where $\nabla V_{r,w}^{n+1}$ is the difference between travelers' expected prospect value, denoted as AV_w^{n+1}, and the perceived prospect value of route r between O–D pair w on day $n+1$; χ_0 is a constant that represents the maximal rerouting probability; And ϖ is a parameter.

If the expected prospect value for the routes between O–D pair w is viewed as the expectation of maximal route prospect value through O–D pair w [40], AV_w^{n+1} is:

$$AV_w^{n+1} = \frac{\ln\sum_{k\in R_w} e^{\theta V_{k,w}^{n+1}}}{\theta}, \quad w \in W, \tag{17}$$

where θ reflects the heterogeneity of travelers. The higher θ is, the greater is the consistency of perception characteristics among travelers.

Proposition 2.1. $\chi_{r,w}^{n+1}\left(\nabla V_{r,w}^{n+1}\right) > 0$ *if* $|R_w| > 1$.

Proof.

$$\begin{aligned} \nabla V_{r,w}^{n+1} &= AV_w^{n+1} - V_{r,w}^{n+1}, \quad \forall r \in R_w, w \in W, \\ &= \frac{\ln\sum_{k\in R_w} e^{\theta V_{k,w}^{n+1}}}{\theta} - V_{r,w}^{n+1}, \\ &= \frac{\ln\left(\sum_{k\in R_w} e^{\theta V_{k,w}^{n+1}}/e^{\theta V_{k,w}^{n+1}}\right)}{\theta} \geq \frac{\ln 1}{\theta}. \end{aligned} \tag{18}$$

For $|R_w| > 1$, $\ln\left(\sum_{k\in R_w} e^{\theta V_{k,w}^{n+1}}/e^{\theta V_{k,w}^{n+1}}\right)/\theta > \ln 1/\theta$ and $\chi_{r,w}^{n+1}\left(\nabla V_{r,w}^{n+1}\right) > 0$. □

We summarize an important property of the rerouting model in Proposition 2.1. It states that if alternatives exist, there always are some travelers choosing to reroute.

2.4. Route Choice. For travelers choosing to reroute, they will choose optimal route to travel, i.e., the route with the maximal prospect value. Because the parameters in CPT would vary across the population, such as risk aversion coefficient, travelers' perceived prospect values are different. The difference can be reflected by adding a random error. If the random error complies with Gumbel distribution, the route choice probability of rerouting travelers, $\widetilde{P}_{r,w}^{n+1}$, can be given in a logit-based formula:

$$\widetilde{P}_{r,w}^{n+1} = \frac{e^{\theta V_{r,w}^{n+1}}}{\sum_{k\in R_w} e^{\theta V_{k,w}^{n+1}}}, \quad \forall r \in R_w, w \in W. \tag{19}$$

For travelers choosing not to reroute, they will continue to travel on the routes of day n. Total chosen probability of route r between O–D pair w on day $n+1$, $P_{r,w}^{n+1}$, is formulated as:

$$P_{r,w}^{n+1} = \sum_{k\in R_w}\left(\chi_{k,w}^{n+1}P_{k,w}^{n}\right)\widetilde{P}_{r,w}^{n+1} + \left(1-\chi_{r,w}^{n+1}\right)P_{r,w}^{n}, \quad \forall r \in R_w, w \in W. \tag{20}$$

The corresponding route flow pattern and link flow patterns are:

$$f_{r,w}^{n+1} = q_w P_{r,w}^{n+1}, \quad \forall r \in R_w, w \in W, \tag{21}$$

$$x_l^{n+1} = \sum_{w\in W}\sum_{r\in R_w} \Lambda_{r,l} f_{r,w}^{n+1}, \quad \forall l \in L, \tag{22}$$

where q_w is the demand of O–D pair w and $f_{r,w}^{n+1}$ is the flow of route r between O–D pair w.

3. Some Mathematical Properties of Proposed DTD Model

3.1. Fixed Point Existence. Supposed a fixed point is formed at day $\overline{n}$, we have $T_{r,w}^{n} = T_{r,w}^{n+1} = T_{r,w}^{*}$ and $\widetilde{T}_{r,w}^{n} = \widetilde{T}_{r,w}^{n+1} = \widetilde{T}_{r,w}^{*}$ for $n \geq \overline{n}$. Further, we infer $\widetilde{T}_{r,w}^{*} = T_{r,w}^{*}$ from $\widetilde{T}_{r,w}^{n+1} = (1-\lambda)\widetilde{T}_{r,w}^{n} + \lambda T_{r,w}^{n}$. At the same time, we have $P_{r,w}^{n} = P_{r,w}^{n+1} = P_{r,w}^{*}$ for $n \geq \overline{n}$. Then, the fixed point of the model can be described as:

$$E(t_l^*) = t_l^{free} + \alpha t_l^{free}(x_l^*)^{\beta} E\left(\frac{1}{(u_l)^{\beta}}\right), \quad \forall l \in L, \tag{23}$$

$$\operatorname{var}(t_l^*) = \alpha^2\left(t_l^{free}\right)^2(x_l^*)^{2\beta}\operatorname{var}\left(\frac{1}{(u_l)^{\beta}}\right), \quad \forall l \in L, \tag{24}$$

$$E(T_{r,w}^*) = \sum_{l\in L}\Lambda_{r,l}E(t_l^*), \quad \forall r \in R_w, w \in W, \tag{25}$$

$$\operatorname{var}(T_{r,w}^*) = \sum_{l\in L}\Lambda_{r,l}\operatorname{var}(t_l^*), \quad \forall r \in R_w, w \in W, \tag{26}$$

$$\begin{aligned} F_{r,w}^*(\tau) &= \Pr(T_{r,w}^* < \tau) \\ &= \int_{\underline{T}_{r,w}}^{\tau} \frac{1}{\sqrt{2\pi\operatorname{var}(T_{r,w}^*)}} e^{\left(-\left(\tau-E\left(T_{r,w}^*\right)\right)^2/2\operatorname{var}\left(T_{r,w}^*\right)\right)} d_\tau, \\ &\forall r \in R_w, w \in W, \end{aligned} \tag{27}$$

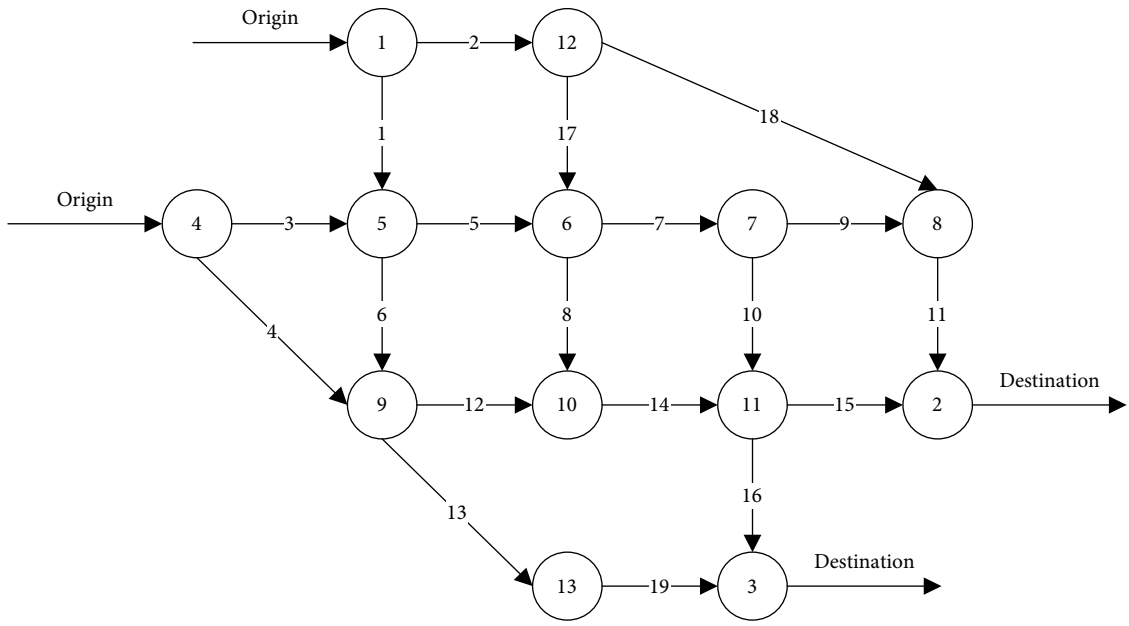

Figure 2: Nguyen–Dupuis network.

Table 1: Network characteristics.

Link	t_l^{free}	$\overline{u}_l$	Link	t_l^{free}	$\overline{u}_l$	Link	t_l^{free}	$\overline{u}_l$
1	5	900	8	5	800	15	5	900
2	7	700	9	7	900	16	6	800
3	7	700	10	6	900	17	7	600
4	12	500	11	9	500	18	20	300
5	6	800	12	6	800	19	10	600
6	5	800	13	12	500			
7	7	900	14	6	800			

Table 2: OD demands and routes.

O–D pair	(1, 2)	(1, 3)	(4, 2)	(4, 3)
Total demand (Veh·h^{-1})	600	1000	800	400
Route	1 (2-18-11)	8 (1-6-13-19)	14 (4-12-14-15)	19 (4-13-19)
	2 (1-5-7-9-11)	9 (1-6-12-14-16)	15 (3-5-7-9-11)	20 (4-12-14-16)
	3 (1-5-7-10-15)	10 (1-5-8-14-16)	16 (3-6-12-14-15)	21 (3-6-13-19)
	4 (1-5-8-14-15)	11 (1-5-7-10-16)	17 (3-5-8-14-15)	22 (3-6-12-14-16)
	5 (1-6-12-14-15)	12 (2-17-8-14-16)	18 (3-5-7-10-15)	23 (3-5-8-14-16)
	6 (2-17-8-14-15)	13 (2-17-7-10-16)		24 (3-5-7-10-16)
	7 (2-17-7-10-15)			
	8 (2-17-7-9-11)			

$$RTT_{r,w}^*(\rho) = \min_{k\in R_w} \left(F_{r,w}^*\right)^{-1}(\rho), \quad \forall r \in R_w, w \in W \tag{28}$$

$$g_{r,w}^*(\tau) = \begin{cases} (RTT_w^*(\rho) - \tau)^{\mu}, \tau \le RTT(\rho) \\ -\eta(\tau - RTT_w^*(\rho))^{\upsilon}, \tau > RTT(\rho) \end{cases}, \quad \forall r \in R_w, w \in W, \tag{29}$$

$$\begin{aligned} V_{r,w}^* = &\int_{\underline{T}_{r,w}}^{RTT_w^*(\rho)} \frac{d_{h\left(F_{r,w}^*(\tau)\right)}}{d_\tau} g_{r,w}^*(\tau) d_\tau \\ &+ \int_{RTT_w^*(\rho)}^{\overline{T}_{r,w}} -\frac{d_{h\left(1-F_{r,w}^*(\tau)\right)}}{d_\tau} g_{r,w}^*(\tau) d_\tau, \\ &\forall r \in R_w, w \in W, \end{aligned} \tag{30}$$

$$\tilde{P}_{r,w}^* = \frac{e^{\theta V_{r,w}^*}}{\sum_{k\in R_w} e^{\theta V_{k,w}^*}}, \quad \forall r \in R_w, w \in W, \tag{31}$$

$$AV_w^* = \frac{\ln \sum_{k\in R_w} e^{\theta V_{k,w}^*}}{\theta}, \quad \forall w \in W, \tag{32}$$

$$\nabla V_{r,w}^* = AV_{r,w}^* - V_{r,w}^*, \quad \forall r \in R_w, w \in W, \tag{33}$$

$$\chi_{r,w}^*\left(\nabla V_{r,w}^*\right) = \frac{\chi_0\left(\nabla V_{r,w}^*\right)^3}{\left(\nabla V_{r,w}^*\right)^3 + \varpi}, \quad \forall r \in R_w, w \in W, \tag{34}$$

Table 3: Basic numerical settings.

Parameter	BRP function	Worst link capacity degradation proportion	Learning rate
Specification	$\alpha = 0.15; \beta = 4$	$\varepsilon_l = 0.6,\ \forall l \in L$	$\lambda = 0.3$
Parameters	CPT	Rerouting choice	Heterogeneity of travelers
Specification	$\rho = 0.7; \mu = 0.37; \nu = 0.59; \eta = 1.51; \gamma = 0.74$	$\chi_0 = 0.3; \varpi = 1$	$\theta = 0.6$

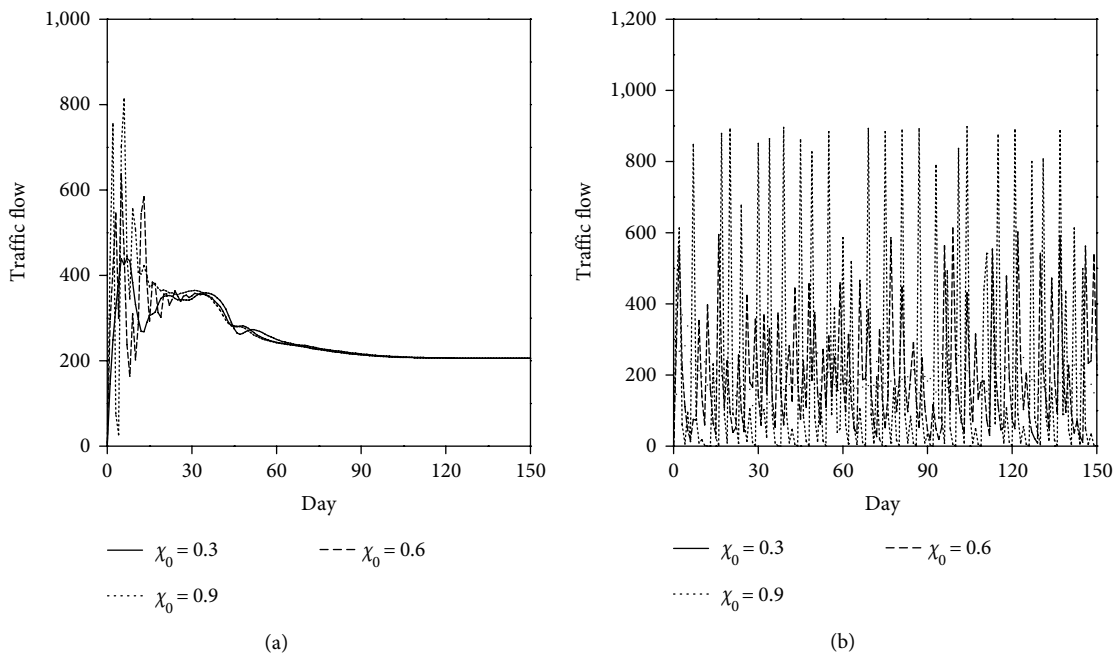

Figure 3: Evolutionary system with different learning rates. (a) $\lambda = 0.1$ and (b) $\lambda = 0.6$.

$$P^*_{r,w} = \begin{cases} \boldsymbol{A}^{-1}\boldsymbol{B} & |R_w| > 1 \\ \widetilde{P}^*_{r,w} & |R_w| = 1 \end{cases}, \quad \forall r \in R_w, w \in W,$$

$$\boldsymbol{A} = \begin{bmatrix} \chi^*_{1,w}\widetilde{P^*_{1,w}} & \chi^*_{2,w}\widetilde{P^*_{1,w}} & \cdots & \chi^*_{|R_w|-1,w}\widetilde{P^*_{1,w}} & \chi^*_{|R_w|,w}\widetilde{P^*_{1,w}} \\ \chi^*_{1,w}\widetilde{P^*_{2,w}} & \chi^*_{2,w}\widetilde{P^*_{2,w}} & \cdots & \chi^*_{|R_w|-1,w}\widetilde{P^*_{2,w}} & \chi^*_{|R_w|,w}\widetilde{P^*_{2,w}} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \chi^*_{1,w}\widetilde{P^*_{|R_w|-1,w}} & \chi^*_{2,w}\widetilde{P^*_{|R_w|-1,w}} & & \chi^*_{|R_w|-1,w}\widetilde{P^*_{|R_w|-1,w}} & \chi^*_{|R_w|,w}\widetilde{P^*_{|R_w|-1,w}} \\ 1 & 1 & \cdots & 1 & 1 \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} \underbrace{0, 0, \cdots 0}_{|R_w|-1}, 1 \end{bmatrix}^{\mathrm{T}}. \tag{35}$$

$$f^*_{r,w} = q_w P^*_{r,w}, \quad \forall r \in R_w, w \in W, \tag{36}$$

$$x^*_l = \sum_{w \in W} \sum_{r \in R_w} \Lambda_{r,l} f^*_{r,w}, \quad \forall l \in L. \tag{37}$$

We can see that $E(T^*_{r,w})$ and $\mathrm{var}(T^*_{r,w})$ are the function of all link flow x^*. Hence, $F^*_{r,w}(\tau)$ are the function of all link flow x^*. Further, we can observe that $RTT^*_{r,w}(\rho)$, $g^*_{r,w}(\tau)$ and $V^*_{r,w}$ all are the function of x^*. Based on Equations (31)–(37), the fixed point x^* defines a map to itself. And the feasible link flow is closed and convex. According to Brouwer's fixed-point theorem, if the self-map about x^* is continuous, there is at least a fixed-point.

Given that $F^*_{r,w}(\tau)$ is continuous with respect to x^*, $V^*_{r,w}$ is continuous with x^* [35]. The logit model ensures that $\widetilde{P}^*_{r,w}$ is continuous with respect to the route prospect value, V^*. Further, we can infer that $AV^*_{r,w}$, $\nabla V^*_{r,w}$ and $\chi^*_{r,w}$ are continuous with $V^*_{r,w}$. Based on them, we can obtain that $P^*_{r,w}$ is continuous with $V^*_{r,w}$. Therefore, the self-map of x^* are continuous and the fixed point exists.

3.2. Fixed Point Uniqueness. Fixed-point uniqueness can be analyzed by investigating the monotonicity of the self-map of x^*. However, the adopted route prospect value function, $V^*_{r,w}$, usually cannot be ensured to be strictly monotone increasing due to endogenous reference point. Therefore, the fixed-point uniqueness cannot be guarantee.

3.3. Fixed Point Stability. For any one fixed point, its stability heavily depends on the parameters adopted in the DTD model.

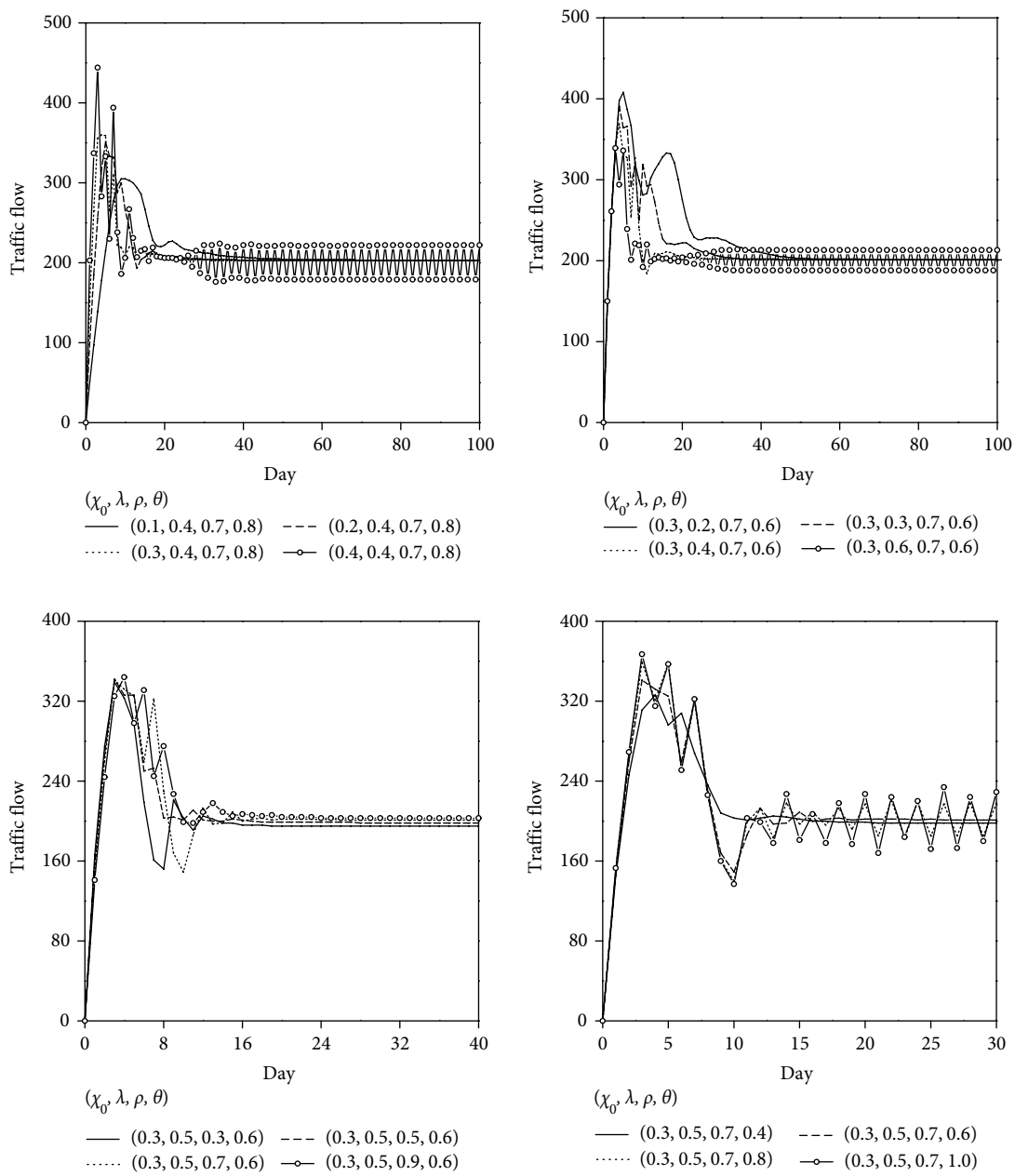

Figure 4: Evolutionary system with different parameter values.

Link flow x_l^{n+1} is the result of updated route choice P_w^{n+1}. P_w^{n+1} relies on new prospect values that are directly determined by means and variances of route travel times, $E\left(\widetilde{T}_{r,w}^{n+1}\right)$ and $\mathrm{var}\left(\widetilde{T}_{r,w}^{n+1}\right)$. Therefore, we can rewrite our model in an abstract way: $\left(E\left(\widetilde{T}_{r,w}^{n+1}\right), \mathrm{var}\left(\widetilde{T}_{r,w}^{n+1}\right), P_{r,w}^{n+1}, x_l^{t+1}\right) = \Psi\left(E\left(\widetilde{T}_{r,w}^{n}\right), \mathrm{var}\left(\widetilde{T}_{r,w}^{n}\right), P_{r,w}^{n}, x_l^{t}\right)$, where Ψ covers all relations formulated in Section 2. Let J denote the Jacobian matrix of the function Ψ. It is known that the evolutionary system defined by our model will converge to a fixed point if the 2R (R refers to the number of routes) eigenvalues of the matrix J are within the unit circle [20, 41]. However, it is very time-consuming, even impossible to derive the eigenvalues. Based on that the work of Cantarella and Cascetta [42], we give a heuristic condition for assuring the stability of the evolutionary system:

The weight λ takes a small enough value, then the evolutionary system defined by our model will likely converge to a fixed point.

For the system with deterministic travel time, the condition is proved, see Appendix.

4. Numerical Experiments

The proposed DTD model is applied to the Nguyen–Dupuis network [43] with 13 nodes and 19 links, whose topology is illustrated in Figure 2 and the link characteristics are displayed in Table 1. There are four O–D pairs in the network, 1-2, 1-3, 4-2 and 4-3. Table 2 gives their demands and all alternative routes. Basic numerical settings are given in Table 3. The traffic

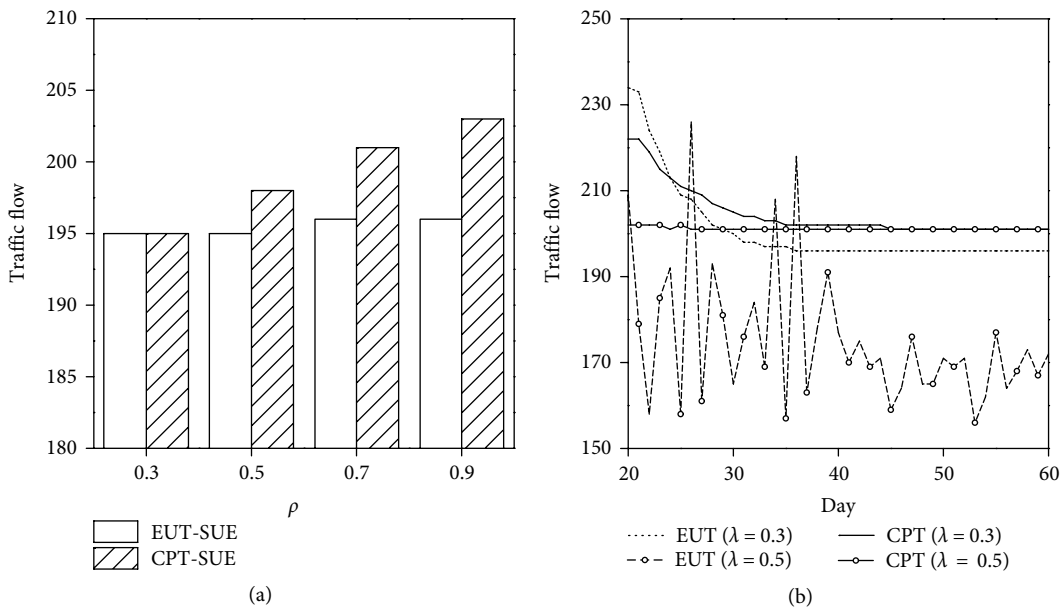

Figure 5: Evolutionary systems under EUT and CPT. (a) Equilibrium state. (b) Sensitivity to parameter λ.

flow on day 0 over traffic network is consistent with the outcome of the all-or-nothing assignment.

4.1. Heuristic Stability Condition. In the section, we validate our heuristic stability condition by comparing two cases. In Case I, the parameter λ is relatively small, i.e., $\lambda = 0.1$; in case II, it is amplified 6 times, i.e., $\lambda = 0.6$. Figure 3 shows the numerical results of these two cases with different maximal rerouting rate, χ_0. For graph simplicity, only the traffic flow on route 13 (Link order: 2-17-7-10-16) is used to illustrate the evolution of the dynamic system, so are the below sections. It can be clearly observed from Figure 3(a) that the route flows gradually evolve to a stable value, 206.0 (Veh·h^{-1}), regardless of the maximal rerouting rate in case I. In contrast with Case I, Figure 3(b) show with a larger learning rate λ, route flows intensively fluctuate in the evolutionary process and the evolutionary system does not converge to a stable state. These results demonstrate that the heuristic stability condition is applicable to our model.

4.2. Sensitivity Analysis. We also conduct sensitivity analyses to investigate the influence of four main parameters $(\lambda, \chi_0, \rho, \theta)$ on dynamic natures of our evolutionary system. In the analyses, we change the value of a parameter while keep the others unchanged each time. Figure 4 displays the route flow over time under different parameters.

We are able to clearly observe that parameters λ, χ_0 and θ affect the stability of the evolutionary system because route flows cannot converge to an equilibrium when they are set to a certain value, which is evidenced by the simplified transformation formula. Although ρ also is included in the transformation formula, it does not affect the stability of the system.

Parameters λ and χ_0 both have positive influence on the speed of evolution. When parameter λ equals 0.2, 0.3 or 0.4, respectively, the system takes 67 days, 46 days or 36 days to evolve the equilibrium. When the parameter χ_0 equals 0.1, 0.2 or 0.3, respectively, the system takes 69 days, 39 days or 32 days to evolve the equilibrium. Convergence duration decreases with the increase in their values because a larger λ represents that travelers are able to more quickly accept a new information and a larger χ_0 means that they more actively react to the information.

Third, there are different equilibrium states when parameters ρ and θ are set to be different values. However, parameter λ does influence the equilibrium state, which is consistence with our mathematical analysis because it is not included in our fixed point model. At the same time, it can also be seen that the equilibrium state has nothing to do with the parameter χ_0. This is because χ_0 in $P^*_{r,w}$ can be eliminated.

4.3. Comparative Analysis of Dynamic Systems Based on EUT and CPT. Under the framework of the proposed DTD model, we also conduct an experiment to investigate the difference between the dynamic system based on CPT and the one based on EUT. It is easy to find that the prospect value recovers to the expected utility when all parameters in the value function $g(\cdot)$ and the weighting function $h(\cdot)$ are set as 1. Then we investigate the impacts of CPT and EUT on the dynamic natures of the proposed system by observing evolutionary processes under different values of parameters ρ and λ.

Figure 5(a) show the effects of CPT and EUT on the equilibrium state of the dynamic system. Compared with the case of EUT, parameter ρ in the case of CPT has more significant influence on the equilibrium flow of route 13. The same phenomenon can also be seen on the other routes. This is explained in the following. Parameter ρ in CPT not only influences perceived gain and loss, but also further affects risk-attitudes while the parameter ρ in EUT just influences the former.

We also check the sensitivity of two systems to parameter λ. As shown in Figure 5(b), when the value of parameter λ changes from 0.3 to 0.5, the system under EUT cannot converge to an equilibrium whereas the system under CPT still

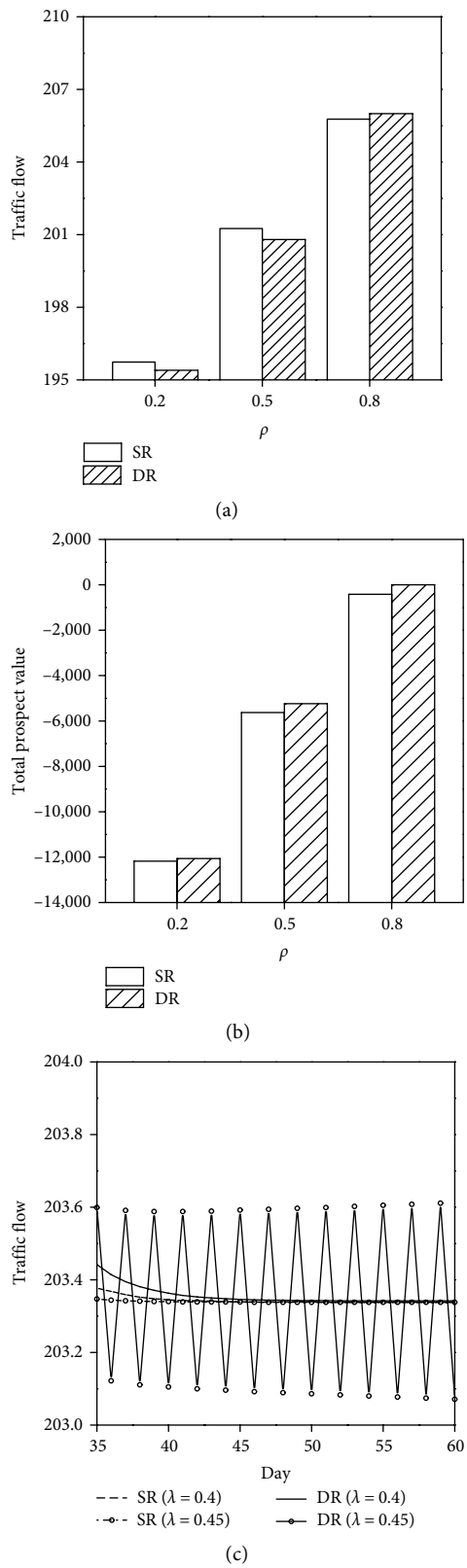

Figure 6: Evolutionary systems with static rerouting and dynamic rerouting behavior. (a) Equilibrium state. (b) System efficiency. (c) Sensitivity to parameter λ.

do. This demonstrates that the dynamic system under EUT is more sensitive to λ than the system under CPT.

4.4. Comparative Analysis of Dynamic Rerouting Behavior and Static Rerouting Behavior. We also investigate the effects of the different rerouting behaviors. In conducting static rerouting experiments, we set the rerouting rate as the maximal rerouting rate and keep it unchanged in the whole evolutionary process. Figure 6 shows the results with different rerouting behaviors.

It can be observed from Figure 6(a) that the equilibrium states of the systems with static rerouting behavior (SR) are different from that of the systems with DR, which is consistent with our theoretical analysis. In the case with SR, the equilibrium route choice probability function is $P^*_{r,w} = e^{\theta V^*_{r,w}} / \sum_{k \in R_w} e^{\theta V^*_{k,w}}$ while it is $P^*_{r,w} = \sum_{k \in R_w} (\chi^*_{k,w} P^*_{k,w}) \widetilde{P}^*_{r,w} + (1 - \chi^*_{r,w}) P^*_{r,w}$, $\forall r \in R_w$, $w \in W$ in the case with DR. As shown in Figure 6(b), the static rerouting behavior leads to the underestimation of the efficiency of the traffic system due to the fact that it overlooks bounded rationality of travelers that they accept the suboptimal route and the bounded rationality makes the equilibrium state close to system equilibrium to some extent.

Figure 6(c) show that the system with DR are more sensitive to the parameter λ than the system with SR. Intuitively, the reason might be that the dynamic rerouting probability depends on parameter λ and with larger value of λ, the influence of new information on rerouting behavior becomes more significant and travelers' behavior more intensively varies, thus leading to a more unstable system.

5. Conclusion

In the framework of DTD deterministic discrete-time traffic assignment model, a new DTD model incorporating risk-taking behavior and dynamic rerouting behavior is established. Based on it, we analytically study the existence and uniqueness of its fixed point and give a heuristic stable condition of the fixed point. Then several numerical experiments are conducted on the Nguyen–Dupuis network to validate the heuristic stability condition, to show the effects of the four main parameters, to investigate the difference between two utility frameworks called EUT and CPT and finally to show the difference between travelers' static rerouting behavior and dynamic rerouting behavior. The results show that:

(1) Parameters λ, χ_0 and θ all affect the stability of the dynamic system.

(2) Parameters λ and χ_0 both have positive influence on the speed of evolution.

(3) Parameters ρ and θ have an influence on the equilibrium state;

(4) CPT considers the change of risk-attitude in the utility function and is superior to EUT in describing travelers' evaluation on attraction of alternatives in uncertain environment.

(5) The dynamic system under CPT are less sensitive to parameter λ than the system under EUT.

(6) Different rerouting behaviors lead to different equilibrium states. Compared with dynamic rerouting, the model with static rerouting underestimates the efficiency of the system and the sensitivity of the system to λ.

In the future, various research directions ensuing from this work can be explored. One of them would be to calibrate and validate the model with empirical data. Another line of research would be to analytically derive the sufficient condition that assures the stability of the dynamic system. The third direction would be to incorporate the effect of traffic information in the model because travelers are able to easily get various traffic information at current times.

Appendix

In the section, we assume that route travel time is deterministic. In the condition, our dynamic system's fixed point is unique.

The system's Jacobian matrix is:

$$\boldsymbol{J}^{t-1} = \begin{vmatrix} T^{n-1} & P^{n-1} \\ (1-\lambda)\boldsymbol{I} & \lambda\boldsymbol{\Lambda}\boldsymbol{J}_{\boldsymbol{T}}\boldsymbol{\Lambda}^{\boldsymbol{T}}\boldsymbol{Q} \\ (1-\lambda)\left(\widetilde{\boldsymbol{P}}\boldsymbol{E}\boldsymbol{P}\boldsymbol{J}_{\boldsymbol{\alpha}} + \boldsymbol{Z}\boldsymbol{J}_{\widetilde{\boldsymbol{P}}} - \boldsymbol{P}\boldsymbol{J}_{\boldsymbol{\alpha}}\right) & \left(\widetilde{\boldsymbol{P}}\boldsymbol{E}\boldsymbol{\alpha} + \boldsymbol{I} - \boldsymbol{\alpha}\right) + \lambda\left(\widetilde{\boldsymbol{P}}\boldsymbol{E}\boldsymbol{P}\boldsymbol{J}_{\boldsymbol{\alpha}} + \boldsymbol{Z}\boldsymbol{J}_{\widetilde{\boldsymbol{P}}} - \boldsymbol{P}\boldsymbol{J}_{\boldsymbol{\alpha}}\right)\boldsymbol{\Lambda}\boldsymbol{J}_{\boldsymbol{T}}\boldsymbol{\Lambda}^{\boldsymbol{T}}\boldsymbol{Q} \end{vmatrix} \begin{matrix} T^n \\ P^n \end{matrix}, \tag{A.1}$$

where

$$\boldsymbol{E} = \begin{vmatrix} \underbrace{\begin{matrix} 1 & \cdots & 1 \\ \cdots & \cdots & \cdots \\ 1 & \cdots & 1 \end{matrix}}_{R_1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \cdots & \begin{matrix} \cdots & \cdots & \cdots \\ 1 & \cdots & 1 \\ \cdots & \cdots & \cdots \\ 1 & \cdots & 1 \end{matrix} \\ & & \underbrace{\qquad\qquad}_{R_w} \end{vmatrix};$$

$$\boldsymbol{Z} = \begin{vmatrix} \underbrace{\begin{matrix} \alpha_1 P_1 + \cdots + \alpha_{R_1} P_{R_1} & \cdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \cdots & \alpha_1 P_1 + \cdots + \alpha_{R_1} P_{R_1} \end{matrix}}_{R_1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \cdots & \begin{matrix} \cdots & \cdots & \cdots \\ \alpha_{R_{w-1}+1} P_{R_{w-1}+1} + \cdots + \alpha_{R_w} P_{R_w} & \cdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \cdots & \alpha_{R_{w-1}+1} P_{R_{w-1}+1} + \cdots + \alpha_{R_w} P_{R_w} \end{matrix} \\ & & \underbrace{\qquad\qquad}_{R_w} \end{vmatrix}. \tag{A.2}$$

In order to simplify the expression of the eigenvalues of $\boldsymbol{J}$, let $\boldsymbol{G} = \left(\widetilde{\boldsymbol{P}}\boldsymbol{E}\boldsymbol{P}\boldsymbol{J}_{\boldsymbol{\alpha}} + \boldsymbol{Z}\boldsymbol{J}_{\widetilde{\boldsymbol{P}}} - \boldsymbol{P}\boldsymbol{J}_{\boldsymbol{\alpha}}\right)\boldsymbol{\Lambda}\boldsymbol{J}_{\boldsymbol{T}}\boldsymbol{\Lambda}^{\boldsymbol{T}}\boldsymbol{Q}$, a $R \times R$ matrix independent of λ. For each of the R eigenvalues (o) of matrix $\boldsymbol{G}$, two eigenvalues of $\boldsymbol{J}$ are defined as a function of parameter λ:

$$\kappa'' = \frac{(1-\lambda) + (1-z) + \lambda z o - \sqrt{\tau}}{2}, \tag{A.3}$$

$$\kappa' = \frac{(1-\lambda) + (1-z) + \lambda z o - \sqrt{\tau}}{2}, \tag{A.4}$$

$$\tau = ((1-\lambda) + (1-z) + \lambda z o)^2 - 4(1-\lambda)(1-z), \tag{A.5}$$

$$z = \left|\boldsymbol{\alpha} - \widetilde{\boldsymbol{P}}\boldsymbol{E}\boldsymbol{\alpha}\right| \leq 1. \tag{A.6}$$

Based on the work of Cantarella and Cascetta, the system definitely converges to the equilibrium if parameter λ is small enough, $\boldsymbol{Q}\left(\widetilde{\boldsymbol{P}^*}\boldsymbol{E}\boldsymbol{P}^*\boldsymbol{J}^*_{\boldsymbol{\alpha}} + \boldsymbol{Z}^*\boldsymbol{J}^*_{\widetilde{\boldsymbol{P}}} - \boldsymbol{P}^*\boldsymbol{J}^*_{\boldsymbol{\alpha}}\right)$ is negative semidefinite and $\boldsymbol{J}^*_{\boldsymbol{T}}$ is positive definite.

By inspection, $\boldsymbol{Q}\left(\widetilde{\boldsymbol{P}^*}\boldsymbol{E}\boldsymbol{P}^*\boldsymbol{J}^*_{\boldsymbol{\alpha}} + \boldsymbol{Z}^*\boldsymbol{J}^*_{\widetilde{\boldsymbol{P}}} - \boldsymbol{P}^*\boldsymbol{J}^*_{\boldsymbol{\alpha}}\right)$ of our model is negative semidefinite and $\boldsymbol{J}^*_{\boldsymbol{T}}$ is positive definite.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work was supported by National Natural Science Foundation of China (No. 51478110) and Science and Technology Program of Jiangsu Province (No. BY2016076-05).

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