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Research Article

Modeling the Morning Commute Problem in a Bottleneck Model Based on Personal Perception

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This paper studies the travel behavior of travelers who drive from the living area through the highway to the work area during the morning rush hours. The bottleneck model based on personal perception travel behavior has been investigated. Based on their willingness to arrive early, travelers can be divided into two categories: active travelers and negative travelers. Three possible situations have been considered based on travelers' personal perception. Travelers' travel choice behaviors are analyzed in detail and equilibrium is achieved with these three situations. The numerical examples show that the departure time choice of the travelers is related not only to the proportion of each type of travelers, but also to personal perceived size.

1. Introduction

The well-known bottleneck model was originally developed by Vickrey [1]. This model is a common situation during the morning rush hour, where a fixed and very large number of commuters travel from home to workplace along the same stretch of road. This road has a single bottleneck with a fixed capacity. If the arrival rate at the bottleneck exceeds its capacity, a queue forms. Although all the commuters wish to arrive at the destination at the same time, this is not physically possible because the bottleneck capacity is finite. Consequently, some travelers may decide to depart earlier or later to avoid the cost of waiting in the queue and pay the penalty cost for doing so. Therefore, each traveler faces a trade-off between travel time cost and the schedule delay cost and chooses an optimal departure time to minimize the total travel cost. At equilibrium, the total travel expenses of all travelers are equal, and no one can reduce his/her commuting cost by changing his/her departure time.

The bottleneck model depicts the commuting behavior of travelers during morning rush hours with a simple and direct way and clearly describes the formation and dissipation of queuing congestion and the departure time choice behavior of travelers. Subsequently, the morning commuting problem

has been expended by many others. Henderson [2] considered the importance of the schedule delay and departure time decisions in a single bottleneck model. Congestion tolls influence the individual commuter's decision of when to finish a trip and change the distribution of traffic flow. Carey and Srinivasan [3] derived system marginal costs, user perceived costs, and user externality costs and obtain a set of optimal congestion tolls. Mun [4] studies a dynamic model of traffic flow which is presented and describes the formation and development of the traffic jam and proposes a coarse toll to alleviate traffic congestion. Mounce [5] considered a dynamic traffic assignment model with deterministic queuing and inelastic demand for each origin-destination (OD) pair in the network.

In previous studies, travelers choose their own departure time by trade-off between waiting time and the schedule delay [6–12]. But, in real life, travelers measure the effect of those practical factors [9–13]; their personal subjective judgment also plays a considerable role.

The perceived judgment is the traveler's psychological judgment of the route travel time, because travelers have their understanding of the route choice. In the static model, the SUE (Stochastic User Equilibrium) is achieved when users can no longer change their perceived utility. This indicates

that traveler's psychological choice plays a key role in the travel process.

For example, there are two highways from the Beijing area to Capital Airport: the Jingcheng Expressway and the Capital Airport Expressway. The toll of Jingcheng Expressway is a little higher than that of the Capital Airport Expressway, but the freeway patency of Capital Airport is much lower than that of Beijing-Chengdu Expressway and the expressway of Capital Airport is congested all day. Generally speaking, this phenomenon is explained by the fact that the underestimation of travelers' time value is the cause of low toll road utilization [14]. However, the perceived choice behavior of travelers is also one of the principal reasons for the low traffic flow. Mahmassani and Jou [15] gave details of the traveler's travel route choice behavior based on the satisfactory decision criterion and believe that as long as the route choice falls within the undifferentiated curve, the traveler will not change the current route choice. Lou et al. [16] studied the traveler's route choice behavior under the bounded rationality, established the bounded rational user equilibrium model (BRUE) based on the route flow and link flow, and analyzed the traffic distribution in the best and worst cases. Guo and Liu [17] considered the route choice behavior of travelers with bounded rationality, established a day-to-day evolutionary dynamic model based on bounded rationality, and simulated the irreversible evolution of the traffic network. Zhao and Huang [18] studied the bounded rational route choice behavior under Simon's satisficing rule when travelers considered perceived travel time costs. Wiseman [19] pointed that the travelers' attitude played an important role in mode choice.

In those studies, the influence of psychological factors on the travelers was investigated in the static model. Few scholars have considered the effects of psychological factors in dynamic models. Based on this situation, the impact of human psychological decision-making behavior in the bottleneck model has been studied. In the bottleneck model, since the travel time on the road does not affect the traveler's departure time choice behavior, it is usually assumed to be zero, and it is not considered. Therefore, each commuter faces a trade-off between travel time cost and the delay cost and chooses an optimal departure time. For schedule delay, on the one hand, travelers need to consider their own early/late time cost; on the other hand, their subjective judgment also plays a certain role in the travel process. How to describe the traveler's travel choice behavior through the subjective consciousness judgment will be the research content of this paper. In the process of psychological decision-making, different travelers have different reactions to arrival early/late. Therefore, this paper divides travelers into two types according to whether they wish to arrive early or not: active and inactive. Next, we will introduce these two kinds of travelers in turn.

For some travelers, the closer he arrives at work, the more he feels nervous and the more he becomes uneasy when he is late. For these travelers, arriving early can reduce their tension, and arriving late can increase their tension. This tension based on personal perception is bound to have an impact on the traveler's choice of departure time. For those who have a strong sense of time, they are called active travelers.

On the contrary, some travelers are not proactive, thinking that arriving at work before the work starting time would cost them some of their own benefits. They are more likely to arrive near the preferred arrival time than arriving early. For this part of the travelers, their travel behavior is not as active as the active travelers. So we call this part of the travelers is negative travelers.

Compared with the previous models which only consider the waiting time and the schedule delay, this paper takes the bottleneck model as a foothold, studies the traveler's departure time choice behavior by considering the travelers' personal perception, and establishes a bottleneck model based on personal perception.

2. The Bottleneck Model Based on Personal Perception

2.1. Model Description. Let us consider a highway between a residential area and a CBD where N travelers commute. If the arrival rate exceeds the capacity of the bottleneck on the highway, a queue develops. All individuals want to arrive at the workplace at work start time. Due to capacity, there always exist some persons with queuing time and arrive early or late. On the other hand, people consider the personal perception when they choose the departure time. Based on the previous description, the bottleneck model based on personal perception has been built by considering the traveler's personal perception. Similar to the classical bottleneck model, the influence of free travel time is not considered.

According to the former description, the general travel cost mainly includes three parts in the bottleneck model based on personal perception: queuing waiting time cost, schedule delay cost, and personal perceived cost. There exist two kinds of generalized travel cost by the active travelers and negative travelers. Here, we suppose that the personal perceived utility is a linear function of time t dependence. The personal perceived utility of active travelers is $f_1(t) = k_1(t^* - t - w(t)), 0 < k_1 < \beta$. The personal perceived utility of negative travelers is $f_2(t) = k_2(t^* - t - w(t)), 0 < k_2 < \beta$. Integrating the travel time, schedule delay, and personal perception utility, traveler's generalized travel cost can be formulated as follows.

The generalized travel cost $C_1(t)$ of active travelers commuting who left home at time t would be expressed:

$$
\begin{aligned}
C_1(t) \\
&= \alpha w(t) \\
&\quad + \max\left\{\beta\left(t^* - w(t) - t\right) - k_1\left(t^* - t - w(t)\right), 0\right\} \\
&\quad + \max\left\{\gamma\left(t + w(t) - t^*\right) - k_1\left(t^* - t - w(t)\right), 0\right\}
\end{aligned}
\tag{1}
$$

The generalized travel cost $C_2(t)$ of negative travelers commuting who left home at time t would be expressed:

$$
\begin{aligned}
C_2(t) \\
&= \alpha w(t)
\end{aligned}
$$

$$+ \max \left\{ \beta \left(t^* - w(t) - t \right) + k_2 \left(t^* - t - w(t) \right), 0 \right\}$$

$$+ \max \left\{ \gamma \left(t + w(t) - t^* \right) + k_2 \left(t^* - t - w(t) \right), 0 \right\}$$

$$(2)$$

where α denotes the value of travel time, β denotes the unit cost of schedule delay early, and γ denotes the unit cost of schedule delay late ($0 < \beta < \alpha < \gamma$). t^* is the preferred arrival time.

According to the former assumption, travelers can be divided into active travelers and negative travelers. If the number of active travelers is θN, then the number of negative travelers is $(1 - \theta)N$. If $\theta=1$, all the travelers are active. If $\theta=0$, all the travelers are negative. If $0 < \theta < 1$, both active travelers and negative travelers are existing. According to the division of θ, there are three situations in the bottleneck model based on personal perception: first, all the travelers are active; second, all the travelers are negative; third, both active travelers and negative travelers are existing. We will talk about these three situations in detail.

2.2. Personal Perceived Bottleneck Model Based on Active Travelers.
In this section, this paper examines the departure time choice behavior of active travelers. At equilibrium, the driver incurs the same travel cost no matter when he leaves home. During the entire operation, the bottleneck has full capacity load operation. This condition implies that all the travelers' generalized travel costs are the same. Let t_{01} and t_{e1} be, respectively, the earliest and the latest times of rush hours. The generalized travel cost of the first active traveler is

$$C_1(t_{01}) = \beta (t^* - t_{01}) - k_1 (t^* - t_{01})$$
$$= (\beta - k_1)(t^* - t_{01}) \tag{3a}$$

The generalized travel cost of the last active traveler is

$$C_1(t_{e1}) = \gamma (t_{e1} - t^*) + k_1 (t_{e1} - t^*)$$
$$= (\gamma + k_1)(t_{e1} - t^*) \tag{3b}$$

During the entire operation, the bottleneck has full capacity load operation, and the length of the period is determined by the following formula:

$$t_{e1} - t_{01} = \frac{N}{s} \tag{4}$$

According to (3a), (3b), (4), and equilibrium conditions, the equilibrium generalized travel cost C_1 is obtained:

$$C_1 = \frac{(\beta - k_1)(\gamma + k_1)}{\beta + \gamma} \frac{N}{s} \tag{5}$$

The total travel cost TC_1 is

$$TC_1 = \frac{(\beta - k_1)(\gamma + k_1)}{\beta + \gamma} \frac{N^2}{s} \tag{6}$$

According to (1), (3a), (3b), and (5), t_{01}, t_{e1}, and t_{n1} (being the departure time at which an individual arrives at work on time t^*) can be obtained:

$$t_{01} = t^* - \frac{\gamma + k_1}{\beta + \gamma} \frac{N}{s} \tag{7}$$

$$t_{e1} = t^* + \frac{\beta - k_1}{\beta + \gamma} \frac{N}{s} \tag{8}$$

$$t_{n1} = t^* - \frac{(\beta - k_1)(\gamma + k_1)}{\alpha(\beta + \gamma)} \frac{N}{s} \tag{9}$$

According to (1), (3a), (3b), (5), and (7)-(9), the waiting time function $w_1(t)$ and the departure time rate $r_1(t)$ are obtained:

$$w_1(t) = \begin{cases} \dfrac{\beta - k_1}{\alpha - \beta + k_1}(t - t_{01}), & t \in [t_{01}, t_{n1}] \\[2ex] \dfrac{\gamma + k_1}{\alpha + \gamma + k_1}(t_{e1} - t), & t \in [t_{n1}, t_{e1}] \end{cases} \tag{10}$$

$$r_1(t) = \begin{cases} \dfrac{\alpha s}{\alpha - \beta + k_1}, & t \in [t_{01}, t_{n1}] \\[2ex] \dfrac{\alpha s}{\alpha + \gamma + k_1}, & t \in [t_{n1}, t_{e1}] \end{cases} \tag{11}$$

2.3. Personal Perceived Bottleneck Model Based on Negative Travelers.
In this section, this paper examines the departure time choice behavior of negative travelers. At equilibrium, the driver incurs the same travel cost no matter when he leaves home. During the entire operation, the bottleneck has full capacity load operation. This condition implies that all the travelers' generalized travel costs are the same. Let t_{02} and t_{e2} be, respectively, the earliest and the latest times of rush hours. The generalized travel cost of the first negative traveler is

$$C_2(t_{02}) = \beta (t^* - t_{02}) + k_2 (t^* - t_{02})$$
$$= (\beta + k_2)(t^* - t_{02}) \tag{12a}$$

The generalized travel cost of the last negative traveler is

$$C_2(t_{e2}) = \gamma (t_{e2} - t^*) + k_2 (t_{e2} - t^*)$$
$$= (\gamma - k)(t_{e2} - t^*) \tag{12b}$$

During the entire operation, the bottleneck has full capacity load operation, and the length of the period is determined by the following formula:

$$t_{e2} - t_{02} = \frac{N}{s} \tag{13}$$

According to (12a), (12b), (13), and equilibrium conditions, the equilibrium generalized travel cost C_2 is obtained:

$$C_2 = \frac{(\beta + k_2)(\gamma - k_2)}{\beta + \gamma} \frac{N}{s} \tag{14}$$

The total travel cost TC_1:

$$TC_2 = \frac{(\beta + k_2)(\gamma - k_2)}{\beta + \gamma} \frac{N^2}{s} \tag{15}$$

According to (2), (12a), (12b), and (14), t_{02}, t_{e2}, and t_{n2} (being the departure time at which an individual arrives at work on time t^*) can be obtained:

$$t_{02} = t^* - \frac{\gamma - k_2}{\beta + \gamma} \frac{N}{s} \tag{16}$$

$$t_{e2} = t^* + \frac{\beta + k_2}{\beta + \gamma} \frac{N}{s} \tag{17}$$

$$t_{n2} = t^* - \frac{(\beta + k_2)(\gamma - k_2)}{\alpha(\beta + \gamma)} \frac{N}{s} \tag{18}$$

According to (12a), (12b), (14), and (16)-(18), the waiting time function $w_2(t)$ and the departure time rate $r_2(t)$ are obtained:

$$w_2(t) = \begin{cases} \dfrac{\beta + k_2}{\alpha - \beta - k_2}(t - t_{02}), & t \in [t_{02}, t_{n2}] \\[2ex] \dfrac{\gamma - k_2}{\alpha + \gamma - k_2}(t_{e2} - t), & t \in [t_{n2}, t_{e2}] \end{cases} \tag{19}$$

$$r_2(t) = \begin{cases} \dfrac{\alpha s}{\alpha - \beta - k_2}, & t \in [t_{02}, t_{n2}] \\[2ex] \dfrac{\alpha s}{\alpha + \gamma - k_2}, & t \in [t_{n2}, t_{e2}] \end{cases} \tag{20}$$

2.4. Personal Perceived Bottleneck Model Based on Active Travelers and Negative Travelers.

In the former two sections, this paper considers two extreme cases when θ is equal to 0 or 1. The travelers' choice behavior would be analyzed when $0 < \theta < 1$. According to the traveler arriving early or late, the traveler's travel choice behavior can be divided into three situations: the first case, all early travelers are active; all late travelers are negative; the second case, some early travelers are active; the rest are negative. In the third case, some late travelers are negative and the others are active. The following is a detailed analysis of traveler's choice behavior in these three situations.

(1) In the first case, all early travelers are active; all late travelers are negative. For the early travelers, the generalized travel cost at time t can be expressed as

$$C_1(t) = \alpha w(t) + \beta(t^* - w(t) - t) \\ - k_1(t^* - t - w(t)) \tag{21a}$$

For the late travelers, the generalized travel cost at time t can be expressed as

$$C_2(t) = \alpha w(t) + \gamma(t + w(t) - t^*) \\ + k_2(t^* - t - w(t)) \tag{21b}$$

At equilibrium, the traveler incurs the same travel cost no matter when he leaves home. During the entire operation, the bottleneck has full capacity load operation. This condition implies that all the travelers' generalized travel costs are equal. Let t_{03} and t_{e3} be, respectively, the earliest and the latest times of rush hours. t_{n3} is the departure time at which a traveler arrives at work on time t^*.

At equilibrium, $C'(t) = 0$ because all the travelers' generalized travel costs are equal. We can get the derivative of the waiting time:

$$w_{31}'(t) = \begin{cases} \dfrac{\beta - k_1}{\alpha - \beta + k_1}, & t \in [t_{03}, t_{n3}] \\[2ex] -\dfrac{\gamma - k_2}{\alpha + \gamma - k_2}, & t \in [t_{n3}, t_{e3}] \end{cases} \tag{22}$$

According to (22), the waiting time function of travelers at time t is

$$w_{31}(t) = \begin{cases} \dfrac{\beta - k_1}{\alpha - \beta + k_1}(t - t_{03}), & t \in [t_{03}, t_{n3}] \\[2ex] \dfrac{\gamma - k_2}{\alpha + \gamma - k_2}(t_{e3} - t), & t \in [t_{n3}, t_{e3}] \end{cases} \tag{23}$$

At equilibrium, the waiting times at time t are equal.

$$\frac{\beta - k_1}{\alpha - \beta + k_1}(t_{n3} - t_{03}) = \frac{\gamma - k_2}{\alpha + \gamma - k_2}(t_{e3} - t_{n3}) \tag{24}$$

In this case, active travelers can only arrive early, and negative travelers are all late. This means that t_{n3} is a critical value. Thus, $C(t_{n3}) = C(t_{03})$, $C(t_{n3}) = C(t_{e3})$. We can obtain

$$C(t_{03}) = C(t_{e3}) \tag{25}$$

The generalized travel costs of the first traveler and the last traveler are

$$C_1(t_{03}) = (\beta - k_1)(t^* - t_{03}) \tag{26}$$

$$C_2(t_{e3}) = (\gamma - k_2)(t_{e3} - t^*) \tag{27}$$

The departure time rate $r_{31}(t)$ is obtained:

$$r_{31}(t) = \begin{cases} \dfrac{\alpha}{\alpha - \beta + k_1}s, & t_{03} \le t < t_{n3} \\[2ex] \dfrac{\alpha}{\alpha + \gamma - k_2}s, & t_{n3} \le t \le t_{e3} \end{cases} \tag{28}$$

The number of active travelers and negative travelers is

$$\frac{\alpha s}{\alpha - \beta + k_1}(t_{n3} - t_{03}) = \overline{\theta}N \tag{29}$$

$$\frac{\alpha s}{\alpha + \gamma - k_2}(t_{e3} - t_{n3}) = (1 - \overline{\theta})N \tag{30}$$

During the entire operation, the bottleneck has full capacity load operation, and the length of the period is determined by the following formula:
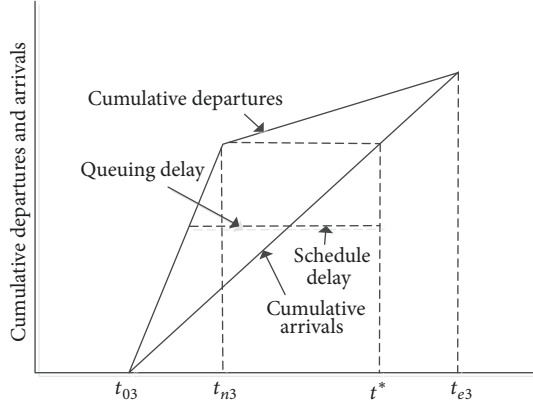
$$t_{e3} - t_{03} = \frac{N}{s} \tag{31}$$

FIGURE 1: Cumulative departures and arrivals at equilibrium in Case 1.

According to (25)-(31), t_{03}, t_{e3}, t_{n3}, and $\overline{\theta}$ can be obtained:

$$t_{03} = t^* - \frac{\gamma - k_2}{\beta + \gamma - k_1 - k_2} \frac{N}{s} \tag{32}$$

$$t_{e3} = t^* + \frac{\beta - k_1}{\beta + \gamma - k_1 - k_2} \frac{N}{s} \tag{33}$$

$$t_{n3} = t^* - \frac{(\beta - k_1)(\gamma - k_2)}{\alpha(\beta + \gamma - k_1 - k_2)} \frac{N}{s} \tag{34}$$

$$\overline{\theta} = \frac{\gamma - k_2}{\beta + \gamma - k_1 - k_2} \tag{35}$$

According to (25), (26), and (32), the equilibrium generalized travel cost of active travelers and negative travelers is obtained:

$$C_1(t_{03}) = C_2(t_{e3}) = \frac{(\beta - k_1)(\gamma - k_2)}{\beta + \gamma - k_1 - k_2} \frac{N}{s} \tag{36}$$

The illustrative departure and arrival profiles are plotted in Figure 1. The horizontal axis represents travelers' departure time while the vertical axis represents the cumulative departures and arrivals. The horizontal distance between the two curves is the queuing time while the vertical distance between the two curves is queue length. The area between the two curves is the total queuing time. Active travelers depart home in the departure time interval $[t_{03}, t_{n3}]$. Negative travelers depart home in the departure time interval $[t_{n3}, t_{e3}]$.

(2) In the second case, some early travelers are active, while others are not. In this situation, $\theta < \overline{\theta}$. Active travelers in the departure time interval $[t_{03}, t_3']$ are early (t_3' is the dividing point between the active travelers and the negative travelers). Negative travelers in the departure time interval $[t_3', t_{n3}]$ are early. Negative travelers in the departure time interval $[t_{n3}, t_{e3}]$ are late.

For the early travelers in the departure time interval $[t_{03}, t_3']$, the generalized travel cost at time t can be expressed as

$$C_1(t) = \alpha w(t) + \beta(t^* - w(t) - t)$$
$$- k_1(t^* - t - w(t)) \tag{37a}$$

For the early travelers in the departure time interval $[t_3', t_{n3}]$, the generalized travel cost at time t can be expressed as

$$C_2(t) = \alpha w(t) + \beta(t^* - w(t) - t)$$
$$+ k_2(t^* - t - w(t)) \tag{37b}$$

For the late travelers in the departure time interval $[t_{n3}, t_{e3}]$, the generalized travel cost at time t can be expressed as

$$C_2(t) = \alpha w(t) + \gamma(t + w(t) - t^*)$$
$$+ k_2(t^* - t - w(t)) \tag{37c}$$

At equilibrium, $C'(t) = 0$ because all the travelers' generalized travel costs are equal. We can get the derivative of the waiting time:

$$w_{32}'(t) = \begin{cases} \dfrac{\beta - k_1}{\alpha - \beta + k_1}, & t_{03} \leq t < t_3' \\ \dfrac{\beta + k_2}{\alpha - \beta - k_2}, & t_3' \leq t < t_{n3} \\ -\dfrac{\gamma - k_2}{\alpha + \gamma - k_2}, & t_{n3} \leq t \leq t_{e3} \end{cases} \quad t_{03} \leq t < t_3' \tag{38}$$

According to (38), the waiting time function of travelers $w_{32}(t)$ at time t is

$w_{32}(t)$

$$= \begin{cases} \dfrac{\beta - k_1}{\alpha - \beta + k_1}(t - t_{03}), & t_{03} \leq t < t_3' \\ \dfrac{\beta + k_2}{\alpha - \beta - k_2}(t - t_3') + \dfrac{\beta - k_1}{\alpha - \beta + k_1}(t_3' - t_{03}), & t_3' \leq t < t_{n3} \\ -\dfrac{\gamma - k_2}{\alpha + \gamma - k_2}(t - t_{e3}), & t_{n3} \leq t \leq t_{e3} \end{cases} \tag{39}$$

The departure time rate $r_{32}(t)$ is obtained:

$$r_{32}(t) = \begin{cases} \dfrac{\alpha}{\alpha - \beta + k_1} s, & t_{03} \leq t < t_3' \\ \dfrac{\alpha}{\alpha - \beta - k_2} s, & t_3' \leq t < t_{n3} \\ \dfrac{\alpha}{\alpha + \gamma - k_2} s, & t_{n3} \leq t \leq t_{e3} \end{cases} \tag{40}$$

The generalized travel cost of the negative travelers at t_3' is

$$C_2(t_3') = (\alpha - \beta - k_2) \frac{\beta - k_1}{\alpha - \beta + k_1}(t_3' - t_{03})$$
$$+ (\beta + k_2)(t^* - t_3') \tag{41}$$

The generalized travel cost of the last traveler is

$$C_2(t_{e3}) = (\gamma - k_2)(t_{e3} - t^*) \tag{42}$$

At equilibrium, all the negative travelers' generalized travel costs are equal. We get

$$(\alpha - \beta - k_2) \frac{\beta - k_1}{\alpha - \beta + k_1} (t_3' - t_{03})$$

$$+ (\beta + k_2)(t^* - t_3') = (\gamma - k_2)(t_{e3} - t^*) \tag{43}$$

According to (40), the number of active travelers is

$$\frac{\alpha}{\alpha - \beta + k_1} s (t_3' - t_{03}) = \theta N \tag{44}$$

The number of negative travelers is

$$\frac{\alpha}{\alpha - \beta - k_2} s (t_{n3} - t_3') + \frac{\alpha}{\alpha + \gamma - k_2} s (t_{e3} - t_{n3})$$

$$= (1 - \theta) N \tag{45}$$

During the entire operation, the bottleneck has full capacity load operation, and the length of the period is determined by the following formula:

$$t_{e3} - t_{03} = \frac{N}{s} \tag{46}$$

According to (41)-(46), t_{03}, t_{e3}, t_{n3}, and t_3' can be obtained:

$$t_{03} = t^* - \frac{(k_1 + k_2) \theta N}{(\beta + \gamma) s} - \frac{(\gamma - k_2) N}{(\beta + \gamma) s} \tag{47}$$

$$t_3' = t^* + \frac{\theta N}{\alpha s} (\alpha - \beta + k_1) - \frac{(k_1 + k_2) \theta N}{(\beta + \gamma) s}$$

$$- \frac{(\gamma - k_2) N}{(\beta + \gamma) s} \tag{48}$$

$$t_{e3} = t^* + \frac{(\beta + k_2) N}{(\beta + \gamma) s} - \frac{(k_1 + k_2) \theta N}{(\beta + \gamma) s} \tag{49}$$

$$t_{n3} = t^* + \frac{N}{\alpha (\beta + \gamma) s} (\gamma - k_2)(\theta k_1 + \theta k_2 - \beta - k_2) \tag{50}$$

According to (47), the equilibrium generalized travel cost of active travelers is obtained:

$$C_1 (t_{03}) = (\beta - k_1)(t^* - t_{03})$$

$$= (\beta - k_1) \left(\frac{(k_1 + k_2) \theta N}{(\beta + \gamma) s} + \frac{(\gamma - k_2) N}{(\beta + \gamma) s} \right) \tag{51}$$

According to (49), the equilibrium generalized travel cost of negative travelers is obtained:

$$C_2 (t_{e3}) = (\gamma - k_2)(t_{e3} - t^*)$$

$$= (\gamma - k_2) \left(\frac{(\beta + k_2) N}{(\beta + \gamma) s} - \frac{(k_1 + k_2) \theta N}{(\beta + \gamma) s} \right) \tag{52}$$

The illustrative departure and arrival profiles are plotted in Figure 2. The horizontal axis represents travelers' departure time while the vertical axis represents the cumulative
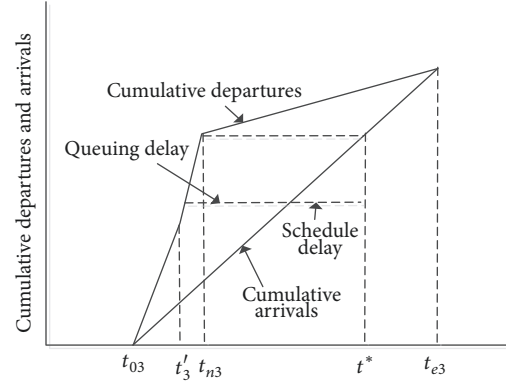

FIGURE 2: Cumulative departures and arrivals at equilibrium in Case 2.

departures and arrivals. The horizontal distance between the two curves is the queuing time while the vertical distance between the two curves is the queue length. The area between the two curves is the total queuing time. Active travelers depart home in the departure time interval $[t_{03}, t_3']$. Negative travelers depart home in the departure time interval $[t_3', t_{e3}]$.

(3) In the third case, some early travelers are negative, while others are not. In this situation, $\theta > \overline{\theta}$. The active travelers in the departure time interval $[t_{03}, t_{n3}]$ are early. Active travelers in the departure time interval $[t_{n3}, t_3'']$ are late (t_3'' is the dividing point between the active travelers and the negative travelers). Negative travelers in the departure time interval $[t_3'', t_{e3}]$ are late.

For the early travelers in the departure time interval $[t_{03}, t_{n3}]$, the generalized travel cost at time t can be expressed as

$$C_1 (t) = \alpha w (t) + \beta (t^* - w (t) - t)$$

$$- k_1 (t^* - t - w (t)) \tag{53a}$$

For the early travelers in the departure time interval $[t_{n3}, t_3'']$, the generalized travel cost at time t can be expressed as

$$C_1 (t) = \alpha w (t) + \gamma (t + w (t) - t^*)$$

$$- k_1 (t^* - t - w (t)) \tag{53b}$$

For the late travelers in the departure time interval $[t_3'', t_{e3}]$, the generalized travel cost at time t can be expressed as

$$C_2 (t) = \alpha w (t) + \gamma (t + w (t) - t^*)$$

$$+ k_2 (t^* - t - w (t)) \tag{53c}$$

At equilibrium, $C'(t) = 0$ because the active travelers' generalized travel costs and the negative travelers' generalized travel costs are equal in their time intervals. We can get the derivative of the waiting time:

$$w'_{33}(t) = \begin{cases} \dfrac{\beta - k_1}{\alpha - \beta + k_1}, & t_{03} \leq t < t_{n3} \\[3mm] -\dfrac{\gamma + k_1}{\alpha + \gamma + k_1}, & t_{n3} \leq t < t''_3 \\[3mm] -\dfrac{\gamma - k_2}{\alpha + \gamma - k_2}, & t''_3 \leq t \leq t_{e3} \end{cases} \quad (54)$$

According to (54), the waiting time function of travelers $w_{33}(t)$ at time t is

$w_{33}(t)$

$$= \begin{cases} \dfrac{\beta - k_1}{\alpha - \beta + k_1}(t - t_{03}), & t_{03} \leq t < t_{n3} \\[3mm] -\dfrac{\gamma + k_1}{\alpha + \gamma + k_1}(t - t''_3) - \dfrac{\gamma - k_2}{\alpha + \gamma - k_2}(t''_3 - t_{e3}), & t_{n3} \leq t < t''_3 \\[3mm] -\dfrac{\gamma - k_2}{\alpha + \gamma - k_2}(t - t_{e3}), & t''_3 \leq t \leq t_{e3} \end{cases} \quad (55)$$

The departure time rate $r_{33}(t)$ is obtained:

$$r_{33}(t) = \begin{cases} \dfrac{\alpha}{\alpha - \beta + k_1}s, & t_{03} \leq t < t_{n3} \\[3mm] \dfrac{\alpha}{\alpha + \gamma + k_1}s, & t_{n3} \leq t < t''_3 \\[3mm] \dfrac{\alpha}{\alpha + \gamma - k_2}s, & t''_3 \leq t \leq t_{e3} \end{cases} \quad (56)$$

Similar to the second case, the waiting time at t''_3 is

$$w(t''_3) = -\dfrac{\gamma - k_2}{\alpha + \gamma - k_2}(t''_3 - t_{e3}) \quad (57)$$

Thus, the generalized travel cost of active travelers at t''_3 is

$$C_1(t''_3) = (\alpha + \gamma + k)\dfrac{\gamma - k_2}{\alpha + \gamma - k_2}(t_{e3} - t''_3) + (\gamma + k_1)(t''_3 - t^*) \quad (58)$$

The first traveler is active, and the generalized travel cost belonging to him is

$$C_1(t_{03}) = (\beta - k_1)(t^* - t_{03}) \quad (59)$$

At equilibrium, all the active travelers' generalized travel costs are equal. Thus, $C_1(t_{03}) = C_1(t''_3)$:

$$(\beta - k_1)(t^* - t_{03})$$
$$= (\alpha + \gamma + k_1)\dfrac{\gamma - k_2}{\alpha + \gamma - k_2}(t_{e3} - t''_3) \quad (60)$$
$$+ (\gamma + k_1)(t''_3 - t^*)$$

According to (54), the number of active travelers is

$$\dfrac{\alpha}{\alpha - \beta + k_1}s(t_{n3} - t_{03}) + \dfrac{\alpha}{\alpha + \gamma + k_1}s(t''_3 - t_{n3}) = \theta N \quad (61)$$

The number of negative travelers is

$$\dfrac{\alpha s}{\alpha + \gamma - k_2}(t_{e3} - t''_3) = (1 - \theta)N \quad (62)$$

During the entire operation, the bottleneck has full capacity load operation, and the length of the period is determined by the following formula:

$$t_{e3} - t_{03} = \dfrac{N}{s} \quad (63)$$

According to (58)-(63), t_{03}, t_{e3}, t_{n3}, and t''_3 can be obtained:

$$t_{03} = t^* + \dfrac{(k_1 + k_2)(1 - \theta)N}{(\beta + \gamma)s} - \dfrac{(\gamma + k_1)N}{(\beta + \gamma)s} \quad (64)$$

$$t_{e3} = t^* + \dfrac{(k_1 + k_2)(1 - \theta)N}{(\beta + \gamma)s} + \dfrac{(\beta - k_1)N}{(\beta + \gamma)s} \quad (65)$$

$$t''_3 = t^* + \dfrac{(k_1 + k_2)(1 - \theta)N}{(\beta + \gamma)s} + \dfrac{(\beta - k_1)N}{(\beta + \gamma)s}$$
$$- \dfrac{(\alpha + \gamma - k_2)(1 - \theta)N}{\alpha s} \quad (66)$$

$$t_{n3} = t^* - \dfrac{(\beta - k_1)(\gamma + \theta k_1 + \theta k_2 - k_2)N}{\alpha(\beta + \gamma)s} \quad (67)$$

According to (64), the equilibrium generalized travel cost of active travelers is obtained:

$$C_1(t_{03}) = (\beta - k_1)(t^* - t_{03})$$
$$= \dfrac{(\gamma + k_1)(\beta - k_1)N}{(\beta + \gamma)s} \quad (68)$$
$$- \dfrac{(k_1 + k_2)(\beta - k_1)(1 - \theta)N}{(\beta + \gamma)s}$$

According to (65), the equilibrium generalized travel cost of negative travelers is obtained:

$$C_2(t_{e3}) = (\gamma - k_2)(t_{e3} - t^*)$$
$$= \left(\dfrac{(k_1 + k_2)(1 - \theta)N}{(\beta + \gamma)s} + \dfrac{(\beta - k_1)N}{(\beta + \gamma)s}\right)(\gamma - k_2) \quad (69)$$

The illustrative departure and arrival profiles are plotted in Figure 3. The horizontal axis represents travelers' departure time while the vertical axis represents the cumulative departures and arrivals. The horizontal distance between the two curves is the queuing time while the vertical distance between the two curves is queue length. The area between the two curves is the total queuing time. Active travelers depart home in the departure time interval $[t_{03}, t''_3]$. Negative travelers depart home in the departure time interval $[t''_3, t_{e3}]$.

We present the following propositions to reveal some interesting properties of the equilibrium solution in this section.

Proposition 1. *When the value of parameter θ approaches zero, the personal perception bottleneck model based on active*
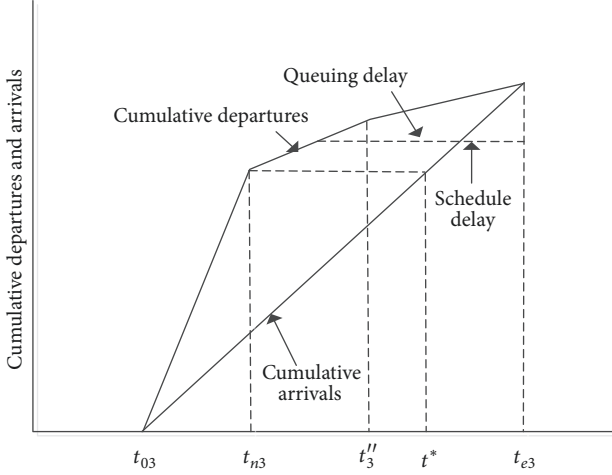
FIGURE 3: Cumulative departures and arrivals at equilibrium in Case 3.

travelers and negative travelers follows the personal perception bottleneck model based on negative travelers.

$$\lim_{\theta \longrightarrow 0} t_{03} = \lim_{\theta \longrightarrow 0} t^* - \frac{(k_1 + k_2)\,\theta N}{(\beta + \gamma)\,s} - \frac{(\gamma - k_2)\,N}{(\beta + \gamma)\,s}$$

$$= t^* - \frac{(\gamma - k_2)\,N}{(\beta + \gamma)\,s}$$

$$\lim_{\theta \longrightarrow 0} t_{e3} = \lim_{\theta \longrightarrow 0} t^* + \frac{N}{s} - \frac{(k_1 + k_2)\,\theta N}{(\beta + \gamma)\,s} - \frac{(\gamma - k_2)\,N}{(\beta + \gamma)\,s}$$

$$= t^* + \frac{(\beta + k_2)\,N}{(\beta + \gamma)\,s}$$

$$\lim_{\theta \longrightarrow 0} t_3'$$

$$= \lim_{\theta \longrightarrow 0} t^* + \frac{\theta N}{\alpha s}(\alpha - \beta + k_1) - \frac{(k_1 + k_2)\,\theta N}{(\beta + \gamma)\,s}$$

$$- \frac{(\gamma - k_2)\,N}{(\beta + \gamma)\,s} = t^* - \frac{(\gamma - k_2)\,N}{(\beta + \gamma)\,s} \qquad (70)$$

$$\lim_{\theta \longrightarrow 0} t_{n3}$$

$$= \lim_{\theta \longrightarrow 0} t^*$$

$$+ \frac{N}{\alpha(\beta + \gamma)\,s}(\gamma - k_2)(\theta k_1 + \theta k_2 - \beta - k_2)$$

$$= t^* - \frac{(\beta + k_2)(\gamma - k_2)}{\alpha(\beta + \gamma)}\frac{N}{s}$$

$$\lim_{\theta \longrightarrow 0} w_{32}(t) = \begin{cases} \dfrac{\beta + k_2}{\alpha - \beta - k_2}(t - t_{03}), & t \in [t_{03}, t_{n3}] \\ \dfrac{\gamma - k_2}{\alpha + \gamma - k_2}(t_{e2} - t), & t \in [t_{n3}, t_{e3}] \end{cases}$$

Thus, we obtain the same traffic flow pattern as the personal perception bottleneck model based on negative travelers.

Proposition 2. *When the value of parameter θ approaches one, the personal perception bottleneck model based on active travelers and negative travelers follows the personal perception bottleneck model based on active travelers.*

$$\lim_{\theta \longrightarrow 1} t_{03} = \lim_{\theta \longrightarrow 1} t^* + \frac{(k_1 + k_2)(1 - \theta)\,N}{(\beta + \gamma)\,s}$$

$$- \frac{(\gamma + k_1)\,N}{(\beta + \gamma)\,s} = t^* - \frac{(\gamma + k_1)\,N}{(\beta + \gamma)\,s}$$

$$\lim_{\theta \longrightarrow 1} t_{e3} = \lim_{\theta \longrightarrow 1} t^* + \frac{(k_1 + k_2)(1 - \theta)\,N}{(\beta + \gamma)\,s}$$

$$+ \frac{(\beta - k_1)\,N}{(\beta + \gamma)\,s} = t^* + \frac{(\beta - k_1)\,N}{(\beta + \gamma)\,s}$$

$$\lim_{\theta \longrightarrow 1} t_{n3} = \lim_{\theta \longrightarrow 1} t^*$$

$$- \frac{(\beta - k_1)(\gamma + \theta k_1 + \theta k_2 - k_2)\,N}{\alpha(\beta + \gamma)\,s} \qquad (71)$$

$$= t^* - \frac{(\beta - k_1)(\gamma + k_1)}{\alpha(\beta + \gamma)}\frac{N}{s}$$

$$\lim_{\theta \longrightarrow 0} t_3'' = \lim_{\theta \longrightarrow 0} t^* + \frac{(k_1 + k_2)(1 - \theta)\,N}{(\beta + \gamma)\,s}$$

$$+ \frac{(\beta - k_1)\,N}{(\beta + \gamma)\,s} - \frac{(\alpha + \gamma - k_2)(1 - \theta)\,N}{\alpha s}$$

$$= t^* + \frac{(\beta - k_1)\,N}{(\beta + \gamma)\,s}$$

$$\lim_{\theta \longrightarrow 0} w_{33}(t) = \begin{cases} \dfrac{\beta - k_1}{\alpha - \beta + k_1}(t - t_{03}), & t \in [t_{03}, t_{n3}] \\ \dfrac{\gamma + k_1}{\alpha + \gamma + k_1}(t_{e3} - t), & t \in [t_{n3}, t_{e3}] \end{cases}$$

Thus, we obtain the same traffic flow pattern as the personal perception bottleneck model based on active travelers.

3. Properties of the Personal Perceived Bottleneck Model

Proposition 3. *At equilibrium, no matter the departure time interval of active travelers or negative travelers, there has no jam phenomenon, which means that it is impossible to have both active travelers and negative travelers.*

Proof (Reversal Law). According to the equilibrium conditions, the equilibrium cost of each traveler will not change at equilibrium. Suppose that at the time t_x and t_y, the traveler arrives at the workplace before the preferred arrival time. Assume that the travelers in the time interval $[t_0, t_1]$ are active

travelers, and there exist two departure times t_x and t_y. If the travelers at time t_x and t_y are active travelers, the generalized travel cost is

$$
\begin{aligned}
C_1(t_x) &= \alpha w(t_x) + \beta(t^* - w(t_x) - t_x) \\
&\quad - k_1(t^* - t_x - w(t_x)) \\
C_1(t_y) &= \alpha w(t_y) + \beta(t^* - w(t_y) - t_y) \\
&\quad - k_1(t^* - t_y - w(t_y))
\end{aligned}
\tag{72}
$$

If the travelers at time t_x and t_y are negative travelers, the generalized travel cost is

$$
\begin{aligned}
C_2(t_x) &= \alpha w(t_x) + \beta(t^* - w(t_x) - t_x) \\
&\quad + k_2(t^* - t_x - w(t_x)) \\
C_2(t_y) &= \alpha w(t_y) + \beta(t^* - w(t_y) - t_y) \\
&\quad + k_2(t^* - t_y - w(t_y))
\end{aligned}
\tag{73}
$$

The wait time of t_x and t_y is

$$
\begin{aligned}
w(t_x) &= \frac{\beta - k_1}{\alpha - \beta + k_1}(t_x - t_{03}) \\
w(t_y) &= \frac{\beta - k_1}{\alpha - \beta + k_1}(t_y - t_{03})
\end{aligned}
\tag{74}
$$

Substituting this result into $C_1(t_x)$, $C_1(t_y)$, $C_1(t_x)$, and $C_1(t_y)$, we get the generalized travel cost:

$$
\begin{aligned}
C_1(t_x) &= \beta(t^* - t_{03}) \\
C_1(t_y) &= \beta(t^* - t_{03}) \\
C_2(t_x) &= (\alpha + \beta + k_2)\frac{\beta - k_1}{\alpha - \beta + k_1}(t_x - t_{03}) \\
&\quad + (\beta + k_2)(t^* - t_x) \\
C_2(t_y) &= (\alpha + \beta + k_2)\frac{\beta - k_1}{\alpha - \beta + k_1}(t_y - t_{03}) \\
&\quad + (\beta + k_2)(t^* - t_y)
\end{aligned}
\tag{75}
$$

We can find $C_1(t_1) = C_1(t_2), C_2(t_1) \neq C_2(t_2)$. There is no jam phenomenon. Similarly, it is possible to prove that there are no active travelers in the departure time interval where there are negative travelers. Thus, Proposition 3 is established. □

Proposition 4. *At equilibrium state, the generalized travel cost for every active and negative traveler is a strictly monotonically decreasing function of the personal perceived cost k_1; that is, $\partial C_1(t_{03})/\partial k_1 < 0$, $\partial C_2(t_{03})/\partial k_1 < 0$.*

Proof. For instance, according to (36), (51), and (68), $\theta < 1$ and $\gamma > \beta$, it is easy to verify the following:

(36):

$$
\frac{\partial C_1(t_{03})}{\partial k_1} = -\frac{(\gamma - k_2)^2}{(\beta + \gamma - k_1 - k_2)^2} < 0
\tag{76}
$$

(51):

$$
\frac{\partial C_1(t_{03})}{\partial k_1} = \frac{(\beta - \gamma - 2k_1)\theta N}{(\beta + \gamma)s} - \frac{(\gamma - k_2)N}{(\beta + \gamma)s} < 0
\tag{77}
$$

(68):

$$
\begin{aligned}
\frac{\partial C_1(t_{03})}{\partial k_1} &= \frac{(\beta - 2k_1)\theta N}{(\beta + \gamma)s} - \frac{(\gamma - (1-\theta)k_2)N}{(\beta + \gamma)s} \\
&= \frac{N}{(\beta + \gamma)s}\left[(\beta - 2k_1)\theta - (\gamma - (1-\theta)k_2)\right] \\
&< \frac{N}{(\beta + \gamma)s}\left[(\beta - 2k_1)\theta - (\beta - (1-\theta)k_2)\right] \\
&= \frac{N}{(\beta + \gamma)s}\left[(\theta - 1)(\beta - k_2) - 2k_1\theta\right] < 0
\end{aligned}
\tag{78}
$$

(36):

$$
\frac{\partial C_2(t_{e3})}{\partial k_1} = -\frac{(\gamma - k_2)^2}{(\beta + \gamma - k_1 - k_2)^2} < 0
\tag{79}
$$

(52):

$$
\frac{\partial C_2(t_{e3})}{\partial k_1} = -\frac{k_1 N}{(\beta + \gamma)s} < 0
\tag{80}
$$

(69):

$$
\frac{\partial C_2(t_{e3})}{\partial k_1} = -\frac{\theta(\gamma - k_2)N}{(\beta + \gamma)s} < 0
\tag{81}
$$

which clearly shows that the active and negative traveler's travel cost decreases with k_1. □

4. Numerical Examples

In this section, we present numerical results for the personal perception bottleneck model. According to Vickrey [1], there must be $\beta < \alpha < \gamma$. Unless otherwise specified, throughout this section, we adopt the following three parameter values from Arnott, de Palma, and Lindsey [20], the unit of travel time cost α=6.4, the unit of schedule delay early β=3.9, and the unit of schedule delay late γ=15.21, and consider the situation with s=3600, N=5000, and t^*=9:00.

Figure 4 depicts the change of the beginning of rush hours when the proportion of active travelers changes. The solid line represents the change of the beginning of rush hours when the proportion of active travelers changes in the personal perception bottleneck model. The black dotted line is the beginning time of the rush hours in the classical bottleneck model. When θ equals 0, it is clear that the beginning of departure time in the personal perception bottleneck model
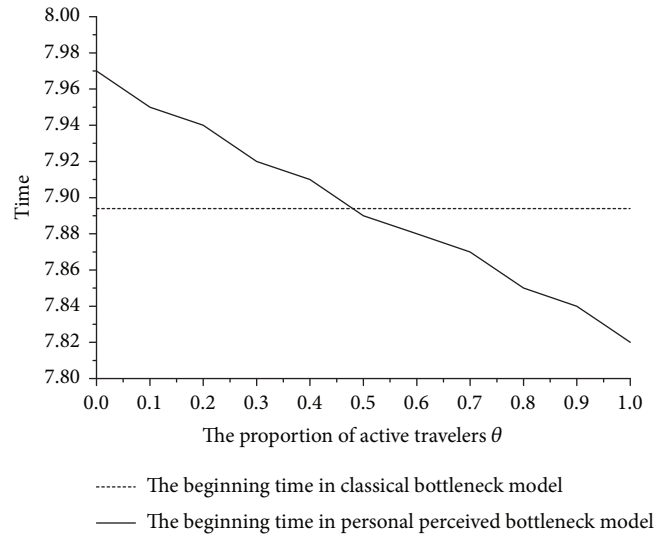
FIGURE 4: Changes of the beginning of rush hours with respect to the θ.
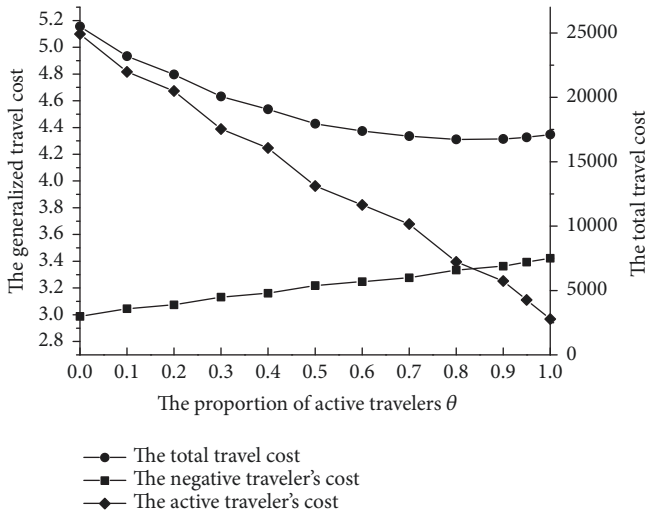
FIGURE 5: Changes of the travel cost with respect to θ.

is later than that in classical bottleneck model. When θ equals 0.5, it is clear that the beginning of departure time in the personal perception bottleneck model is equal to that in classical bottleneck model. According to the previous theoretical calculation, the beginning of rush hours in the mixed state is equal to that in the classical bottleneck model, and this is only when θ equals $k_2/(k_1 + k_2)$. In this example, $k_1 = k_2 = 1$ is assumed. That is why the beginning of departure time in the personal perceived bottleneck model equal to that in classical bottleneck model happens. When θ is more than 0.5, it is found that the starting time of the rush hours is earlier than that of the classical bottleneck model. This is because as the proportion of active travelers θ becomes larger and larger, their influence becomes greater and greater in the system. Eventually, rush hours occurred earlier and became earlier.

Figure 5 depicts the change of the generalized travel cost and total travel cost when the proportion of active travelers changes. With the growth of θ, the cost of active travelers is increasing, while the negative traveler's cost is decreasing. The increase rate of active traveler's cost is less than that of the negative travelers. That is to say, the influence of the active travelers on system is greater than that of negative travelers on system. When θ is equal to 0.84, it is easy to find that the active traveler's cost is equal to the negative traveler's cost. The reason may be that under such circumstances all active travelers arrive early and all negative travelers arrive late. From the numerical example, we also verify the above theory. At the same time, the figure also depicts the influence of the total travel cost when the proportion of active travelers changes. The total travel costs initially decreased, but, with the number of active travelers increasing, the speed of total travel costs effectively slowed down, even to a certain extent, but the total travel costs increased.

Figure 6 depicts the change of the generalized travel cost when k_1 and k_2 change. Figure 6(a) depicts the change of active and negative traveler's cost: when k_1 changes, k_2 is fixed. The active and negative traveler's cost decreases with k_1 increasing. This shows that active travelers can choose departure time more accurately to reduce their cost when the unit personal perception cost of active traveler increases. When active travelers change their departure time, negative travelers change their departure time choice behavior, making travel cost lower. Figure 6(b) depicts the change of active and negative traveler's cost: when k_2 changes, k_1 is fixed. Active traveler's cost is reduced slightly, and negative traveler's cost is increased slightly. This means that the influence of k_2 is not as great as we think, because negative travelers are starting after active travelers. This also means that the unit personal perception cost of negative traveler has less effect on the system than that of the active traveler.

5. Conclusions

Based on the classical bottleneck model, this paper establishes a bottleneck model based on personal perception by

(a)

(b)
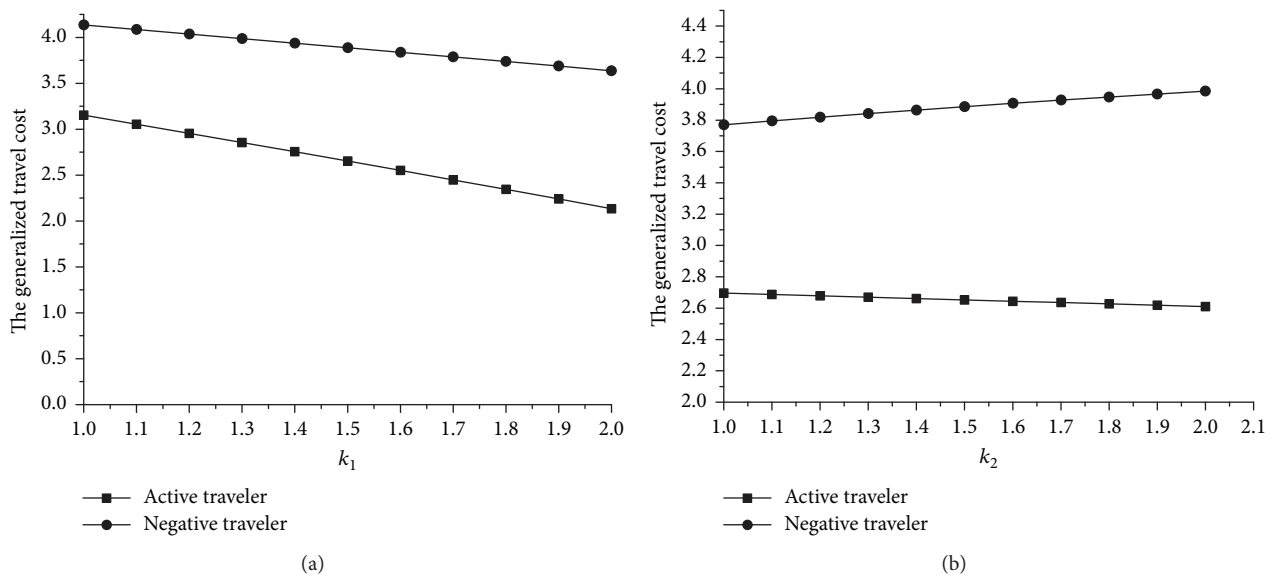
FIGURE 6: Changes of the generalized travel cost with respect to k_1 and k_2.

considering the influence of personal perception on departure time selection. For travelers, some prefer to arrive early, called active travelers, while some are not likely to arrive early, called negative travelers. This paper mainly studies the departure time choice behavior of travelers under these two travel attitudes. It is concluded that the bottleneck model based on personal perception can accurately describe the departure time choice behavior of travelers, which are not only related to the proportion of each type of travelers, but also related to the size of travel perception. This paper mainly considers the departure time choice behavior of travelers but does not take into consideration the impact of congestion pricing and other related strategies on travelers. These will be the contents of our next research.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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