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Research Article

Adaptive Model Predictive Control for Cruise Control of High-Speed Trains with Time-Varying Parameters

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The cruise control of high-speed trains is challenging due to the presence of time-varying air resistance coefficients and control constrains. Because the resistance coefficients for high-speed trains are not accurately known and will change with the actual operating environment, the precision of high speed train model is lower. In order to ensure the safe and effective operation of the train, the operating conditions of the train must meet the safety constraints. The most traditional cruise control methods are PID control, model predictive control, and so on, in which the high-speed train model is identified offline. However, the traditional methods typically suffer from performance degradations in the presence of time-varying resistance coefficients. In this paper, an adaptive model predictive control (MPC) method is proposed for cruise control of high-speed trains with time-varying resistance coefficients. The adaptive MPC is designed by combining an adaptive updating law for estimated parameters and a multiply constrained MPC for the estimated system. It is proved theoretically that, with the proposed adaptive MPC, the high-speed trains track the desired speed with ultimately bounded tracking errors, while the estimated parameters are bounded and the relative spring displacement between the two neighboring cars is stable at the equilibrium state. Simulations results validate that proposed method is better than the traditional model predictive control.

1. Introduction

In recent years, the high-speed railway transportation has played a more and more important role in modern society. High-speed train has many more advantages such as high speed, large volume, and safe and comfortable environment than traditional railway traffic. With the speed of the high-speed trains rising, it is extremely difficult for human drivers to guarantee the safety of the operation of high-speed trains. In order to ensure the safe and effective operation of high-speed trains, the automatic train control (ATC) system is proposed, which is used to monitor, control, and adjust the train operations to guarantee safety, punctuality, and comfort [1–3].

One of the demanding control problems associated with the automatic train control (ATC) is cruise control problem in which the speed of the train is automatically controlled to follow a desired trajectory. The methods proposed for cruise control of high-speed trains which are developed based on a motion model obtained from Newton's second law can be classified into two categories. One is to model the whole train that consists of multiple cars as a single point mass [4, 5], while the interaction force between the two cars of the train is ignored. Considering the relative movement between the two cars, the other one is to construct the high-speed train model by a cascade of masses connected by flexible couplers, which provides much more accuracy in characterizing the dynamics of the high-speed trains [6, 7].

In the most existing literature of the high-speed train, the resistance coefficients of train were often assumed to be constant [8–10]. However, for a high-speed train, the aerodynamic resistance will change large, when the train is traveling at high speed. So, the dynamic motion model of the train is a time-varying model dependent on the operating conditions. The robust adaptive tracking control method was derived for a multiple-mass-points high-speed train dynamics model with unknown and time varying resistance coefficients [11]. Besides, the robust output feedback cruise control is developed for speed tracking with the unknown parameters [12]. Moreover, some more complex train operating conditions are considered in paper [13], and an adaptive controller is developed to deal with the problem of the uncertainty of the air resistance coefficients of the piecewise model. However, the studies of the robust adaptive controller neglected the state constraints and control input constraints. The safe speed and the saturation characteristics of traction and braking units are very important for online operation of high-speed train. The model predictive control has an advantage that fully considers the input and state constrains of the system [14, 15]. However, the influence of time-varying air resistance parameters on the system model is neglected in [15], resulting in low system model precision.

Model predictive control cannot only deal with multi-objective constraint problem, but its dynamic response is fast [16]. The goal of cruise control of high-speed trains is to track the desired target speed quickly and accurately, so, model predictive control is very suitable for the high-speed train cruise controller. According to the principle of model predictive control, it is known that model predictive control requires a prediction model with high precision. The accuracy of the prediction model determines the performance of the controller. Therefore, the starting point of this paper is how to improve the accuracy of the dynamics model of high-speed. In this paper, a multibody model of the high-speed train with time-varying air resistance coefficients and control constraints is considered. The train dynamics model set up in this paper contains time-varying resistance coefficients, and the relative movements among the connected cars of the train are considered, so the dynamics model in this paper is more accurate than that in paper [15]. This paper designs an adaptive model predictive control for cruise control of high-speed trains. Based on Lyapunov's stability theory, an adaptive updating law is given for estimated system model parameters. The closed-loop system is capable of tracking the desired speed, and the relative spring displacements between the two neighbored cars are stable at the equilibrium state.

Compared to the existing work, the proposed adaptive MPC not only solves the cruise control problem with time-varying resistance coefficients but also ensures the train operations within the range of safety constraints. The model complexity is equivalent to the traditional model. The main contributions in this paper can be summarized as follows:

(1) By considering the time-varying air resistance coefficients, this paper firstly constructs a linear multiple-mass-points dynamical model of high-speed trains with time-varying resistance coefficients.

(2) In order to improve the accuracy of the constructed model, an estimated system is proposed based on the proposed model. Then, an adaptive updating law is designed for time-varying parameters of estimated system. Based on the estimated system model, an adaptive model predictive control framework is introduced and the estimated system model is used as the prediction model. Based on the estimated system prediction model, the control problem can be formulated as an objective optimization problem with multiple constraints.

(3) According to the practical requirement, the objective optimization problem with multiple constraints is transformed into the linear quadratic programming problem, which determines the optimal cruise control for the high-speed train with time-varying resistance coefficients and control constraints to improve the safety and energy efficiency of the operation of the train.

The rest of this paper is arranged as following. In Section 2, the linear dynamical model of high-speed train with time-varying parameters is introduced. In Section 3, an adaptive model predictive controller is developed for the train with time-varying parameters to speed tracking. In Section 4, a simulation study is presented to show the performance of the proposed method. Finally, some conclusions are given in Section 5.

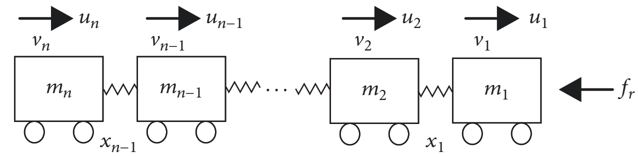

Figure 1: The multiple mass-point model structure of high-speed train.

2. Dynamic Model of High-Speed Train

In this section, the nonlinear multiple mass point dynamic model of high-speed trains with time-varying resistance coefficients is established by analyzing their dynamical characteristics. It is difficult to design the system controller because of the complex characteristics of the nonlinear model. So, the linear error dynamic model of high-speed train is constructed around the equilibrium point.

2.1. The Dynamic Model of the High-Speed Train. Figure 1 presents the multiple mass-point model structure for high-speed train. A high-speed train contains n cars; these cars are connected by flexible couplers. In high-speed train operation, the flexible couplers play an important role in the connected cars and transmit interaction force between two connected cars. As the couplers between two adjacent cars are not perfectly rigid, the function of couplers can be described by a spring model such that the coupler force is a function of the relative displacement ξ between two connected cars.

$$f(\xi) = k\xi \tag{1}$$

where k is the stiffness coefficient, which is positive. The running resistance f_r of high-speed train consists of the rolling mechanical resistance and aerodynamic drag, which is commonly expressed as

$$f_r = c_0 + c_1 v + c_2 v^2 \tag{2}$$

where v is the car velocity and c_0, c_1, and c_2 are the resistance coefficients and they are bounded time-varying parameters.

The parameters change near the nominal value [17]; they are can be described as $c_0 = c_0^* + \Delta c_0$, $c_1 = c_1^* + \Delta c_1$, and $c_2 = c_2^* + \Delta c_2$. c_0^*, c_1^*, and c_2^* and Δc_0, Δc_1, and Δc_2 are the nominal value of the resistance coefficients and the deviation value near the nominal value, respectively, where the deviation value is assumed to be time-varying and bounded, i.e., $|\Delta c_0| \leq \bar{c}_0$, $|\Delta c_1| \leq \bar{c}_1$, and $|\Delta c_2| \leq \bar{c}_2$. c_0 defines the train's rolling resistance component, c_1 defines the train's linear resistance coefficient, and c_2 defines the train's nonlinear resistance coefficient. $c_0 + c_1 v$ represents the rolling mechanical resistance, and $c_2 v^2$ denotes the aerodynamic drag.

The multiple mass-point dynamic equation of a train can be described as

$$\begin{aligned}
&\dot{x}_i = v_i - v_{i+1}, \quad i = 1, 2, 3 \ldots, n-1 \\
&m_1 \dot{v}_1 = u_1 - kx_1 - m_1 (c_0 + c_1 v) - \left(\sum_{i=1}^{n} m_i\right) c_2 v_1^2 \\
&m_i \dot{v}_i = u_i + kx_{i-1} - kx_i - m_i (c_0 + c_1 v), \\
&\qquad\qquad i = 1, 2, 3, \ldots, n-1 \\
&m_n \dot{v}_n = u_n(t) + kx_{n-1} - m_n (c_0 + c_1 v + c_2 v^2)
\end{aligned} \tag{3}$$

where $x_i(t)$ is the relative spring displacement between the neighboring cars i and $i+1$, $v_i(t)$ is the speed of i-th car, and u_i represents the traction force provided by i-th car.

It can be seen from $c_2 v^2$ that the aerodynamic drag is proportional to the square of the speed of high-speed train. When the speed of high-speed train increases, the nonlinear characteristic between aerodynamic drag and speed becomes stronger. The nonlinearity of high-speed train model presents difficulty in solving the optimization problem. It is desirable to linearize the train dynamical equation to facilitate the controller design.

Assume that when the velocity of the high-speed train reaches the desired velocity, the current state of the train is the equilibrium state. The velocity of a high-speed train at equilibrium state is denoted as $\bar{v}_1 = \bar{v}_2 = \cdots = \bar{v}_n = v_0$, and the relative displacements between neighboring cars are zero at the equilibrium state, which are given as $\bar{x}_1 = \bar{x}_2 = \cdots \bar{x}_{n-1} = 0$. Obviously, the acceleration of each car is zero at the equilibrium state, which is given as $\dot{\bar{v}}_1 = \dot{\bar{v}}_2 = \cdots = \dot{\bar{v}}_n = 0$, so the traction or braking force $\bar{u}_i$ in the equilibrium state can be derived from (3) that

$$\begin{aligned}
&\bar{u}_1 = c_0 m_1 + c_1 m_1 v_0 + c_2 \left(\sum_{i=1}^{n} m_i\right) {v_0}^2 \\
&\bar{u}_i = c_0 m_i + c_1 m_i v_0, \quad i = 2, 3, \ldots, n
\end{aligned} \tag{4}$$

So we define the error displacement variable as $\hat{x}_i = x_i - \bar{x}_i$, the error speed variable as $\hat{v}_i = v_i - \bar{v}_i$, and the error control variable as $\hat{u}_i = u_i - \bar{u}_i$. According to (3) and (4), the linearized error dynamic equation around the equilibrium state is obtained as

$$\begin{aligned}
&\dot{\hat{x}}_i = \hat{v}_i - \hat{v}_{i+1} \\
&m_1 \dot{\hat{v}}_1 = \hat{u}_1 - k\hat{x}_1 - c_1 v_1 m_1 - 2c_2 v_0 \left(\sum_{i=1}^{n} m_i\right) \hat{v}_1 \\
&m_i \dot{\hat{v}}_i = \hat{u}_i + k\hat{x}_{i-1} - k\hat{x}_i - c_1 v_i m_i, \quad i = 2, 3 \ldots n-1 \\
&m_n \dot{\hat{v}}_n = \hat{u}_n + k\hat{x}_{n-1} - c_1 v_n m_n
\end{aligned} \tag{5}$$

Choosing $x = [\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_{n-1}, \hat{v}_1, \hat{v}_2, \ldots, \hat{v}_n]^T$ as the state variable and $u = [\hat{u}_1, \hat{u}_2, \ldots, \hat{u}_n]^T$ as the control variable, the error dynamic equation (5) can be written as

$$\dot{x} = A_p x + B_p u \tag{6}$$

where $A_p = \begin{bmatrix} 0_{n\times n} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$ and $B_p = \begin{bmatrix} 0_{n\times n} \\ B_{21} \end{bmatrix}$.

To be exact, $B_{21} = \operatorname{diag}\{1/m_1, 1/m_2, \ldots, 1/m_n\}$,

$$\begin{aligned}
A_{12} &= \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & -1 \end{bmatrix} \\
A_{21} &= \begin{bmatrix} -\dfrac{k}{m_1} & 0 & \cdots & 0 \\ \dfrac{k}{m_2} & -\dfrac{k}{m_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \dfrac{k}{m_n} \end{bmatrix} \\
A_{22} &= \begin{bmatrix} -c_1 - \dfrac{2c_2 v_0 \left(\sum_{i=1}^{n} m_i\right)}{m_1} & 0 & \cdots & \cdots & 0 \\ 0 & -c_1 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & -c_1 & 0 \\ 0 & \cdots & 0 & \cdots & -c_1 \end{bmatrix}
\end{aligned} \tag{7}$$

Then the above continuous time-domain state-space equation is discretized by the zero-order hold method with sampling period T_s to have the following form:

$$x(k+1) = Ax(k) + Bu(k) \tag{8}$$

where $A = e^{A_p T_s}$ and $B = \int_0^{T_s} e^{A_p \tau} d\tau B_p$. This discrete model is then used in the following controller design in an MPC framework.

3. Controller Design

As shown in Figure 2, an adaptive MPC controller is proposed to achieve the closed loop stability and speed tracking

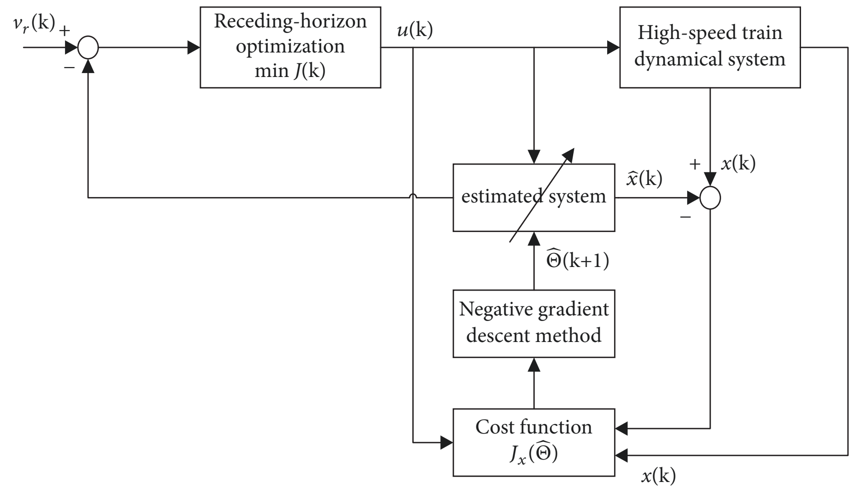

Figure 2: Adaptive model predictive control scheme.

accurately for the high-speed trains. The adaptive model predictive control scheme mainly consists of three parts: formulation of the optimal control problem, adaptive updating law design for time-varying parameters of the estimation model of high-speed train, and MPC design for the estimated train system.

3.1. Formulation of the Optimal Control Problem. Cruise control of high-speed train must track the desired velocity profile quickly so that the train arrives at its destination on time. With the development of high-speed railway, energy-saving driving and safe driving are of much concern. So, the optimization objective function we set up includes energy consumption, velocity tracking, and the relative displacements between neighboring cars. In this paper, the control input is used to express energy consumption.

Consequently, the optimization objective function can be established as follows:

$$J = \sum_{i=0}^{N_p-1} \left(u\left(k+i\right)|k\right)^2 + \left(x_i\left(k+i\right)|k\right)^2 + \left(v\left(k+i\right)|k - v_r\left(k+i\right)|k\right)^2 \quad (9)$$

where k is the current sampling time and N_p represents the predictive horizon for MPC design. $v_r((k+i)|k)$ represents the reference speed. $v((k+i)|k)$ represents the actual speed and it denotes the predicted speed value of v at step i. $u(k+i)|k = [u_1(k)|k, u_2(k+1)|k, \ldots, u_n(k+N_p-1)|k]^T$ represents the control input and $x(k+i)|k = (x_1(k)|k, x_2(k+1)|k, \ldots, x_{n-1}(k+N_p-1)|k)$ represents the relative displacements between neighboring cars.

As a practical system, in order to ensure the safe and efficient operation of high-speed trains, some specific constraints must be satisfied as follows.

First, the traction and brake forces are bounded because of the nature physical characteristics of the traction motor. Second, the maximum allowable speed of high-speed trains is affected not only by line conditions and operating conditions, but also by their physical characteristics. Third, coupler force must be manipulated to vary in an acceptable range in order to ensure the train's run safety. In this paper, the coupler deformation is used to represent intrain forces characteristic. Then, the constraints can be illustrated by the following inequality:

$$\begin{gathered} u_i^l\left(k\right) \le u_i\left(k\right) \le u_i^u\left(k\right) \\ 0 \le v_i\left(k\right) \le v_{\max} \\ x_i^l\left(k\right) \le x_i\left(k\right) \le x_i^u\left(k\right) \end{gathered} \quad (10)$$

where u_i^l and u_i^u are the lower and upper bounds of the ith J in this objective function are a general measure of the "cost" of the train's operation affected the optimization problem is to minimize the objective function (9) subject to (10).

3.2. Adaptive Updating Law for Time-Varying Parameters. For (8), because $\operatorname{rank}[B, AB \ldots A^{n-1}B] = 2n$, the pair (A,B) is controllable. Denoting $\Theta = [A, B]$, the matrix A contains time-varying parameters c_1 and c_2, so the matrix A is a time varying matrix. Denoting $A = A_m + \Delta A$, A_m is the nominal matrices and ΔA is the time-varying matrices which are used to describe the parameter uncertainties, $[\Delta A] \le \overline{A}$. So there exists a conservative bound $\|\Theta\| \le \overline{\Theta}$.

Design an estimated system for (8)

$$\hat{x}\left(k+1\right) = \widehat{A}x\left(k\right) + Bu\left(k\right) \quad (11)$$

where $\widehat{A}$ is time-varying estimated parameters for uncertain constant matrices A; $\hat{x}(k)$ is the estimated state for the actual high speed-trains state $x(k)$.

The actual high-speed trains state and the estimated system state can be rewritten into another forms

$$\begin{aligned} x\left(k+1\right) &= \Theta X\left(k\right) \\ \hat{x}\left(k+1\right) &= \widehat{\Theta} X\left(k\right) \end{aligned} \quad (12)$$

where $\widehat{\Theta}(k) \triangleq [\widehat{A}(k), B(k)]$ and $X(k) \triangleq [x(k)^T, u(k)^T]^T$. Subtracting the above two equations yields

$$\widetilde{x}(k+1) = \widetilde{\Theta} x(k) \tag{13}$$

where $\widetilde{\Theta} \triangleq \Theta - \widehat{\Theta}$ and $\widetilde{x} \triangleq x - \widehat{x}$.

Define a cost function for the estimated error $\widetilde{x}$

$$\begin{aligned} J_x &= \widetilde{x}(k+1)^T \widetilde{x}(k+1) \\ &= \left(x(k+1) - \widehat{\Theta} X(k)\right)^T \left(x(k+1) - \widehat{\Theta} X(k)\right) \end{aligned} \tag{14}$$

Its gradient with respect to $\widehat{\Theta}$ can be calculated by

$$\begin{aligned} \nabla J_x\left(\widehat{\Theta}\right) &= \frac{\partial J_x}{\partial \widehat{\Theta}} = -X(k)\left(x(k+1) - \widehat{\Theta} X(k)\right)^T \\ &= X(k)\widetilde{x}(k+1)^T \end{aligned} \tag{15}$$

Consequently, the updating law for $\widehat{\Theta}(k+1)$ can be designed by

$$\begin{aligned} \widehat{\Theta}(k+1) &= \widehat{\Theta}(k) - \lambda \nabla J_x{}^T \\ &= \widehat{\Theta}(k) + \lambda \widetilde{x}(k+1) X(k)^T \end{aligned} \tag{16}$$

where $\lambda > 0$ is the updating rate to be assigned. It can be proved that [18], with the proposed updating law (17), estimated parameters converge to their actual values, if $X(k)$ is persistently exciting and satisfies the following constraint:

$$X(k)^T X(k) < \frac{2-\alpha}{\lambda} \tag{17}$$

where $0 < \alpha < 2$, In this brief, (19) is treated as an additional constraint. To guarantee that $\widehat{\Theta}$ converges to Θ exponentially fast, a strategy to determine λ can be suggested as [19].

3.3. Adaptive MPC Design for the Estimated System. In the MPC framework, N_p is the predictive horizon and N_c the control horizon. The optimization problem is to compute a trajectory of a future manipulated variable u to optimize the future behavior of the train. At the sampling time k, the current state variable $\widehat{x}(k)$ of estimated high-speed trains system can be measured, and the predicted state variable $\widehat{x}(k+1)$ can be calculated according to the predictive equations of the estimated system. The state-space equations of the estimated system (11) can be given by

$$\begin{aligned} \widehat{x}(k+1 \mid k) &= \widehat{A}(k) x(k) + Bu(k) \\ \widehat{x}(k+2 \mid k) &= \widehat{A}(k) x(k+1 \mid k) + Bu(k+1 \mid k) \\ &= \widehat{A}^2 x(k) + \widehat{A}Bu(k) + Bu(k+1) \\ &\vdots \\ \widehat{x}\left(k+N_p \mid k\right) &= \widehat{A}(k) x(k) + \widehat{A}^{N_p-1} Bu(k) \\ &\quad + \widehat{A}^{N_p-2} Bu(k+1) + \cdots + \widehat{A}^{N_p-N_c} Bu\left(k+N_c+1\right) \end{aligned} \tag{18}$$

Suppose that the output of the estimated system is given by

$$\widehat{y}(k) = C\widehat{x}(k) \tag{19}$$

where $C = \left[0_{(n-1)\times n} \;\; I_{(n-1)\times(n-1)}\right]$ and the output value of the estimated system is the predicted speed of the high-speed trains. From the predicted state variables, the predicted output variables are by substitution

$$\begin{aligned} \widehat{y}(k+1 \mid k) &= C\widehat{A}(k) x(k) + CBu(k) \\ \widehat{y}(k+2 \mid k) &= C\widehat{A}^2(k) x(k+1 \mid k) + C\widehat{A}Bu(k+1 \mid k) \\ \widehat{y}(k+3 \mid k) &= C\widehat{A}^3 x(k) + C\widehat{A}^2 Bu(k) \\ &\quad + C\widehat{A}Bu(k+1) + CBu(k+2) \\ &\vdots \\ \widehat{y}\left(k+N_p \mid k\right) &= C\widehat{A}^{N_p} x(k) + C\widehat{A}^{N_p-1} Bu(k) \\ &\quad + C\widehat{A}^{N_p-2} Bu(k+1) + \cdots \\ &\quad + C\widehat{A}^{N_p-N_c} Bu\left(k+N_c+1\right) \end{aligned} \tag{20}$$

Define $\widehat{Y}(k) = [\widehat{y}^T(k+1|k), \widehat{y}^T(k+2|k), \ldots, \widehat{y}^T(k+N_p|k)]$. According to classical MPC design [18], the predictive equations for (11) can be written into a compact form

$$\widehat{Y}(k) = \widehat{F}(k) x(k) + \widehat{\Phi}(k) U \tag{21}$$

where $\widehat{F}(k) = \begin{bmatrix} C\widehat{A} \\ C\widehat{A}^2 \\ \vdots \\ C\widehat{A}^{N_p} \end{bmatrix}$ and

$$\widehat{\Phi}(k) = \begin{bmatrix} CB & 0 & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & & & \vdots \\ C\widehat{A}^{N_p-1}B & C\widehat{A}^{N_p-2}B & \cdots & C\widehat{A}^{N_p-N_c}B \end{bmatrix}. \tag{22}$$

where $U = [u^T(k), u^T(k+1), \ldots, u^T(k+N_c-1)]^T$ and the desired speed signals are given by $V_r(k) = [v_r^T(k+1), v_r^T(k+2), \ldots, v_r^T(k+N_p)]^T$.

The optimization objective function can be written as

$$\begin{aligned} J &= \left(\widehat{Y}(k) - V_r(k)\right)^T \left(\widehat{Y}(k) - V_r(k)\right) \\ &\quad + U^T(k) R U(k) \end{aligned} \tag{23}$$

where $R = \mathrm{diag}(r)_{N_c \times N_c}$ is a diagonal weight matrix with $r > 0$. To formulate the optimization problem, the cost function is further calculated by

$$\begin{aligned} J &= \left(\widehat{F}x + \widehat{\Phi}U - V_r\right)^T \left(\widehat{F}x + \widehat{\Phi}U - V_r\right) \\ &\quad + U(k)\, RU(k) \\ &= \left(\widehat{F}x - V_r\right)^T \left(\widehat{F}x - V_r\right) + 2U^T \widehat{\Phi}^T \left(\widehat{F}x - V_r\right) \\ &\quad + U^T \left(\widehat{\Phi}^T \widehat{\Phi} + R\right) U \end{aligned} \tag{24}$$

In order to minimize objective function (24) and optimal control input $U(k)$, we just need the formula related to $U(k)$. Ultimately, the train operation optimization problem in the MPC framework, which can be uniformly solved by a quadratic programming (QP) approach, is given as

$$J = U^T H U + 2U^T f \tag{25}$$

where $\mathrm{H} = \widehat{\Phi}^T \Phi + R$ and $f = \widehat{\Phi}^T(\widehat{F}x - V_r)$. The formula (17) can be written as follows:

$$\begin{gathered} X(k)^T X(k) < \frac{2-\alpha}{\lambda} \\ \Updownarrow \\ x(k)^T x(k) + u(k)^T u(k) < \frac{2-\alpha}{\lambda} \\ \Updownarrow \\ u(k)^T u(k) < \frac{2-\alpha}{\lambda} - x(k)^T x(k) \\ \Updownarrow \\ \|u(k)\| < \sqrt{\frac{2-\alpha}{\lambda} - x(k)^T x(k)} \end{gathered} \tag{26}$$

Each $u(k+i|k)$ in the predictive control vector $U(k)$ should satisfy (26). It follows that U should satisfy

$$M_1 U \leq \gamma_1 \tag{27}$$

where $M_1 = \left[\begin{smallmatrix} I_{m\times m} & 0 & \cdots & 0 \\ -I_{m\times m} & 0 & \cdots & 0 \end{smallmatrix}\right]$ and

$$\begin{aligned} \gamma_1 &= \frac{\sqrt{m\left((2-\alpha)/\lambda - x(k)^T x(k)\right)}}{m} \begin{bmatrix} l_m \\ l_m \end{bmatrix}, \\ l_m &= [1, \dots, 1]^T. \end{aligned} \tag{28}$$

To facilitate the MPC design, constraints (10) should be transformed into a form with respect to predictive control vector U. The constraints on the amplitude of the control signals $u_i^l(k) \leq u_i(k) \leq u_i^u(k)$ can be formulated as

$$\begin{bmatrix} I_{(nN_c\times nN_c)} \\ -I_{(nN_c\times nN_c)} \end{bmatrix} U \leq \begin{bmatrix} U^U_{(nN_c\times 1)} \\ U^L_{(nN_c\times 1)} \end{bmatrix} \tag{29}$$

where U^U and U^L are the upper and lower limits of traction or braking force containing N_c upper and lower limit vectors (u^u and u^l). The output value of estimated system (21) represents the speed. So, the speed signals $0 \leq v_i(k) \leq v_{\max}$ can be written as

$$\begin{bmatrix} \widehat{\Phi} \\ \widehat{\Phi} \end{bmatrix} U \leq \begin{bmatrix} \widehat{Y}^U - \widehat{F}x(k) \\ \widehat{Y}^L - \widehat{F}x(k) \end{bmatrix} \tag{30}$$

where $\widehat{Y}^U$ and $\widehat{Y}^L$ are the upper and lower limits of speed containing N_c upper and lower limit vectors (0 and $v_{\max}$).

The relative displacements constraints between neighboring cars are included in constraints of the state variable x(k). Designing a matrix Z for obtaining the relative displacements, it is constructed as $Z = \left[I_{(n-1)\times(n-1)} \quad 0_{(n-1)\times n}\right]$. Because of the relationship $x_i(k) = Zx(k)$, the predicted $x_i(k)$ within the control horizon N_c can be obtained as

$$\widehat{X}_i(k) = \widehat{F}_x(k)\, x(k) + \widehat{\Phi}_x(k)\, U \tag{31}$$

where $\widehat{X}_i(k) = [x_i(k+1|k)^T, x_i(k+2|k)^T, \dots, x_i(k+N_c|k)^T]^T$ and

$$\begin{aligned} \widehat{F}_x(k) &= \begin{bmatrix} Z\widehat{A} \\ Z\widehat{A}^2 \\ \vdots \\ Z\widehat{A}^{N_p} \end{bmatrix}, \\ \widehat{\Phi}_x(k) &= \begin{bmatrix} ZB & 0 & \cdots & 0 \\ Z\widehat{A}B & ZB & \cdots & 0 \\ \vdots & & & \vdots \\ Z\widehat{A}^{N_p-1} & Z\widehat{A}^{N_p-2}B & \cdots & Z\widehat{A}^{N_p-N_c}B \end{bmatrix} \end{aligned} \tag{32}$$

Consequently, the relative displacements constraints $x_i^l(k) \leq x_i(k) \leq x_i^u(k)$ can be defined as

$$\begin{bmatrix} \widehat{\Phi}_x \\ \widehat{\Phi}_x \end{bmatrix} \leq U \leq \begin{bmatrix} X_i^U - \widehat{F}_x x(k) \\ X_i^L - \widehat{F}_x x(k) \end{bmatrix} \tag{33}$$

where X_i^U and X_i^L are the upper and lower limits of relative displacements between neighboring cars containing N_c upper and lower limit vectors (x_i^u and x_i^u).

Combining constraints (27), (29), (30), and (33) yield constraints for MPC design, and the constraints in this optimization problem can be eventually constructed as

$$MU \leq \gamma \tag{34}$$

where $M = \begin{bmatrix} M_1 \\ I_{(nN_c\times N_c)} \\ -I_{(nN_c\times nN_c)} \\ \widehat{\Phi} \\ -\widehat{\Phi} \\ \widehat{\Phi}_x \\ -\widehat{\Phi}_x \end{bmatrix}$ and $\gamma = \begin{bmatrix} \gamma_1 \\ U^U_{(nN_c)} \\ U^L_{(nN_c)} \\ \widehat{Y}^U - \widehat{F}x(k) \\ \widehat{Y}^L - \widehat{F}x(k) \\ X_i{}^U - \widehat{F}_x x(k) \\ X_i{}^L - \widehat{F}_x x(k) \end{bmatrix}$

3.4. Adaptive MPC Algorithm. The proposed adaptive MPC algorithm can be summarized as follows:

(1) Select a positive λ according to [19]; the predictive horizon and the control horizon satisfy $N_c = N_p = N$.

(2) Calculate optimal $U(k)$. With the linear state-space equations (11), the optimization problem is to minimize (25) subject to (19) and (34). This is an optimization problem with a quadratic objective function, which can be uniformly solved by a QP approach [20]. The standard quadratic programming problem has been extensively studied in the literature [21, 22]; this is a field of study in its own right; it requires a considerable effort to completely understand the relevant theory and algorithms. Optimization Toolbox in MATLAB provides functions for finding parameters that minimize or maximize objectives while satisfying constraints. The toolbox includes solvers for linear programming (LP), mixed-integer linear programming (MILP), quadratic programming (QP), nonlinear programming (NLP), constrained linear least squares, nonlinear least squares, and nonlinear equations. If the optimal control problem can be transformed into a quadratic programming problem, the quadratic programming problem can be solved by MATLAB quadprog toolbox more easily. The instructions for the toolbox can be found at https://www.mathworks.com/help/optim/quadratic-programming.html.

(3) Find $u(k)$ by using receding horizon scheme: $u(k) = [I_{m\times m}, 0, \ldots, 0]U(k)$.

(4) Update the estimated parameters $\widehat{A}(k+1)$ by using the adaptive updating law (16).

(5) Make $k = k + 1$, and update system states, inputs and outputs with control $u(k)$, and state-space equations (11). Repeat steps (1)-(4).

4. Simulation and Discussion

A simulation study on a high-speed train is presented to demonstrate the effectiveness of the proposed adaptive MPC algorithm. The simulation in this paper is to solve the optimization problem of model prediction with quadprog toolbox of MATLAB simulation software version 2016b under the system environment of Windows 10 operating system. The parameters of the train model are from the CRH-3 high speed train in China, which are given in Table 1. This paper investigates the advantages of using MPC to optimize the train's performance by comparing its performance under different prediction horizons. The variables N_p and N_c are set to be equal in order to investigate the prediction's impacts on the performance of the high-speed train.

4.1. Simulation Parameter Selecting. In order to evaluate the performance of the controller, the desired velocity curve including accelerating, decelerating, velocity step increase, velocity step decrease, and constantvelocity stages, the speed command of high-speed train is given in Table 2. Here we focus on the dynamic characteristic and performance of the high-speed train in the cruise phases. The considered time horizon is $T = 1200s$. In this scenario, we choose the number of trains as $n = 4$, and the train is comprised of all locomotives. The predictive horizon and the control horizon are given by $N_P = 10$ and $N_c = 10$. $\lambda = 0.01$ and $R = \text{diag}(0.1)$ are assigned. Additionally, the control input u_i is subjected to the constraints $-30kN \le u_i \le 30kN$, the coupler deformation is subjected to the constraints $-0.02 \le x_i \le 0.02$ and the bounded of the deviation value near the nominal value $\bar{c}_0 = 0.01$, $\bar{c}_1 = 0.00035$, and $\bar{c}_2 = 0.000008$.

Table 1: Parameters of the CRH-3 high-speed train.

Symbol	Value	Unit
m_i	47.5	t
c_0^*	7.75×10^{-3}	Nkg^{-1}
c_1^*	2.28×10^{-4}	$Ns(mkg)^{-1}$
c_2^*	1.66×10^{-5}	$Ns^2(m^2kg)^{-1}$
k	1×10^4	kNm^{-1}

Table 2: Speed Command of high-speed train.

Phase	Time(s)	Velocity(m/s)
acceleration	$0 \longrightarrow 100$	$0 \longrightarrow 40$
cruise	$100 \longrightarrow 400$	40
acceleration	$400 \longrightarrow 450$	40-70
cruise	$450 \longrightarrow 850$	70
deceleration	$850 \longrightarrow 900$	70-50
cruise	$900 \longrightarrow 1200$	50

4.2. Simulation Results. Figure 3 shows the velocity curve for each car, where the abscissa is the simulation time and the ordinate is the velocity of the vehicle. From the figure, we can see that the running speed of each car almost stays the same whether in the accelerating phase or in the decelerating phase, and each car can track the reference speed well during the operation time. From $t = 0s$ to $t = 100s$, each car operate at an $0.4m/s^2$ acceleration. When $t = 100s$, the actual speeds of each car of the high-speed train are closed to the reference speed $40m/s$. From $t = 400s$ to $t = 450s$, the high-speed train is running with $0.6m/s^2$ acceleration, From the zoomed-in figure, we can see that the each car moves with the almost same velocity, and the speed error is negligible. From $t = 440s$ to $t = 460s$, the speed of each car gradually achieved stability, and the velocity of each car is the same as the reference velocity basically. Based on the above simulation analysis, we can make a conclusion that the high speed train can track the target speed quickly and maintain a small steady-state tracking error, which verifies the effectiveness of proposed control method.

y is output of (8), and we define estimated output errors $\tilde{y} = y - \hat{y}$, which is shown in Figure 4. From the figure, we can see that the estimated output errors converge to zeros during the operation time, which verifies the accuracy of the estimation model is proved.

The curves for the tracking and braking forces of each car in the cruise phases are plotted in Figure 5, and the dashed line is the upper bound of traction or the lower bound of brake force. From $t = 0s$ to $t = 100s$, in order to keep

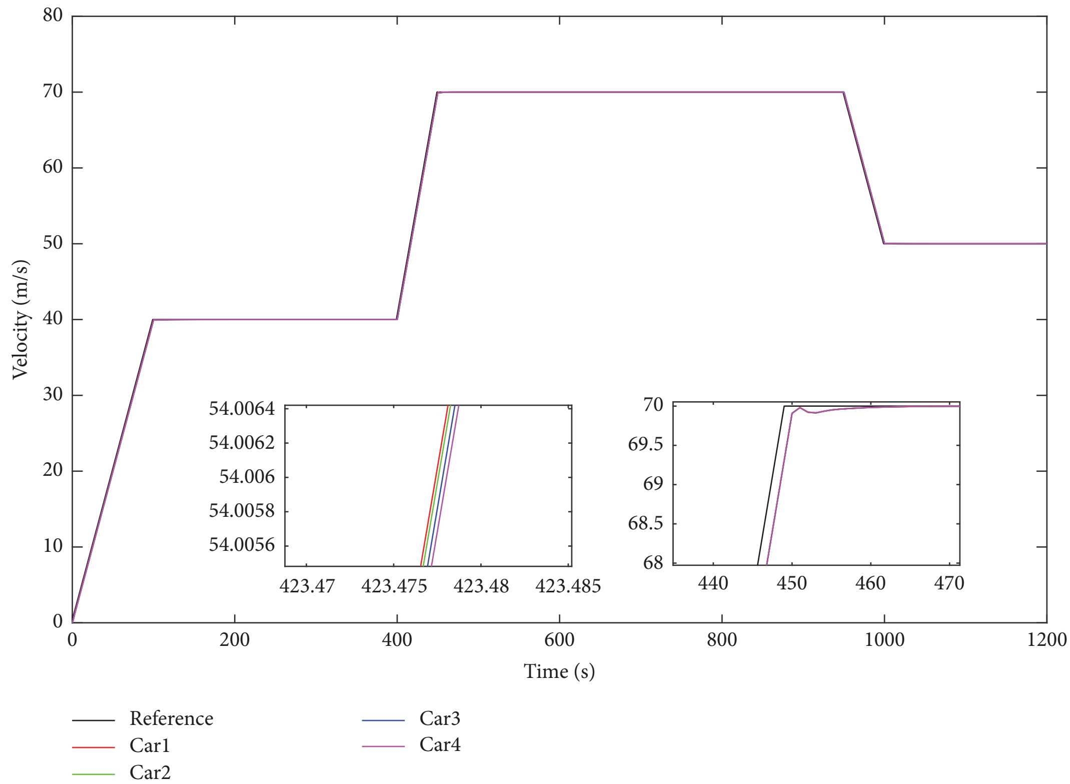

Figure 3: Speed curves for each car of high-speed train in the cruise phases.

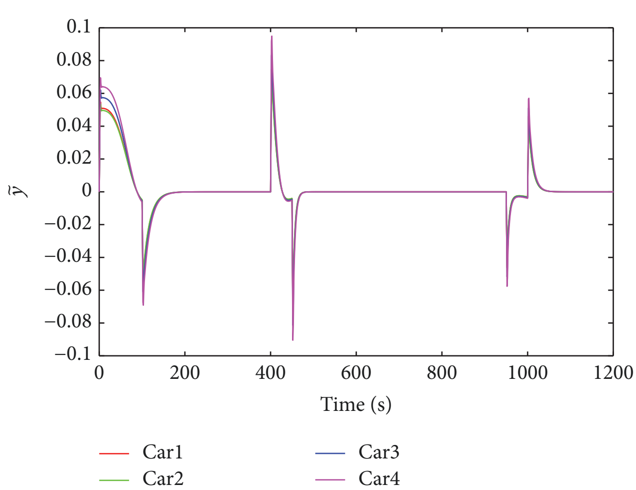

Figure 4: Estimated output errors.

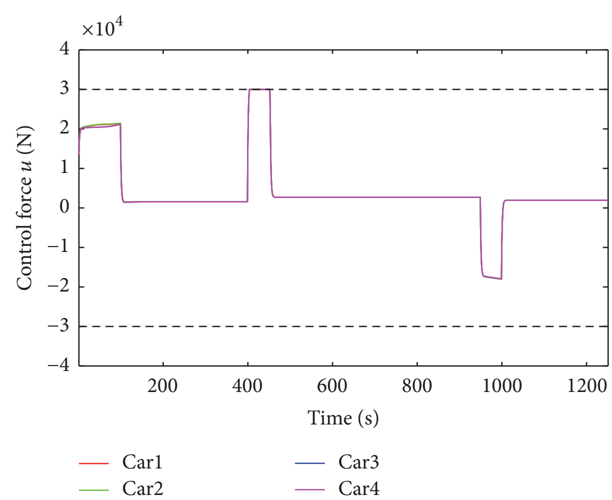

Figure 5: The force output of each car.

the acceleration constant, the tracking forces of high-speed train increase rapidly, because the train's resistance increases with speed. At $t = 400s$, the high-speed train reaches its maximum speed and the maximum tracking force of each car of the train are $30kN$. From $t = 850s$ to $t = 900s$, the braking forces of high-speed train increase rapidly, and maintain about $20s$ to decrease to the desired velocity. From Figure 5, we can observe that control outputs for each car of the high speed train are almost the same. Based on the above simulation analysis, we can make a conclusion that each car of the high-speed train can regulate the tracking force and braking force quickly based on actual speed commands and the magnitude of the tracking force and braking force is in the picture satisfies the constraints, which verifies the effectiveness of proposed control method.

The curves of relative spring displacements between the two neighboring cars are plotted in Figure 6, which shows that, under adaptive model predictive cruise control, the relative spring displacements between the two neighboring

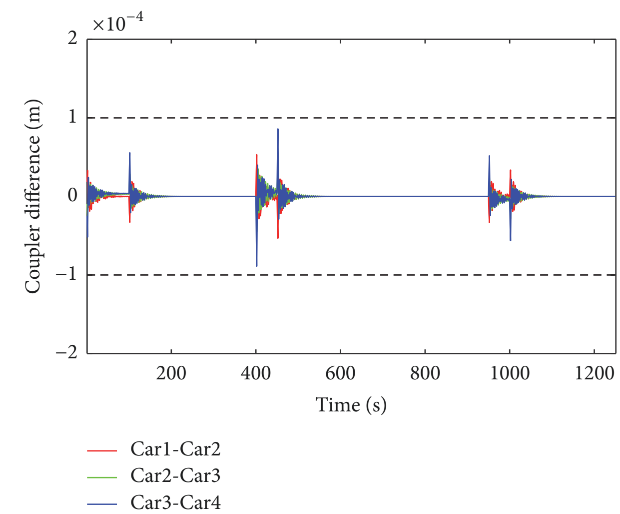

FIGURE 6: The coupler deformation between neighboring cars.

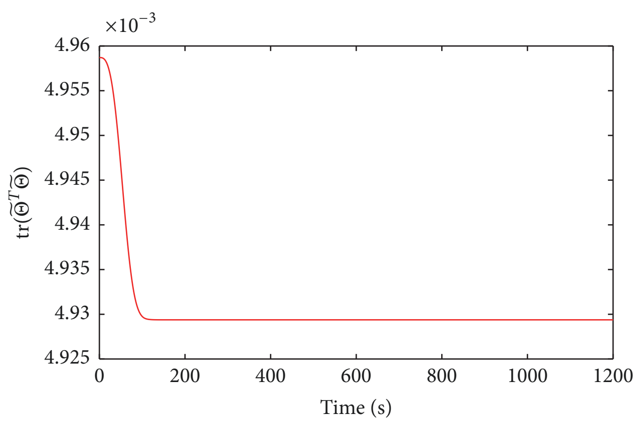

FIGURE 7: Norm of estimation errors.

cars converge to the zero point. During the train operation stage, the relative spring displacements change in a small range, which ensures the safety and comfort of the operating of high-speed train. Additionally, from the figure, we can find that the change of relative displacement is within the constraint range in the acceleration and deceleration stages, and the change of relative displacement is 0 at the equilibrium state.

The norm of estimation errors is defined by $\mathrm{tr}(\widetilde{\Theta}^T\widetilde{\Theta})$, and the variation of estimated parameters are defined by $\Delta\Theta(k+1) = \widehat{\Theta}(k+1) - \widehat{\Theta}(k)$. Because the estimated parameter $\widehat{\Theta}$ is a bounded matrix, the estimation error is a bounded matrix. Figure 7 shows the norm of estimation errors, which is bounded. The variation of estimated parameters is shown in Figure 8; from the figure, we can see that the variation of estimated parameters converges to 0 at around $t = 100s$.

4.3. Further Discussions. In this subsection, we further discuss the performance of the proposed adaptive model predictive controller in terms of superiority and computation efficiency.

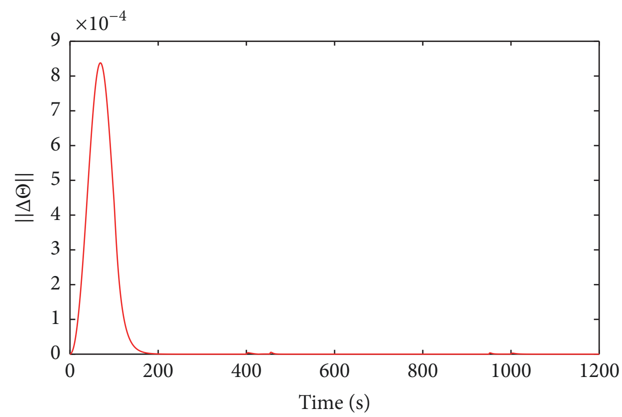

FIGURE 8: The variation of estimated parameters.

4.3.1. Superiority. In order to verify the superiority of the method proposed in this paper, we make a simulation comparison with the method in literature [15]; the system in [15] is a nonadaptive control system; it does not consider the impact of model parameter changes on the system model. The parameters of controller are the same of adaptive control system and nonadaptive control system. The purpose of this paper is to improve the accuracy of the prediction model, so that high-speed trains can follow the desired target speed quickly and accurately. The velocity prediction error represents the difference between the predicted model's velocity and the expected velocity; the smaller the error, the higher the accuracy of the prediction model. The velocity error of each car is plotted in the following figures. Figures 9(a) and 9(b) show the simulation results under adaptive control system and nonadaptive control system, respectively. From Figure 9(a), the car of four locomotives can track the reference velocity accurately. The maximum velocity error is no more than $2m/s$. From the zoomed in figure, there is little difference in the tracking velocity error of each car, because the parameters of each car are same. This paper's goal is to make sure that each car tracks the reference velocity very well with the coupling force in mind. The method of [15] presents poor control performance when the parameters of the model are uncertain; the maximum velocity error is about $8m/s$ and the velocity of each car has severe chattering; this control performance is not conducive to the safe operation of the train.

The coupler force of each car is plotted in Figure 10. Figures 10(a) and 10(b) show the simulation results under adaptive control system and nonadaptive control system, respectively. The coupler can be damaged by too much force and excessive coupler force is not conducive to the safe operation of trains.so, when high-speed trains are running; the less coupling force between vehicles, the better. From Figure 10(a), the coupler of each car is very small; from the zoomed in Figure 10(a), the coupling force is no more than 2KN. The method of [15] presents poor control performance, and the coupler of each car is very powerful. This control performance is very unfavorable to the safe operation of the train.

4.3.2. Computational Cost. In this subsection, by considering the different prediction horizons, this paper considers the

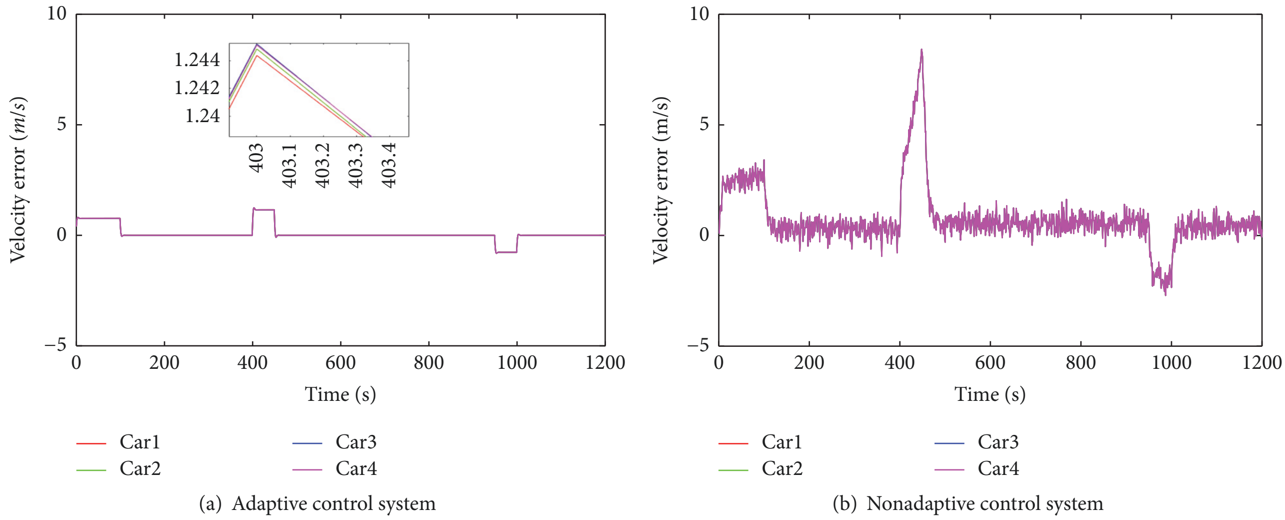

FIGURE 9: Comparison of adaptive control system with nonadaptive control system in predicted output velocity and the desired velocity error.

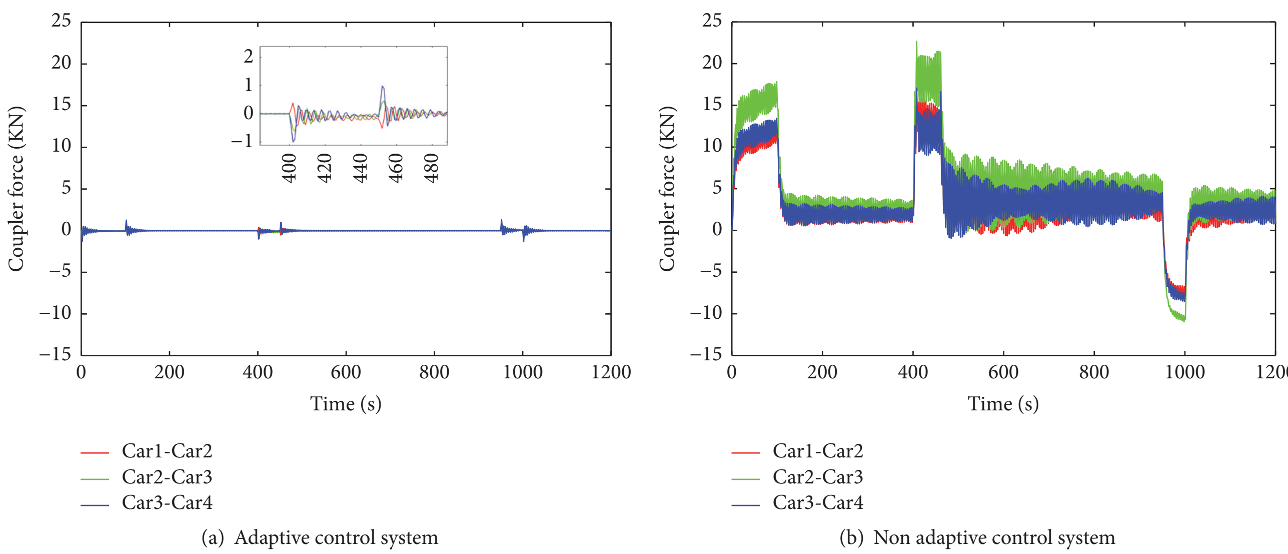

FIGURE 10: Comparison of adaptive control system with nonadaptive control system in predicted output velocity and the desired velocity error.

computation complexity of the adaptive model predictive control with different prediction horizons. Within different prediction horizons, the adaptive MPC and the traditional MPC are respectively implemented on (8). The simulations are carried out under Windows 10 operating system with Intel Core i5-4460 CPU, 8GB RAM on a notebook computer. The computation time is shown in Table 3. It can be seen that the running time of our proposed adaptive MPC is less than the traditional MPC and the computational time of the algorithm will increase with the increase of the predicted horizons from Table 3.

TABLE 3: Computation time of adaptive MPC and traditional MPC.

Algorithm	$N_p = 4$	$N_p = 6$	$N_p = 10$
Adaptive MPC	9.12s	10.45s	11.96s
Traditional MPC	10.08s	11.06s	13.29s

5. Conclusion

In this paper, the optimal cruise control of high-speed trains with time-varying air resistance coefficients and control constraints is investigated. The control objective is accurate speed tracking control with minimum energy consumption and safe relative displacement between two neighbored cars. First, a multiple-mass-point model of high-speed trains is built. By considering multiple constraints and performance metrics, an adaptive MPC method is proposed to design the cruise control controller. In order to improve the accuracy of the method, a dynamic estimated system model of high-speed trains with time-varying parameters is proposed. Also, an adaptive updating law for estimated system parameters by the Lyapunov stability theory is designed. Then the optimization objective and operation constraints are analyzed in detail. In addition, the cruising control problem is transformed into a constrained finite-time optimal control problem with aquadratic objective function, which can be uniformly solved by a quadratic programming approach. Using the method in this paper, the high-speed trains track the desired speed quickly and precisely, and the relative spring displacement between the two neighbored cars is stable at the equilibrium state. Performance of the closed-loop system is substantiated by simulation results.

Data Availability

The simulation result data used to support the findings of this study have been deposited in the https://github.com/xuxiaokang1123/Adaptive-Model-Predictive-Control-for-Cruise-Control-of-High-Speed-Trains-with-Time-varying-Paramete.git. The simulation result data includes the velocity curve data, the force output of each car data, the coupler deformation between neighboring car data and norm of estimation errors, and the variation of estimated parameters data. Everyone can download it through the internet.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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