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*Research Article*

# A Traffic Flow Evolution Process toward Mixed Equilibrium with Multicriteria of Route Choice Behaviour

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Based on the price-quantity adjustment behaviour principle of disequilibrium theory, the route choices of travellers are also affected by a quantity signal known as traffic flow, while the route cost is considered as a price signal in economics. Considering the quantity signal's effect among travellers, a new route comfort choice behaviour criterion and its corresponding equilibrium condition are established. The network travellers are classified into three groups according to their route choice behaviour: travellers in the first group choose the shortest route following the route rapidity behaviour criterion with complete information forming the UE (user equilibrium) pattern, travellers in the second group choose the most comfortable route following the route comfort behaviour criterion with complete information forming the QUE (quantity adjustment user equilibrium) pattern, and travellers in the third group choose a route according to their perceived travel time with incomplete information forming the SUE (stochastic user equilibrium) pattern. The traffic flows of all three groups converge to a new UE-QUE-SUE mixed equilibrium flow pattern after interaction. To depict the traveller-diversified choice behaviour and the traffic flow interaction process, a mixed equilibrium traffic flow evolution model is formulated. After defining the route comfort indicator and the corresponding user equilibrium state, the equilibrium conditions of the three group flows are given under a mixed equilibrium pattern. In addition, an equivalent mathematical programming of the mixed equilibrium traffic flow evolution model is proposed to demonstrate that the developed model converges to the mixed equilibrium state. Finally, numerical examples are examined to evaluate the effect of route comfort proportions on the traffic network flow evolution and analyse the performance of the proposed model.

## 1. Introduction

The dynamic evolution process of traffic flow has been a hot issue in the transportation field, and various traffic flow dynamic models based on travellers' route adjustment behaviours have been examined. These models can depict the process of the traffic flow as it evolves from a disequilibrium state to an equilibrium state [1, 2]. Obviously, the network flow forms a Wardrop UE (user equilibrium) pattern if all travellers choose the actual shortest route in the route decision process and converges to a SUE (stochastic user equilibrium) pattern if all travellers adjust their routes according to the perceived route travel time. When a SO (system optimal) state is reached, all travellers select the route that can minimize the system travel time.

Although these classical traffic flow equilibrium conditions differ in the form of travel time, they share the same route rapidity choice criterion, which can embody the travellers' preference of route rapidity and reflect the price signal affection. It is apparent that all existing models in the literature were formulated based on a single-route rapidity choice preference, where the route travel time is the core indicator. This approach has been widely applied in various network traffic flow models, whereas with the rapid economic growth and social development, people-oriented considerations, such as traffic safety and travel satisfaction, have been the dominant research direction [3–8].

With the development of intelligent transportation systems and technologies, travellers can make their route choices according to the accurate information of the traffic

network [9, 10]. By relaxing the restrictions on the behavioural hypothesis conditions of the Smith model, Guo et al. established a discrete dynamic system to study the day-to-day evolution process of traffic flow from nonequilibrium to equilibrium. This assumed that travellers choose a travel route based on the self-estimated travel time, and the number of adjusted travellers is related to the route adjustment ratio [11, 12]. Zhao and Huang adopted the concept of satisfaction and established a parallel network traffic distribution model based on satisfaction criteria [13]. Tang et al. established a route-based traffic flow model based on road network disturbance factors and studied the influence of two typical factors, a bus station and an accident, on the traffic flow [14].

Ma et al. provided a methodology to derive the critical point from free flow to crowded flow and proposed a new capacity allocation strategy that can improve the capacity of a traffic network [15]. Cantarella and Watling formulated a discrete stochastic deterministic process model to study the impact of travellers' route selection behaviours on traffic flow evolution. The model considered travellers' travel habits and provided a general method to portray the traffic flow distribution [16]. Considering the social interaction of travellers, Wei et al. utilized a traffic flow evolution model in order to describe travellers' route choice behaviours and studied the impact of individual travellers' route choice decisions and their interactions on the traffic flow pattern of a road network [17].

Considering capacity constraints, Hoang et al. established the linear programming of a UE-DTA (user equilibrium dynamic traffic assignment), connected the UE solution to the SO solution, and proposed an incremental loading method that effectively reduced the difficulty in obtaining a UE solution [18]. Liu et al. analysed the interaction between travellers and traffic information providers through a network evolution model that considered the influence of user inertia on travellers' route decisions [19].

Different traveller route choice behaviour criteria lead to different network traffic flow distribution patterns [20, 21]. A network-mixed equilibrium is formed through the interaction of various traffic flows following different route choice behaviour criteria. At present, mixed equilibrium traffic flow studies have mainly examined the mixed equilibrium between UEs and other specific equilibrium traffic flows, including UE-SUE, UE-CN (Cournot–Nash), UE-SO, and UE-CN-SO. Zhou et al. established a discrete dynamic evolution model of mixed equilibrium traffic flow that describes travellers' route adjustment behaviours and simulates the evolution trajectory of traffic flow converging to a UE-SUE mixed equilibrium state with a given ATIS (advance traveller information system) market penetration [22].

Zhang et al. divided travellers into two categories according to travel behaviour principles: the UE principle and CN principle (CN is the intermediate state between perfect competition and perfect coordination, manifested as internal coordination and external competition). The interaction of traveller's route choice behaviours finally produces a result between the competition equilibrium and

monopoly equilibrium: the UE-CN-mixed equilibrium [23]. Proble et al. classified travellers into two types (perfect cooperation and perfect competition) and proposed a UE-SO mixed equilibrium model in which perfectly cooperative travellers obey the SO criterion and perfectly competitive travellers obey the UE criterion [24]. Site et al. separated travellers into three categories: (a) travellers equipped with predictive ATIS, (b) travellers equipped with static ATIS and are subjected to it, and (c) travellers not equipped with ATIS or are not subjected to it. The researchers established a mixed equilibrium behaviour models with predictive ATIS and static ATIS [25].

After elaborating on the traffic behavioural implications of the price-quantity adjustment behaviour principle in economics, this paper summarizes a new route comfort behaviour, which has not been investigated so far, to simulate the route decision process of travellers' adjustment behaviours. Unlike the existing studies under a single-route rapidity choice behaviour criterion, a new route comfort choice behaviour criterion is proposed in this study to analyse the route adjustment process and to accurately model the traffic flow evolution from disequilibrium to mixed equilibrium. Specifically, given complete information, travellers in the first group, following the route rapidity choice behaviour criterion, are likely to choose the shortest route under current conditions, while travellers in the second group, following the route comfort choice behaviour criterion, are supposed to choose the most comfortable route with minimum traffic flow. Travellers in the third group with incomplete information follow the logit-based SUE principle based on the perceived travel time.

The resultant equilibrium of travellers under a predefined penetration of route decision behaviour criteria and complete information is referred to as a mixed equilibrium. A route flow adjustment model is proposed to analyse the mixed traffic flow of travellers with the multicriteria of route choice behaviours. The convergence of our day-to-day flow adjustment model to the mixed equilibrium state is demonstrated. In addition, we present a list of route flow evolution trajectories under different behaviour criteria percentages and show specific cases to refine the evolution process toward the mixed equilibrium state.

In previous studies that did not consider the existence of quantity signals in the traffic market, it was customary to assume that all travellers make route selection decisions based on the travel price signal, known as travel cost. This is difficult to achieve in a real traffic travel process because in addition to inaccuracies and partial network information, the behaviours of the travellers are heterogeneous. While some travellers choose a route according to the travel time and speed, some travellers may pay more attention to the comfort of travel or safety and other factors. Therefore, the mixed equilibrium traffic flow evolution model under the multicriteria of route choice behaviour not only takes into account the heterogeneity and bounded rationality of the traveller but also vividly explains the various traffic travel shift behaviours of the traveller. In addition, the model describes the dynamic evolution process of the actual network traffic flow more flexibly and objectively, which

provides the basis for the formulation of a network traffic flow control strategy and traffic construction plan.

## 2. Traffic Flow Equilibrium State and Route Choice Behaviour

*2.1. Price-Quantity Adjustment Principle and Route Choice Behaviour.* Based on the price-quantity adjustment behaviour principle of disequilibrium theory, individuals accept both commodity price signals and the trading quantity signals from the market in economics. This affects product decisions and furthermore influences the commodity demand and supply. As a specific travel market, residential travel route choice behaviour is influenced by the route travel time (price signal) and the route flow (quantity signal) as well. This makes a difference on traffic demand and supply in an origin-destination pair [26].

Since the shortest route choice reflects the travellers' pursuit of travel rapidity, the route rapidity choice criterion is widely used to depict the route choice behaviour that uses the route travel time as the only decision basis. This expresses the price signal effect in the traffic market, such as in the traditional Wardrop UE or SO assignment. Since a UE or SO assignment focuses on the route travel time decision behaviour, they are unable to describe the travel route choice behaviour that was influenced by the route flow.

As an important supplement to the traditional route rapidity choice criterion, this paper proposes a new route comfort choice criterion to portray the route adjustment behaviour that seeks a relatively comfortable route. The route comfort choice criterion assumes that the route flow is the decision basis of travellers, and travellers shift from a congested route to a low-flow route based on the experienced comfort. Hence, according to the price-quantity adjustment behaviour principle, the general pursuit of travellers in the travel route decision process is to choose a route that is both fast and comfortable. Applying the route rapidity and comfort adjustment criteria stimulatingly to individual travel route selection behaviour shows that individuals will comprehensively consider the route cost and the route surplus capacity to make a route decision. When applied to the traveller group, the aggregated effect performs as some travellers choose the shortest route and some travellers choose the most comfortable route, which is discussed in this study.

In the travel route decision process, travellers have some human properties that affect route decision behaviours and cause diversity in travellers' route choice behaviours. To provide variety, a navigation system in real life, such as Amap or Baidu Maps, provides three kinds of route choice in route planning: the shortest time, the shortest distance, and the minimum number of traffic lights. All these inform the different travel preference of travellers between travel rapidity and comfort. As the route choice behaviour affects the network traffic flow distribution pattern directly, it is more convincing to model the equilibrium traffic flow under the rapidity and comfort route decision criteria than the single rapidity or comfort preferences.

*2.2. Route Rapidity Choice Criterion.* Given a network $G = (N, A)$, where $N$ is the set of nodes and $A$ is the set of links, let the set of origin-destination pairs be $W$. The set of routes in OD pair $w \in W$ is denoted by $R_w$.

The traditional UE, SUE, and SO assignment models assume a perfect perception of travel cost and develop traffic flow adjustment processes over a single-route rapidity choice behaviour criterion. Based on this criterion, travellers select the fastest route that can minimize their travel time under the current alternative route information.

The flow of travellers on the route $r \in R_w$ is denoted by $f_w^r$. The traffic flow on the link $a \in A$, denoted by $x_a$, is given by

$$x_a = \sum_{w \in W} \sum_{r \in R_w} \delta_w^{ra} f_w^r, \quad a \in A, \tag{1}$$

where $\delta_w^{ra} = 1$ if route $r \in R_w$ contains link $a$ and 0 otherwise. $t_a = t_a(x_a)$ is the travel time function on link $a \in A$, which is assumed positive, additive, and strictly increasing with respect to link flow $x_a$. Thus, the travel time function on the route is expressed by

$$c_w^r = \sum_{a \in A} \delta_w^{ra} t_a(x_a), \quad r \in R_w, r \in W. \tag{2}$$

Travellers in the first group equipped with complete network information will follow the UE route choice behaviour assumption, in which all travellers are supposed to shift to the alternative shorter route to reduce their actual travel time given the current information. The traffic flow evolution formed by this shift movement will converge toward a Wardrop UE equilibrium state where all routes of the OD pair share the same actual travel time. The equilibrium condition is

$$\begin{cases} f_w^r > 0, c_w^r = u_w, \\ f_w^r = 0, c_w^r \geq u_w, \quad r \in R_w, w \in W, \end{cases} \tag{3}$$

where $u_w$ denotes the minimal travel time between OD pair $w$.

By contrast, travellers in the third group with incomplete network information will choose their routes in a logit-based SUE manner. All travellers in the third group will choose to shift their routes according to the perceived route travel time, which may result in multiple possible route adjustment trajectories in the same situation owing to differences in perception. The SUE equilibrium state is reached when the route perceived travel time of all alternative routes is equal, and the following condition holds

$$f_w^r = p_w^r \cdot d_w = \frac{\exp(-\theta c_w^r)}{\sum_{k \in R_w} \exp(-\theta c_w^r)} d_w, \quad r \in R_w, w \in W, \tag{4}$$

where $p_w^r$ represents the probability that route $r$ between OD pair $w$ is chosen, $\theta$ is a perception parameter, and $d_w$ is the traffic demand of OD pair $w$.

*2.3. Route Comfort Choice Criterion.* Travellers in the second group are assumed to choose the most comfortable route in the OD pair under the route comfort choice criterion. Comfort is a kind of physiological experience and is the comprehensive evaluation of the satisfaction degree of the objective reality environment in both physiology and psychology. This can be affected by various factors and differs between individuals owing to their disparate perceptions. There is no uniform definition of route comfort at present, so is the main measure indicators.

By applying the price-quantity adjustment principle in route choice behaviour research, the route surplus capacity is proposed as the indicator of the route comfort degree in this study. This is a general expression form of the traffic flow that reflects the impact of the quantity signal on the route decision process. The route surplus capacity is the difference between the route maximum capacity and the route flow, which concerns not only the physical capacity of the network route but also indicates the travel comfort degree. In addition, the route with a larger surplus capacity indicates a higher degree in the route service level, road infrastructure facilities, environmental satisfaction, travel fluency, and experience of comfort than a lower surplus capacity, and vice versa.

Let $K_a$ denotes the maximum traffic capacity of link $a$. Then, the maximum traffic capacity of route $r \in R_w$ is expressed as

$$K_r = \min\left(\delta_w^{ra} K_a\right). \tag{5}$$

As the study subject of this research is the traffic evolution short-term behavior, the traffic capacity of route is assumed to be constant. The surplus capacity of route $r \in R_w$ is given by

$$s_w^r\left(f_w^r\right) = K_r - f_w^r. \tag{6}$$

The maximum surplus capacity in OD pair $w \in W$ is defined as

$$v_w = \max_{r \in R_w}\{s_w^r\left(f_w^r\right)\}. \tag{7}$$

When the traffic network travel demand is low, all of the route capacities are relatively high, so travellers in the second group select the maximum surplus capacity route, that is, the most comfortable travel route. With an increase in the network travel demand, the surplus capacities of all routes are reduced since the network gradually evolves to a congested state, as does the route comfort degree, apparently. Under this circumstance, travellers in the second group are supposed to shift to an alternative route whose surplus capacity is greater than that of the current route. In addition, the traffic flow will be stable in the equilibrium state where all route surplus capacities are the same and are equal to the maximum surplus capacity of the OD pair. This route shift behaviour is defined as route comfort choice behaviour, and the formed network equilibrium state is the quantity adjustment user equilibrium.

For the quantity adjustment use equilibrium, the route surplus capacities of all used routes between each OD pair are equal to the maximum surplus capacity and greater than (or equal to) the other routes with no flows. The corresponding equilibrium condition is given by

$$\begin{cases} f_w^r > 0, s_w^r\left(f_w^r\right) = v_w, \\ f_w^r = 0, s_w^r\left(f_w^r\right) \le v_w, \quad r \in R_w, w \in W. \end{cases} \tag{8}$$

## 3. Mixed Equilibrium Evolution

The traffic network travel information includes all alternative routes and their travel times, as well as the volume of traffic flow and degree of crowdedness. The route travel time is the rapidity indicator, while the route surplus capacity is used as the comfort indicator to represent the traffic volume and crowding degree of the network. Assuming that there are three groups of travellers in the network, the first group of travellers has complete travel information and obeys the route rapidity choice behaviour criterion, the second group has complete travel information and follows the route comfort choice behaviour criterion, and the third group chooses its routes in accordance with the logit-based route choice probability based on their incomplete travel information. The third group also behaves under the route rapidity decision criterion.

The multicriteria behaviour of the three groups of travellers affects the travel time and the distribution pattern of the traffic flow. The interaction between these traffic flows, caused by various travel decision behaviours, forms a new mixed equilibrium flow. To simulate the evolution trajectory of the mixed equilibrium flow toward the stable state, a day-to-day route flow adjustment process is presented as follows.

*3.1. Route Adjustment Process Model.* The proportion of travellers with complete information is denoted by $\alpha$, in which travellers follow the route rapidity choice criterion represented by $\beta$, and the rest are assumed to choose the comfortable route rather than the shortest one. Hence, as the total traffic demand is denoted as $D_w$ for the OD pair $w$, the travel demand of travellers in the first group is expressed as $d_w = \alpha\beta \cdot D_w$, the travel demand of travellers in the second group is denoted by $\overline{d}_w = \alpha(1 - \beta) \cdot D_w$, and the travel demand of travellers in the third group is calculated by $\widehat{d}_w = (1 - \alpha) \cdot D_w$.

The flows of travellers on route $r \in W$ in these groups are represented by $f_w^r$, $\overline{f}_w^r$, and $\widehat{f}_w^r$. These route flows are grouped into three vectors and can be expressed by

$$\begin{cases} \mathbf{f} = \left(f_w^r : w \in W, r \in R_w\right), \\ \overline{\mathbf{f}} = \left(\overline{f}_w^r : w \in W, r \in R_w\right), \\ \widehat{\mathbf{f}} = \left(\widehat{f}_w^r : w \in W, r \in R_w\right). \end{cases} \tag{9}$$

The traffic flow on link $a \in A$ is the aggregated link flow from both groups, which is

$$x_a = \sum_{w \in W} \sum_{r \in R_w} \delta_w^{ra} \big( f_w^r + \overline{f}_w^r + \widehat{f}_w^r \big), \quad a \in A. \tag{10}$$

Based on the general framework of the discrete day-to-day route flow dynamic model, the evolution process of the route flow from disequilibrium to the equilibrium state is presented as follows [27]:

$$\begin{pmatrix} \mathbf{f}^{(n+1)} \\ \overline{\mathbf{f}}^{(n+1)} \\ \widehat{\mathbf{f}}^{(n+1)} \end{pmatrix} = (1 - \eta) \begin{pmatrix} \mathbf{f}^{(n)} \\ \overline{\mathbf{f}}^{(n)} \\ \widehat{\mathbf{f}}^{(n)} \end{pmatrix} + \eta \begin{pmatrix} \mathbf{y}^{(n)} \\ \overline{\mathbf{y}}^{(n)} \\ \widehat{\mathbf{y}}^{(n)} \end{pmatrix}, \tag{11}$$

where $(\mathbf{f}^{(n)}, \overline{\mathbf{f}}^{(n)}, \widehat{\mathbf{f}}^{(n)})$ is the route flow on day $n$, $(\mathbf{y}^{(n)}, \overline{\mathbf{y}}^{(n)}, \widehat{\mathbf{y}}^{(n)})^{\mathrm{T}}$ is the adjusted route flow on the next day $n + 1$, and $\eta (0 \le \eta \le 1)$ is the route flow adjust ratio. Apparently, $(\mathbf{f}^{(n+1)}, \overline{\mathbf{f}}^{(n+1)}, \widehat{\mathbf{f}}^{(n+1)})^{\mathrm{T}}$, the route flow on day $n + 1$, consists of two parts: the travellers who choose the current route, and the rest of the travellers, who shift to the alternative route. The existing models differ in their choices of target flows and adjustment ratios.

*3.2. Excepted Route Flow.* The adjusted route flow of first group $\mathbf{y}^{(n)}$ is given by the rational behavior adjustment process proposed (RBAP) by Yang and Zhang who assumed that a traveller's route adjustment mechanism urges the traveller to avoid shifts to a route whose travel cost is higher than the current cost. Thus, the aggregate travel cost of the system will decrease with the traveller's route adjustments. That is, with the dynamic evolution of traffic flow, the aggregate travel cost of the system will keep decreasing until it reaches an equilibrium state [2]. The mathematical expression of the rational behavior adjustment process is

$$\begin{cases} \Gamma \ne \phi, \dot{f}(t) \in \Gamma, \\ \Gamma \ne \phi, \dot{f}(t) = 0, \\ \Gamma = \left\{ z(t) : \sum_{r \in R_w} z_w^r(t) = 0, c(t)^{\mathrm{T}} z(t) < 0 \right\}, \end{cases} \tag{12}$$

where $\dot{f}(t)$ denotes the derivative of the traffic flow with respect to time and $\Gamma$ is the set of all feasible directions that can reduce the total travel time based on the current route travel time. $\mathbf{z}(t)$ is the vector of a slight change in the traffic flow. $\mathbf{c}(t)^{\mathrm{T}} \mathbf{z}(t) < 0$ indicates that in order to reduce their travel costs, travellers will make rational behavior decisions according to the traffic information of the route, which reduces the system travel time (see [2] for more details).

The travellers in the second group choose a travel route on the basis of the surplus capacity, and the route choice adjustment mechanism will encourage travellers to shift to the alternative route whose surplus capacity is higher than the current one. Travellers are assumed to make route choice decision in accordance with their current travel quantity information and their experienced expected travel comfort degree. To depict this quantity-oriented adjustment behavior,

we assume that the expected route surplus capacity of travellers for $w$ on day $n + 1$, expressed as $\overline{\mathbf{v}}_w^{r,n+1}$, is the weighted sum of the expected route surplus capacity $\overline{\mathbf{v}}_w^r$ and the actual route surplus capacity $\mathbf{s}_w^{r,n}$ on day $n$. It is easy to see that travellers followed the route comfort choice behaviour criterion and adjusted their travel route based on the previous travel route surplus capacity in this study. Mathematically, this can be expressed as

$$\begin{aligned} \overline{\mathbf{v}}_w^{r,n+1} &= \rho \cdot \mathbf{s}_w^{r,n}(x) + (1 - \rho) . \overline{\mathbf{v}}_w^{r,n} \\ &= \rho \cdot \mathbf{s}_w^{r,n}(x) + (1 - \rho) \cdot \big( \rho \cdot \mathbf{s}_w^{r,n-1}(x) + (1 - \rho) \overline{\mathbf{v}}_w^{r,n-1} \big) \\ &= \rho \cdot \mathbf{s}_w^{r,n}(x) + (1 - \rho) \cdot \big( \rho \cdot \mathbf{s}_w^{r,n-1}(x) + (1 - \rho)^2 \overline{\mathbf{v}}_w^{r,n-1} \big) \\ &= \cdots = \sum_{k=0}^{n} \rho \cdot (1 - \rho)^k \cdot \mathbf{s}_w^{r,n-k}(x), \end{aligned} \tag{13}$$

where $\rho (0 < \rho \le 1)$ is the preference parameter that reflects the preference between actual route surplus capacity and expected route surplus capacity by travellers, and it is obvious that $\mathbf{s}_w^r = \overline{v}_w$ holds if $\rho = 1$, which means that the equilibrium state is reached when all current route surplus capacities are equal to the expected surplus capacity, which is the maximum surplus capacity of the OD pair.

It is assumed that the travellers' behavior in the third group is a dynamic route adjustment behavior based on historical travel experience, where the predicted route travel cost of the travellers is a linear weight of the travellers' historical-experience travel costs, and the value of the weight decreases exponentially with time and distance. According to this assumption, the predicted travel time of route $r$ on day $n + 1$ of an OD pair can be expressed as the weighted sum of the predicted travel time and the actual travel time on day $n$ [28].

$$\widetilde{\mathbf{c}}_w^{r,n+1} = (1 - \kappa) \widetilde{\mathbf{c}}_w^{r,n} + \kappa \mathbf{c}_w^{r,n}(x^n). \tag{14}$$

It can be rewritten as

$$\widetilde{\mathbf{c}}_w^{r,n+1} = \sum_{l=0}^{n} \kappa (1 - \kappa)^l \cdot \mathbf{c}_w^{r,n-l}, \tag{15}$$

where $\kappa \; (0 < \kappa \le 1)$ is the experience preference parameter, which reflects the memory decay characteristic of travellers with regard to historical travel experience information (see [28] for more details).

In conclusion, in order to depict the route adjustment behavior of the three groups of travellers, a rational-expected traffic flow assumption (REFA) is proposed based on the rational target flow assumption (RTFA) of Zhou et al. [22]. It is assumed that the expected traffic flow pattern of travellers in different groups is given by the optimal solution of the following minimization problem:

$$\min_{\mathbf{f}} \sum_{w \in W} \sum_{r \in R_w} \mathbf{c}_w^{r,n} \cdot f_w^r, \tag{16}$$

$$\min_{\overline{\mathbf{f}}} \sum_{w \in W} \sum_{r \in R_w} \big( \mathbf{v}_w^n(\overline{f}_w^r) - \mathbf{s}_w^{r,n}(\overline{f}_w^r) \big) \cdot \overline{f}_w^r, \tag{17}$$

$$\min_{\widehat{\mathbf{f}}} \sum_{w \in W} \sum_{r \in R_w} \widetilde{c}_w^{r,n} \cdot \widehat{f}_w^r + \frac{1}{\theta} \sum_{w \in W} \sum_{r \in R_w} \widehat{f}_w^r \cdot \ln \widehat{f}_w^r, \qquad (18)$$

where equation (16) is the 0-1 assignment problem under a given route travel cost, indicating that travellers in the first group will choose the shortest route, which is subjected to the UE criterion. Equation (17) indicates that travellers from the second group will choose the route with the largest surplus capacity, and its optimal solution is attained in the quantity-adjusted user equilibrium state. Equation (18) is the classic logit-based stochastic assignment problem, where travellers will choose the route with the minimum perceived travel cost, and the expected traffic flow of the third group of travellers can be expressed as

$$\widehat{\mathbf{y}}_w^{r,n} = \widehat{\mathbf{d}}_w \cdot \widehat{\mathbf{p}}\left(\widetilde{c}_w^{r,n}, \theta\right). \qquad (19)$$

It is easily seen from equation (18) that the REFA model is not characterized by a unique solution. Thus, multiple solutions may be obtained from the minimization problem (16)–(18), which inherits the multiple-evolution-trajectory problem (see [22] for more details).

## 4. Property Analysis

*4.1. Equivalency.* An equivalent linear programming model is demonstrated below to avoid the multiple-evolution-trajectory problem. The mixed equilibrium conditions $(\mathbf{f}^*, \overline{\mathbf{f}}^*, \widehat{\mathbf{f}}^*)^{\mathrm{T}}$ are the optimal solution of the following mixed-behaviour linear programming model:

$$\min_{\mathbf{f},\overline{\mathbf{f}},\widehat{\mathbf{f}}} Z(\mathbf{f}, \overline{\mathbf{f}}, \widehat{\mathbf{f}}) = \min_{(\mathbf{f},\overline{\mathbf{f}},\widehat{\mathbf{f}})} \begin{bmatrix} \mathbf{c}(\mathbf{f}) \\ -\mathbf{s}(\overline{\mathbf{f}}) \\ \widehat{\mathbf{c}}(\widehat{\mathbf{f}}) \end{bmatrix}^{\mathrm{T}} \begin{pmatrix} \mathbf{f} \\ \overline{\mathbf{f}} \\ \widehat{\mathbf{f}} \end{pmatrix}, \qquad (20)$$

$$\Lambda \mathbf{f} - \mathbf{d} = 0, \qquad (21)$$

$$\Lambda \overline{\mathbf{f}} - \overline{\mathbf{d}} = 0, \qquad (22)$$

$$\Lambda \widehat{\mathbf{f}} - \widehat{\mathbf{d}} = 0, \qquad (23)$$

$$\begin{aligned} \mathbf{f} &\geq 0, \\ \overline{\mathbf{f}} &\geq 0, \\ \widehat{\mathbf{f}} &\geq 0, \end{aligned} \qquad (24)$$

where $Z(\mathbf{f}, \overline{\mathbf{f}}, \widehat{\mathbf{f}})$ is the comprehensive travel cost function, $\Lambda$ is the OD-route incidence matrix, $\mathbf{c}(\mathbf{f})$ is the route travel time of the first group, $\mathbf{s}(\overline{\mathbf{f}})$ is the route surplus capacity of the second group, and $\widehat{\mathbf{c}}(\widehat{\mathbf{f}})$ denotes the modified route travel time concerned with individual memory recession and perception difference, which satisfies

$$\widehat{\mathbf{c}}(\widehat{\mathbf{f}}) = \mathbf{c}(\widehat{\mathbf{f}}) + \frac{1}{\theta} (\ln(\widehat{\mathbf{f}}) + I). \qquad (25)$$

*Proof.* Clearly, the following necessary and sufficient optimality conditions for optimization problems (20)–(24) can be obtained according to the KKT (Karush–Kuhn–Tucker) optimality conditions:

$$\begin{bmatrix} \mathbf{c}(\mathbf{f}) - \Lambda \pi^* \\ \Lambda \overline{\pi}^* - \mathbf{s}(\overline{\mathbf{f}}) \\ \widehat{\mathbf{c}}(\widehat{\mathbf{f}}) - \Lambda \widehat{\pi}^* \end{bmatrix}^{\mathrm{T}} \begin{pmatrix} \mathbf{f} \\ \overline{\mathbf{f}} \\ \widehat{\mathbf{f}} \end{pmatrix} = 0, \qquad (26)$$

$$\mathbf{c}(\mathbf{f}) - \Lambda \pi^* \geq 0, \qquad (27)$$

$$\Lambda \overline{\pi}^* - \mathbf{s}(\overline{\mathbf{f}}) \geq 0, \qquad (28)$$

$$\widehat{\mathbf{c}}(\widehat{\mathbf{f}}) - \Lambda \widehat{\pi}^* \geq 0, \qquad (29)$$

$$\Lambda \mathbf{f} - \mathbf{d} = 0, \qquad (30)$$

$$\Lambda \overline{\mathbf{f}} - \overline{\mathbf{d}} = 0, \qquad (31)$$

$$\Lambda \widehat{\mathbf{f}} - \widehat{\mathbf{d}} = 0, \qquad (32)$$

$$\begin{aligned} \mathbf{f} &\geq 0, \\ \overline{\mathbf{f}} &\geq 0, \\ \widehat{\mathbf{f}} &\geq 0, \end{aligned} \qquad (33)$$

where $\pi^*$, $\overline{\pi}^*$, and $\widehat{\pi}^*$ are the optimal Lagrange multipliers corresponding to the flow conservation constraints shown in equations (21)–(23), respectively.

Since the route flow is strictly positive in the logit-based stochastic assignment model, which is $\widehat{f}_w^r > 0, r \in R_w$, then equation (26) becomes

$$\begin{bmatrix} \mathbf{c}(\mathbf{f}) - \Lambda \pi^* \\ \Lambda \overline{\pi}^* - \mathbf{s}(\overline{\mathbf{f}}) \end{bmatrix}^{\mathrm{T}} \begin{pmatrix} \mathbf{f} \\ \overline{\mathbf{f}} \end{pmatrix} = 0, \qquad (34)$$

$$\widehat{\mathbf{c}}(\widehat{\mathbf{f}}) - \Lambda \widehat{\pi}^* = 0. \qquad (35)$$

Combining equations (27), (28), and (34), we obtain

$$\begin{cases} \mathbf{f} > 0, & \mathbf{c}(\mathbf{f}) = \Lambda \pi^*, \\ \mathbf{f} = 0, & \mathbf{c}(\mathbf{f}) \geq \Lambda \pi^*, \end{cases} \qquad (36)$$

$$\begin{cases} \overline{\mathbf{f}} > 0, & \mathbf{s}(\mathbf{f}) = \Lambda \overline{\pi}^*, \\ \overline{\mathbf{f}} = 0, & \mathbf{s}(\mathbf{f}) \leq \Lambda \overline{\pi}^*. \end{cases} \qquad (37)$$

Therefore, the multipliers $\pi^*$ and $\overline{\pi}^*$ signify the minimum route travel time and the maximum route surplus capacity, respectively.

Substituting equations (25) into (35), we have

$$\widehat{\mathbf{f}} = \exp\left(-\theta\left(\mathbf{c} - \widehat{\pi}^*\right) - 1\right),$$

$$\widehat{f}_w^r = \frac{\exp\left(-\theta c_w^r\right)}{\sum_{k \in R_w} \exp\left(-\theta c_w^r\right)} \widehat{d}_w, \quad r \in R_w. \qquad (38)$$

It can be seen that equations (36)–(38) are actually the equilibrium conditions of the travellers in the three groups. □

*4.2. Uniqueness.* Based on the variational inequality (VI) theory, it is clear that a route flow pattern $(\mathbf{f}^*, \overline{\mathbf{f}}^*, \widehat{\mathbf{f}}^*)^{\mathrm{T}}$ is a solution to the equivalent linear programming problem in (20)–(24) if and only if it solves the following VI problem:

$$\nabla Z\left(\mathbf{f}^*, \overline{\mathbf{f}}^*, \widehat{\mathbf{f}}^*\right)^{\mathrm{T}} \cdot \begin{pmatrix} \mathbf{f} - \mathbf{f}^* \\ \overline{\mathbf{f}} - \overline{\mathbf{f}}^* \\ \widehat{\mathbf{f}} - \widehat{\mathbf{f}}^* \end{pmatrix} = \begin{bmatrix} \mathbf{c}\left(\mathbf{f}^*, \overline{\mathbf{f}}^*, \widehat{\mathbf{f}}^*\right) \\ -\mathbf{s}\left(\mathbf{f}^*, \overline{\mathbf{f}}^*, \widehat{\mathbf{f}}^*\right) \\ \widehat{\mathbf{c}}\left(\mathbf{f}^*, \overline{\mathbf{f}}^*, \widehat{\mathbf{f}}^*\right) \end{bmatrix}^{\mathrm{T}} \begin{pmatrix} \mathbf{f} - \mathbf{f}^* \\ \overline{\mathbf{f}} - \overline{\mathbf{f}}^* \\ \widehat{\mathbf{f}} - \widehat{\mathbf{f}}^* \end{pmatrix} \geq 0.$$

(39)

This can be rewritten as follows:

$$Z\left(\mathbf{f}(n), \overline{\mathbf{f}}(n), \widehat{\mathbf{f}}(n)\right) - Z\left(\mathbf{f}^*, \overline{\mathbf{f}}^*, \widehat{\mathbf{f}}^*\right)$$

$$= \begin{bmatrix} \mathbf{c}\left(\mathbf{f}^*, \overline{\mathbf{f}}^*, \widehat{\mathbf{f}}^*\right) \\ -\mathbf{s}\left(\mathbf{f}^*, \overline{\mathbf{f}}^*, \widehat{\mathbf{f}}^*\right) \\ \widehat{\mathbf{c}}\left(\mathbf{f}^*, \overline{\mathbf{f}}^*, \widehat{\mathbf{f}}^*\right) \end{bmatrix}^{\mathrm{T}} \begin{pmatrix} \mathbf{f}(n) - \mathbf{f}^* \\ \overline{\mathbf{f}}(n) - \overline{\mathbf{f}}^* \\ \widehat{\mathbf{f}}(n) - \widehat{\mathbf{f}}^* \end{pmatrix} \geq 0.$$

(40)

It can be readily seen that $(\mathbf{f}^*, \overline{\mathbf{f}}^*, \widehat{\mathbf{f}}^*)^{\mathrm{T}}$ is the extreme point of $Z(\mathbf{f}, \overline{\mathbf{f}}, \widehat{\mathbf{f}})$, which means that $(\mathbf{f}^*, \overline{\mathbf{f}}^*, \widehat{\mathbf{f}}^*)^{\mathrm{T}}$ is the mixed equilibrium route flow pattern. The comprehensive travel cost achieves its minimum $Z(\mathbf{f}^*, \overline{\mathbf{f}}^*, \widehat{\mathbf{f}}^*)$ and cannot be further reduced. Since $Z(\mathbf{f}, \overline{\mathbf{f}}, \widehat{\mathbf{f}})$ is a monotonically increasing function of route flow $(\mathbf{f}, \overline{\mathbf{f}}, \widehat{\mathbf{f}})$, it can be concluded that $(\mathbf{f}^*, \overline{\mathbf{f}}^*, \widehat{\mathbf{f}}^*)^{\mathrm{T}}$ is the optimal solution to the linear programming problem in equations (20)–(24).

*4.3. Stability of Solution.* Suppose that $(\mathbf{f}^*, \overline{\mathbf{f}}^*, \widehat{\mathbf{f}}^*)^{\mathrm{T}}$ and $(\mathbf{f}, \overline{\mathbf{f}}, \widehat{\mathbf{f}})^{\mathrm{T}}$ satisfy the VI problem (39). Then, we have

$$\nabla Z^*\left(\mathbf{f}^*, \overline{\mathbf{f}}^*, \widehat{\mathbf{f}}^*\right)^{\mathrm{T}} \cdot \begin{pmatrix} \mathbf{f}' - \mathbf{f}^* \\ \overline{\mathbf{f}}' - \overline{\mathbf{f}}^* \\ \widehat{\mathbf{f}}' - \widehat{\mathbf{f}}^* \end{pmatrix}$$

$$= \begin{bmatrix} \mathbf{c}\left(\mathbf{f}^*, \overline{\mathbf{f}}^*, \widehat{\mathbf{f}}^*\right) \\ -\mathbf{s}\left(\mathbf{f}^*, \overline{\mathbf{f}}^*, \widehat{\mathbf{f}}^*\right) \\ \widehat{\mathbf{c}}\left(\mathbf{f}^*, \overline{\mathbf{f}}^*, \widehat{\mathbf{f}}^*\right) \end{bmatrix}^{\mathrm{T}} \begin{pmatrix} \mathbf{f}' - \mathbf{f}^* \\ \overline{\mathbf{f}}' - \overline{\mathbf{f}}^* \\ \widehat{\mathbf{f}}' - \widehat{\mathbf{f}}^* \end{pmatrix} \geq 0,$$

(41)

$$\nabla Z(\mathbf{f}, \overline{\mathbf{f}}, \widehat{\mathbf{f}})^{\mathrm{T}} \cdot \begin{pmatrix} \mathbf{f}' - \mathbf{f} \\ \overline{\mathbf{f}}' - \overline{\mathbf{f}} \\ \widehat{\mathbf{f}}' - \widehat{\mathbf{f}} \end{pmatrix} = \begin{bmatrix} \mathbf{c}(\mathbf{f}, \overline{\mathbf{f}}, \widehat{\mathbf{f}}) \\ -\mathbf{s}(\mathbf{f}, \overline{\mathbf{f}}, \widehat{\mathbf{f}}) \\ \widehat{\mathbf{c}}(\mathbf{f}, \overline{\mathbf{f}}, \widehat{\mathbf{f}}) \end{bmatrix}^{\mathrm{T}} \begin{pmatrix} \mathbf{f}' - \mathbf{f} \\ \overline{\mathbf{f}}' - \overline{\mathbf{f}} \\ \widehat{\mathbf{f}}' - \widehat{\mathbf{f}} \end{pmatrix} \geq 0.$$

(42)

Rewriting equation (41) for $(\mathbf{f}', \overline{\mathbf{f}}', \widehat{\mathbf{f}}')^T = (\mathbf{f}, \overline{\mathbf{f}}, \widehat{\mathbf{f}})^T$ and equation (42) for $(\mathbf{f}', \overline{\mathbf{f}}', \widehat{\mathbf{f}}')^T = (\mathbf{f}^*, \overline{\mathbf{f}}^*, \widehat{\mathbf{f}}^*)^T$ and adding the resulting inequalities, then



FIGURE 1: Test traffic network.

TABLE 1: Link-route incidence matrix.

$$\Delta^T = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

TABLE 2: Parameters in link travel time functions.

| Link | $t_a^0$ | $c_a$ |
| --- | --- | --- |
| 1 | 8 | 60 |
| 2 | 16 | 60 |
| 3 | 10 | 80 |
| 4 | 8 | 60 |
| 5 | 8 | 40 |
| 6 | 10 | 40 |
| 7 | 14 | 60 |
| 8 | 14 | 80 |
| 9 | 8 | 60 |
| 10 | 10 | 80 |
| 11 | 8 | 40 |
| 12 | 8 | 60 |

$$\Delta F(t) \cdot \Delta(t) = \left[\nabla Z^*\left(\mathbf{f}^*, \overline{\mathbf{f}}^*, \widehat{\mathbf{f}}^*\right)^{\mathrm{T}} - \nabla Z(\mathbf{f}, \overline{\mathbf{f}}, \widehat{\mathbf{f}})^{\mathrm{T}}\right] \cdot \begin{pmatrix} \mathbf{f}^* - \mathbf{f} \\ \overline{\mathbf{f}}^* - \overline{\mathbf{f}} \\ \widehat{\mathbf{f}}^* - \widehat{\mathbf{f}} \end{pmatrix}$$

$$= \begin{bmatrix} \mathbf{c}\left(\mathbf{f}^*, \overline{\mathbf{f}}^*, \widehat{\mathbf{f}}^*\right) - \mathbf{c}(\mathbf{f}, \overline{\mathbf{f}}, \widehat{\mathbf{f}}) \\ \mathbf{s}(\mathbf{f}, \overline{\mathbf{f}}, \widehat{\mathbf{f}}) - \mathbf{s}\left(\mathbf{f}^*, \overline{\mathbf{f}}^*, \widehat{\mathbf{f}}^*\right) \\ \widehat{\mathbf{c}}\left(\mathbf{f}^*, \overline{\mathbf{f}}^*, \widehat{\mathbf{f}}^*\right) - \widehat{\mathbf{c}}(\mathbf{f}, \overline{\mathbf{f}}, \widehat{\mathbf{f}}) \end{bmatrix}^{\mathrm{T}} \cdot \begin{pmatrix} \mathbf{f}^* - \mathbf{f} \\ \overline{\mathbf{f}}^* - \overline{\mathbf{f}} \\ \widehat{\mathbf{f}}^* - \widehat{\mathbf{f}} \end{pmatrix} \leq 0.$$

(43)

*4.4. Algorithm*

Step 1 (initialization): let $n = 0$. Set the model parameters $\alpha, \beta, \eta, \rho, \kappa,$ and $\theta$ and the convergence

Figure 2: Flow evolution of test traffic network with different quantity adjustment ratios.

Table 3: Route flow and cost in equilibrium.

| Route | Link | $f$ | $c$ |
|---|---|---|---|
| 1 | 1, 2, 9, 12 | 78 | 218 |
| 2 | 1, 4, 8, 12 | 1 | 221 |
| 3 | 1, 6, 8, 11 | 79 | 218 |
| 4 | 3, 4, 7, 12 | 106 | 218 |
| 5 | 3, 6, 7, 11 | 1 | 226 |
| 6 | 5, 6, 7, 10 | 35 | 218 |

parameter $\varepsilon$. Input the link-route incidence matrix $\Delta$, all-one matrix $I$, and the OD demand $D$.

Step 2: make an initial allocation. Solve the traffic flow assignment problem by using the shortest route loading method and 0-1 assignment based on the route surplus

capacity and logit stochastic assignment method for the three groups of travellers. Hence, get the initial route flow pattern $f^{(0)}, \overline{f}^{(0)}$, and $\widehat{f}^{(0)}$.

Step 3: calculate the route flow $F^{(n)} = f^{(n)} + \overline{f}^{(n)} + \widehat{f}^{(n)}$, the link flow $x_a^{(n)} = \Delta' f^{(n)}$, and the link travel time by the BPR (Bureau of Public Roads) function. Update the route travel time, route surplus capacity, and perceived route travel time according to equations (1), (7), and (25).

Step 4: update the adjustment route flows $y^{(n)}, \overline{y}^{(n)}$, and $\widehat{y}^{(n)}$ in accordance with equations (16)–(18).

Step 5: Reassign the traffic route flow based on equation (11).

Step 6 (convergence examination): terminate the iteration and output the route flow pattern $f^{(n+1)}$,

FIGURE 3: Evolutionary trajectories of route flows.

TABLE 4: Route flow and cost in equilibrium.

| Route | Link | $\bar{f}$ | $s$ |
| --- | --- | --- | --- |
| 1 | 1, 2, 9, 12 | 50 | 10 |
| 2 | 1, 4, 8, 12 | 50 | 10 |
| 3 | 1, 6, 8, 11 | 30 | 10 |
| 4 | 3, 4, 7, 12 | 50 | 10 |
| 5 | 3, 6, 7, 11 | 30 | 10 |
| 6 | 5, 6, 7, 10 | 30 | 10 |



FIGURE 4: Evolutionary trajectories of route flows.

TABLE 5: Route flow and cost in mixed equilibrium for travellers in the first group.

| Route | Link | $f$ | $c$ |
|---|---|---|---|
| 1 | 1, 2, 9, 12 | 27 | 217 |
| 2 | 1, 4, 8, 12 | 0 | 226 |
| 3 | 1, 6, 8, 11 | 34 | 217 |
| 4 | 3, 4, 7, 12 | 38 | 217 |
| 5 | 3, 6, 7, 11 | 0 | 220 |
| 6 | 5, 6, 7, 10 | 21 | 217 |



FIGURE 5: Evolutionary trajectories of route flows (a) and route travel costs (b) for travellers in the first group.

TABLE 6: Route flow and surplus capacity in mixed equilibrium for travellers in the second group.

| Route | Link | $\overline{f}$ | $s$ |
|---|---|---|---|
| 1 | 1, 2, 9, 12 | 30 | 30 |
| 2 | 1, 4, 8, 12 | 30 | 30 |
| 3 | 1, 6, 8, 11 | 10 | 30 |
| 4 | 3, 4, 7, 12 | 30 | 30 |
| 5 | 3, 6, 7, 11 | 10 | 30 |
| 6 | 5, 6, 7, 10 | 10 | 30 |

$\overline{f}^{(n+1)}$, and $\widehat{f}^{(n+1)}$. if $|(f^{(n+1)} - f^{(n)})/f^{(n+1)}| \le \varepsilon \cap |(\overline{f}^{(n+1)} - \overline{f}^{(n)})/\overline{f}^{(n+1)}| \le \varepsilon \cap |(\widehat{f}^{(n+1)} - \widehat{f}^{(n)})/\widehat{f}^{(n+1)}| \le \varepsilon$     holds. Otherwise, let $n = n + 1$, and go to Step 2.

## 5. Numerical Experiments

*5.1. Proportion of Route Comfort Behaviour.* In this subsection, we study the effects of the comfort criterion ratio on the traffic flow of each route in a network under the multicriteria behavior, where the tested network is shown in Figure 1. The incidence matrix of routes and links for the network is tabulated in Table 1, and a simplified link travel time function that is often used in practice is the equation developed by the U.S. BPR (Bureau of Public Roads), with free-flow travel time and link capacity given in Table 2.

$$t_a(x_a) = t_a^0 \left[ 1 + 0.15 \left( \frac{x_a}{c_a} \right)^4 \right], \quad \forall a \in A. \qquad (44)$$

The traffic demand pattern between the OD pairs is assumed to be $d = 300$. Assume that 80% of the travellers are equipped with complete information. The dispersion parameter $\theta$ is set to be 1, and the route adjustment flow ratio $\eta = 1/n$.

As we stated above, $\beta$ represents the proportion of travellers who follow the route comfort behavior criterion. Keeping the other parameters constant, the route flow evolution trajectories in the test network are depicted in Figure 2 when $\beta$ is set to 20%, 50%, 70%, and 90%. This shows that a larger $\beta$ will make the corresponding trajectory smoother and steadier and the fluctuation smaller. This

FIGURE 6: Evolutionary trajectories of route flows (a) and route surplus capacity (b) for travellers in the second group.

TABLE 7: Route flow and perceived cost in mixed equilibrium for travellers in the third group.

| Route | Link | $\widehat{f}$ | $\widehat{c}$ |
|---|---|---|---|
| 1 | 1, 2, 9, 12 | 14 | 224 |
| 2 | 1, 4, 8, 12 | 2 | 225 |
| 3 | 1, 6, 8, 11 | 13 | 225 |
| 4 | 3, 4, 7, 12 | 15 | 224 |
| 5 | 3, 6, 7, 11 | 5 | 226 |
| 6 | 5, 6, 7, 10 | 11 | 226 |



FIGURE 7: Evolutionary trajectories of route flows (a) and perceived travel costs (b) for travellers in the third group.

FIGURE 8: Test traffic network.

TABLE 8: Parameters in link travel time functions.

| Link no. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $t_a$ | 6 | 5 | 6 | 7 | 6 | 8 | 5 | 10 | 11 | 16 | 12 | 8 | 6 | 7 | 5 | 9 | 10 | 8 | 5 |
| $c_a$ | 200 | 200 | 150 | 200 | 100 | 100 | 150 | 150 | 200 | 100 | 200 | 150 | 150 | 100 | 250 | 200 | 150 | 200 | 100 |

TABLE 9: OD-route-link incidence matrix.

| OD | Route | Link |
|---|---|---|
| | 1 | 1, 10, 19 |
| | 2 | 2, 6, 9, 16, 19 |
| | 3 | 2, 6, 9, 15, 17 |
| (1, 2) | 4 | 2, 6, 14, 11, 17 |
| | 5 | 2, 5, 7, 11, 17 |
| | 6 | 1, 13, 9, 16, 19 |
| | 7 | 1, 13, 9, 15, 17 |
| | 8 | 1, 13, 14, 11, 17 |
| | 9 | 2, 5, 8, 12 |
| | 10 | 2, 6, 9, 15, 18 |
| (1, 3) | 11 | 2, 6, 14, 11, 18 |
| | 12 | 2, 5, 7, 11, 18 |
| | 13 | 1, 13, 9, 15, 18 |
| | 14 | 1, 13, 14, 11, 18 |
| | 15 | 4, 7, 11, 17 |
| | 16 | 3, 6, 9, 16, 19 |
| (4, 3) | 17 | 3, 6, 9, 15, 17 |
| | 18 | 3, 6, 14, 11, 17 |
| | 19 | 3, 5, 7, 11, 17 |
| | 20 | 4, 8, 12 |
| | 21 | 4, 7, 11, 18 |
| (4, 2) | 22 | 3, 5, 8, 12 |
| | 23 | 3, 6, 9, 15, 18 |
| | 24 | 3, 6, 14, 11, 18 |
| | 25 | 3, 5, 7, 11, 18 |

TABLE 10: Route flow and cost in mixed equilibrium for travellers in the first group.

| OD | Route | $f$ | $c$ |
|---|---|---|---|
| | 1 | 120 | 110 |
| | 2 | 0 | 114 |
| | 3 | 0 | 114 |
| (1, 2) | 4 | 0 | 116 |
| | 5 | 0 | 114 |
| | 6 | 0 | 114 |
| | 7 | 0 | 114 |
| | 8 | 0 | 116 |
| | 9 | 65 | 82 |
| | 10 | 3 | 82 |
| (1, 3) | 11 | 0 | 84 |
| | 12 | 0 | 83 |
| | 13 | 52 | 82 |
| | 14 | 0 | 84 |
| | 15 | 73 | 117 |
| | 16 | 37 | 117 |
| (4, 3) | 17 | 10 | 117 |
| | 18 | 0 | 119 |
| | 19 | 0 | 118 |
| | 20 | 109 | 85 |
| | 21 | 0 | 86 |
| (4, 2) | 22 | 11 | 85 |
| | 23 | 0 | 86 |
| | 24 | 0 | 87 |
| | 25 | 0 | 86 |

means that with an increase in the number of travellers who choose the travel route in accordance with the route comfort degree during the evolution process, which is reflected as a larger route surplus capacity in this study, the influence of the route travel cost decreases.

5.2. Single-Criterion Adjustment Evolution Process Research. The traffic demand pattern between the OD pairs is assumed to be $d = 300$. Assume that all travellers are equipped with complete information, and none of them follow the route

FIGURE 9: Evolutionary trajectories of route flows (a) and route travel costs (b) for travellers in the first group.

comfort decision behaviour criterion, the route adjustment flow ratio $\eta = 1/n$. Under this condition, all travellers with complete information choose the shortest route following the route rapidity behaviour criterion, forming a single UE equilibrium state. The route flow pattern and correspondent route travel time in the single UE equilibrium state are tabulated in Table 3, while Figure 3 shows the single rapidity criterion adjustment evolution process that converges to the UE equilibrium state after a fluctuant period.

The traffic demand pattern between the OD pairs is assumed to be $d = 240$. Assume that all travellers are equipped with complete information, and all of them follow the route comfort decision behaviour criterion, the route adjustment flow ratio $\eta = 1/n$. Under this condition, all travellers choose the most comfortable route following the route comfort behaviour criterion with complete information forms a single QUE equilibrium state. The route flow pattern and correspondent route travel time in the single QUE equilibrium state are tabulated in Table 4, while Figure 4 shows the single comfort criterion adjustment evolution process that converges to the QUE equilibrium state after a fluctuant period.

### 5.3. Mixed Equilibrium Evolution Process Research.
In this section, numerical examples are presented to illustrate the application of the proposed route flow dynamic evolution model. The traffic demand pattern between the OD pairs is assumed to be $d = 300$. Assume that 80% of the travellers are equipped with complete information, and 50% of them follow the route comfort decision behaviour criterion. The dispersion parameter $\theta$ is set to be 0.5, and the route adjustment flow ratio is set to be $\eta = 1/n$.

TABLE 11: Route flow and surplus capacity in mixed equilibrium for travellers in the second group.

| OD | Route | $\overline{f}$ | $s$ |
|---|---|---|---|
| (1, 2) | 1 | 9 | 91 |
| | 2 | 9 | 91 |
| | 3 | 9 | 91 |
| | 4 | 9 | 91 |
| | 5 | 9 | 91 |
| | 6 | 9 | 91 |
| | 7 | 59 | 91 |
| | 8 | 9 | 91 |
| (1, 3) | 9 | 12 | 88 |
| | 10 | 12 | 88 |
| | 11 | 12 | 88 |
| | 12 | 12 | 88 |
| | 13 | 62 | 88 |
| | 14 | 12 | 88 |
| (4, 3) | 15 | 64 | 86 |
| | 16 | 14 | 86 |
| | 17 | 14 | 86 |
| | 18 | 14 | 86 |
| | 19 | 14 | 86 |
| (4, 2) | 20 | 53 | 97 |
| | 21 | 53 | 97 |
| | 22 | 3 | 97 |
| | 23 | 3 | 97 |
| | 24 | 3 | 97 |
| | 25 | 3 | 97 |

The route flow pattern and correspondent route travel time in the mixed equilibrium state for travellers in the first group are tabulated in Table 5, while Figure 5 shows the specific evolution process wherein the traffic flow gradually converges to the equilibrium state after a fluctuant period.

FIGURE 10: Evolutionary trajectories of route flows (a) and route surplus capacity (b) for travellers in the second group.

TABLE 12: Route flow and perceived cost in mixed equilibrium for travellers in the third group.

| OD | Route | $\widehat{f}$ | $\widehat{c}$ |
|---|---|---|---|
| | 1 | 50 | 114 |
| | 2 | 1 | 114 |
| | 3 | 2 | 114 |
| (1, 2) | 4 | 1 | 115 |
| | 5 | 1 | 114 |
| | 6 | 3 | 114 |
| | 7 | 1 | 115 |
| | 8 | 1 | 115 |
| | 9 | 20 | 86 |
| | 10 | 10 | 86 |
| (1, 3) | 11 | 1 | 86 |
| | 12 | 9 | 86 |
| | 13 | 18 | 86 |
| | 14 | 2 | 86 |
| | 15 | 19 | 120 |
| | 16 | 15 | 120 |
| (4, 3) | 17 | 12 | 120 |
| | 18 | 2 | 120 |
| | 19 | 12 | 120 |
| | 20 | 21 | 89 |
| | 21 | 10 | 89 |
| (4, 2) | 22 | 13 | 89 |
| | 23 | 8 | 89 |
| | 24 | 1 | 89 |
| | 25 | 7 | 89 |

The above observation indicates that travellers in first group will evolve to a UE state, in which the travel costs of all used routes between the same OD pairs are equal and minimal.

Meanwhile, the route flow pattern and correspondent route surplus capacity in the mixed equilibrium state for

travellers in the second group are tabulated in Table 6, while Figure 6 shows the specific evolution process in which the traffic flow gradually converges to the equilibrium state after a fluctuant period. The above observation indicates that travellers in the second group will evolve to a quantity-adjusted user equilibrium state in which the route surplus capacities of all used routes between the same OD pairs are equal and maximum.

The route flow pattern and perceived route travel time in the mixed equilibrium state for travellers in the third group are tabulated in Table 7, while Figure 7 shows the specific evolution process wherein the traffic flow gradually converges to the equilibrium state after a fluctuant period. The above observation indicates that travellers in the third group will evolve to an SUE state where no travellers can unilaterally change routes to reduce his/her perceived travel costs.

To better analyse the performance of the proposed mixed equilibrium route flow dynamic model, a network is shown in Figure 8 [29]. The link travel time functions follow the BPR form, with the free-flow travel time and link capacity given in Table 8. The incidence matrix of routes and links for the network are tabulated in Table 9. The traffic demand pattern between the four OD pairs is assumed to be $(\overline{d}_{12}, \overline{d}_{13}, \overline{d}_{42}, \overline{d}_{42}) = (300, 300, 300, 300)$, the parameter $\theta$ is set to be 1, $\alpha = 80\%$, and $\beta = 50\%$. The route adjustment flow ratio is $\eta = 1/n$ to guarantee the convergence.

The route flow pattern and corresponding route travel time in mixed equilibrium state for travellers in the first group are tabulated in Table 10, while Figure 9 shows the specific evolution trajectories. The route flow pattern and correspondent route surplus capacity in the mixed equilibrium state for travellers in the second group are tabulated in Table 11, while Figure 10 shows the specific evolution process. The route flow pattern and perceived route travel

FIGURE 11: Evolutionary trajectories of route flows (a) and perceived travel costs (b) for travellers in the third group.

time in the mixed equilibrium state for travellers in the third group are tabulated in Table 12, while Figure 11 shows the specific evolution trajectories.

Clearly, from the above results, we can observe that traffic flows all converge to the stable state after a fluctuant period and that the proposed route adjustment process simulates the ideal traffic flow evolution of the three groups of travellers. Table 10 shows that for travellers in the first group, the demand is entirely loading in the routes with the minimum travel times in the stable state, and Table 11 shows the flows in the second group are stable because all of the surplus capacities of the routes are equal. Since different travellers have different perception errors, Table 12 proves that all the routes have the same minimum perceived travel cost and are eventually selected by the third group of travellers. Hence, the stable state is exactly the mixed equilibrium state formed by the different behaviours of these three groups of travellers.

## 6. Conclusions

Based on the non-Walrasian equilibrium theory proposed by R.W. Clower, a new route comfort choice behaviour criterion was proposed considering the quantity signal influence of a traffic network through the route surplus capacity indicator. A traveller who follows this travel route decision criterion is defined as a quantity adjustment traveller. This study divided the traffic network travellers into three categories: travellers in the first group receive complete travel information and choose the shortest route as assumed in the classical research, the second group of travellers follows the proposed travel route decision criterion and chooses the most comfortable route based on complete travel information, and the third group of travellers chooses its routes in

accordance with the logit-based route choice probability owing to incomplete travel information.

This paper analysed the common route rapidity criterion and the proposed route comfort choice behaviour criterion with their corresponding equilibrium states and established a route flow adjustment process to depict the flow adjustment process of the interacting flows of the three groups of travellers that converge to a mixed equilibrium state. This mixed equilibrium not only considers the diversity of the route selection criteria of the travellers but also elaborates the interaction between the different travellers' groups. This means that all route choices made by the travellers from the three groups are determined on the basis of the comprehensive route cost generated by the sum of the flows of the three groups of travellers.

With the rapid development of science and technology and the urbanization process, new traffic patterns have emerged from large-scale urban infrastructure construction, road network expansion, and so on. To rationally characterize the traffic flow dynamic evolution process from disequilibrium to equilibrium, this paper classified the travellers and simulated the dynamic evolution of the network traffic flow by using the principles of economics. This deepens the understanding of network traffic flows and improves the level of urban traffic management.

Several issues have been left unsolved and are directions worthy of further study. First, it will be interesting to focus on a situation in which an information provider of an intelligent transportation system uses the optimal system as the decision-making principle. Another future research direction is choosing the most suitable quantity signal considering the effects of the road grade, functions, and service level of the traffic network. Moreover, the influence of the proportion of travellers who follow the quantity

adjustment path selection criteria in mixed equilibrium evolution is also a challenge for future research.

## Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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