Research Article

Multimodal Capacitated Hub Location Problems with Multi-Commodities: An Application in Freight Transport

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Hub location problems (HLPs) support decision making on multimodal transport strategic planning. It is related to the location of hubs and the allocation of origin/destination (O/D) flows in a system. Classical formulations assume that these flows are predefined paths and direct delivery is not available. This applied research presents a mixed integer linear programming (MILP) model for a capacitated multimodal, multi-commodity HLP. Furthermore, an application on the export process in a Latin American country is detailed. The new proposed model, unlike the traditional HLP, allows direct shipment, and its O/D flows are part of the decision model. Situations with up to 100 nodes, six products, and two transport modes are used, working with initial and projected flows. All instances can be solved optimally using the commercial solver, Gurobi 7.5.0, in computational times less than a minute. Results indicate that only one hub is profitable for the case study, both for the initial and projected scenarios. The installation of a hub generates transport savings over 1% per year. Two factors affect the location decision: low concentration and distance between the hubs and destinations. Long distances involve an exhaustive use of trains instead of trucks, which leads to lower transport cost per unit.

1. Introduction

A hub is a facility where materials flow and ship around the world. The target is to achieve an economic profit using a hub instead of shipping direct. The location of these nodes is a decisive factor that largely determines the system’s efficiency. In this context, Hub Location Problem (HLP) is a tool that helps to make strategic decisions. It helps to locate one or more hubs, allocate origin/destination (O/D) flow in a network, and minimize the cost of a system [1].

An HLP consists of a set of nodes: origins, destinations, and hub candidates. Between the nodes exists an O/D flow for shipping, which is expected to be cheaper through a hub than direct shipment. Therefore, one or more nodes are selected as hubs to improve system performance. Research on HLPs started with O’Kelly [2, 3], who proposed the first mathematical formulations. Over the years, many authors have developed new formulations and solution methods, with applications in different areas such as logistics [4–6], airlines [7–10], telecommunications [11], health services [12], and transport [13]. There are several classifications for HLP according to the attributes of each model. Classical HLP models decide the number of hubs to install as part of the solution. Nevertheless, when the number of facilities to locate is predefined by a parameter, it corresponds to a $p$-HLP. If the model allows for a nonhub node to send through only one hub, it is a single allocation HLP; otherwise, the problem is called a multiple allocation HLP. Some formulations restrict the amount of flow that can be managed by hubs, which is known as capacitated HLP. It is important to consider that capacity may not only be at the hub but also in arcs. A comprehensive survey of the description, features, and classifications of HLP can be found in references [14] and [15].

One of the principal assumptions of classical HLP models is that direct shipment is prohibited. However, some papers propose models relaxing this assumption. In [16], the authors present a mixed network topology model for freight transport, which allows delivery, either directly, or through a hub. Another model that incorporates direct shipment is presented in [17], which uses an efficient metaheuristic to solve the
model proposed in [18]. In [19], a mixed network is introduced to take advantage of the economies of scale of intermodal transport and short travel times of direct shipments.

Solution methods have been the principal focus of HLP research in recent years [15]. In this sense, commercial solvers such as CPLEX have been used to solve mixed integer linear programming (MILP) models with a small network and low complexity [6, 20, 21]. Nevertheless, to solve more complex and/or larger HLPS, exact, heuristic, and metaheuristic algorithms have been used. Some of these use fuzzy integer linear programming approaches [22], Benders decompositions [23–25], Branch and Price with Lagrangian relaxation [26], genetic algorithms [9, 27], and local search and evolutionary algorithms in [28]. For a recent state of the art of solution methods, please read [29].

Freight transport is one of several applications of HLP models. According to the review presented in [30], HLP is the main tool used to make strategic decisions on multimodal freight transport. The concept of multimodal transport is intuitively related to the use of different transport modes to maximize profit. This implies that each mode could have its own cost structure, network connectivity, and other features [1]. Classical HLP formulations do not incorporate different transport modes; however, there are several variants that present multimodal models. In [31], the authors propose a mixed integer nonlinear problem model, which considers different cost structures and travel times for each transport mode. Another multimodal MILP model is introduced in [20], where the authors present a model with independent fixed and variable cost structures for each mode. Another facility location problem in multimodal networks is shown in [32], where the authors propose multiobjective mixed integer nonlinear programming (MINLP) that investigates minimizing costs and environmental taxes in a four-level supply chain network.

Freight movement can be applied to different types of products. Similar to multimodal cases, these may have different transportation and handling costs. Furthermore, the origins and destinations could be specialized for different kinds of products, implying that they are capable of attending only specific markets. In these cases, the formulation of multi-commodity models is appropriate to deal with the problem and its features. Multi-commodity models are proposed in [33, 34] and more recently in [20]. In the literature, we found few articles on multimodal multi-commodity HLP models in [20] and [35].

This paper presents a new MILP model for capacitated multimodal multi-commodity HLP and its application in freight transport. A novel of the proposed model is that direct shipment is possible. O/D flow determined by offer and demand constraints is a part of the model decision. Handling cost is incorporated in the objective function with the transport cost. This, associated with the composition of the balance constraint, allows for modal changes.

2. Materials and Methods

An MILP model is proposed to represent a system with a set of origins, where the flow of different products is shipped to a set of destinations, either directly or through a hub, and is able to use several transport modes. Given that, a hub can act as an origin and destination, and sets of Super Origins and Super Destinations are defined to incorporate functionality of the hubs. Furthermore, destinations, hubs, and arcs have a limited capacity. According to [15] it is classified as a capacitated (in arcs, destinations, and hubs), multiple allocation, and discrete domain HLP. Furthermore, the proposed model allows direct shipment. Unlike traditional models, the proposed model has free allocation, that is, there is not a preset O/D flow, but the model decides it in favor of system performance.

The sets that compose the model are the following:

Sets

$I$: Origins, $i = 1, 2, \ldots, I$.

$H$: Hubs, $h = 1, 2, \ldots, H$.

$J$: Destinations, $j = 1, 2, \ldots, J$.

$SO$: Super origins, $SO = I \cup H$.

$SD$: Super destination, $SD = H \cup J$.

$M$: Transport modes, $m = 1, 2, \ldots, M$.

$K$: Products, $k = 1, 2, \ldots, K$.

It is important to note that the separation between Origins, Destinations, and Hubs implies that each node type has its own parameters for the modeled system. It is assumed that origins can only send flow, destinations can only receive, and hubs can function as a sender and receiver. Furthermore, hubs and other nodes are different entities, because the hub works as a facility while origins are productive centers and destinations are ports. Another essential feature of the proposed model is the incorporation of set $K$. This allows the differentiation of cost structures by product, as well as the representation of the specialization of each origin, destination, and hub through offer, demand, and hub capacity parameters.

Parameters grouped in capacities, flows, costs, and bounds are part of the MILP, and are listed below:

Parameters

$q_{hi}^k$: Capacity of hub $h$, t/year.

$q_{k}^j$: Capacity of hub $h$ for product $k$, t/year.

$T_{ij}^d$: Capacity of destination $j$, t/year.

$A_{ij}^m$: Capacity of arc $i, j$ for the $m$ transport mode, t/year.

$q_{ij}^k$: Flow of product $k$ that must be sent from the origin $i$ (offer), t/year.

$d_{ij}^m$: Flow of product $k$ that must be received by the destination $j$ (demand), t/year.

$c_{ij}^m$: Transport cost for shipping a unit of product $k$ from node $i$ to $j$ using the transport mode $m$. If $i$ is equal to $j$ it corresponds to a handling cost associated to the modal change into the hub, $/$/t.

$F_h$: Fixed cost to install the $h$ Hub, $$/year.

$f_{ij}^m$: Fixed cost to create the arc between the nodes $i$ and $j$ for the transport mode $m$, $$/year.

$E$: Big M, t/year.

$p$: Maximum number of hubs to install.
Decision variables

\[ X_{ij}^{km} \]: Flow of product \( k \) sends from \( i \) to \( j \) using the transport mode \( m \), \( t/\text{year} \).

\[ Y_k \]: \begin{align*}
1 & \text{ if hub } h \text{ is located} \\
0 & \text{ otherwise.}
\end{align*}

\[ W_{ij}^{m} \]: \begin{align*}
1 & \text{ if arc from } i \text{ to } j \text{ is created for the } m \text{ transport mode} \\
0 & \text{ otherwise.}
\end{align*}

The proposed model's formulation corresponds to (1)–(13):

\[
\begin{align*}
\min & \sum_{i \in SO} \sum_{j \in SD} \sum_{m \in M} C_{ij} X_{ij}^{km} + \sum_{i \in SO} \sum_{j \in SD} \sum_{m \in M} W_{ij}^{m} f_{ij}^{m} + \sum_{h \in H} Y_h F_h.
\end{align*}
\] (1)

s.t:

\[
\sum_{h \in H} Y_h \leq p.
\] (2)

\[
\sum_{i \in SO} X_{ih}^{km} \leq a_i^k, \forall i \in I, \forall k \in K.
\] (3)

\[
\sum_{i \in SO} X_{ij}^{km} \geq d_j^k, \forall j \in J, \forall k \in K.
\] (4)

\[
\sum_{i \in SO} \sum_{h \in H} X_{hj}^{km} \leq Y_h Q_h, \forall h \in H.
\] (5)

\[
\sum_{i \in SO} \sum_{h \in H} X_{ih}^{km} \leq q_i^k, \forall h \in H, \forall k \in K.
\] (6)

\[
\sum_{i \in SO} \sum_{h \in H} X_{ij}^{km} \leq T_j, \forall j \in J.
\] (7)

\[
X_{ij}^{km} \leq A_{ij}^m, \forall i \in SO, \forall j \in SD, \forall m \in M.
\] (8)

\[
X_{ij}^{km} \leq W_{ij}^m E, \forall i \in SO, \forall j \in SD, \forall m \in M, \forall k \in K.
\] (9)

\[
X_{ij}^{km} \geq 0, \forall i \in SO, \forall j \in SD, \forall m \in M, \forall k \in K.
\] (10)

\[
Y_h \in \{0, 1\}, \forall h \in H.
\] (11)

\[
W_{ij}^m \in \{0, 1\}, \forall i \in SO, \forall j \in SD, \forall m \in M.
\] (12)

Note that if neither of the hubs is profitable for the system, the model decides to send all flow directly.

### 3. Results and Discussion

The proposed model was coded in AMPL and solved with the commercial solver, Gurobi 7.5.0, using a computer with the following configuration: Intel® Core™ i7-6560U, 2.2 GHz processor, 16 GB RAM, and 64-bit Windows 10 OS. Each experiment was solved optimally in computational times less than a minute, and, for this reason, we do not present time and gap tables.

#### 3.1. Context

The context is the export process of a region in a Latin American country with a long and narrow geography. The study focuses on freight transport between productive centers and destinations (ports, airports, and others). The problem is where, to locate one or more intermodal hubs. Fifty districts act as origins, which need to export products through one or more destinations. There are six types of products, but one district may not necessarily produce all of them. Infrastructure corresponds to railways and roads. All the nodes of the system can use roads, but only two origins 
(25 and 7) have access to a train mode. In contrast, hubs can use both modes, but only while paying for the handling cost if there is a modal change. The available railway network is a long pathway with few branches. In this model, arcs could be created for hubs that do not have direct access to trains, but only if it is profitable for the system. There are seven destinations: four maritime ports, two border crossings, and an airport. Only the maritime ports have access to trains. It is important to note that the experiments analyze 54 hubs, where each origin is a candidate, in addition to 4 other locations (in Figure 1 nodes from 1 to 50 are both origins and hub candidates, while nodes from 51 to 54 are only hub candidates). Instances are an initial situation and a 20-year projection. Analyzing the geographical distribution of freight is possible to note the high flow concentrations. It is noted that the north of the study area shows a high production of commodities A, B, and C, the center-north
is associated with product D, and the south zone typically generates commodities E and F. Figure 1 represents the context situation; the intensity of the color represents the flow that each origin produces. In Figure 1(a), all the nodes that make up the system are shown, including the available roads. On the other side, Figure 1(b) presents the study area with the railroad connectivity. As mentioned previously, the road network is fully connected, with the number of arcs for this mode being $|SO| \times |SD| = 6,344$. In the case of the railroad system, only 2 origins, 34 hubs, and 4 destinations have access to the network; therefore, the number of existing arcs is 1,368. It is important to clarify that the model works with real distances instead of Euclidian or another type of length, to create the cost structure.

3.2. Experiment 1. A ranking of the best hubs is generated to determine the optimal market for each and the associated transport cost savings. Transport savings represent the maximum value that the system will be able to pay to keep

![Figure 1: Context situation. (a) The complete system with roads. (b) The study area with rail connectivity.](image-url)

![Table 1: Vector of variables $Y_{Hc}$](table-url)
With these vectors, we can compare the performance of each hub candidate working individually. A summary of the results of Experiment 1 are available in Table 2, where the five best locations are presented.

In analyzing Table 2, it is easy to note a pattern in the results, which shows that all of the good solutions (hubs) the hub as a profitable investment. All the candidates compete in equally, that is, the investment is the same for each candidate (USD 413,400), and the specific hub capacity constraint (7) is relaxed. The ranking is made by introducing a vector of variables $Y_h$ for each $h$ in $H$, taking the respective values as shown in Table 1.

**Table 2: Ranking of hubs.**

<table>
<thead>
<tr>
<th>Ranking</th>
<th>Hub</th>
<th>Flow (t/year)</th>
<th>Total cost (USD/year)</th>
<th>Total marginal savings (USD/year)</th>
<th>Transport cost (USD/year)</th>
<th>Transport marginal savings (USD/year)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Initial</td>
<td>Projection</td>
<td>Initial</td>
<td>Projection</td>
<td>Initial</td>
</tr>
<tr>
<td>1</td>
<td>49</td>
<td>273,894</td>
<td>495,676</td>
<td>42,293,900</td>
<td>82,974,301</td>
<td>97,135</td>
</tr>
<tr>
<td>2</td>
<td>54</td>
<td>319,924</td>
<td>518,077</td>
<td>42,388,910</td>
<td>83,178,340</td>
<td>2125</td>
</tr>
<tr>
<td>3</td>
<td>42</td>
<td>312,955</td>
<td>514,765</td>
<td>42,425,286</td>
<td>83,235,241</td>
<td>−34,251</td>
</tr>
<tr>
<td>4</td>
<td>53</td>
<td>298,066</td>
<td>514,629</td>
<td>42,514,822</td>
<td>83,374,770</td>
<td>−123,787</td>
</tr>
<tr>
<td>5</td>
<td>51</td>
<td>53,001</td>
<td>228,536</td>
<td>42,546,521</td>
<td>83,429,141</td>
<td>−155,486</td>
</tr>
</tbody>
</table>

**Figure 2:** Different hubs, allocation comparison. (a) The best hub. (b) Hub 24, a hub close to seaports.

With these vectors, we can compare the performance of each hub candidate working individually. A summary of the results of Experiment 1 are available in Table 2, where the five best locations are presented.

In analyzing Table 2, it is easy to note a pattern in the results, which shows that all of the good solutions (hubs)
are located in the south of the study area. Furthermore, all the presented hubs have access to the rail mode, implying there is no arc creation. The use of the best candidate implies 1.2% and 1.1% in transport savings annually, for the initial and projected scenarios, respectively. This situation can be explained by the concentration of flow (Product E) in this zone. Nevertheless, the north zone presents flow concentration too, but none of the profitable candidates are shown in this area. This implies that volume is necessary but not a major condition in the model. It is possible to see that the distance traveled is directly proportional to the savings.

Finally, it is possible to say that there are substitute hubs according to both their geographic location and the market they each serve. Indeed, as all the candidates listed in Table 1 are located in the south (see Figure 1), they are attending mainly to commodity E and, to a lesser extent, product F. The situation explained above is presented in Figure 3.

3.3. Experiment 2. This experiment analyzes the possibility of two or more hubs working together. In this case, we force the model to open a particular number of hubs, but the model is not forced to open specific hubs. Then, the model indicates the best combination of hubs for each number of hubs to open.

Figure 4 shows that only one hub is profitable for the system in the initial situation and the 20-year projection. The increase in transportation savings for two or more hubs does not cover the investment costs. Transportation savings increase by 9% for two hubs compared to one hub, and it grows at decreasing rates; the increase is equal to 6% and 3% from four to five hubs in initial situation and projection at 20-years, respectively. When a hub well attends a market, substitutes will not be open (Figure 3); this situation is visible comparing the results of this experiment with experiment 1. It is important to note that, for the initial situation, three out of the five hubs opened attend product D; nevertheless, these hubs report a low transport saving. The following reason can explain this situation; the origin 25 is one of the only two who has access to train. However, arcs originated at this point are capacitated; therefore, as the capacity is less than the offer, the model sends all the flow that the arc capacity constraints allow in a direct way to the destinations (using train). Then, hubs focused on product D start to be opened, and the rest of the flow is shipped to these centers using rail mode, to finally go to the destinations using the same modality. This situation implies that there is no modal change and therefore, no handling cost. In practice, this situation does not generate a significant saving for the system because the opened hubs are too close to the destinations, that is, the travel distance is not enough to be profitable. In Figure 1, it is possible to verify the explained situation; hubs 28, 29, and 30 are located in the center of the study area, close to three ports. The results of this experiment indicate that when we force our model to open five hubs, for example, the combination includes the same four previously selected hubs. The reader should understand that these results could not be the same in other instances. This situation can
be explained because of the particular features of the study area (long and narrow geography of the country, supply and demand flows, and cost structures), where only the first open hub is profitable, and none of the selected hubs needed to create new railway arcs.

It is important to clarify that the solution of the model, not forcing it to open some quantity of hubs, indicated that only one hub is profitable for the system.

### 4. Conclusions

A new MILP model for capacitated HLP is proposed. This model is capacitated, which means it is a multimodal and multi-commodity formulation that allows direct shipment between origins and destination, and whose O/D flows are not predefined but are part of the model solution. The experiments prove that it is a valid tool to support the strategic decision-making process in multimodal transport.

For the case study, it was found that the location decision is strongly influenced by two factors, the volumes of load and, the second and most important condition, the distance between hubs and destinations. In practice, the economies of scale come from the use of train instead of trucks, so that travel distance is a key indicator for greater savings derived from the system. Furthermore, only one hub is profitable, both for the initial and projected situation. If a hub is installed, it should be located geographically in the south of the study area (the long distances from these candidates to destinations imply an intensive use of trains instead of trucks, that is, more savings are achieved).

Future works could be in the direction of adding new components to the objective function, such as pollution, social costs, and other externalities. Moreover, flow, costs, capacities, and other parameters may change over time; therefore, adding uncertainty into the model is a good way to represent the variability of the system.

### Data Availability

The distance matrix, supply and demand flows by product, and cost (US$/t-km) by type of product used to support the findings of this study are included within the supplementary information file.

### Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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### Supplementary Materials

Worksheets: DISTANCE MATRIX-ROAD here we present a distance matrix for road mode (measured in km). Each row represents an origin or a hub candidate, and each column represents destinations or a hub candidate. Worksheets: DISTANCE MATRIX-RAIL Here we present a distance matrix for railway mode (measured in km). Each row represents a hub candidate (are the only who have access to railway mode), and each column represents destinations. Nevertheless, as the rail network is not fully connected, only the available arcs are shown. Worksheets: DEMAND Here, the reader can find the annual demand by-product of each destination (measured in ton/year). Data available correspond to the initial and 20-year projected flow. Worksheets: OFFER similar to demand, we show the annual offer by-product of each origin (measured in
Data available correspond to the initial and 20-year projected flow. Worksheets: COST here we present, the cost matrix by product type and transport mode, and the modal change cost. Products A-B-C-E and F are freeze, while D is general freight. These factors are measured in USD/ton-km. (Supplementary Materials)

References


