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Research Article

Reliability Optimization Model of Stop-Skipping Bus Operation with Capacity Constraints

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In a bus line with high passenger demand, the stop-skipping operation can benefit both users and operators as well as improve the service level of bus line, but it usually cannot be effectively adopted by operators because of the unordered operation of buses and the discomfort of waiting passenger at stops. Therefore, the stop-skipping operation requires higher reliability for stop serving. This paper proposes a reliable stop-skipping service design with holding strategy by taking into account bus capacity constraints. Under a stop-skipping service, the holding strategy is used to balance the interval of stop serving rather than the headway of buses. Meanwhile, to reflect the actual boarding process of waiting passengers, the number of left-over passengers and the waiting time are calculated during serving time and holding time, respectively. The objective function is to minimize the total costs of bus operation system. Besides, a Genetic Algorithm combined with Monte Carlo Simulation method is defined and implemented to solve the reliability optimization model. Finally, a numerical example based on a bus route in Changchun city is carried out to test the reliability optimization model. Results showed that the reliability optimization strategy can improve the stability of stop service and then save cost of passengers' travel time.

1. Introduction

In public transit systems, the frequency setting is usually used to change the service level of a bus line which means that high frequency means high level. Frequency setting is easily adopted to schedule and manage bus operation for bus operator, expected to meet the high passenger demand. Unfortunately, the frequency setting may not reach good effect because of the different and unbalanced distribution of the passenger demand at each stop along the bus line [1]. Therefore, the stop-skipping strategies are used to deal with the unbalanced passenger demand distribution so as to provide different service frequencies at each stop. Under the stop-skipping strategy, buses serve the high-demand stops, in which passengers get aboard the bus and skip the low-demand stops. Because of the differentiation of stop serving, the stop-skipping bus operation can reduce the waiting time and travel time of passengers at stops with high demand. Accordingly, in the limited transportation capacity, the stop-skipping operation is conducive to the improvement of serving level of transit system.

Generally, there are some uncertainties for the stop-skipping operation. On the one hand, since buses do not need to serve the skipped stop, departure time may be ahead of the schedule. Thus, the adjacent headway will become unequal and break the stable operation of buses with balanced headway, in which the departure intervals of buses are usually the same at the starting stop. On the other hand, there are many factors affecting the running time of buses such as road congestion, signal control at signalized intersections between stops, and speed differences between drivers driving buses. Speed difference between buses causes reliability degradation. To improve the reliability of bus operation, some control actions and operational strategies are presented. Their purpose is to ensure the balance of headway. However, with regard to waiting passengers, balanced headway does not mean balanced stop service under the stop-skipping bus operation.

Passengers cannot get aboard the skipped buses, and their waiting time will be extended. Therefore, there is a potential need to develop a reliability optimization for the stop-skipping bus operation.

1.1. Literature Review. Along a bus route, passenger demands of each stop present the unbalanced distribution, where the frequency of services should be different for each stop. The bus operational stopping problem is put forward to handle the unbalanced demand and to improve the operating efficiency of buses. By adopting a stop-skipping operation, stops with high demand can be provided with higher service frequency than other stops in a bus line. Given a bus route, the optimization objective of stop-skipping operational strategy is to save costs of both users and operators with the main decision to skip those appropriate stops.

The stop-skipping strategy was first proposed to reduce the travel time and shorten operation cycle. With a static demand, several operational stopping strategies such as short-turn lines, deadheading, and limited-stop lines were implemented to improve the efficiency in a corridor [2]. Suh et al. [3] set up a skip-stop system to save the travel time for a subway. The information required included origin–destination (O–D) demand matrix, distances between stations, headways, and maximum link speeds. Under those operational stopping strategies, a subset of the stops which were skipped would be selected and passengers' travel time and buses' running time could be shortened [4–8, 13–15]. Chen et al. [9] used a hybrid artificial bee colony (ABC) and Monte Carlo method to solve the optimal stopping strategy, considering the effect of vehicle capacity and stochastic travel time on actual operation. Instead of designing stop-skipping service in tactical planning, a rolling time horizon approach was adopted to achieve real-time optimization [10–16]. Specifically, Ould Sidi et al. [17] proposed a multi-objective optimization approach to determine the dynamic stopping pattern and the departure time when a disruption occurred.

There are a number of studies on the transit reliability optimization with focus on the normal bus service. Benefited from the application of intelligent transportation system (ITS) technologies, bus location and speed can be obtained [18–20]. They provided a potential to maintain system stability under the influence of uncertain information. Holding strategy is one of the control approaches, which balances the headway of buses and thus provides provide reliable service [21, 22]. Daganzo [23] adopted a time control point strategy to achieve regular headways and, based on real-time information, determined bus holding times at a time control points. Saez et al. [24] proposed a hybrid predictive control strategy including holding and expressing actions to deal with a triggered event. Yan et al. [25] considered the bus travel time uncertainty and bus drivers' schedule recovery efforts and developed a optimization model to improve the reliability of a bus route.

Two gaps can be identified from the previous studies. Firstly, most of the stop-skipping strategies in the above literature review were just proposed to determine the proper stops that should be skipped while the stability of operation was not considered, and the models about reliability optimization were only applied to the normal bus service without stop-skipping operation. However, under the stop-skipping strategies, bus operation is more irregular and easily to cause bus bunching. Secondly, existing holding control strategy as an effective method of reliability optimization did not consider the boarding process of passengers during bus dwell time. With high boarding demand, holding strategies have a great influence on the waiting time of passengers while passengers cannot get aboard buses because of the bus capacity. This is also one of the reasons why holding strategies are limited in wide application.

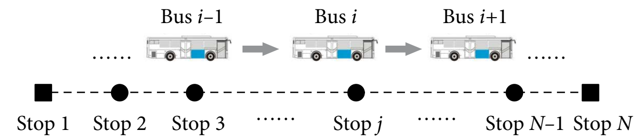

Figure 1: Illustration of a bus route and the bus stops.

1.2. Objectives and Contributions. The objective of this study is to propose a model to design a reliable strategy for stop-skipping bus operation on a fixed route, considering the boarding process of waiting passengers with bus capacity constraints. First, one index (stop service interval) is proposed to express the interval of serving buses at the same stop. It will be used to reflect the reliability of waiting time for passengers at a stop based on its variance, rather than simply through headways in the existing literature. And the reliability of stop-skipping bus operation will be improved by a holding strategy. Second, the boarding process of passengers is analyzed, taking account of the capacity constraints of buses during bus dwell time including a holding time. Since a harmful influence of holding strategy that the passengers' travel time has to be extended, the waiting time and the in-vehicle time of passenger should be clearly shown in the optimization.

The remainder of this paper is organized as follows: Section 2 describes the reliability optimization problem and explains related variables. Section 3 provides the expressions for the calculation of holding time and the objective function. A Genetic Algorithm combined with Monte Carlo Simulation method is designed to solve the objective function in Section 4. Numerical examples are presented in Section 5 based on a bus route in Changchun city, China. Finally, Section 6 concludes this paper.

2. Notation and Problem Description

We now consider a typical bus route, and there are N bus stops, as shown in Figure 1. Stop 1 and stop N are the starting stop and the terminal stop respectively. According to a given frequency-based operation, buses dispatch from stop 1. This paper proposes a reliability optimization strategy under the stop-skipping bus operation, in which the holding control and the stop-skipping scheme are generated at the same time. Note that the reliability optimization strategy is an operational planning decision that has been determined before buses are dispatched from the starting stop. In order to keep passengers from waiting too long, it is assumed that two successive buses cannot skip

Table 1: List of notation.

Variable	Description
i	Index of buses, $i = 1, 2, \ldots$
j	Index of stops, $j = 1, 2, \ldots, N$
$y_{i,j}$	Variable that indicates the type of stop j for bus i; if bus i serves stop j, then $y_{i,j} = 1$; if bus i skips stop j, then $y_{i,j} = 0$
$t^h_{i,j}$	Holding time of bus i at stop j
m	The number of buses in a bus fleet
$\overline{h}$	Departure interval of buses
$T^A_{i,j}$	Arrival time of bus i at stop j
$T^D_{i,j}$	Departure time of bus i at stop j
t_{ac}	Bus acceleration time
t_{de}	Bus deceleration time
$t^R_{j-1,j}$	Running time of buses between stop $j-1$ and stop j
$t^E_{i,j}$	Serving time of bus i at stop j
$h_{i,j}$	Headway between bus i and bus $i-1$ at stop j
$\lambda_{i,j\to k}$	Arrival rate of passengers from stop j to stop k
$A^+_{i,j}$	The number of arriving passengers during the interval of consecutive buses
$A^{h+}_{i,j}$	The number of arriving passengers during the holding time

the same stop which could be seen in the above literature review. Meanwhile, we assume that waiting passengers are allowed to board the buses during a holding time, considering the travel habits of passengers. For the sake of presentation, the following notations are introduced in Table 1.

The objectives of reliability strategy proposed in this paper are to determine the value of two variables. One is a binary variable $y_{i,j}$ that indicates the stop-skipping scheme of bus i at stop j. If bus i skips the stop j, $y_{i,j} = 0$, passengers at stop j cannot board the bus. Meanwhile, this skipping service is not included in the stop service interval at this time and no holding control is required, in order not to affect the operation efficiency of buses. Passengers can get the information about stop skipping through an electronic bulletin board at each stop. The two is holding time $t^h_{i,j}$ that indicates a dwell time of bus i at stop j after all waiting passengers board the bus. Holding time is expected to terminate when the consecutive intervals of stop service are balanced. Thus, output variables of the reliability optimization model can be expressed as

$$\begin{Bmatrix} t^h_{i,1} & t^h_{i,2} & \cdots & t^h_{i,j} & \cdots & t^h_{i,N} \\ y_{i,1} & y_{i,2} & \cdots & y_{i,j} & \cdots & y_{i,N} \end{Bmatrix}, \quad t^h_{i,j} \geq 0, y_{i,j} = [0,1]. \tag{1}$$

Since a stop-skipping operation of a bus is interacted with other bus operation schemes, we assume that m buses constitute a bus fleet, in which optimization schemes of m buses will be presented. The bus queue of a fleet is $\{m+1, m+2, \ldots, i, \ldots, 2+m\}$. The buses are dispatched from starting stop with an interval $\overline{h}$. After buses disport the stop, there is an acceleration time t_{ac}, and when the next stop is about to arrive, buses start to slow down. The running time between adjacent stops includes the acceleration time t_{ac}, the travel time $t^R_{j-1,j}$ between adjacent stops and the deceleration time t_{de}. Thus, the arrival time of bus at each stop along the bus route can be written as

$$\begin{aligned} T^A_{m+1,1} &= T^D_{m,1} + \overline{h}, \\ T^A_{m+1,2} &= T^D_{m+1,1} + t_{ac} \cdot y_{m+1,1} + t^R_{1,2} + t_{de} \cdot y_{m+1,2}, \\ &\vdots \\ T^A_{i,j} &= T^D_{i,j-1} + t_{ac} \cdot y_{i,j-1} + t^R_{j-1,j} + t_{de} \cdot y_{i,j}, \\ &\vdots \\ T^A_{2m,N} &= T^D_{2m,N-1} + t_{ac} \cdot y_{2m,N-1} + t^R_{N-1,N} + t_{de} \cdot y_{2m,N}. \end{aligned} \tag{2}$$

When buses arrive at stops, waiting passengers board the bus and in-vehicle passenger alight. The total dwell time at a stop includes a serving time and a holding time. Thus, the departure time $T^D_{i,j}$ can be expressed as

$$\begin{aligned} T^D_{m+1,1} &= T^A_{m+1,1} + y_{m+1,1} \cdot \left(t^E_{m+1,1} + t^h_{m+1,1}\right), \\ T^D_{m+1,2} &= T^D_{m+1,1} + t_{ac} \cdot y_{m+1,1} + t_{de} \cdot y_{m+1,2} + y_{m+1,2} \cdot \left(t^E_{m+1,2} + t^h_{m+1,2}\right) + t^R_{1,2}, \\ &\vdots \\ T^D_{i,j} &= T^D_{i,j-1} + t_{ac} \cdot y_{i,j-1} + t_{de} \cdot y_{i,j} + y_{i,j} \cdot \left(t^E_{i,j} + t^h_{i,j}\right) + t^R_{j-1,j}, \\ &\vdots \\ T^D_{2m,N} &= T^D_{2m,N-1} + t_{ac} \cdot y_{2m,N-1} + t_{de} \cdot y_{2m,N} + y_{2m,N} \cdot \left(t^E_{2m,N} + t^h_{2m,N}\right) + t^R_{N-1,N}. \end{aligned} \tag{3}$$

Because the purpose of reliability operation strategy is to balance the service interval of stops, it is necessary to calculate the time–distance gaps between buses based on arrival time and departure time. We take the departure time of the bus as the deductive node; according to the calculation of the arrival time and departure time of the bus at each stop in Equations (2) and (3), the headway between consecutive buses should be:

$$h_{i,j} = T^D_{i,j} - T^D_{i-1,j}. \tag{4}$$

With regard to changes of passenger-flow at stops, the number of waiting passengers will be reduced after boarding and alighting end. There is no guarantee that a stop is emptied of passengers when buses dispatch from the stop. The skipped passengers are correspondingly left over since the bus skips the stop. Those passengers have to wait for the next bus and the waiting time will be greatly extended. There is another situation that waiting passengers cannot board the serving buses because of bus capacity constraints. When a serving bus arrives at stop, some passengers begin to board the bus and another part has to be left over at stop if the load of the bus reaches the bus capacity constraint. In such a case, it is necessary to estimate the number of waiting passengers and in-vehicle passengers and to determine whether the boarding is restricted by capacity at a serving time or a holding time. During the interval of consecutive buses and the holding time, the number of arriving passengers can be, respectively, expressed as

$$A^+_{i,j} = \left(T^A_{i,j} - T^D_{i-1,j}\right) \cdot \sum_{k=j+1}^{N} \lambda_{i,j\to k} \cdot y_{i,j} \cdot y_{i,k}, \tag{5}$$

$$A^{h+}_{i,j} = t^h_{i,j} \cdot \sum_{k=j+1}^{N} \lambda_{i,j\to k} \cdot y_{i,j} \cdot y_{i,k}, \tag{6}$$

where the interval of consecutive buses at stop j is the arrival time of bus i minus the departure time of bus $i-1$.

According to changes of passenger-flow at stops, the number of waiting passengers left over and their waiting time can be classified into the following categories:

(1) The waiting passengers left over by stop-skipping.

When bus i skips the stop j, arriving passengers between the departure times of bus $i+1$ $T^{D}_{i+1,j}$ and bus i $T^{D}_{i,j}$ cannot board the bus i and will be left over to wait until the bus $i-1$ arrive. The number of waiting passengers left over by stop-skipping can be calculated as follows:

$$\Delta S^{+}_{i,j}=\begin{cases}\left(T^{A}_{i+1,j}-T^{D}_{i,j}\right)\cdot\sum\limits_{k=j+1}^{N}\lambda_{i,j\rightarrow k}\cdot\left(1-y_{i,j}\right)\cdot\left(1-y_{i,k}\right), & \text{if } y_{i,j}=0, y_{i+1,j}=1,\\ 0 & \text{otherwise.}\end{cases} \quad (7)$$

The average waiting time for these passengers is equal to the interval between the bus i and bus i+1. Thus, the total waiting time of passengers left over is

$$T^{W,\Delta S}_{i,j}=\begin{cases}\Delta S^{+}_{i,j}\cdot\left(T^{D}_{i,j}-T^{A}_{i+1,j}\right) & \text{if } y_{i,j}=0, y_{i+1,j}=1,\\ 0 & \text{otherwise.}\end{cases} \quad (8)$$

(2) The waiting passengers left over by capacity constraint.

If bus i serve the stop j, the load of bus i plus the waiting passengers at stop j may exceed the bus capacity. It can be expressed as $L_{i,j-1}+A^{+}_{i,j}-N^{-}_{i-1,j}\geq Q_c$, and extra waiting passengers are left over to the bus $i+1$. Under the proposed reliability optimization strategy, the serving buses still need to hold for some time after passenger board and alight. Thus, the overload phenomenon may occur during these two periods: the serving time $t^{E}_{i,j}$(i.e., $T^{A}_{i,j}\leq T\leq T^{A}_{i,j}+t^{E}_{i,j}$) or holding time $t^{h}_{i,j}$(i.e., $T^{A}_{i,j}+t^{E}_{i,j}<T\leq T^{D}_{i,j}$), as shown in Figure 2. Figure 2(a) shows overload appears during the serving time $t^{E}_{i,j}$ and the moment of triggering overload is defined as $T\left(L_{i,j-1}+A^{+}_{i,j}-N^{-}_{i-1,j}\geq Q_c\right)$. Figure 2(b) shows overload appears during the holding time $t^{h}_{i,j}$ when the load of bus i at the end of serving time is less than bus capacity and that at the end of holding time is larger than capacity, i.e., $L_{i,j-1}+A^{+}_{i,j}+A^{h+}_{i,j}-N^{-}_{i1,j}\geq Q_c\geq L_{i,j-1}+A^{+}_{i,j}-N^{-}_{i,j}$, and the moment of triggering overload is defined as $T\left(L_{i,j-1}+A^{+}_{i,j}+A^{h+}_{i,j}-N^{-}_{i1,j}\geq Q_c\geq L_{i,j-1}+A^{+}_{i,j}-N^{-}_{i,j}\right)$. According to the moment that overload appears, the left-over waiting passengers is discussed from two aspects.

(a) Overload appears during the serving time.

When overload appears during the serving time, only a few passengers arriving within the interval of consecutive buses can board the bus, while other passengers and the passengers arriving within holding time are left over. Thus, in such a case, the number of waiting passengers left over can be computed by

$$\Delta P^{1+}_{i,j}=\begin{cases}L_{i,j-1}+A^{+}_{i,j}-N^{-}_{i,j}-Q_c+A^{h+}_{i,j}, & \text{if } L_{i,j-1}+A^{+}_{i,j}-N^{-}_{i,j}\geq Q_c,\ y_{i,j}=1,\\ 0 & \text{otherwise.}\end{cases} \quad (9)$$

where the load of bus i dispatched from stop j $L_{i,j}$ can be deduced as

$$L_{i,k}=\sum_{j=1}^{k}\left(N^{+}_{i,j}-N^{-}_{i,j}\right). \quad (10)$$

$$N^{-}_{i,j}=\sum_{k=1}^{j-1}N^{+}_{i,k\rightarrow j}=\sum_{k=1}^{j-1}N^{+}_{i,k}\cdot\frac{\lambda_{i,k\rightarrow j}}{\sum_{k=1}^{j-1}\lambda_{i,k\rightarrow j}}. \quad (11)$$

The waiting time for these passengers includes the holding time and the interval between the bus i and bus $i+1$. Thus, the waiting time of passengers left over is

$$T^{W,\Delta P1}_{i,j}=\begin{cases}\left(L_{i,j-1}+A^{+}_{i,j}-N^{-}_{i,j}-Q_c\right)\cdot\left(t^{h}_{i,j}+T^{D}_{i,j}-T^{A}_{i+1,j}\right)\\ +A^{h+}_{i,j}\cdot\left[\frac{1}{2}\cdot t^{h}_{i,j}+\left(T^{D}_{i,j}-T^{A}_{i+1,j}\right)\right], & \text{if } L_{i,j-1}+A^{+}_{i,j}-N^{-}_{i,j}\geq Q_c, y_{i,j}=1,\\ 0 & \text{otherwise.}\end{cases} \quad (12)$$

(b) Overload appears during the holding time.

When overload appears during the holding time, all of the passengers arriving within the interval of consecutive buses can board the bus, while the passengers arriving within holding time are left over. Thus, in such a case, the number of waiting passengers left over can be computed by

$$\Delta P^{2+}_{i,j}=\begin{cases}L_{i,j-1}+A^{+}_{i,j}+A^{h+}_{i,j}-N^{-}_{i,j}-Q_c, & \text{if } L_{i,j-1}+A^{+}_{i,j}+A^{h+}_{i,j}-N^{-}_{i,j}\geq Q_c\geq L_{i,j-1}+A^{+}_{i,j}-N^{-}_{i,j},\ y_{i,j}=1,\\ 0 & \text{otherwise.}\end{cases} \quad (13)$$

The waiting time for these passengers consists of two parts: a part of the holding time and the interval between the bus i and bus i+1. Thus, the waiting time of passengers left over is

$$T^{W,\Delta P2}_{i,j}=\begin{cases}\Delta P^{2+}_{i,j}\cdot\left\{\frac{1}{2}\cdot\left[t^{h}_{i,j}-\left(L_{i,j-1}+A^{+}_{i,j}+A^{h+}_{i,j}-N^{-}_{i,j}-Q_c\right)\cdot\frac{t^{h}_{i,j}}{A^{h+}_{i,j}}\right]+\left(T^{D}_{i,j}-T^{A}_{i+1,j}\right)\right\}, & \text{if } L_{i,j-1}+A^{+}_{i,j}+A^{h+}_{i,j}-N^{-}_{i,j}\geq Q_c\geq L_{i,j-1}+A^{+}_{i,j}-N^{-}_{i,j}, y_{i,j}=1\\ 0 & \text{otherwise.}\end{cases} \quad (14)$$

In summary, considering all the above, the total number of arriving passengers that can board the bus i at stop j can be computed by

$$N^{+}_{i,j}=\begin{cases}0 & \text{if } y_{i,j}=0,\\ Q_c-N^{-}_{i,j}+L_{i,j-1} & \text{if } L_{i,j-1}+A^{+}_{i,j}+A^{h+}_{i,j}-N^{-}_{i,j}\geq Q_c, y_{i,j}=1,\\ \Delta S^{+}_{i-1,j}+\Delta P^{1+}_{i-1,j}+\Delta P^{2+}_{i-1,j}+A^{+}_{i,j}+A^{h+}_{i,j} & \text{otherwise.}\end{cases} \quad (15)$$

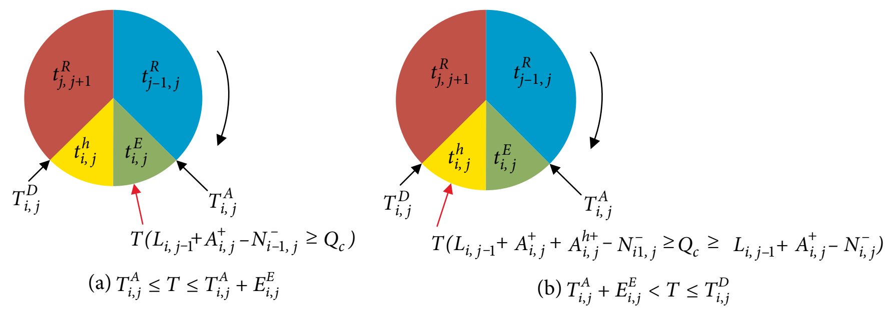

Figure 2: Time window when passengers are left over by capacity constraint (a) and (b).

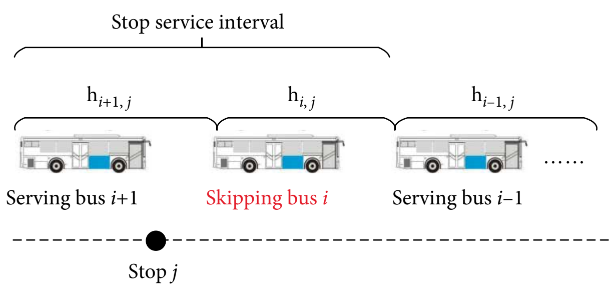

Figure 3: An example of the stop service interval.

Among them, the number of arriving passengers that can board the bus i at stop j during the interval between the bus i and bus $i+1$, which is used to calculate travel time separately, can be computed by

$$N^{A+}_{i,j}=\begin{cases}0 & \text{if } y_{i,j}=0,\\ Q_c-N^-_{i,j}+L_{i,j-1} & \text{if } L_{i,j-1}+A^+_{i,j}-N^-_{i,j}\geq Q_c,\, y_{i,j}=1,\\ \Delta S^+_{i-1,j}+\Delta P^{1+}_{i-1,j}+\Delta P^{2+}_{i-1,j}+A^+_{i,j} & \text{otherwise.}\end{cases} \quad (16)$$

3. Reliability Optimization Formulation

Under the reliability optimization strategy, a holding control is used to reduce the deviation of service intervals at stop that was caused by the stop-skipping operation and the random running time of buses, while too long holding time will extend the in-vehicle time of passengers and the running time of buses. Therefore, the purpose of the reliability optimization strategy is to ensure the reliability of stop-skipping bus operation and avoid significant increases in bus system costs. Based on the changes of bus timetable and passenger-flow at stops, the proper holding times can be calculated for the purpose of balancing the stop service interval. And, thereby, by minimizing the total cost of the bus system, the reliability optimization model can be established.

3.1. Estimation of the Holding Time. The holding strategy is used to improve the reliability of stop service and thereby enables passengers to make a more accurate estimation of travel time. The calculation of a holding time is related to a stop service interval, while the stop service interval is different from the bus headway. It refers to the time interval between two consecutive serving buses. For example, a bus skips the stop, where the waiting passengers need to wait for the next bus, and the service interval of the stop is lengthened accordingly, as shown in Figure 3. Thus, the holding time can be calculated by the gap between consecutive service intervals. Meanwhile, to avoid a too long holding time, a maximum time $\theta\cdot\overline{h}$ is set for the holding time, where θ is adjustment factor for the departure interval of buses $\overline{h}$.

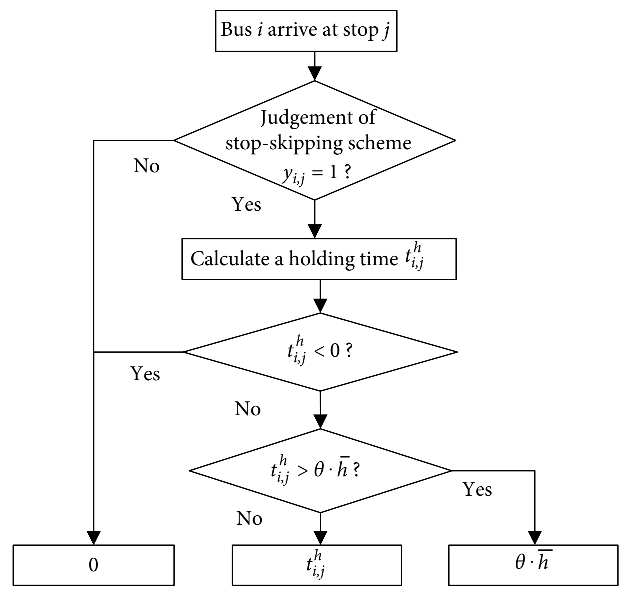

Figure 4: Judgement process of holding control.

Figure 4 shows a judgement process whether holding control is adopted at a stop, based on the above analysis. When bus i skips the stop j ($y_{i,j}=0$), holding control is not adopted, i.e., $\hat{t}^h_{i,j}=0$, when bus i serves the stop j ($y_{i,j}=1$), holding control is adopted and the holding time can be calculated by the stop service interval; if the calculated holding time is negative, the holding time is set to 0; if the calculated holding time exceeds the maximum time $\theta\cdot\overline{h}$, the holding time is set to $\theta\cdot\overline{h}$, otherwise the holding time is calculated value $t^h_{i,j}$.

The stop service interval is set as the difference between the departure time of two consecutive buses and that of two buses serving the same stop. The service interval of this stop can be expressed as

$$H_{i,j} = \begin{cases} h_{i,j} & y_{i,j} = 1, y_{i-1,j} = 1 \\ h_{i,j} + h_{i-1,j} & y_{i,j} = 1, y_{i-1,j} = 0, y_{i-2,j} = 1. \end{cases} \quad (17)$$

The conception of holding control method is to postpone the departure time of buses when serving time is end, so that the current service interval is equal to the previous service interval. According to Equation (3), it can be thus calculated by

$$T^D_{i-1,j} - \left(T^A_{i,j} + t^E_{i,j} + t^h_{i,j} + t_{de} \cdot y_{i,j}\right) = \begin{cases} H_{i-1,j} & \text{if } H_{i-1,j} > H_{i,j}, y_{i,j} = 1 \\ 0 & \text{otherwise} \end{cases}. \quad (18)$$

We thus obtain the holding time:

$$t^h_{i,j} = \begin{cases} H_{i-1,j} - T^D_{i-1,j} + T^A_{i,j} - t^E_{i,j} - t_{de} \cdot y_{i,j} \\ \quad \text{if } H_{i-1,j} > H_{i,j}, y_{i,j} = 1 \\ 0 \quad \text{otherwise} \end{cases}, \quad (19)$$

where the serving time $t^E_{i,j}$ is used to meet passengers' boarding and alighting demand, and, according to Equations (11) and (16), it is equal to the maximum value between boarding time and alighting time:

$$t^E_{i,j} = \begin{cases} \max\left[a \cdot N^{A+}_{i,j}, b \cdot N^-_{i,j}\right] & \text{if } y_{i,j} = 1 \\ 0 & \text{if } y_{i,j} = 0 \end{cases}. \quad (20)$$

3.2. Mathematical Model. The objective of the reliability optimization model is to achieve the minimization of total costs of bus operation system under the stop-skipping and holding strategy. The optimal stopping scheme $y_{i,j}$ and the corresponding holding time $t^h_{i,j}$ will be found out. Total costs of bus operation system include the costs of waiting time and in-vehicle time of passengers and the cost of the running time of buses.

For the costs of waiting time, the waiting time of passengers includes the waiting time of passengers arriving during the interval of consecutive buses and the waiting time of passengers left over. Thus, the costs of waiting time can be expressed as

$$C_W = c_W \cdot \sum_i^m \sum_j^N \left\{ \sum_{k=j+1}^{N} \left[\left(T^A_{i,j} - T^D_{i-1,j}\right) \cdot \lambda_{i,j\to k} \cdot \frac{T^A_{i,j} - T^D_{i-1,j}}{2} \right] + \left(T^{W,\Delta S}_{i,j} + T^{W,\Delta P1}_{i,j} + T^{W,\Delta P2}_{i,j}\right) \right\}, \quad (21)$$

where c_W is the value of waiting time.

Since the waiting time of the passengers on the bus for the departure is taken as the in-vehicle time during a holding time, the in-vehicle time can be obtained by

$$C_{IN} = c_{IN} \cdot \sum_i^m \sum_j^N \left[L_{i,j-1} \cdot \left(t_{ac} \cdot y_{i,j-1} + t^R_{j-1,j} + t_{de} \cdot y_{i,j}\right) + T^{INS}_{i,j} \cdot y_{i,j} \right], \quad (22)$$

where c_{IN} is the value of waiting time.

In the right-hand-side of Equation (22), the first term $L_{i,j-1} \cdot \left(t_{ac} \cdot y_{i,j-1} + t^R_{j-1,j} + t_{de} \cdot y_{i,j}\right)$ is the total travel time of passengers between stops, which is achieved by multiplying the travel time between bus stops and the load of buses. The second term $T^{INS}_{i,j} \cdot y_{i,j}$ is the total waiting time of the passengers on the bus for the departure, which is obtained by multiplying the average in-vehicle waiting time and the load of buses during a dwell time. Meanwhile, according to the change of bus load during serving time and holding time, the number of in-vehicle passengers can be divided into three categories, as shown in Equation (23): (a) the load of bus does not reach the vehicle capacity when bus i dispatched from stop $j\left(L_{i,j} \le Q_c\right)$, and at this time, all arriving passengers can board the bus; (b) the load of bus reaches the vehicle capacity during a serving time ($L_{i,j-1} + A^+_{i,j} - N^-_{i,j} \ge Q_c$), and at this time, part of arriving passengers during a stop service interval can board the bus; (c) the load of bus reaches the vehicle capacity during a holding time ($L_{i,j-1} + A^+_{i,j} + A^{h+}_{i,j} - N^-_{i,j} \ge Q_c \ge L_{i,j-1} + A^+_{i,j} - N^-_{i,j}$), and at this time all arriving passengers during a stop service interval and part of the arriving passengers during a holding time can board the bus. Thus, the waiting time of in-vehicle passengers to departure during a dwell time $T^{INS}_{i,j}$ can be calculated by

$$T^{INS}_{i,j} = \begin{cases} (L_{i,j-1} + N^{A+}_{i,j} + N^-_{i,j}) \cdot (t^E_{i,j} + t^h_{i,j}) + A^{h+}_{i,j} \cdot \frac{t^h_{i,j}}{2} & \text{if } L_{i,j} \le Q_c \\ L_{i,j} \cdot (t^E_{i,j} + t^h_{i,j}) & \text{if } L_{i,j-1} + A^+_{i,j} - N^-_{i,j} \ge Q_c \\ (L_{i,j-1} + N^{A+}_{i,j} + N^-_{i,j}) \cdot (t^E_{i,j} + t^h_{i,j}) + (L_{i,j} - N^{A+}_{i,j}) \cdot \\ [t^h_{i,j} - \frac{1}{2}(N^+_{i,j} - N^{A+}_{i,j}) \cdot \frac{t^h_{i-1,j}}{A^{h+}_{i-1,j}}] & \text{if } L_{i,j-1} + A^+_{i,j} + A^{h+}_{i,j} - N^-_{i,j} \ge Q_c \ge L_{i,j-1} + A^+_{i,j} - N^-_{i,j} \end{cases}. \quad (23)$$

For the operator, the costs of running time of buses include the travel time cost between stops, serving time cost and holding time cost. Thus, the costs of running time of m buses in a bus fleet can be modified as

$$C_R = \sum_i^m \sum_j^N \left\{ c_R \cdot \left[t_{ac} \cdot y_{i,j-1} + t^R_{j-1,j} + t_{de} \cdot y_{i,j} + \left(t^E_{i,j} + t^h_{i,j}\right) \cdot y_{i,j} \right] \right\} \quad (24)$$

where c_R is the value of running time of buses.

According to Equations (21)–(24), minimizing the total costs of bus system is then taken as the objective of the reliability optimization model:

$$\min C = C_W + C_{IN} + C_R. \quad (25)$$

4. Solution Algorithm

The output variables of the reliability optimization model for stop-skipping operation include the stop-skipping scheme $y_{i,j}$

$$\begin{array}{r} \text{Stop-skipping scheme} \\ \text{Holding time} \end{array} \left[\underbrace{\begin{matrix} y_{1,2}, y_{1,3}, \ldots\ldots, y_{1,N} \\ t^h_{1,2}, t^h_{1,3}, \ldots\ldots, t^h_{1,N} \end{matrix}}_{\text{bus } 1}, \ldots, \underbrace{\begin{matrix} y_{i,1}, y_{i,2}, \ldots\ldots, y_{i,N} \\ t^h_{i,1}, t^h_{i,2}, \ldots\ldots, t^h_{i,N} \end{matrix}}_{\text{bus } i}, \ldots, \underbrace{\begin{matrix} y_{m,1}, y_{m,2}, \ldots\ldots, y_{m,N} \\ t^h_{m,1}, t^h_{m,2}, \ldots\ldots, t^h_{m,N} \end{matrix}}_{\text{bus } m} \right]$$

Figure 5: A chromosome coding.

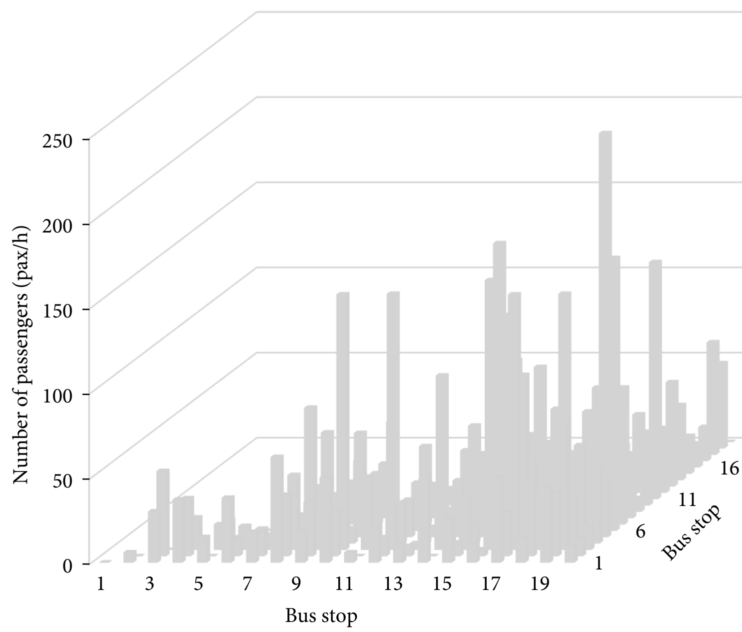

Figure 6: Average passenger demand during the early peak period.

and the proper holding time $t^h_{i,j}$ at serving stops. Among them, the optimization of stop-skipping scheme is a 0-1 nonlinear programming problem, in which a Genetic Algorithm is used to solve [26, 27]. Meanwhile, the optimization strategy considers the randomness of the bus travel time caused by road traffic situations and driver operating habits, and a Monte Carlo Simulation method is adopted to deal with uncertain travel time [28, 29]. The specific steps of this solution method are as follows:

Step 1. Initialization.

Step 1.1. Set parameters of Genetic Algorithm: the number of iterations N_G, the crossover probability P_c, the mutation probability P_m and population size M_G. Set parameters of Monte Carlo method: the counter of simulations n_{mc}, the maximum number of simulations N_{mc} and the variance of travel time σ_j.

Step 1.2. Input the passenger demand $\lambda_{i,j\to k}$ and the number of buses in a fleet m.

Step 1.3. Initiate skipping schemes $y_{i,j}$. Randomly generate a population and set up the holding time of the corresponding stops ($y_{i,j} = 1$). A chromosome coding in population can be expressed as shown in Figure 5.

Step 2. Deal with random travel time of buses by Monte Carlo Simulation method.

Step 2.1. Perform travel time sampling and generate the travel time of buses $t^R_{j-1,j}$ based on its distribution function.

Step 2.2. Deduce the arrival time $T^A_{i,j}$ and departure time $T^D_{i,j}$ of buses at each stop, and calculate the load of buses $L_{i,j}$ and the number of boarding passengers $N^+_{i,j}$.

Step 2.3. According to Equation (19), calculate and update the holding time $t^{h,(n_{mc})}_{i,j}$ and the final output of holding time is determined by the average value of simulation samples:

$$\bar{t}^{h,n_{mc}}_{i,j} = \frac{t^{h,n_{mc}}_{i,j} + (n_{mc} - 1) \cdot \bar{t}^{h,n_{mc}-1}_{i,j}}{n_{mc}}. \tag{26}$$

Step 2.4. Increase the number of simulations by 1 and determine whether the counter of simulations nmc is less than the maximum number of simulations N_{mc}. If $n_{mc} < N_{mc}$, return to step 2.1; otherwise, stop and output the holding time.

Step 3. Genetic operator.

Step 3.1. According to Equation (25), calculate the value of the objective function C and the fitness of each chromosome, where the fitness is equal to the reciprocal of the objective function 1/C.

Table 2: Statistical analysis of bus running time.

Bus stop	Running time (min) Mean	Minimum	Maximum	Standard deviation (σ)
1 (Fuqiang street)–2	0.5	0.4	0.9	0.11
2-3	1.1	1.0	2.1	0.23
3-4	1.0	0.8	1.5	0.19
4-5	1.1	0.8	1.6	0.21
5-6	1.5	1.3	1.9	0.29
6-7	1.2	0.9	1.6	0.25
7-8	3.6	3.2	4.5	0.71
8-9	1.0	0.9	1.3	0.23
9-10	2.1	1.9	2.9	0.44
10-11	1.4	1.3	2.0	0.26
11-12	1.0	0.8	1.5	0.21
12-13	2.2	1.9	2.8	0.42
13-14	3.5	3.1	4.4	0.68
14-15	1.8	1.6	2.2	0.38
15-16	2.9	2.6	3.5	0.53
16-17	3.5	3.1	4.8	0.72
17-18	2.0	1.7	2.7	0.41
18-19	3.8	3.3	5.2	0.75
19-20 (Changchun Station)	3.5	2.9	4.5	0.69

Step 3.2. If the maximal number of generations is exceeded, then stop; otherwise, go to step 2.

Step 4. Output the optimization program.

5. Numerical Test

5.1. Experimental Surroundings. We take the bus route number 6 of Changchun City in China as a numerical example to test the validity of the reliability optimization model. This bus route runs between uptown and downtown, from the southern terminal in Fuqiang Street to northern terminal at Changchun Railway Station. In actual operation, buses serve each stop without stop-skipping or holding strategy along the route with 9.9km length and 20 stops. At the early peak, the departure interval of buses $\bar{h}$ is 4 min. The number of a bus fleet m is set as 4.

Historical data of passenger demand were collected during the early peak period (7:00 AM–9:00 AM) on weekdays. Via on board surveys of the entire bus route during the peak time period, the historical OD data are collected, which record the boarding and alighting stop of passengers along bus route. The average passenger demand between stops is deduced from historical data, as shown in Figure 6. The running speed of buses is 21 km/h and the statistical results of the running time between adjacent stops based on operation data of 34 buses are shown in Table 2. It is assumed that the running time between stops follows a random distribution. According to Table 2, the mean value and the variance of running time between stops are denoted as $t^R_{j-1,j}$ and σ,

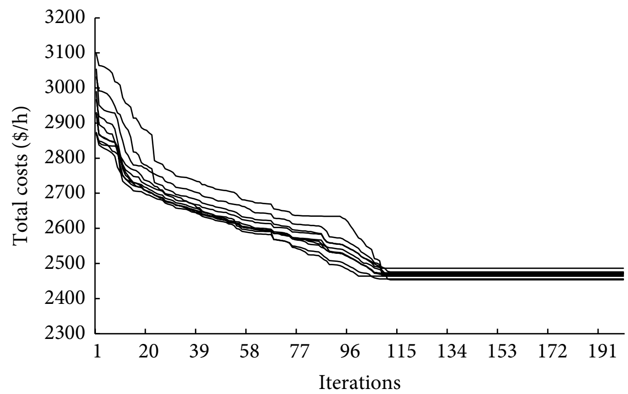

Figure 7: Convergence trend of the solution algorithm.

respectively. Namely, the running time follow a normal distribution $N\left(\hat{t}^R_{j-1,j}\sigma^2_j\right)$. The accelerated speed of buses is 0.77 m/s^2, and the passenger boarding a and alighting times b are 3 s/pax and 1.5 s/pax, respectively. The adjustment factor θ in judgement process of holding control is set as 0.15. According to the practicalities of everyday life and other studies, time values in Equations (21)–(23) are assumed to be $4/pax-h for waiting time c_W, $4/pax-h for in-vehicle time c_{IN}, and $50/veh-h for running time c_R. Noted that, in order to reduce the influence of in-vehicle time during a holding period on total costs, in this paper, the value of waiting time is set equal to that of in-vehicle time.

A mathematical software MATLAB R2011a is applied to process the proposed solution algorithm on a Computer Intel Core I5, 2.4 GHz with 8 GB RAM. The parameters in Genetic Algorithm are taken as $N_G = 200$, $P_c = 0.45$, $P_m = 0.04$, and $M_G = 200$. The maximum number of simulations in Monte Carlo method N_{mc} is set to 100. Figure 7 depicts the convergence of the calculation with experimenting 10 times. The algorithm has good convergence and the optimal solutions of the objective function are found within 106 iterations.

5.2. Experimental Results and Analysis. To evaluate the performance of the proposed reliability optimization strategy, we test three alternative strategies: (1) normal operation (without stop-skipping or holding strategy), (2) stop-skipping only strategy (without holding), and (3) stop-skipping strategy with holding control. The performances of three different strategies that use identical parameters have been evaluated. The first strategy requires all buses to serve each stop, in which $y_{i,j} = 1$ and $t^h_{i,j} = 0$. The second strategy allows buses to skip several stops, in which $t^h_{i,j} = 0$. The third strategy is adopted to improve the reliability for the second one. Under the above three strategies, simulation programs are run ten times and test results are shown in Figure 8. It can be seen that, in terms of optimization of total costs, the stop-skipping bus operation can effectively reduce the total costs of operation system, while the strategy with holding control has a more obvious effect.

The average of test results of each strategy is calculated to evaluate effect on the waiting time cost, in-vehicle time cost and running time cost, as shown in Table 3. Under the stop-skipping only strategy, the costs of waiting time,

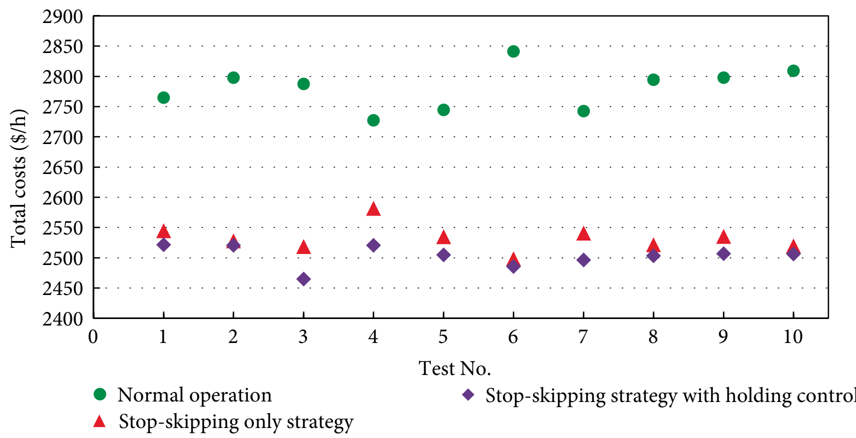

FIGURE 8: Test results of three strategies.

TABLE 3: Effects of the three strategies.

Strategies	Waiting time cost ($/h)	In-vehicle time cost ($/h)	Running time cost ($/h)	Total costs ($/h)	Change of total costs
Normal operation	411.7	1748.6	636.3	2796.6	-
Stop-skipping only strategy	358.4	1600.9	569.1	2528.4	−9.59%
Stop-skipping strategy with holding control	232.4	1673.7	597.1	2503.2	−10.49%

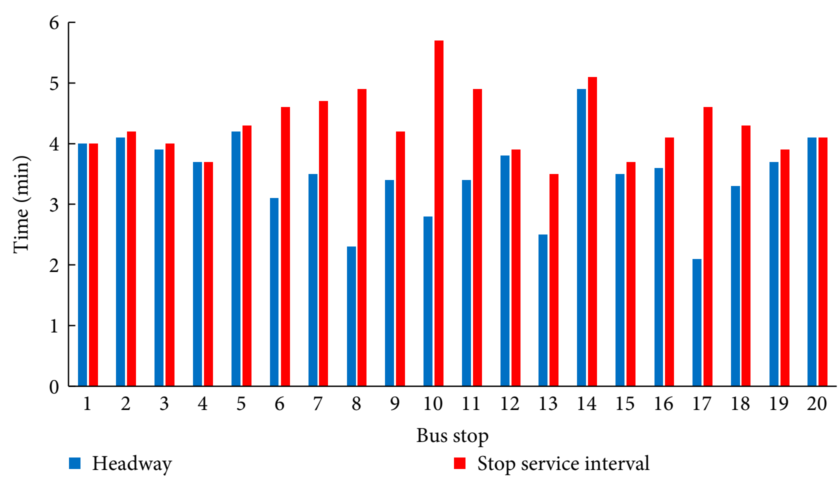

FIGURE 9: Average headway and average stop service interval at a stop.

in-vehicle time and running time all decline, compared with normal operation. Total costs are reduced from 2796.6 $/h to 2528.4 $/h, which can save 9.59%. Meanwhile, the stop-skipping only strategy can shorten the running time of buses, thus decreasing passengers' travel time accordingly, in which in-vehicle time cost drops from 1748.6 $/h to 1600.9 $/h. Furthermore, the holding control is helpful to shorten waiting time of passengers and greatly decrease waiting time cost, although the in-vehicle time cost has gone up slightly. Compared with normal operation, stop-skipping strategy with holding control reduce the total costs by 10.49%.

The proposed holding strategy regards equilibrium of stop service intervals as optimization goal. In order to reflect the effect of holding control on balancing stop service intervals and the relationship between headway and stop service interval, the headway and service interval at each stop are statistically analyzed in the reliability optimization strategy, as shown in Figure 9. It can be seen that the stop service is generally larger than headway of buses at stops, such as stop 8, stop 10, and stop 17, since a percentage of buses is allowed to skip stops. The greater difference between the headway and the stop service interval at a stop, the higher frequency skipping stop will have. Figure 10 shows a comparison of the

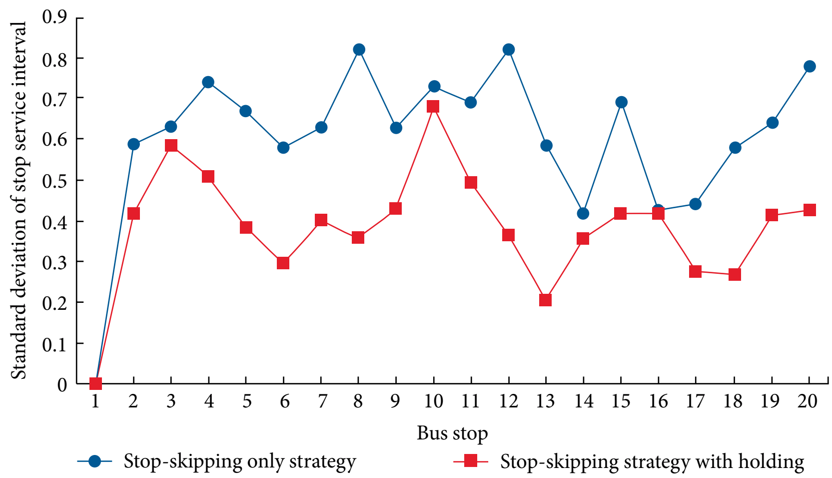

Figure 10: Standard deviation of stop service intervals at a stop.

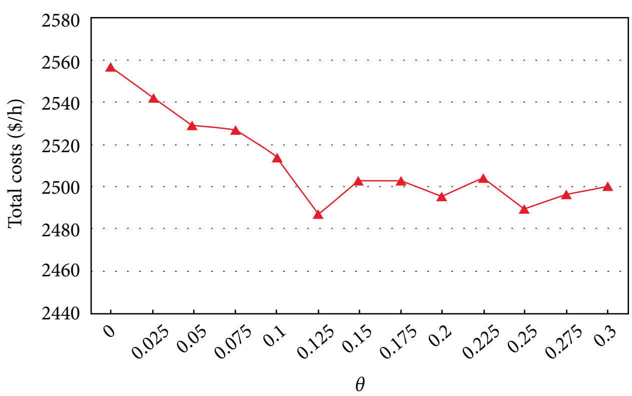

Figure 11: Relationship between total costs and θ.

standard deviation of stop service intervals with or without holding controls and all data are collected under a stop-skipping bus operation. The standard deviation of stop service intervals with a holding control is significantly lower than that without holding control, which indicates that the service interval of each stop under holding strategy is more balanced. Balanced service interval can make the number of passengers gathered at a stop relatively stable and then help to reduce the waiting time of passengers, which is also verified that the waiting time of passengers in Table 3 is reduced under holding control.

In addition, this paper sets a maximum time $\theta \cdot \overline{h}$ in the calculation model of holding time and, by adjusting the value of parameter θ, we can achieve different values of the maximum time. A sensitivity analysis of the total costs with different value of parameter θ is shown in Figure 11. According to Figure 11, the curve is not smooth because running time samples of buses are randomly selected and there is a certain change in the result of each test. When the parameter θ is 0, total costs for buses run with stop-skipping only strategy are the highest. With the increase of the value of parameter θ, total costs of bus operation system gradually decrease. When the value of parameters θ is between 0.125 and 0.3, the total costs tend to be the minimum, which indicates that within this range ($0.125 \leq \theta \leq 0.3$), a suitable holding time can be obtained to balance the service interval of bus stops and save the total cost of bus operation system.

6. Conclusions

In transit system, the reliability of bus operation has attracted more and more attention from operators and users. This paper develops a reliability optimization model to improve the service stability of waiting passengers under the stop-skipping bus operation. Considering the influence of stop-skipping operation on waiting passengers at skipped stops, the stop service interval is proposed. And by a holding control it is balanced to achieve the reliability of bus operation instead of the headway. Then, the boarding process of passengers during serving time and holding time is analyzed with the effect of bus capacity constraints, and the number of left-over passengers and their waiting time are determined in two cases of overload. By minimizing total costs of bus operation system, the objective function is established. Numerical test is conducted by selecting a bus route in Changchun City. Results of the simulation test show that the proposed strategy can reduce the variance of stop service interval and improve the reliability of stop-skipping operation, thereby saving the total costs. Future research for this study may focus on considering the influence of capacity limitation of bus platform on holding time and optimizing holding strategy.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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