Research Article

Doping the Buckminsterfullerene by Substitution: Density Functional Theory Studies of C$_{59}$X (X = B, N, Al, Si, P, Ga, Ge, and As)

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The heterofullerenes C$_{59}$X (X = B, N, Al, Si, P, Ga, Ge, and As) were investigated by quantum chemistry calculations based on density functional theory. These hybrid cages can be seen as doping the buckminsterfullerene by heteroatom substitution. The geometrical structures, relative stabilities, electronic properties, vibrational frequencies, dielectric constants, and aromaticities of the doped cages were studied systematically and compared with those of the pristine C$_{60}$ cage. It is found that the doped cages with different heteroatoms exhibit various electronic, vibrational, and aromatic properties. These results imply the possibility to modulate the physical properties of these fullerene-based materials by tuning substitution elements.

1. Introduction

Fullerenes and related materials have aroused considerable attention since the discovery of buckminsterfullerene C$_{60}$ [1]. During the last two decades, a great number of studies have been carried out to investigate the structures and physical properties of the carbon cages and the derivatives [2–7]. Among various nanostructures derived from fullerenes, the heterofullerenes, in which one or more carbon atoms of the cage are substituted by heteroatoms, have especially caught the eyes of the researchers. The heterofullerenes exhibit unique structural, electronic, and nonlinear optical properties due to the existence of the heteroatom, which are considerably different from those of the pure carbon cages [8–13]. Therefore, heterofullerenes should be interesting new nanoscaled materials to be expected in the future.

In 1991, six years after the experimental discovery of the buckminsterfullerene, Chai et al. [14] claimed the spectroscopic observation of gas-phase formation of heterofullerene ions, indicating that the synthesis of heterofullerenes was achieved. And then, the doped cages obtained by N, B, Si, P, Ge, As, and transition-metal (such as Pt, Fe, and Co) substitution have been reported by several research groups [15–21]. Most recently, N-, P-, and Si- doped single-walled carbon nanotubes (SWCNTs) are also synthesized using chemical vapor deposition method [22].

As for the theoretical side, several literatures have paid attention to the heterofullerenes [9, 11–13, 23–28]. However, most of the studies mainly focus on the geometries and ordinary electronic structures of the doped fullerenes. Up until now a systematic study on the relationship of structure and property for C$_{60}$-based heterofullerenes by a single approach has not been reported according to our best knowledge. Furthermore, the doped carbon cages are good candidates of materials for hydrogen storage, optical device, and molecular sensor [13, 29, 30], and they have become the state-of-the-art research subjects in recent years. Additionally, the heterofullerenes are of prominent importance since they are the building blocks of various polymerized fullerenes structures [9, 11, 21]. Thus, in order to achieve a further understanding
of structure-property relationship of carbon cages and the derivates, it is desirable to study heterofullerenes.

In this paper, we carried out systematic calculations on the heterofullerenes obtained by doping C_{60} cage with B, N, Al, Si, P, Ga, Ge and As atoms by means of the Kohn-Sham self-consistent field method under the framework of density functional theory (DFT).

2. Models and Computational Methods

It is known that the synthesized C_{60} have I_h symmetry and all the 60 carbon atoms are equivalent. The heterofullerene structure C_{59}X, obtained by only one carbon atom of the C_{60} cage substituted by other atoms, is studied in this paper. The atom of several main group elements, including III (B, Al, and Ga), IV (Si and Ge), and V (N, P, and As) subgroups, are considered as the heteroatom to replace the carbon atom of the buckminsterfullerene cage. The obtained C_{59}X cage only reserves a mirror plane, and the symmetry is reduced to C_s. Different spin states for the doped cages are also considered in our calculations. The ground state is treated as the lowest energy structure.

The DFT hybrid functional B3LYP method [31] is adopted to calculate C_{59}X cages. Both the geometrical optimizations and the electronic property calculations through out this paper are all performed using the Kohn-Sham self-consistent field method at B3LYP/6-31G* level with Gaussian 09 program [32]. In the DFT calculations, symmetry constraint is always adopted, and default values of convergence criteria in Gaussian 09 program are used. According to the previous calculations, the B3LYP method has been successfully applied to the theoretical studies on fullerene-based nanostructures [2–5, 8, 13, 23, 27], and the methods used here could give rather good results compared with those obtained by various different functionals and basis sets [33].

3. Results and Discussion

3.1. Structures. The optimized structures of the C_{59}X (X = B, N, Al, Si, P, Ga, Ge and As) and C_{60} cages are shown in Figure 1, and the bond lengths and atomic coordinates are listed in Tables S1 and S2 (see Supplementary Material available online at doi:http://dx.doi.org/10.1155/2013/571709). From

![Figure 1: Structures of C_{59}X (X = B, N, Al, Si, P, Ga, Ge, and As) and C_{60} Cages.](image-url)
The sphericity parameter (SP), cohesive energy ($E_{coh}$), $\Delta E$, replacing energy ($E_{replace}$), relaxing energy ($E_{relax}$) and asphericity parameter (ASP) of C$_{60}$ and the doped cages. ($SP$ in GHz$^{-1}$; $E_{coh}$ in eV/atom; $\Delta E$, $E_{replace}$ and $E_{relax}$ in eV).

<table>
<thead>
<tr>
<th>Cages</th>
<th>SP</th>
<th>$E_{coh}$</th>
<th>$\Delta E$</th>
<th>$E_{replace}$</th>
<th>$E_{relax}$</th>
<th>ASP</th>
</tr>
</thead>
<tbody>
<tr>
<td>C$_{59}$B</td>
<td>0.126</td>
<td>6.779</td>
<td>−0.932</td>
<td>−0.170</td>
<td>−0.762</td>
<td>0.0045</td>
</tr>
<tr>
<td>C$_{59}$Al</td>
<td>1.198</td>
<td>6.697</td>
<td>−1.926</td>
<td>11.839</td>
<td>−13.765</td>
<td>0.0645</td>
</tr>
<tr>
<td>C$_{59}$Ga</td>
<td>3.183</td>
<td>6.692</td>
<td>−1.905</td>
<td>13.122</td>
<td>−15.027</td>
<td>0.0659</td>
</tr>
<tr>
<td>C$_{60}$</td>
<td>0.000</td>
<td>6.813</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.0000</td>
</tr>
<tr>
<td>C$_{59}$Si</td>
<td>1.169</td>
<td>6.733</td>
<td>−1.373</td>
<td>7.958</td>
<td>−9.331</td>
<td>0.0509</td>
</tr>
<tr>
<td>C$_{59}$Ge</td>
<td>3.546</td>
<td>6.724</td>
<td>−1.437</td>
<td>12.550</td>
<td>−13.987</td>
<td>0.0685</td>
</tr>
<tr>
<td>C$_{59}$N</td>
<td>0.055</td>
<td>6.807</td>
<td>0.020</td>
<td>0.144</td>
<td>−0.124</td>
<td>0.0013</td>
</tr>
<tr>
<td>C$_{59}$P</td>
<td>1.273</td>
<td>6.759</td>
<td>−1.235</td>
<td>6.879</td>
<td>−8.114</td>
<td>0.0502</td>
</tr>
<tr>
<td>C$_{59}$As</td>
<td>3.637</td>
<td>6.753</td>
<td>−1.402</td>
<td>13.273</td>
<td>−14.675</td>
<td>0.0736</td>
</tr>
</tbody>
</table>

Figure 1 we can see that all the doped cages undergo some distortions due to the heteroatoms, though they still preserve closed cage structures. Here to evaluate the sphericity of the doped cages, the sphericity parameter (SP) is calculated through the equation [34, 35]:

$$SP = \left[ \left( \frac{1}{A} - \frac{1}{B} \right)^2 + \left( \frac{1}{A} - \frac{1}{C} \right)^2 + \left( \frac{1}{B} - \frac{1}{C} \right)^2 \right]^{1/2},$$

where $A$, $B$, and $C$ are the rotational constants (in GHz) of the corresponding cages. The structure with larger SP value is distorted more away from perfect sphere. As shown in Table 1, SP of C$_{60}$ is zero, and it comes without surprise because C$_{60}$ with $I_h$ symmetry has the perfect sphere shape. As for the doped cages, the values of SP are in the range of 0.055–3.637 GHz$^{-1}$, indicating that deformations of the cage are occurred when the heteroatom is introduced into the pristine cage. It can be seen that SP are about 0.1 GHz$^{-1}$ for C$_{59}$X with X = B and N, while about 1.2 GHz$^{-1}$ for X = Al, Si, and P, and the values of SP are even larger than 3.1 GHz$^{-1}$ for X = Ga, Ge, and As. Thus, it is clearly that the cage with larger heteroatom gives larger SP and more obvious distortion.

Then we pay attention to the bond lengths of the heterofullerenes. It is well-known that there are two kinds of C–C bond in C$_{60}$ cage, the [6, 6] bond and the [5, 6] bond. The bond lengths are 1.395 and 1.454 Å for [6, 6] and [5, 6] bonds, respectively, based on our DFT calculations, which agree well with 1.391 (or 1.39) and 1.455 (or 1.46) Å by neutron diffraction experiments [36, 37]. When the carbon cage is doped by the heteroatom, the C–X bonds are presented. From Table S1, it can be seen that the C–X bond lengths are in the range of 1.404–1.950 Å. The bond lengths increased obviously for X = B, Al, Si, P, Ga, Ge, and As, ranking from 1.526 Å to 1.950 Å. However, the C–N bonds in C$_{59}$N are 1.408 and 1.424 Å, and thus the original [5, 6] bond is even decreased by 0.03 Å compared with that in the pristine cage. It is also found that the C–X bond lengths increase more significantly for the larger heteroatoms. For instance, the C–B bonds are 1.526 and 1.549 Å, while the C–X bonds (X = Al, Si, P) are calculated to be within 1.796–1.904 Å, but the values are even 1.876–1.950 Å for X = Ga, Ge, As. These results also agree with our SP analyses as well as the previous studies [25–28]. Now we focus on the C–C bonds in the doped fullerenes. As shown in Table S1, the original [6, 6] and [5, 6] bonds exhibit slight changes, with the lengths in the range of 1.385–1.427 Å and 1.433–1.517 Å, respectively. Moreover, it also can be seen that the C–C bond lengths change more significantly near the region of the heteroatom, but almost inert in the region away from the heteroatom.

### 3.2. Energies and Relative Stabilities

The doped cages with different spin-multiplicity states are calculated with open-shell DFT B3LYP/6-31G$^*$ method to determine the ground state, and their energies are listed in Table S3 of supplementary material. It can be seen that the high-spin state structures always exhibit higher energies than those of the low-spin states according to the obtained energies (both corrected and uncorrected with zero-point vibrational energies). Thus, the spin multiplicity of the ground state of C$_{59}$X is 1 for IV group elements, and 2 for III and V elements, respectively.

In order to study the thermodynamic stabilities of the doped cages, the cohesive energy ($E_{coh}$) per atom are calculated with the energies corrected with zero-point energy (ZPE), and the obtained results are listed in Table 1 and shown in Figure 2(a). Here the system with larger $E_{coh}$ is more stable. We can see that $E_{coh}$ of C$_{60}$ is calculated to be 6.813 eV/atom, and agrees with the previous results [9, 38]. $E_{coh}$ of the heterofullerene cages are in the range of 6.692–6.807 eV/atom, and slightly smaller than those of the pristine cage. Thus the introduced heteroatoms would decrease the thermodynamic stability of the cages from viewpoint of cohesive energy. From Figure 2(a), we can see that $E_{coh}$ of C$_{59}$X with X = B and N are larger than those of C$_{59}$X with X = Al, Si, P, and even a bit more larger than those of C$_{59}$X with X = Ga, Ge, and As. Therefore, the doped cage with smaller heteroatom is more stable.

The formation of the C$_{59}$X cage can be seen in reaction (2).
Then the energy difference of the above process, $\Delta E$, can be calculated by:

$$\Delta E = E\left(\text{C}_{59}\text{X}\right) + E\left(\text{corannulene}\right) - E\left(\text{C}_{60}\right) - E\left(X-\text{corannulene}\right),$$

(3)

where $E(\text{C}_{59}\text{X})$, $E(\text{C}_{60})$, $E(\text{corannulene})$ and $E(X-\text{corannulene})$, are the energies of the species with the minimum structure, respectively. Furthermore, from theoretical point of view, the formation reaction of $\text{C}_{59}\text{X}$ cage from $\text{C}_{60}$ cage can be considered as two processes. In the first step, one carbon atom of the carbon cage is directly replaced by the heteroatom from the doped corannulene to form a hybrid structure, for which the skeleton of the hybrid cage is still the same as that of the pristine $\text{C}_{60}$ cage. In the next step the hybrid cage is then relaxed to reach its minimum structure. Therefore, the replacing energy ($E_{\text{replace}}$) and the relaxing energy ($E_{\text{relax}}$) for the two processes are defined as follows:

$$E_{\text{replace}} = E\left(\text{C}_{59}\text{X}^*\right) + E\left(\text{corannulene}\right) - E\left(\text{C}_{60}\right) - E\left(X-\text{corannulene}\right),$$

(4)

$$E_{\text{relax}} = E\left(\text{C}_{59}\text{X}\right) - E\left(\text{C}_{59}\text{X}^*\right),$$

where $E(\text{C}_{59}\text{X}^*)$ is the energy of a $\text{C}_{59}\text{X}$ cage with the skeleton the same as that of the pristine $\text{C}_{60}$ cage. Based on (3)-(4), it is easy to get $\Delta E = E_{\text{replace}} + E_{\text{relax}}$. The calculated $\Delta E$, $E_{\text{replace}}$, and $E_{\text{relax}}$ are shown in Table 1 and Figures 2(b)–2(d).

We can see that $\Delta E$ of the doped cages are all negative except for that of $\text{C}_{59}\text{N}$, indicating the formations of the most doped cages are exothermic. Even for $\text{C}_{59}\text{N}$, the obtained $\Delta E$ is only 0.020 eV. Thus it is energetically favorable to form the $\text{C}_{59}\text{X}$ cages from viewpoint of total energy change. The obtained $\Delta E$ are ranged from $-1.926$ to 0.020 eV for the heterofullerenes studied in this paper. From Figure 2(b), we can see that the curves of $\Delta E$ have the same trend as those of the $E_{\text{coh}}$. Thus, contrary to the cohesive energy results, it seems that the formation of the doped cage with larger heteroatom is energetically more favorable from viewpoint of the total energy change of the reaction.

As for $E_{\text{replace}}$ of the heterofullerenes, $\text{C}_{59}\text{B}$ gives $-0.170$ eV, but others all exhibit positive values in the range of 0.144–13.273 eV. This fact means that the directly replacement of a carbon atom by a heteroatom is an endothermic process for most of the doped cages. From Figure 2(c) for $E_{\text{replace}}$ of $\text{C}_{59}\text{X}$ where $X$ belongs to III, IV and V Groups, it can be seen that $E_{\text{replace}}$ decrease monotonically for each group. Furthermore, $E_{\text{replace}}$ of $\text{C}_{59}\text{B}$ and $\text{C}_{59}\text{N}$ are the smallest among the eight doped cages studied in this paper. This is because the C–B and C–N bond lengths are more close to that of the C–C bond compared with those of other C–X bonds. Thus $\text{C}_{59}\text{N}$ has the least $E_{\text{replace}}$ and $\text{C}_{59}\text{B}$ even exhibits exothermic replace process.

Now we turn to the relaxing energy, $E_{\text{relax}}$. From the obtained $E_{\text{relax}}$ shown in Table 1 and Figure 2(d), it can be seen that all obtained $E_{\text{relax}}$ of the doped cages are negative, indicating that the relaxing effect is exothermic. The calculated $E_{\text{relax}}$ are ranged from $-0.124$ to $-15.027$ eV for the heterofullerenes.

Since the doped cage becomes distorted structure from a perfect ball in the relaxing process, the asphericity parameter, ASP, is calculated to evaluate the geometrical distortion for the doped cages. ASP is introduced by Fowler et al. and can be calculated by [39]:

$$\text{ASP} = \sum \frac{(R_i - R_0)^2}{R_0^2},$$

(5)

where $R_i$ is the radial distance of atom $i$ from the cage central of mass, and $R_0$ is the average radius. Here the structures with smaller ASP values are more close to a perfect sphere shape. The obtained ASP is listed in Table 1. It can be seen that the calculated ASP are all nonzero for the heterofullerenes, with the values in the range of 0.001–0.074. This result also confirms the distortions of the cage away from the perfect sphere. Furthermore, it seems that ASP of the cages has something to do with $E_{\text{relax}}$. Here the doped cage with larger ASP values gives more negative $E_{\text{relax}}$. For instance, $\text{C}_{59}\text{N}$ and $\text{C}_{59}\text{B}$ have the smallest ASP (less than 0.005) and the $E_{\text{relax}}$ are only $-0.124$ and $-0.762$ eV, respectively. $\text{C}_{59}\text{Si}$ and $\text{C}_{59}\text{P}$ have larger ASP (about 0.05) and the $E_{\text{relax}}$ are also more negative (about $-8$ and $-9$ eV). For $\text{C}_{59}\text{Al}$, $\text{C}_{59}\text{Ga}$, $\text{C}_{59}\text{Ge}$, and $\text{C}_{59}\text{As}$, they have the largest ASP (larger than 0.065) and also the most negative $E_{\text{relax}}$ (less than $-13$ eV).

3.3. Electronic Properties. It is well-known that the frontier orbitals, the highest occupied molecular orbital (HOMO), and the lowest unoccupied molecular orbital (LUMO) play...
an important role in chemical reaction for the reactant molecule, thus the frontier orbital analysis of the doped cages is necessary. In Table 2, we summarize the HOMO and LUMO energy levels of the heterofullerenes. It can be seen that HOMO levels of the doped cages are all increased compared with that of C\(_{60}\). However, the HOMO levels all vary not very large, except for that of C\(_{59}\)_N, which is increased by 1.4 eV. As for LUMO levels, they are decreased by about 0.5 eV when doping with Si and Ge atoms, but nearly unchanged for doping with other atoms compared with that of the pristine cage.

Figure S2 in Supplementary Material shows the distributions of HOMO and LUMO for the cages studied in this paper. It can be seen that the frontier orbitals of C\(_{60}\) cage are rather delocalized and spread over the whole surface of the cage. However, the frontier orbitals of several doped cages become localized obviously due to the present of the heteroatoms. For example, the contributions from the Si, Ga, and Ge atoms are as large as 27.4%, 16.4%, and 35.3% to the HOMO, while 24.3%, 42.6% and 22.4% to LUMO, respectively according to our quantitative evaluations.

It is known that both the thermodynamic stability and kinetic stability have crucial influence on the relative abundances of different fullerene structures. It has been pointed out that higher kinetic stability is usually related with a larger HOMO-LUMO energy gap (\(E_{\text{g}}\)) [40], because exciting electrons from a low HOMO to a high LUMO is energetically unfavorable, which would be necessary to activate a reaction. The calculated \(E_{\text{g}}\) of the doped cages are listed in Table 2. It can be found that all the doped cages present smaller \(E_{\text{g}}\) than that of C\(_{60}\) cage. Thus kinetic stability of the cage is decreased by substitution from viewpoint of HOMO-LUMO gap.

Since the charge transport is one of the central issues for the performance of organic electronic devices, here the exciton binding energy (\(E_{\text{b}}\)) is calculated to understand more about the transport properties of the doped fullerenes. Physically, the exciton binding energy can be seen as the energy required to decompose an exciton into a free electron and hole in the solid, and is defined as follows:

\[
E_{\text{b}} = E_{\text{t}} - E_{\text{opt}},
\]

where \(E_{\text{t}}\) is the transport gap and \(E_{\text{opt}}\) is the optical gap. \(E_{\text{t}}\) can be treated as the orbital energy difference between the LUMO and the HOMO [41]. As for \(E_{\text{opt}}\), it is calculated to be as the allowed lowest singlet optical transition energy with nonzero oscillator strength obtained by the time-dependent DFT (TD-DFT) calculations at B3LYP/6-31G* theory level in this paper. Here the obtained \(E_{\text{opt}}\) of C\(_{60}\) is 2.099 eV, which agrees with 1.95 eV by experiment of optical absorption spectrum.
Table 2: The obtained HOMO, LUMO, $E_g$, $E_{opt}$, $E_b$, $\alpha$, $\nu$, $\varepsilon$ and NICS of C$_{60}$ and the doped cages (HOMO, LUMO, $E_g$, $E_{opt}$ and $E_b$ in eV, and $\alpha$ and $\nu$ in $\AA^2$).

<table>
<thead>
<tr>
<th>Cages</th>
<th>HOMO</th>
<th>LUMO</th>
<th>$E_g$</th>
<th>$E_{opt}$</th>
<th>$E_b$</th>
<th>$\alpha$</th>
<th>$\nu$</th>
<th>$\varepsilon$</th>
<th>NICS</th>
</tr>
</thead>
<tbody>
<tr>
<td>C$_{59}$B</td>
<td>−5.660</td>
<td>−3.225</td>
<td>2.435</td>
<td>0.807</td>
<td>1.629</td>
<td>71.75</td>
<td>631.96</td>
<td>3.72</td>
<td>−5.94</td>
</tr>
<tr>
<td>C$_{59}$Al</td>
<td>−5.377</td>
<td>−3.120</td>
<td>2.257</td>
<td>1.018</td>
<td>1.239</td>
<td>75.48</td>
<td>640.92</td>
<td>3.92</td>
<td>−5.62</td>
</tr>
<tr>
<td>C$_{59}$Ga</td>
<td>−5.442</td>
<td>−3.240</td>
<td>2.202</td>
<td>0.964</td>
<td>1.238</td>
<td>75.77</td>
<td>640.09</td>
<td>3.95</td>
<td>−5.65</td>
</tr>
<tr>
<td>C$_{60}$</td>
<td>−5.988</td>
<td>−3.223</td>
<td>2.765</td>
<td>2.099</td>
<td>0.666</td>
<td>69.45</td>
<td>622.93</td>
<td>3.63</td>
<td>−2.72</td>
</tr>
<tr>
<td>C$_{59}$Si</td>
<td>−5.817</td>
<td>−3.649</td>
<td>2.168</td>
<td>1.646</td>
<td>0.523</td>
<td>74.98</td>
<td>641.32</td>
<td>3.88</td>
<td>−2.80</td>
</tr>
<tr>
<td>C$_{59}$Ge</td>
<td>−5.887</td>
<td>−3.769</td>
<td>2.118</td>
<td>1.509</td>
<td>0.608</td>
<td>75.81</td>
<td>648.08</td>
<td>3.88</td>
<td>−3.18</td>
</tr>
<tr>
<td>C$_{59}$N</td>
<td>−4.584</td>
<td>−3.295</td>
<td>1.290</td>
<td>0.664</td>
<td>0.626</td>
<td>71.38</td>
<td>616.53</td>
<td>3.82</td>
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<tr>
<td>C$_{59}$P</td>
<td>−5.202</td>
<td>−3.252</td>
<td>1.950</td>
<td>1.275</td>
<td>0.675</td>
<td>73.12</td>
<td>632.85</td>
<td>3.81</td>
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</tr>
<tr>
<td>C$_{59}$As</td>
<td>−5.247</td>
<td>−3.220</td>
<td>2.027</td>
<td>1.339</td>
<td>0.688</td>
<td>73.70</td>
<td>637.16</td>
<td>3.82</td>
<td>−5.25</td>
</tr>
</tbody>
</table>

that the shapes of IR absorption spectra are different for the different doped cages. These characteristic features in the IR spectra could be helpful to identify these heterofullerenes from the experimental spectra.

3.5. Dielectric Constant. The dielectric constant is one of the important parameters for the materials of organic solid. In this paper a simple model based on the Clausius-Mossotti equation [49] is adopted. This model has been used to successfully evaluate the dielectric constant of C$_{60}$ and conjugated organic molecules [49, 50]. Under the framework of the Clausius-Mossotti model, the dielectric constant, $\varepsilon$, can be expressed as:

\[ \varepsilon = \frac{1 + 8\pi\alpha/3\nu}{1 - 4\pi\alpha/3\nu}, \]  

where $\alpha$ is the first order polarizability with $\alpha = (1/3) \sum \alpha_i$, in which $\alpha_i$ are the diagonal matrix elements of the tensor. $\nu$ is the volume occupied by a single molecule (tight option was taken for better accuracy). $\varepsilon$ of C$_{60}$ is calculated to be 3.63 in this paper, which is comparable with previous experimental [51] and theoretical results [49]. From Table 2, we can see that $\varepsilon$ of the doped cages are in the range of 3.72–3.95. Thus the substituted doping could increase $\varepsilon$ of the cages. It is also found that for C$_{60}$X cage $\varepsilon$ is increased by 2.8–9.2% by substituted doping compared with that of C$_{60}$. However, doping with N atom even decreased $\varepsilon$ by 1.0%, and for other heteroatoms $\varepsilon$ is only increased by 1.5–4.0%. Recall that larger $\varepsilon$ is related with larger $\alpha$ and smaller $\nu$ values according to (7). As a result, all the doped cages exhibit larger dielectric constant than that of C$_{60}$.

3.6. Aromaticity and Nuclear Independent Chemical Shift. Aromaticity can be explained by the ring current theory and is a significant concept in chemistry. In this paper the aromaticity of the cages is evaluated by using the nuclear independent chemical shift (NICS), which has proven to be a simple and efficient aromaticity probe [52–54]. The NICS is defined as the negative of the isotropic magnetic shielding constant of a ghost atom located at the central of the cage. Negative NICS value means the aromaticity of the cage. In
In this study, the NICS values listed in Table 2 is computed with the gauge-including atomic orbital (GIAO) method at B3LYP/6-31G* theory level. NICS of \( C_{60} \) we obtained is \(-2.72\), which indicates the weak aromaticity of \( C_{60} \) and also agrees well with \(-2.8 \) by previous DFT calculation \([55]\). From Table 2, it can be seen that all the doped cages in this paper give negative NICS. Thus the eight doped cages studied are all aromatic. Among them, \( C_{59}\)N cage has the NICS of \(-2.41\), indicating it is slightly less aromatic than \( C_{60} \). As for other doped cages, the obtained NICS are more negative and thus they are more aromatic than the pristine cage, though the difference is small. Additionally, the \( C_{59}\)B cage gives the most negative NICS, though it is only a bit distorted from the perfect sphere shape according to the SP and ASP analysis. Therefore, it seems that there is no uniform correlation between the aromaticity and the sphere shape for the doped fullerenes.

Since it has been pointed that NICS at the cage centers have essentially the same values as the endohedral helium chemical shifts \([55, 56]\), these obtained values are also helpful for the possible characterization of these doped fullerene cages.

4. Conclusion

Theoretical studies of the \( C_{59}\)X (X = B, N, Al, Si, P, Ga, Ge, and As) have been performed systemically based on the DFT calculations. The results of the geometrical structures, relative stabilities, electronic properties, vibrational frequencies, dielectric constants, and the aromaticities of the doped cages were discussed to achieve a further understanding of structure-property relationship of the doped cages. It is found that the hybrid cages undergo some distortions due to the substitution of the heteroatoms. According to the calculated cohesive energies, the \( C_{59}\)X cage with smaller heteroatom is more stable. HOMOs of the heterofullerenes are all increased, but the HOMO-LUMO gaps are decreased compared with those of the \( C_{60} \). As for the exciton binding energy, the cages doped with III Group elements are obviously larger than other cages. The calculations also indicate that doping \( C_{60} \) by substitution would give larger dielectric constant due to the increased polarizability. The obtained NICS show that most of the doped fullerenes are lightly more aromatic than the pristine cage. However, no correlation between the aromaticity and the sphere shape is found for the doped cages.

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