Research Article

Effect of Matrix-Wellbore Flow and Porosity on Pressure Transient Response in Shale Formation Modeling by Dual Porosity and Dual Permeability System

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A mathematical dual porosity and dual permeability numerical model based on perpendicular bisection (PEBI) grid is developed to describe gas flow behaviors in shale-gas reservoirs by incorporating slippage corrected permeability and adsorbed gas effect. Parametric studies are conducted for a horizontal well with multiple infinite conductivity hydraulic fractures in shale-gas reservoir to investigate effect of matrix-wellbore flow, natural fracture porosity, and matrix porosity. We find that the ratio of fracture permeability to matrix permeability approximately decides the bottom hole pressure (BHP) error caused by omitting the flow between matrix and wellbore and that the effect of matrix porosity on BHP is related to adsorption gas content. When adsorbed gas accounts for large proportion of the total gas storage in shale formation, matrix porosity only has a very small effect on BHP. Otherwise, it has obvious influence. This paper can help us understand the complex pressure transient response due to existence of the adsorbed gas and help petroleum engineers to interpret the field data better.

1. Introduction

Shale formation has both free gas and adsorbed gas [1], which usually is described by Langmuir isotherm equation and affects pressure behaviors and production performance significantly. Another characteristic of shale formation is the ultralow permeability. Because of that, flow in shale-gas reservoir leads to several transport mechanisms, such as slip, transition, and molecular diffusion, which depends on the value of Knudsen number [2–7].

Due to the incomplete development of the natural fracture network in shale, natural fracture network is not well interconnected and cannot form effective flow channels. In most cases, economic production is possible only if a very complex hydraulic fracture network is created that effectively connects a huge reservoir surface area to the wellbore [8]. Therefore, the shale formation consists of matrix, natural fracture, and hydraulic fracture. Many researchers use analytical or semianalytical solution of dual porosity model to study the pressure transient behaviors or production data analysis for fractured horizontal wells [9–13]. Clarkson et al. [14] and Yan et al. [15] use numerical simulation for flow behaviors in single porosity or three porosities model to study flow in shale gas. In order to decrease capillary pressure to improve gas flow, wettability alteration by chemical treatments is proposed [16].

In the above studies, the dual porosity and dual permeability (DPDP) model, in which there are flow in matrix blocks and fracture blocks and flow between them, is seldom mentioned.

In this paper, we first establish a mathematical dual porosity and dual permeability shale-gas flow model incorporating the gas retention and slip flow mechanism. Then the nonlinear pressure equation is solved by numerical way based on PBEI grid. Lastly, we use the numerical model to study the effect of flow between matrix and fracture, effect of natural fracture porosity, and effect of matrix porosity on pressure transient response for wells in shale-gas reservoir.
2. Flow Model Scheme of Shale Gas

2.1. Flow Model. Taking the adsorbed gas as accumulation term and using apparent permeability to describe the slip flow, we can obtain the following equation for matrix system and fracture system, respectively:

\[
\nabla \left[ \frac{k_m}{\mu_g B_g} \left( \nabla p_m - \gamma_0 \nabla Z \right) \right] - \alpha k_m \frac{p_m - p_f}{\mu_g B_g} = \frac{\partial}{\partial t} \left( \phi_m \right) + \frac{\partial}{\partial t} \left( \rho_3 V_{\text{ads}} \right) + q_{gsc,m},
\]

(1)

\[
\nabla \left[ \frac{k_f}{\mu_g B_g} \left( \nabla p_f - \gamma_0 \nabla Z \right) \right] + \alpha k_m \frac{p_m - p_f}{\mu_g B_g} = \frac{\partial}{\partial t} \left( \phi_f \right) + q_{gsc,f},
\]

(2)

where subscript \(m\) denotes matrix system, subscript \(f\) denotes natural fracture system, \(\rho_3\) is shale density in Kg/m\(^3\), \(B_g\) is formation volume factor of gas in fraction, \(\phi\) is porosity in fraction, \(V_{\text{ads}}\) is adsorption content in m\(^3\)/Kg, and \(q_{gsc,m}\) and \(q_{gsc,f}\) are volume flow rate of the well divided by the volume of the well grid in 1/s.

Many attempts have been made in the past to define the shape factor. The classical formula is given by Kazemi [17] as follows:

\[
\alpha = 4 \left( \frac{1}{L_x^2} + \frac{1}{L_y^2} + \frac{1}{L_z^2} \right),
\]

(3)

where \(L_x\), \(L_y\), and \(L_z\) are the natural fracture spacing in \(x\), \(y\), and \(z\) directions in meter, respectively, and \(\alpha\) is the shape factor to describe the interporosity flow between matrix and natural fracture. In this paper, for simplicity, we suppose that \(L_x\), \(L_y\), and \(L_z\) are equal and just use \(L_x\) to express the natural fracture spacing.

Let \(Q\) in m\(^3\)/s be the production of the well at surface condition, and considering wellbore storage, the flow equation in the wellbore is given by

\[
\frac{2\pi kh}{\mu_g B_g \left[ \ln \left( \frac{r_f}{r_w} \right) + S \right]} \left( p_i - p_{\text{wi}} \right) - \frac{C}{\Delta t} \left( p_{m,i} - p_{m,i}^n \right) = Q,
\]

(4)

where \(p_{\text{wi}}\) is BHP in Pa, \(r_f\) is equivalent well radius in m and \(r_w\) is actual well radius in m, \(C\) is wellbore storage in m\(^3\)/Pa, and \(n\) is time step.

2.2. Apparent Permeability. In order to describe the flow mechanism caused by the ultralow permeability, the intrinsic permeability in (1) and (2) is replaced by apparent permeability, which can be described with Knudsen number for the slip flow [18] as follows:

\[
k = k_0 \left( 1 + \frac{4Kn}{1 + Kn} \right).
\]

(5)

According to the definition of mean free path of gas molecules, Knudsen number is expressed as

\[
Kn = \frac{\mu}{dp} \sqrt{\frac{\pi RT}{2M}}
\]

(6)

where \(\mu\) is the gas viscosity in Pa·s, \(R = 8314\) J/kmol/K is the universal gas constant, \(T\) is the absolute temperature in K, and \(M\) is the molecular mass of gas in kg/kmol. Civan [4] estimates the equivalent hydraulic radius:

\[
d = 2\sqrt{2\tau} \frac{k}{\phi}
\]

(7)

where \(\tau\) is the tortuosity and \(\phi\) is the porosity of porous media.

According to Schaaf and Chambre [19], free molecular flow happens if \(Kn > 10\), when collision between the gas molecules and the pore wall dominates. For \(0.1 \leq Kn \leq 10\), it is considered as transition regime. For \(10^{-3} < Kn < 0.1\), the flow at the wall cannot be neglected, and slip flow exists. For \(Kn < 10^{-3}\), collision between molecules and the pore wall can be neglected, and the flow behaviors can be described by Darcy's law. In this paper, the flow regime is limited to slip flow.

2.3. Isotherm Sorption of Gas. The amount of adsorption in (1) can be modeled by the Langmuir isotherm equation:

\[
V_{\text{ads}} = \frac{V_L p}{p_L + p},
\]

(8)

where \(V_{\text{ads}}\) is the standard volume of gas adsorbed per unit shale mass in m\(^3\)/Kg under the pressure \(p\), \(V_L\) denotes the ultimate adsorption amount in m\(^3\)/Kg, \(p\) is the gas pressure in Pa, and \(p_L\) is the pressure when adsorption reaches half of the ultimate adsorption amount in Pa.

3. Numerical Calculation Scheme

PEBI grids can overcome structure grid disadvantages, such as inflexibility in description of geologic features, inaccurate representation of multiwell locations, and grid orientation, and are especially widely used in numerical well test [20–23]. PEBI grid is actually Voronoi block, which is defined as the region of space that is closer to its grid point than to any other grid points, and its boundaries are normal to lines joining the nodes on the two sides of each boundary.

Figure 1(a) gives the grids of horizontal well with 5 hydraulic fractures. Figure 1(b) gives the fracture grids and horizontal well grids in detail. In order to capture the characteristics of the transient flow, the grid points are distributed by radial growth near the well and fractures. The well and fractures are modeled as grids and treated as infinite conductivity [24].

Equations (1), (2), and (4) form an equation system. There are a total of 2 variables of \(p_f\), \(p_m\) for a grid and one variable for one well producing under constant flow rate.

Nonlinear pressure equations (7) and (8) are implicitly solved by using the matrix solver GMRES (generalized minimal residual method) [25].
Fractured horizontal well
(a) Grid
Hydraulic fractures
Horizontal well
(b) Part of the horizontal well and fractures

Figure 1: Hydraulically fractured horizontal well grid without cell points in 2D form.

Table 1: Data used in validation.

<table>
<thead>
<tr>
<th>Name</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir size</td>
<td>810 × 810</td>
<td>m</td>
</tr>
<tr>
<td>Boundary condition</td>
<td>Closed</td>
<td></td>
</tr>
<tr>
<td>Initial pressure</td>
<td>20</td>
<td>MPa</td>
</tr>
<tr>
<td>Reservoir thickness</td>
<td>30</td>
<td>m</td>
</tr>
<tr>
<td>Rock compressibility</td>
<td>0.0000145</td>
<td>1/MPa</td>
</tr>
<tr>
<td>Temperature</td>
<td>122</td>
<td>°F</td>
</tr>
<tr>
<td>Gas gravity</td>
<td>0.7</td>
<td></td>
</tr>
<tr>
<td>Half of hydraulic fracture</td>
<td>75</td>
<td>m</td>
</tr>
<tr>
<td>Natural fracture space</td>
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<td>m</td>
</tr>
<tr>
<td>Fracture</td>
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<td>Permeability</td>
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<td>mD</td>
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<tr>
<td>Porosity</td>
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<tr>
<td>Matrix</td>
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<td></td>
</tr>
<tr>
<td>Permeability</td>
<td>0.1 mD</td>
<td></td>
</tr>
<tr>
<td>Porosity</td>
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<td></td>
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<tr>
<td>Horizontal well</td>
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<td></td>
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<tr>
<td>Skin factor</td>
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<td></td>
</tr>
<tr>
<td>Well storage</td>
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<td>m³/MPa</td>
</tr>
<tr>
<td>Constant rate (STC)</td>
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<td>Day</td>
</tr>
<tr>
<td>Production time</td>
<td>200 Day</td>
<td></td>
</tr>
<tr>
<td>Deviation factor Z</td>
<td>Dranchuk-Abou-Kassem equation</td>
<td></td>
</tr>
<tr>
<td>Gas viscosity</td>
<td>Lee equation</td>
<td></td>
</tr>
</tbody>
</table>

4. Simulation and Analysis

4.1. Validation. We use the IMEX model of STARS software to validate our core codes. STARS is a commercial software developed by CMG and is widely accepted in petroleum industry. The code we used here to compare with STARS (2012 version) is only the conventional mass balance equation containing gas phase flowing in DPDP and without adsorption and apparent permeability correction. The conventional balance equation is the core part of our model. Our code will be convincing if the core part of our model is validated.

The horizontal wellbore only connects the fractures in this paper; thus, horizontal well with one fracture is equivalent to fractured vertical well. In our numerical code, the well and hydraulic fracture are treated as inner boundary.

The grid number is 81 × 81 in IMEX model. The size of grid is 10 m. The blocks including the hydraulic fracture are subdivided into 11 × 3 grids. The main parameters used in the validation are shown in Table 1.

The well is in the middle of reservoir and produces gas at a constant flow rate of 12000 m³/day in a closed boundary reservoir. The BHP is compared between our code and STARS as shown in Figure 2. The almost overlap of the two output curves validates our code.

4.2. Reservoir Parameters and Grid. Table 2 gives the main parameters used for the simulation.

Table 3 gives parameters about the DPDP and hydraulic fractures. The total simulation time is 20000 days (about 54.8 years). Gas viscosity and volume formation factor are
Table 2: Simulation parameters (if not specified additionally).

<table>
<thead>
<tr>
<th>Name</th>
<th>Value</th>
<th>Unit</th>
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</thead>
<tbody>
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<td>Reservoir size</td>
<td>80000 × 80000</td>
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<tr>
<td>Initial pressure</td>
<td>20</td>
<td>MPa</td>
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<tr>
<td>Reservoir thickness</td>
<td>30</td>
<td>m</td>
</tr>
<tr>
<td>Rock compressibility</td>
<td>0.00015</td>
<td>1/MPa</td>
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<tr>
<td>Langmuir adsorption</td>
<td>0.02</td>
<td>Std m³/Kg</td>
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<tr>
<td>Langmuir pressure</td>
<td>7.5</td>
<td>MPa</td>
</tr>
<tr>
<td>Horizontal well</td>
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<td></td>
</tr>
<tr>
<td>Skin factor</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Well storage</td>
<td>10</td>
<td>m³/MPa</td>
</tr>
<tr>
<td>Constant rate</td>
<td>10000</td>
<td>m³/day</td>
</tr>
<tr>
<td>Production time</td>
<td>10000</td>
<td>Day</td>
</tr>
<tr>
<td>Shut time</td>
<td>10000</td>
<td>Day</td>
</tr>
<tr>
<td>Deviation factor Z</td>
<td>Dranchuk-Abou-Kassem equation</td>
<td></td>
</tr>
<tr>
<td>Gas viscosity</td>
<td>Lee equation</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: DPDP data (if not specified additionally).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_m$</td>
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</tr>
<tr>
<td>$\phi_m$</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>$k_f$</td>
<td>0.1 mD</td>
<td></td>
</tr>
<tr>
<td>$\phi_f$</td>
<td>0.001</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Error percentage of BHP without flow between wellbore and matrix.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Q (m³/day)</th>
<th>$L_x$</th>
<th>$k_f$</th>
<th>Error percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 3(a)</td>
<td>10000</td>
<td>1 m</td>
<td>10k_m</td>
<td>9.03%</td>
</tr>
<tr>
<td>Figure 4(a)</td>
<td>10000</td>
<td>10 m</td>
<td>10k_m</td>
<td>9.26%</td>
</tr>
<tr>
<td>Figure 5(a)</td>
<td>50000</td>
<td>10 m</td>
<td>10k_m</td>
<td>12.6%</td>
</tr>
<tr>
<td>Figure 3(b)</td>
<td>10000</td>
<td>1 m</td>
<td>100k_m</td>
<td>1.004%</td>
</tr>
<tr>
<td>Figure 4(b)</td>
<td>10000</td>
<td>1 m</td>
<td>100k_m</td>
<td>1.028%</td>
</tr>
<tr>
<td>Figure 5(b)</td>
<td>50000</td>
<td>10 m</td>
<td>100k_m</td>
<td>1.19%</td>
</tr>
</tbody>
</table>

The BHP error is calculated with PVT [26, 27] and are changing with pressure and obtained through iterative process.

4.3. Effect of Flow between Matrix and Wellbore with Adsorption. In Figure 3, ultimate adsorption capacity ($V_L$) is 0.02 m³/Kg, and constant flow rate (Q) is 10000 m³/day. The other parameters are shown in the figures.

Figure 3(a) shows that when fracture permeability is 10 times of the matrix permeability, the BHP difference at 10000th day is about 0.11 MPa. The pressure change is about 1.157 MPa under dual well model. If well-matrix flow is omitted, the BHP error is 9.03% (Table 4).

When the fracture permeability is 100 times of the matrix permeability (Figure 3(b)), the BHP difference at 10000th day is 0.0146 MPa. If matrix-wellbore flow is omitted, the BHP error is 1.004% (Table 4).

By comparison of the BHP error with the ratio of $k_f/k_m$, we know that the ratio of the matrix permeability to the fracture permeability approximately decides the BHP error caused by omitting the matrix-wellbore flow.

The corresponding pressure change and pressure derivative comparison is shown in Figures 3(c) and 3(d).

In Figure 4, except the natural fracture spacing of $L_x = 10$ m, other data are the same as data in Figure 3, from which we can see the influence of interporosity flow ability on the BHP error. The BHP error for $k_f = 10k_m$ in Figure 4(a) is 9.26% and BHP error for $k_f = 100k_m$ in Figure 4(b) is 1.028%, which is shown in Table 4. The BHP errors in Figure 4 are correspondingly larger than that in Figure 3, which is caused by smaller interporosity ability.

When interporosity ability is small, due to the large permeability of fracture system, some gas needs to flow from matrix system to fracture system. If the interporosity ability is not large enough, larger pressure difference between matrix and fracture is required, which causes larger error if the matrix-wellbore flow is omitted. Therefore, the low interporosity flow ability can enlarge the error when we omit the matrix-wellbore flow.

In Figure 5, the constant flow rate increases from 10000 m³/day to 50000 m³/day compared to the parameters in Figure 4. In Figure 5(a), the fracture permeability is 10 times of the matrix permeability, and the BHP difference at 10000th day is 0.77247 MPa. The BHP error is 12.6% if matrix-wellbore flow is neglected. In Figure 5(b), the fracture permeability is 100 times of the matrix permeability, the BHP difference at 10000th day is about 0.09457 MPa, and the BHP error is 1.1924% if the flow between wellbore and matrix is omitted.

Compared to the value of BHP error in Figure 4 (see Table 4), the BHP error under $Q = 50000$ m³/day increases, which shows that large flow rate leads to the large BHP error. We also find that the increase of BHP error under $k_f = 10k_m$ is larger than the increase of BHP error under $k_f = 100k_m$, which shows that large ratio of $k_f/k_m$ decreases BHP error if matrix-wellbore flow is omitted.

Table 4 gives the BHP error comparison for Figures 3, 4, and 5. When natural fracture spacing increases from 1 m to 10 m under $Q = 10000$ m³/day, the BHP error increases from 9.03% to 9.26% under $k_f = 10k_m$, which means that BHP error increases with decrease of interporosity flow ability.

When constant flow rate increases 5 times, the BHP error increases from 9.26% to 12.6% under $k_f = 100k_m$, while the BHP error only increases from 1.028% to 1.19% under $k_f = 100k_m$, which has a smaller increase rate than that under $k_f = 10k_m$. Therefore, when ratio of $k_f/k_m$ is large, neglecting of the matrix-wellbore flow only causes very small BHP error. However, if ratio of $k_f/k_m$ is relatively large, the matrix-wellbore flow should not be omitted.
4.4. Effect of Porosity on Transient Pressure Response with Adsorption Gas

4.4.1. Effect of Fracture Porosity on Transient Pressure Response. Figure 6 shows the effect of natural fracture porosity on pressure transient responses for a horizontal well with 3 hydraulic fractures with ultimate adsorption capacity of $V_L = 0.1 \text{ m}^3/\text{Kg}$ and natural fracture spacing of $L_x = 1 \text{ m}$.

The effect of fracture porosity on BHP is so small that it is not easy to identify the difference in Figure 6(a). From pressure derivative curves in Figure 6(b), we can see the difference of the early linear flow regime. However, the time of the difference only lasts 0.01 days. When fracture porosity is 0.0001, the early linear flow regime is obvious, while the early linear flow regime is not clear under fracture porosity of 0.001. This is because the natural fracture system is the main flow channel. Larger fracture porosity can make the linear flow in nature fracture last longer and linear flow behavior becomes obvious.

4.4.2. Effect of Matrix Porosity on Transient Pressure Response. Figure 7 shows the effect of matrix porosity on pressure transient responses of a horizontal well with 3 hydraulic fractures for ultimate adsorption capacity of $V_L = 1 \text{ m}^3/\text{Kg}$, natural fracture spacing of $L_x = 1 \text{ m}$.

Figure 7 shows that the matrix porosity has minimal effect on BHP when the gas ultimate adsorption capacity is $1 \text{ m}^3/\text{Kg}$. Matrix system is the storage space and its porosity should affect behaviors of BHP. However, in shale gas, the situation is different. When pressure drops, the adsorbed
gas desorbed. If the ultimate adsorption capacity is big, the magnitude of the free gas obtained from the adsorption gas is much bigger than the free gas from the matrix porosity, which 

reduces the effect of the matrix porosity.

When the ultimate adsorption capacity is small, the desorbed gas will have less effect on the BHP, and the effect of the matrix porosity on BHP will appear. Figure 8 gives effect of matrix porosity on pressure response with ultimate adsorption capacity of 0.05 m$^3$/Kg. With small adsorption content, the BHP difference in Figure 8(a) is 0.28 MPa at the end of the production period due to the porosity difference between matrix systems. When adsorption content decreases, the free gas will account for more proportion of the flow rate. Therefore, matrix porosity has more influence on the pressure transient response.

Figure 9 gives the case of no adsorption gas. The BHP difference in Figure 9(a) is 0.46 MPa, which is much larger than the BHP difference under $V_L = 0.05$ m$^3$/Kg in Figure 8(a). The pressure change and pressure derivative in Figures 7, 8, and 9 together clearly show the effect of matrix porosity under different ultimate adsorption capacities.

5. Conclusion

In this paper, we first developed a dual porosity and dual permeability model for a multistage fractured horizontal well in shale-gas reservoirs incorporating the slippage corrected gas permeability and gas adsorption. Then we developed fully implicit numerical simulation model using PEBI grid and finally conducted parametric studies to quantify
the transient pressure behaviors for flow in shale-gas formation. Our conclusions could be summarized as below.

(1) The ratio of fracture permeability to matrix permeability approximately decides the BHP error caused by omitting the flow between the matrix and the wellbore. The BHP error is also related to the interporosity flow ability. The smaller interporosity flow ability is, the larger BHP error is.

(2) In shale-gas formation with adsorption gas, the effect of matrix porosity on BHP is related to adsorption gas content. The desorbed free gas in matrix can offset the effect of the matrix porosity. When adsorbed gas accounts for a big part of total gas storage in shale formation, the change of the matrix porosity only has a very small effect of the total gas, which leads to a very small effect on BHP.

(3) Although effect of natural fracture system porosity on BHP is not discernable, it influences the flow behaviors during the early stage significantly.
This paper can help us understand the complex pressure transient response due to existence of adsorbed gas and help petroleum engineers to interpret the field data better.

**Conflict of Interests**  
The authors declare that there is no conflict of interests regarding the publication of this paper.

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