Research Article

Research on Control-Oriented Modeling for Turbocharged SI and DI Gasoline Engines

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In order to analyze system performance and develop model-based control algorithms for turbocharged spark ignition and direct injection (SIDI) gasoline engines, a control oriented mean value model is developed and validated. The model is constructed based on theoretical analysis for the different components, including the compressor, turbine, air filter, intercooler, throttle, manifold, and combustion chamber. Compressor mass flow and efficiency are modeled as parameterized functions. A standard nozzle model is used to approximate the mass flow through the turbine, and the turbine efficiency is modeled as a function of blade speed ratio (BSR). The air filter is modeled as a tube for capturing its pressure drop feature. The effectiveness number of transfer units (NTU) modeling method is utilized for the intercooler. The throttle model consists of the standard nozzle model with an effective area regressed to throttle position. Manifolds are modeled for their dynamically varying pressure state. For the cylinder, the air mass flow into cylinders, fuel mass, torque, and exhaust temperature are modeled. Compared to the conventional lookup table approach, transient dynamics error can be improved significantly through using the model from this work.

1. Introduction

With the development of advanced combustion gasoline engines under Chinese intellectual property, advanced combustion engines are becoming an important research focus [1–3]. The path for improving the engine performance is to utilize advanced technologies such as turbocharging and direct injection and use advanced combustion concepts like stratified combustion and homogenous charge compression ignition (HCCI). With the increasing complexity of engine systems, control of engine is becoming a complex task. Thus, dynamic simulation models and model-based designs are increasingly used for designing and optimizing engine control strategies. The objective of this paper is to develop a mean value engine model which describes properties of spark ignition and direct injection (SIDI) engines.

The reference [4] constructed turbocharger models which focused on compressor flow rate by a curve fitting method. A component based modeling methodology [5] was utilized for developing the models for the compressor efficiency,

compressor flow, and turbine flow. A simple model for the gas exchange process in diesel engine was developed in [6]. Based on thermodynamic analysis of compressor stage, a novel model-based approach was developed to predict the compressor behavior [7] which overcame the sparse nature of available compressor maps and characterized the flow and efficiency outputs of centrifugal compressors. Model predictive control was presented to coordinate throttle and turbocharger wastegate actuation for engine airflow and boost pressure control [8]. The model utilized the mass equation along with an isothermal manifold assumption. In order to design a controller for regulating speed of diesel engine, the nonlinear model was linearized and represented in a state-space form [9]. Reference [10] presented a control-oriented model for predicting major turbine variables in a turbocharged spark ignition engine. In the model, the turbine was simulated as a two-nozzle chamber.

In this paper, a complete mean value model is presented including compressor, turbine, air filter, intercooler, manifold, throttle, and cylinder. The proposed submodels

FIGURE 1: Sketch of the turbocharged SIDI engine.

capture key features of their corresponding component. All submodels are verified through the experimental data collected from a four-cylinder SIDI engine. Compressor and turbine models are developed as parameterized functions to describe the performance. The air filter is modeled as a simple pressure drop. The intercooler is modeled for its effect on intake temperature. The throttle is modeled to capture its flow rate. The manifolds are modeled with their pressure dynamics. For the cylinders, air mass flow into cylinders, fuel mass, and torque and exhaust temperature are modeled.

2. Engine Overview

The sketch of the turbocharged SIDI engine is displayed in Figure 1. The components to be modeled include compressor, turbine, air filter, intercooler, manifolds, throttle, and cylinders. Ambient air filtered through air filter is boosted by compressor. The boosted air is cooled down by intercooler and then flows into cylinder through throttle and intake manifold. A specified amount of fuel is injected into cylinder according to the mass of air. After combustion, the majority of exhaust gas exits, passing through the turbine and generating power for the compressor. The rest of the exhaust gas flows out through wastegate depending on the wastegate position. After being treated in the catalyst, exhaust gas returns back to the environment.

The specifications of the SIDI engine are shown in Table 1.

3. Modeling

3.1. Compressor. In order to supply the cylinder with air of high density, compressor pressurizes the air and directs it to the intake manifold. Compressor maps are usually described by corrected compressor mass flow, expansion ratio, modeled efficiency lines, and modeled speed lines. A compressor model should include the features from the map. In this paper, the compressor model consists of two submodels for compressor mass flow and compressor efficiency.

TABLE 1: Engine specifications.

Specification	Value	Unit
Displacement volume	2	Liter
Number of cylinders	4	—
Bore diameter	86	mm
Stroke length	86	mm
Compression ratio	10.75	—
Maximum torque	340@3500 rpm	N·m
Maximum power	178@5500 rpm	Kw

FIGURE 2: Velocity triangles of impeller eye and tip.

Based on analysis of specific energy transfer of compressor [11], Figure 2 shows the velocity triangles at the impeller eye and tip of the compressor with prewhirl:

- C_1 : absolute velocity of air at impeller eye (inducer),
- C_{x1} : axial component of C_1 ,
- $C_{\theta 1}$: tangential component of C_1 ,
- U_1 : tangential component of blade tip speed,
- W_1 : air velocity relative to blade (impeller eye),
- W_{1b} : optimal value of W_1 ,
- $W_{\theta 1}$: destruction of tangential component of W_1 ,
- β_c : optimal angle,
- U_2 : impeller tip speed,
- C_2 : absolute velocity of air leaving impeller tip,
- C_{r2} : radial component of C_2 ,
- $C_{\theta 2}$: tangential component of C_2 ,

W_2 : air velocity relative to blade (tip),

ω_{tc} : turbo revolution speed.

According to geometrical relationships in Figure 2,

$$W_{\theta 1} = U_1 - C_{\theta 1} - \frac{C_{x1}}{\tan \beta_c}. \quad (1)$$

Assuming turbocharger compressors without stationary prewhirl, the air approaching the impeller does not include a tangential component; that is, $C_{\theta 1} = 0$.

Thus, (1) is rewritten as

$$W_{\theta 1} = U_1 - \frac{C_{x1}}{\tan \beta_c} = \omega_{tc} r_{c1} - \frac{C_{x1}}{\tan \beta_c} \quad (2)$$

$$C_{x1} = C_1 = \frac{\phi_c}{\rho_c A_c}, \quad (3)$$

where ϕ_c is inlet mass flow, A_c is the inlet inducer cross-sectional area, ρ_c is air static density, and r_{c1} is inlet radius of impeller.

As the air adapts to the blade direction, the kinetic energy associated with the tangential component $W_{\theta 1}$ is destroyed. Thus, the incidence loss is given by

$$h_{inc} = \frac{W_{\theta 1}^2}{2}. \quad (4)$$

From (2) and (4), (5) is derived:

$$h_{inc} = \frac{1}{2} \left(\omega_{tc} r_{c1} - \frac{\phi_c}{\rho_c A_c \tan \beta_c} \right)^2. \quad (5)$$

According to [11], the loss of useful energy due to friction is given by

$$h_{fric} = \delta_c \phi_c^2, \quad (6)$$

where δ_c is an empirical constant.

Applying the Euler turbine equation and assuming $C_{\theta 1} r_{c1} \ll C_{\theta 2} r_{c2}$, the change of air angular momentum is given by

$$h_{total} = \omega_{tc} (r_{c2} C_{\theta 2} - r_{c1} C_{\theta 1}) = (\omega_{tc} r_{c2})^2, \quad (7)$$

where r_{c2} is outlet radius of impeller.

The specific energy for isentropic compression is given by

$$\Delta h_c = h_{total} - h_{inc} - h_{fric}. \quad (8)$$

Substituting (5), (6), and (7) into (8),

$$\Delta h_c = (\omega_{tc} r_{c2})^2 - \frac{1}{2} \left(\omega_{tc} r_{c1} - \frac{\phi_c}{\rho_c A_c \tan \beta_c} \right)^2 - \delta_c \phi_c^2. \quad (9)$$

The amount of energy required by the adiabatic compression process is given by

$$\Delta h_c = C_p T_{cin} \left(\frac{p_{cout}}{p_{cin}} \right)^{1-1/\gamma_c}, \quad (10)$$

FIGURE 3: Mass flow model for compressor at stationary engine condition (the unit of compressor speed is rpm).

where p_{cout} and p_{cin} are the pressure out of compressor and into compressor, respectively, T_{cin} is the temperature of air into compressor, C_p is air heat capacity, and γ_c is specific heat capacity ratio.

Combining (9) with (10), the mass flow ϕ_c through the compressor is derived:

$$\Phi_c = k_{n1} \omega_{tc} - k_{n2} \sqrt{k_{n3} \omega_{tc}^2 + k_{n4} \left[\left(\frac{p_{cout}}{p_{cin}} \right)^{1-1/\gamma_c} - 1 \right]}, \quad (11)$$

where $k_{n1} = 2\rho_c A_c \gamma_c \tan \beta_c / (2\delta_c (\rho_c A_c \tan \beta_c)^2 - 1)$, $k_{n2} = \rho_c A_c \gamma_c \tan \beta_c / (-2\delta_c (\rho_c A_c \tan \beta_c)^2 - 1)$, $k_{n3} = 2r_{c2}^2 - 2(\rho_c A_c \tan \beta_c)^2 \delta_c r_{c1}^2 + 4(\rho_c A_c \tan \beta_c)^2 \delta_c r_{c2}^2$, and $k_{n4} = -2C_p T_{cin} ((\rho_c A_c \tan \beta_c)^2 \delta_c + 1)$.

For all parameters of (11), only β_c needs to be adjusted. This is different from other models. As shown in Figure 3, measured data of the mass flow agree with model simulation results accurately over the whole operating range. The average error is 0.12%. Figure 4 shows the results of fine-tuned parameter β_c .

The efficiency of the compressor is defined as the ratio of isentropic to the actual total input work:

$$\eta_c = \frac{h_{total} - h_{inc} - h_{fric}}{h_{total}}. \quad (12)$$

Substituting (5), (6), and (7) into (12), the efficiency is rewritten as

$$\eta_c = \frac{k_{c1}}{\omega_{tc}^2} \phi_c^2 + \frac{k_{c2}}{\omega_{tc}} \phi_c + k_{c3}, \quad (13)$$

where $k_{c1} = (2\delta_c \rho_c A_c \tan \beta_c - 1) / 2(\rho_c A_c \tan \beta_c)^2 r_{c2}^2$ and $k_{c2} = r_{c2} / r_{c2}^2 \rho_c A_c \tan \beta_c$, $k_{c3} = 1 - (R_{c2}^2 / 2r_{c2}^2)$.

FIGURE 4: Optimal angle as parameter of compressor mass flow model (the unit of compressor speed is rpm).

FIGURE 5: Efficiency model for compressor at stationary engine condition.

As shown in Figure 5, with the fine-tuned parameter β_c , measured data of the efficiency agree with model simulation results accurately over the whole operating range. Figure 6 shows the results of parameter β_c .

In addition, based on (13), compressor efficiency can also be modeled as a polynomial function, as shown in

$$\eta_c = f_{(a1,b1,c1,d1)} \left(\frac{\phi_c}{\omega_{tc}} \right)^4 + f_{(a2,b2,c2,d2)} \left(\frac{\phi_c}{\omega_{tc}} \right)^3 + f_{(a3,b3,c3,d3)} \left(\frac{\phi_c}{\omega_{tc}} \right)^2 + f_{(a4,b4,c4,d4)} \frac{\phi_c}{\omega_{tc}} + f_{(a5,b5,c5,d5)}, \quad (14)$$

where $f_{(a,b,c,d)} = a\omega_{tc}^3 + b\omega_{tc}^2 + c\omega_{tc} + d$.

The values of coefficients a , b , c , and d are shown in Table 2.

Figures 7, 8, and 9 show the simulation and measured results. The maximum relative error is about 7% which

FIGURE 6: Optimal angle as parameter of compressor efficiency model.

FIGURE 7: Efficiency model in polynomial form at stationary engine conditions (turbo speeds: 76130 rpm and 49600 rpm).

TABLE 2: Coefficient values of function $f_{(a,b,c,d)}$.

	a	b	c	d
$f_{(a1,b1,c1,d1)}$	$-1.49 * 10^6$	$2.66 * 10^{11}$	$-1.58 * 10^{16}$	$2.56 * 10^{20}$
$f_{(a2,b2,c2,d2)}$	50.71	$-8.53 * 10^6$	$4.56 * 10^{11}$	$-6.82 * 10^{15}$
$f_{(a3,b3,c3,d3)}$	$-6.75 * 10^{-4}$	111.88	$-5.64 * 10^6$	$7.52 * 10^{10}$
$f_{(a4,b4,c4,d4)}$	$4.15 * 10^{-9}$	$-7.06 * 10^{-4}$	36.22	$-4.52 * 10^5$
$f_{(a5,b5,c5,d5)}$	$-9.62 * 10^{-15}$	$1.67 * 10^{-9}$	$-8.62 * 10^{-5}$	1.55

occurs at speeds of 172119 rpm and 102600 rpm. For other compressor speeds, the relative error is less than 0.4%.

3.2. Turbine. Turbine is driven by the exhaust gas to provide power for compressor. The turbine model consists of sub-models for the turbine mass flow and the turbine efficiency.

Turbine mass flow is modeled as exhaust gas flowing through a nozzle [11, 12]. In particular, the pressure ratio at chock pr_{crit} of turbine is much higher than that of an adiabatic nozzle. The choke pressure ratio of turbine is measured

FIGURE 8: Efficiency model in polynomial form at stationary engine conditions (turbo speeds, 122800 rpm and 102600 rpm).

FIGURE 9: Efficiency model in polynomial form at stationary engine conditions (turbo speeds: 172119 rpm and 152590 rpm).

through experiments. At the condition in which $p_{\text{tout}}/p_{\text{tin}} < \text{pr}_{\text{crit}}$, choke occurs, and the calculation formula of turbine mass flow is shown in (15). When $p_{\text{tout}}/p_{\text{tin}} > \text{pr}_{\text{crit}}$, the calculation formula is shown in (16). In addition, turbine mass flow has a significant speed dependence, which is shown in Figure 10. Consider

$$\phi_{tb} = \frac{p_{ic}}{\sqrt{RT_{\text{tin}}}} A_t \sqrt{\frac{2\gamma}{\gamma-1} \left[(\text{pr}_{\text{crit}})^{2/(\gamma-1)} - (\text{pr}_{\text{crit}})^{(\gamma+1)/(\gamma-1)} \right]} \quad (15)$$

$$\phi_{tb} = \frac{p_{\text{tin}}}{\sqrt{RT_{\text{tin}}}} A_t \sqrt{\frac{2\gamma}{\gamma-1} \left[\left(\frac{p_{\text{tout}}}{p_{\text{tin}}} \right)^{2/\gamma} - \left(\frac{p_{\text{tout}}}{p_{\text{tin}}} \right)^{(\gamma+1)/\gamma} \right]}, \quad (16)$$

where p_{tin} is the turbine entry pressure, p_{tout} is the turbine exit pressure, T_{tin} is the entry temperature of air, $\gamma = 1.4$ is the specific heat capacity ratio, and A_t is the effective

TABLE 3: Coefficient values of function A_t .

Coefficient	Value
a_{tm1}	3.32×10^{-13}
b_{tm1}	-1.07×10^{-7}
c_{tm1}	0.0101
a_{tm2}	-2.69×10^{-13}
b_{tm2}	9.93×10^{-8}
c_{tm2}	-0.0044

FIGURE 10: Turbine mass flow model.

cross-sectional area of turbine which varies with tangential component of velocity. A_t is given by

$$A_t = (a_{tm1}n_{tc}^2 + b_{tm1}n_{tc} + c_{tm1}) \frac{p_{\text{tin}}}{p_{\text{tout}}} + (a_{tm2}n_{tc}^2 + b_{tm2}n_{tc} + c_{tm2}), \quad (17)$$

where n_{tc} is the turbine revolution speed (in units of revolutions per minute) and a_{tm1} , b_{tm1} , c_{tm1} , a_{tm2} , b_{tm2} , and c_{tm2} are coefficients, whose values are shown in Table 3.

Measured and model simulation values of turbine mass flow for different speed are shown in Figure 10. The average error is 0.16%.

Blade speed ratio (BSR) [11] is defined as the ratio of speed at the mean blade height (U_t) to the velocity. The velocity is calculated assuming isentropic expansion from the inlet conditions to the pressure at the exit from the turbine (total to static). The function is given by

$$\text{BSR} = \frac{U_t}{\sqrt{2c_p T_{\text{tin}} \left[1 - (p_{\text{tout}}/p_{\text{tin}})^{(\gamma-1)/\gamma} \right]}}. \quad (18)$$

TABLE 4: Coefficient values of function η_{tb} .

Coefficient	Value
a_{te1}	$-1.74 * 10^{-14}$
b_{te1}	$4.16 * 10^{-9}$
c_{te1}	$-2.64 * 10^{-4}$
d_{te1}	-1.9538
a_{te2}	$2.99 * 10^{-14}$
b_{te2}	$-7.33 * 10^{-9}$
c_{te2}	$4.97 * 10^{-4}$
d_{te2}	0.2828
a_{te3}	$-1.32 * 10^{-14}$
b_{te3}	$3.33 * 10^{-9}$
c_{te3}	$-2.43 * 10^{-4}$
d_{te3}	2.1765

FIGURE 11: Turbine efficiency model.

Turbine efficiency is modeled as a polynomial function:

$$\eta_{tb} = (a_{te1}n_{tc}^3 + b_{te1}n_{tc}^2 + c_{te1}n_{tc} + d_{te1})BSR^2 + (a_{te2}n_{tc}^3 + b_{te2}n_{tc}^2 + c_{te2}n_{tc} + d_{te2})BSR + (a_{te3}n_{tc}^3 + b_{te3}n_{tc}^2 + c_{te3}n_{tc} + d_{te3}), \quad (19)$$

where a_i , b_i , c_i , and d_i ($i = te1, te2, te3$) are coefficients, whose values are shown in Table 4.

The measured and simulation results are shown in Figure 11. The average error is 1.2%.

3.3. Air Filter. The most important feature of the air filter is the pressure drop that it causes. Air filter is modeled as a simple tube. According to pressure drop calculation for a tube

[13], air filter model can be described by (20). This model for pressure drop is also utilized by [5]:

$$\Delta P_{af} = C_{af} \frac{T_{af} \phi_{af}^2}{P_{af}}, \quad (20)$$

where ΔP_{af} is pressure drop, C_{af} is a coefficient which depends on Reynolds number R_e , length and diameter of air filter, and gas constant R , and T_{af} , P_{af} , and ϕ_{af} are upstream temperature, pressure, and mass flow of incoming air, respectively.

3.4. Intercooler. In order to cool down the compressed intake air, increase its density, and decrease knock probability, an intercooler is used in the air path of the engine.

Cengel [13] described a method called the effectiveness number of transfer units (NTU) method for intercooler heat exchanger analysis. The method is based on a dimensionless parameter called the heat transfer effectiveness ε , defined as

$$\varepsilon = \frac{T_{ic,in} - T_{ic,out}}{T_{ic,in} - T_{cool}}, \quad (21)$$

where $T_{ic,in}$ is the temperature of hot air that enters into intercooler, $T_{ic,out}$ is the temperature of cooled air that outflows intercooler, and T_{cool} is the temperature of ambient air that blows towards intercooler. According to (21), the temperature of cooled air $T_{ic,out}$ that flows out of intercooler is given by

$$T_{ic,out} = T_{ic,in} - \varepsilon (T_{ic,in} - T_{cool}). \quad (22)$$

A detailed derivation for calculating the heat transfer effectiveness ε is presented in [13]. The final equations are shown below:

$$\varepsilon = 1 - \exp \left\{ \frac{NTU^{0.22}}{c} \left[\exp(-c \cdot NTU^{0.78}) - 1 \right] \right\},$$

$$NTU = \frac{U \cdot A_s}{C_p \phi_{ic}}, \quad (23)$$

$$c = \frac{\phi_{ic}}{\phi_{cool}},$$

where U is the overall heat transfer coefficient, A_s is the intercooler surface area, C_p is air heat capacity, ϕ_{ic} is the mass of compressed air from compressor, and ϕ_{cool} is the mass of ambient air.

The advantage of NTU is that the model includes heat transfer physics. However, it is not accurate enough and the calculation of ε is complex. Based on measured data, Eriksson [14] developed a regression model for the calculation of ε

$$\varepsilon = a_0 + a_1 \left(\frac{T_{ic,in} + T_{cool}}{2} \right) + a_2 \phi_{ic} + a_3 \frac{\phi_{ic}}{\phi_{cool}}. \quad (24)$$

It is assumed that there is no pressure loss for intercooler, which gives the following equation:

$$P_{ic,out} = P_{ic,in}. \quad (25)$$

TABLE 5: Coefficient values of the effective opening area.

Coefficient	a_{th2}	a_{th1}	a_{th0}
Value	-0.1514	0.7816	0.1635

3.5. Throttle. The throttle controls the amount of air flowing into cylinders in SI engine at part load. The butterfly type throttle is regarded as a nozzle with compressible gas flow. Heywood [15] described the model for the nozzle and [14, 16] used this method to develop the throttle model. Formulae are presented below.

When the gas velocity flowing through the throat of throttle is equal to or larger than the velocity of sound, that is, $p_{imf}/p_{ic} < (2/(\gamma + 1))^{\gamma/(\gamma-1)}$, the maximum mass flow occurs. The formula of calculating mass flow ϕ_{th} is shown in

$$\phi_{th} = C_d A_{th} \frac{p_{ic}}{\sqrt{RT_{ic,out}}} \cdot \sqrt{\frac{2\gamma}{\gamma-1} \left[\left(\frac{2}{\gamma+1} \right)^{2/(\gamma-1)} - \left(\frac{2}{\gamma+1} \right)^{(\gamma+1)/(\gamma-1)} \right]}. \quad (26)$$

When the gas velocity is less than the velocity of sound, the formula of calculating mass flow is shown in

$$\phi_{th} = C_d A_{th} \frac{p_{ic}}{\sqrt{RT_{ic,out}}} \sqrt{\frac{2\gamma}{\gamma-1} \left[\left(\frac{p_{imf}}{p_{ic}} \right)^{2/\gamma} - \left(\frac{p_{imf}}{p_{ic}} \right)^{(\gamma+1)/\gamma} \right]}, \quad (27)$$

where A_{th} is the throttle plate opening area, C_d is the discharge coefficient, p_{imf} is the air pressure in intake manifold, and $\gamma = 1.4$ is the specific heat capacity ratio. The effective opening area $C_d A_{th}$ is a function of throttle opening angle α . The function is shown below:

$$C_d A_{th} = A_1 (1 - \cos(a_{th2}\alpha^2 + a_{th1}\alpha + a_{th0})) + A_0, \quad (28)$$

where A_1 is the throttle bore area, A_0 is leakage area, and a_{th2} , a_{th1} , and a_{th0} are fit coefficients based on measured data, whose values are shown in Table 5.

Model simulation results versus measured values of effective area are shown in Figure 12 and results of mass flow through throttle are shown in Figure 13. The models capture the main characters of effective area and mass flow through throttle.

3.6. Manifold. For the intake and exhaust manifolds, pressure in manifold depends on the change rate of the total air mass, which is equal to the sum of the air mass flows into and out of the manifold. Manifold models can be created according to

FIGURE 12: Effective area of throttle.

FIGURE 13: Air mass flow through throttle.

isothermal model [15, 17]. The intake and exhaust manifolds are modeled as follows:

$$\begin{aligned} \frac{dp_{imf}}{dt} &= \frac{R_a T_{imf}}{V_{imf}} (\phi_{th} - \phi_{ein}) \\ \frac{dp_{emf}}{dt} &= \frac{RT_{emf}}{V_{emf}} (\phi_{eout} - \phi_{tb}), \end{aligned} \quad (29)$$

where p_{imf} and p_{emf} are the pressure of intake and exhaust manifolds, respectively, T_{imf} is the temperature of air flowing into intake manifold, which is assumed to be equal to the temperature of cooled air that exits intercooler, T_{emf} is the temperature of exhaust manifold, V_{imf} and V_{emf} are volumes

FIGURE 14: Volumetric efficiency for engine speeds 4500 rpm and 3000 rpm.

of intake and exhaust manifolds, ϕ_{ein} is the mass flow from the intake manifold into the cylinders, and ϕ_{eout} is the total mass flow out from cylinder. The ideal gas constant R_a of intake manifold and ideal gas constant R of exhaust manifold are different because T_{emf} is much higher than T_{imf} .

3.7. Cylinder. The total mass flow ϕ_{ei} into the cylinder is modeled using the standard model based on volumetric efficiency:

$$\phi_{ei} = \frac{\eta_{\text{vol}}(p_{\text{imf}}, n_e, \alpha) p_{\text{imf}} n_e V_d}{120 R_a T_{\text{imf}}}, \quad (30)$$

where p_{imf} is the air pressure in the intake manifold. T_{imf} is the air temperature in the intake manifold, n_e is engine speed, V_d is displacement volume, and η_{vol} is volumetric efficiency which is a function of p_{imf} , n_e , and throttle opening angle α . η_{vol} is a function of engine speed, throttle position, and intake manifold pressure and the results are shown in Figures 14 and 15. For a given engine and throttle position, volumetric efficiency varies with intake manifold pressure as the cylinder intake and exhaust valve timings are changed. For example, in Figure 14, at the condition in which engine speed is equal to 4500 rpm and throttle position is equal to 26%, the volumetric efficiency varies from 0.63 to 0.42.

The fuel mass flow ϕ_{ef} injected into cylinder varies with air fuel ratio. The fuel mass flow can be modeled [7] with fuel mass flow variable μ_δ which has units of milligrams, engine speed n_e , and the number of cylinder n_{cyl} :

$$\phi_{ef} = \frac{10^{-6}}{120} \mu_\delta n_e n_{\text{cyl}}. \quad (31)$$

The mass flow out from the cylinder is the sum of the total mass flow into cylinders ϕ_{ei} and the fuel mass injected into cylinders ϕ_{ef} .

For the engine torque M_e , based on the engine friction definition in reference [15], it can be calculated from gross

FIGURE 15: Volumetric efficiency for engine speeds 2000 rpm and 1200 rpm.

indicated torque M_{ig} and total friction torque M_{tf} . The total friction torque includes friction torques produced by pumping work, rubbing friction work, and accessory work. The engine torque is then

$$M_e = M_{ig} - M_{tf}. \quad (32)$$

The gross indicated torque is calculated from fuel consumption:

$$M_{ig} = \frac{30 \phi_{ef} Q_{\text{HV}} \eta_{ig}}{\pi n_e}, \quad (33)$$

where $\eta_{ig} = 1 - (1/r_c^{\gamma-1})$ (this assumes instantaneous adiabatic combustion at top dead center), where r_c is the compression ratio and γ is the specific heat capacity ratio, and Q_{HV} is the fuel heating value.

The total friction torque of SI engine is modeled as a function of engine speed:

$$M_{tf} = \frac{V_d 10^5}{4\pi} \left(C_{f1} + C_{f2} \frac{n_e}{1000} + C_{f3} \left(\frac{n_e}{1000} \right)^2 \right). \quad (34)$$

Exhaust temperature T_{exh} is important to model turbocharged engines. Models for exhaust temperature in reference [17] have two components, which are an exhaust manifold temperature and a temperature drop in a straight pipe.

The exhaust manifold temperature is modeled as

$$T_{\text{cyl}} = e^{a_{\text{cy}} + (b_{\text{cy}}/\phi_{\text{exh}})}, \quad (35)$$

where T_{cyl} is temperature of the fluid delivered by the cylinder to the exhaust manifold, ϕ_{exh} is the exhaust mass flow, and a_{cy} and b_{cy} are tuning parameters, whose values are 7.239 and -0.005, respectively.

The temperature at the turbine inlet is modeled as (36), which is based on temperature drop in a straight pipe:

$$T_{\text{exh}} = T_{\text{amb}} + (T_{\text{cyl}} - T_{\text{amb}}) e^{-h_{\text{cv}} \pi d_{\text{pipe}} l_{\text{pipe}} / \phi_{\text{exh}} C_p}, \quad (36)$$

FIGURE 16: Model of the temperature at the turbine inlet.

where T_{amb} is the pipe wall temperature which is equal to ambient temperature, h_{cv} is the total transfer coefficient, d_{pipe} and l_{pipe} are the diameter and length of exhaust pipe, respectively, and C_p is the heat capacity. The model validated results are shown in Figure 16. T_{cyl} is predicted utilizing (35) and the parameters of T_{cyl} are tuned.

4. Experiment and Simulation

All engine models above are parameterized to experimental data that are collected on a 2.0 L four-cylinder turbocharged SIDI engine at steady state conditions. Sensors like in-cylinder pressure sensors, k-type thermocouples, and air flow meter are equipped on the test bench. The models are validated at engine speeds from 1000 RPM up to 4500 RPM.

The whole simulation models are implemented in Simulink according to the physical engine system, as shown in Figure 17. The parameters and coefficients of the models are listed in Section 3. The completed model includes inter-cooler, throttle, intake manifold, cylinder, air filter, compressor, turbo, turbine, and exhaust manifold.

The engine was operated at around 2200 r/min with gasoline 97#. Figures 18 and 19 show the other engines' working conditions. The throttle position was stepped from 33.7% open to 45.5% open, after holding for 35 seconds, and then fell back to 38.4% open. The varying tendency of injection duration is almost the same.

In Figures 20–23, the simulated results are compared to the experimental measurements. The model is able to capture the transient dynamics and estimate the outputs with desirable accuracy. In Figure 20, the modeled intake mass flow results are a little ahead of experimental measurements at the rising phase. After that, the model retains its good tracking performance. The transient error is a result of the response time of the air mass flow transducer. In Figure 21, the model tracks the transient dynamics well at the rising

FIGURE 17: Complete control-oriented model.

FIGURE 18: Throttle position of engine operation conditions.

FIGURE 19: Injection duration of engine operation conditions.

FIGURE 20: Measured and modeled results of intake air mass flow.

FIGURE 21: Measured and modeled results of fuel mass flow.

FIGURE 22: Measured and modeled results of boost pressure.

phase, but, in stable phase, the estimation error is 0.11, about 1.3%. The boost pressure and intake manifold pressure results shown in Figures 22 and 23 have the same characteristics with fuel mass flow.

FIGURE 23: Measured and modeled results of intake manifold pressure.

5. Conclusions

A complete mean value model based on component sub-models has been developed and validated. The intended applications of the models are developments of model-based control strategies and system analysis. Several submodels were described and novel models for compressor flow and compressor efficiency were developed. The accuracy of the new compressor model has an average error of 0.12%, which is a significant improvement over conventional mapping approaches. The experimental results confirm that the developed model is capable of tracking transient dynamics. Compared to the conventional lookup table approach, our validation results show that transient dynamics error can be improved significantly through using the model from this work.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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