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Research Article

Topology-Based Analysis of OTIS (Swapped) Networks O_{K_n} and O_{P_n}

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In the fields of chemical graph theory, topological index is a type of a molecular descriptor that is calculated based on the graph of a chemical compound. In this paper, M-polynomial O_{K_n} and O_{P_n} networks are computed. The M-polynomial is rich in information about degree-based topological indices. By applying the basic rules of calculus on M-polynomials, the first and second Zagreb indices, modified second Zagreb index, general Randić index, inverse Randić index, symmetric division index, harmonic index, inverse sum index, and augmented Zagreb index are recovered.

1. Introduction

Cheminformatics is a new branch of science which relates chemistry, mathematics, and computer sciences. Quantitative structure-activity relationship (QSAR) and quantitative structure-property relationship (QSPR) are the main components of cheminformatics which are helpful to study physicochemical properties of chemical compounds [1–3].

A topological index is a numeric quantity associated with the graph of chemical compound, which characterizes its topology and is invariant under graph automorphism [4–6]. There are numerous applications of graph theory in the field of structural chemistry. The first well-known use of a topological index in chemistry was by Wiener in the study of paraffin boiling points [7–9]. After that, in order to explain physicochemical properties, various topological indices have been introduced and studied [10, 11].

A computer network is a digital telecommunications network which allows nodes to share resources. In computer networks, computing devices exchange data with each other using connections (data links) between nodes. These data links are established over cable media such as wires or optic cables, or wireless media such as WiFi. Optical transpose interconnection system (OTIS) networks were initially contrived to give productive network to new optoelectronic computer models that profit by both optical and electronic advancements [12]. In OTIS networks, processors are orchestrated into groups. Electronic inter-connects are used between processors within the same cluster, while optical links are used for intercluster communication. Various algorithms have been produced for directing, determination/arranging, certain numerical calculations, Fourier transformation [13], matrix multiplication [14], image processing [15], and so on [16, 17]. The structure of an interconnection system can be scientifically modeled by a graph. The vertices of this graph are the processor nodes and the edges are the connections between the processors. The topology of a graph decides the manner by which vertices are associated by edges. From the topology of a system, certain properties can be decided. The diameter of a graph is the maximum distance between any two vertices of the graph.

Definition 1 (OTIS (swapped) network $O_{K_{n+2}}$). The OTIS (swapped) network is derived from the graph Ω, which is a graph with vertex set $V(O_\Omega) = <g, p> \mid g, p \in V(\Omega)$ and edge set $E(O_\Omega) = <g, p_1>, <g, p_2> g \in V(\Omega), (p_1, p_2) \in E(\Omega) \cup (<g, p>, <g, p, g>) \mid g, p \in V(\Omega)$ and $g \neq p$.

The graph of OTIS (swapped) network $O_{K_{n+2}}$ given in Figure 1 has $(n^3/2) + 3n^2 + (11n/2) + 3$ edges and $(n+2)^2$ vertices.

Definition 2 (OTIS (swapped) network O_{P_n}). Let P_n be path of n vertices and OP_n be OTIS (swapped) network with basis

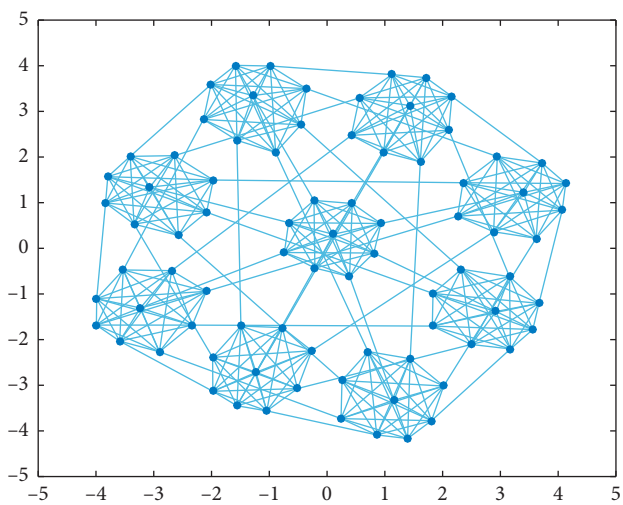

Figure 1: O_{K_7}.

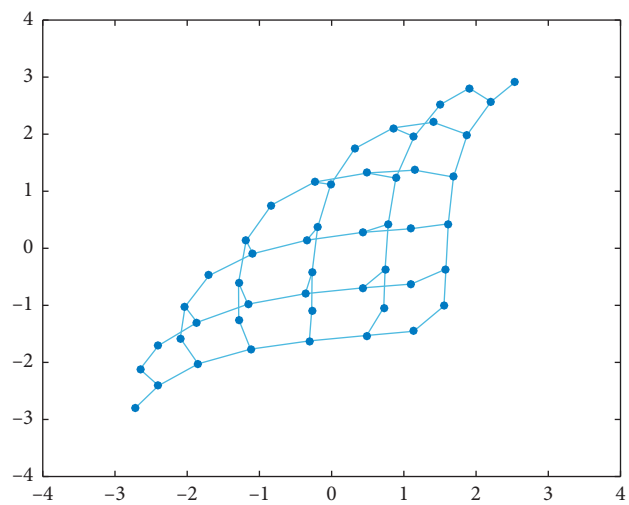

Figure 2: OP_7.

network P_n. An OTIS (swapped) network with the basis network P_6 is shown in Figure 2.

The OTIS (swapped) network graph O_{p_n} given in Figure 2 has $(3/2)(n^2 - n)$ edges and n^2 vertices.

In this paper, we aim to compute degree-dependent topological indices of OTIS (swapped) networks O_{K_n} and O_{P_n}. In Section 2, we give definitions and literature review about TIs. In Sections 3 and 4, we present the methodology and application of our results in chemistry, respectively. Section 5 contains our main results, and Section 6 concludes our paper.

2. Topological Indices (TIs)

TIs are numbers which depend on the molecular graph and are helpful in deciding the properties of the concerned molecular compound [18–20]. We can consider TI as a function which assigns a real number to each molecular graph, and this real number is used as a descriptor of the concerned molecule. From the TIs, a variety of physical and chemical properties like heat of evaporation, heat of formation, boiling point, chromatographic retention, surface tension, and vapor pressure of understudy molecular compound can be identified. A TI gives us the mathematical language to study a molecular graph. There are three types of TIs:

(1) Degree-based TIs

(2) Distance-based TIs

(3) Spectrum-based TIs

The first type of TI depends upon the degree of vertices, second one depends upon the distance of vertices and the third type of TI depends upon the spectrum of graph.

2.1. Zagreb Indices. To compute total π-electron energy, the following TI is defined:

$$M_1(G) = \sum_{v \in V(G)} d_v^2. \tag{1}$$

But soon it was observed that this index increases with increases in branching of the skeleton of carbon atoms. After 10 years, Balaban et al. wrote a review [21], in which he declared M1 and M2 are among the degree-based TIs and named them as Zagreb group indices. The name Zagreb group indices was soon changed to Zagreb indices (ZIs), and nowadays, M_1 and M_2 are abbreviated as first Zagreb index and second Zagreb index.

In 1975, Gutman et al. gave a remarkable identity [22]. Hence, these two indices are among the oldest degree-based descriptors, and their properties are extensively investigated. The mathematical formulae of these indices are

$$\begin{aligned} M_1(G) &= \sum_{uv \in E(G)} d_u + d_v, \\ M_2(G) &= \sum_{uv \in E(G)} d_u d_v. \end{aligned} \tag{2}$$

For detailed survey about these indices, we refer [23–26]. Doslic et al. [27] gave the idea of augmented ZI, whose mathematical formula is

$$\mathrm{AZI}(G) = \sum_{uv \in E(G)} \left(\frac{d_u d_v}{d_u + d_v - 2} \right)^3. \tag{3}$$

2.2. Randić or Connectivity Index. Historically, ZIs are the very first degree-based TIs, but these indices were used for completely different purposes; therefore, the first genuine degree-based TI is the Randić index (RI) which was given in 1975 by Randić [28] as

$$R_{-1/2}(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_u d_v}}. \tag{4}$$

Firstly, Randić named it as a branching index, which was soon named as connectivity index, and nowadays, it is called as RI. The RI is the most popular degree-based TI and has been extensively studied by both mathematicians and chemists. Randić himself wrote two reviews [29, 30], and

many papers and books on this topological invariant are present in the literature; few of them are [31–34]. Researchers recognized the importance of the Randić index in drug design. Bollbás and Erdos, famous mathematicians of that time investigated some hidden mathematical properties of RI [35]; after that, RI was worth studying, and a surge of publications began [36–39]. An unexpected mathematical quality of the Randić index was discovered recently, which tells us about the relation of this topological invariant with the normalized Laplacian matrix [38, 40, 41]. The GRI known as general RI [42] is defined as

$$R_\alpha(G) = \sum_{uv\in E(G)} (d_u d_v)^\alpha. \tag{5}$$

2.3. M-Polynomial. The mathematical formula of M-polynomial is

$$M(G; x, y) = \sum_{uv\in E(G)} \left(x^{d_u} y^{d_v}\right). \tag{6}$$

Detailed survey about the definitions of other TIs computed in this paper and relation of TIs with M-polynomial can be seen in [43, 44], where

$$\begin{aligned} D_x(f(x,y)) &= x\frac{\partial}{\partial x}(f(x,y)),\\ D_y(f(x,y)) &= y\frac{\partial}{\partial y}(f(x,y)),\\ S_x(f(x,y)) &= \int_0^x \frac{f(t,y)}{t}\,\mathrm{d}t,\\ S_y(f(x,y)) &= \int_0^y \frac{f(x,t)}{t}\,\mathrm{d}t,\\ J(f(x,y)) &= J(f(x,x)),\\ Q_\alpha(f(x,y)) &= x^\alpha(f(x,y)). \end{aligned} \tag{7}$$

3. Methodology

To compute the M-polynomial of a graph G, we need to compute the number of vertices and edges in it and divide the edge set into different classes with respect to the degrees of end vertices. From the M-polynomials, we can recover many degree-dependent indices by applying some differential and integral operators.

4. Applications in Chemistry

A topological description is used to depict the features of the studied compounds, and indices of graph-theoretical origin are used to investigate the correlations between structure and biological activity [45–48]. For example, the Randić index demonstrates great relationship with the physical property of alkanes. The geometric arithmetic index has a similar role as that of the Randić index. The sum-connectivity index is helpful in guessing the melting point of compounds. Zagreb indices are used to calculate π-electron energy.

5. Main Results

In this section, we compute M-polynomials of understudy networks and recover nine TIs from these polynomials.

5.1. Results for OP_n

Theorem 1. *Let OP_n be the swapped network; then,*

$$\begin{aligned} M(G; x, y) &= 2xy^3 + 3x^2y^2 + (6n-14)x^2y^3\\ &\quad + \left(\frac{3(n-2)(n-3)}{2}\right)x^3y^3. \end{aligned} \tag{8}$$

Proof. The OP_n network has the following four types of edges based on the degree of end vertices:

$$\begin{aligned} E_{\{1,3\}}(OP_n) &= \{uv \in E(OP_n) : d_u = 1, d_v = 3\},\\ E_{\{2,2\}}(OP_n) &= \{uv \in E(OP_n) : d_u = 2, d_v = 2\},\\ E_{\{2,3\}}(OP_n) &= \{uv \in E(OP_n) : d_u = 2, d_v = 3\},\\ E_{\{3,3\}}(OP_n) &= \{uv \in E(OP_n) : d_u = 3, d_v = 3\}, \end{aligned} \tag{9}$$

such that

$$\begin{aligned} \left|E_{\{1,3\}}(OP_n)\right| &= 2,\\ \left|E_{\{2,2\}}(OP_n)\right| &= 3,\\ \left|E_{\{2,3\}}(OP_n)\right| &= (6n-14),\\ \left|E_{\{3,3\}}(OP_n)\right| &= \left(\frac{3(n-2)(n-3)}{2}\right). \end{aligned} \tag{10}$$

Now, from the definition of M-polynomial, we have

$$\begin{aligned} M(G; x, y) &= \sum_{\delta\le i\le j\le\Delta} m_{ij}x^iy^j\\ &= \sum_{1\le 3} m_{13}x^1y^3 + \sum_{2\le 2} m_{22}x^2y^2 + \sum_{2\le 3} m_{23}x^2y^3\\ &\quad + \sum_{3\le 3} m_{33}x^3y^3\\ &= \left|E_{\{1,3\}}(OP_n)\right|x^1y^3 + \left|E_{\{2,2\}}(OP_n)\right|x^2y^2\\ &\quad + \left|E_{\{2,3\}}(OP_n)\right|x^2y^3\\ &\quad + \left|E_{\{3,3\}}(OP_n)\right|x^3y^3\\ &= 2xy^3 + 3x^2y^2 + (6n-14)x^2y^3\\ &\quad + \left(\frac{3(n-2)(n-3)}{2}\right)x^3y^3. \end{aligned} \tag{11}$$

□

Corollary 1. *Let OP_n be the swapped network; then,*

$$M_1(OP_n) = 9n^2 - 15n + 4. \tag{12}$$

Proof. Let

$$f(x,y) = M(G;x,y) = 2xy^3 + 3x^2y^2 + (6n-14)x^2y^3 + \left(\frac{3(n-2)(n-3)}{2}\right)x^3y^3. \tag{13}$$

Then,

$$\begin{aligned} D_x(f(x,y)) &= 2xy^3 + 6x^2y^2 + 2(6n-14)x^2y^3 + \left(\frac{9(n-2)(n-3)}{2}\right)x^3y^3, \\ D_y(f(x,y)) &= 6xy^3 + 6x^2y^2 + 3(6n-14)x^2y^3 + \left(\frac{9(n-2)(n-3)}{2}\right)x^3y^3. \end{aligned} \tag{14}$$

Now, we have

$$\begin{aligned} M_1(OP_n) &= D_xf + D_yf\big|_{x=y=1} \\ &= 9n^2 - 15n + 4. \end{aligned} \tag{15}$$

□

Corollary 2. *Let OP_n be the swapped network; then,*

$$M_2(OP_n) = \frac{27n^2}{2} - \frac{63n}{2} + 15. \tag{16}$$

Proof. Let

$$f(x,y) = M(G;x,y) = 2xy^3 + 3x^2y^2 + (6n-14)x^2y^3 + \left(\frac{3(n-2)(n-3)}{2}\right)x^3y^3. \tag{17}$$

Then,

$$\begin{aligned} D_y(f(x,y)) &= 6xy^3 + 6x^2y^2 + 3(6n-14)x^2y^3 + \left(\frac{9(n-2)(n-3)}{2}\right)x^3y^3, \\ D_xD_y(f(x,y)) &= 6xy^3 + 12x^2y^2 + 6(6n-14)x^2y^3 + \left(\frac{27(n-2)(n-3)}{2}\right)x^3y^3. \end{aligned} \tag{18}$$

Now, we have

$$\begin{aligned} M_2(OP_n) &= D_xD_yf\big|_{x=y=1} \\ &= \frac{27n^2}{2} - \frac{63n}{2} + 15. \end{aligned} \tag{19}$$

□

Corollary 3. *Let OP_n be the swapped network; then,*

$${}^mM_2(OP_n) = \frac{n^2}{6} + \frac{n}{6} + \frac{1}{12}. \tag{20}$$

Proof. Let

$$f(x,y) = M(G;x,y) = 2xy^3 + 3x^2y^2 + (6n-14)x^2y^3 + \left(\frac{3(n-2)(n-3)}{2}\right)x^3y^3. \tag{21}$$

Then,

$$\begin{aligned} S_y(f(x,y)) &= \left(\frac{2}{3}\right)xy^3 + \left(\frac{3}{2}\right)x^2y^2 + \left(\frac{(6n-14)}{3}\right)x^2y^3 + \left(\frac{(n-2)(n-3)}{2}\right)x^3y^3, \\ S_xS_y(f(x,y)) &= \left(\frac{2}{3}\right)xy^3 + \left(\frac{3}{4}\right)x^2y^2 + \left(\frac{(6n-14)}{6}\right)x^2y^3 + \left(\frac{(n-2)(n-3)}{6}\right)x^3y^3. \end{aligned} \tag{22}$$

Now, we have

$$\begin{aligned} {}^mM_2(OP_n) &= S_xS_yf\big|_{x=y=1} \\ &= \frac{n^2}{6} + \frac{n}{6} + \frac{1}{12}. \end{aligned} \tag{23}$$

□

Corollary 4. *Let OP_n be the swapped network; then,*

$$R_\alpha\left((OP_n) = 6^\alpha + 6^\alpha 2^\alpha + 3^\alpha 2^\alpha(6n-14) + \frac{9^\alpha 3^\alpha(n-2)(n-3)}{2^\alpha}\right). \tag{24}$$

Proof. Let

$$f(x,y) = M(G;x,y) = 2xy^3 + 3x^2y^2 + (6n-14)x^2y^3 + \left(\frac{3(n-2)(n-3)}{2}\right)x^3y^3. \tag{25}$$

Then,

$$D_y^\alpha(f(x,y)) = 6^\alpha xy^3 + 6^\alpha x^2y^2 + 3^\alpha(6n-14)x^2y^3 + \left(\frac{9^\alpha(n-2)(n-3)}{2^\alpha}\right)x^3y^3,$$

$$D_x^\alpha D_y^\alpha\left((f(x,y)) = 6^\alpha xy^3 + 6^\alpha 2^\alpha x^2y^2 + 3^\alpha 2^\alpha(6n-14)x^2y^3 + \left(\frac{9^\alpha 3^\alpha(n-2)(n-3)}{2^\alpha}\right)x^3y^3\right). \tag{26}$$

Now, we have

$$R_\alpha\left((OP_n)\right) == D_x^\alpha D_y^\alpha f|_{x=y=1} = 6^\alpha + 6^\alpha 2^\alpha + 3^\alpha 2^\alpha(6n-14) + \frac{9^\alpha 3^\alpha(n-2)(n-3)}{2^\alpha}. \tag{27}$$

□

Corollary 5. *Let OP_n be the swapped network; then,*

$$RR_\alpha\left((OP_n) = \frac{2^\alpha}{3^\alpha} + \frac{3^\alpha}{2^{2\alpha}} + \frac{6n-14}{3^\alpha 2^\alpha} + \frac{(n-2)(n-3)}{2^\alpha 3^\alpha}\right). \tag{28}$$

Proof. Let

$$f(x,y) = M(G;x,y) = 2xy^3 + 3x^2y^2 + (6n-14)x^2y^3 + \left(\frac{3(n-2)(n-3)}{2}\right)x^3y^3. \tag{29}$$

Then,

$$S_y^\alpha(f(x,y)) = \left(\frac{2^\alpha}{3^\alpha}\right)xy^3 + \left(\frac{3^\alpha}{2^\alpha}\right)x^2y^2 + \left(\frac{(6n-14)}{3^\alpha}\right)x^2y^3 + \left(\frac{(n-2)(n-3)}{2^\alpha}\right)x^3y^3,$$

$$S_x^\alpha S_y^\alpha\left((f(x,y)) = \left(\frac{2^\alpha}{3^\alpha}\right)xy^3 + \left(\frac{3^\alpha}{2^{2\alpha}}\right)x^2y^2 + \left(\frac{(6n-14)}{3^\alpha 2^\alpha}\right)x^2y^3 + \left(\frac{(n-2)(n-3)}{2^\alpha 3^\alpha}\right)x^3y^3\right). \tag{30}$$

Now, we have

$$RR_\alpha\left((OP_n) == S_x^\alpha S_y^\alpha f|_{x=y=1} = \frac{2^\alpha}{3^\alpha} + \frac{3^\alpha}{2^{2\alpha}} + \frac{6n-14}{3^\alpha 2^\alpha} + \frac{(n-2)(n-3)}{2^\alpha 3^\alpha}. \tag{31}$$

□

Corollary 6. *Let OP_n be the swapped network; then,*

$$\mathrm{SSD}(OP_n) = 3n^2 - 2n + \frac{1}{3}. \tag{32}$$

Proof. Let

$$f(x,y) = M(G;x,y) = 2xy^3 + 3x^2y^2 + (6n-14)x^2y^3 + \left(\frac{3(n-2)(n-3)}{2}\right)x^3y^3. \tag{33}$$

Then,

$$S_xD_y(f(x,y)) = 6xy^3 + 3x^2y^2 + \left(\frac{3(6n-14)}{2}\right)x^2y^3 + \left(\frac{3(n-2)(n-3)}{2}\right)x^3y^3,$$

$$D_xS_y(f(x,y)) = \left(\frac{2}{3}\right)xy^3 + 3x^2y^2 + \left(\frac{2(6n-14)}{3}\right)x^2y^3 + \left(\frac{3(n-2)(n-3)}{2}\right)x^3y^3. \tag{34}$$

Now, we have

$$\mathrm{SSD}(OP_n) = \left(D_xS_yf + S_xD_yf\right)\Big|_{x=y=1} = 3n^2 - 2n + \frac{1}{3}. \tag{35}$$

□

Corollary 7. *Let OP_n be the swapped network; then,*

$$H(OP_n) = 2\left(\frac{n^2}{4} - \frac{n}{20} - \frac{1}{20}\right). \tag{36}$$

Proof. Let

$$f(x,y) = M(G;x,y) = 2xy^3 + 3x^2y^2 + (6n-14)x^2y^3 + \left(\frac{3(n-2)(n-3)}{2}\right)x^3y^3. \tag{37}$$

Then,

$$J(f(x,y)) = 5x^4 + (6n-14)x^5 + \left(\frac{3(n-2)(n-3)}{2}\right)x^6,$$

$$2S_xJ(f(x,y)) = 2\left(\frac{5x^4}{4} + \frac{(6n-14)x^5}{5} + \frac{(n-2)(n-3)x^6}{4}\right). \tag{38}$$

Now, we have

$$H(OP_n) = 2S_x J f\big|_{x=1} = 2\left(\frac{n^2}{4} - \frac{n}{20} - \frac{1}{20}\right). \tag{39}$$

□

Corollary 8. *Let OP_n be the swapped network; then,*

$$I(OP_n) = \frac{9n^2}{4} - \frac{81n}{20} - \frac{129}{5}. \tag{40}$$

Proof. Let

$$f(x,y) = M(G;x,y) = 2xy^3 + 3x^2y^2 + (6n-14)x^2y^3 + \left(\frac{3(n-2)(n-3)}{2}\right)x^3y^3. \tag{41}$$

Then,

$$\begin{aligned}
D_y(f(x,y)) &= 6xy^3 + 6x^2y^2 + 3(6n-14)x^2y^3 + \left(\frac{9(n-2)(n-3)}{2}\right)x^3y^3,\\
D_xD_y(f(x,y)) &= 6xy^3 + 12x^2y^2 + 6(6n-14)x^2y^3 + \left(\frac{27(n-2)(n-3)}{2}\right)x^3y^3,\\
JD_xD_y(f(x,y)) &= 18x^4 + 6(6n-14)x^5 + \left(\frac{27(n-2)(n-3)}{2}\right)x^6,\\
S_xJD_xD_y(f(x,y)) &= \frac{18x^4}{4} + \left(\frac{6(6n-14)}{5}\right)x^5 + \left(\frac{27(n-2)(n-3)}{12}\right)x^6.
\end{aligned} \tag{42}$$

Now, we have

$$I(OP_n) = S_xJD_xD_yf\big|_{x=1} = \frac{9n^2}{4} - \frac{81n}{20} - \frac{129}{5}. \tag{43}$$

□

Corollary 9. *Let OP_n be the swapped network; then,*

$$A(OP_n) = \frac{729n^2}{64} - \frac{573n}{64} - \frac{413}{32}. \tag{44}$$

Proof. Let

$$f(x,y) = M(G;x,y) = 2xy^3 + 3x^2y^2 + (6n-14)x^2y^3 + \left(\frac{3(n-2)(n-3)}{2}\right)x^3y^3. \tag{45}$$

Then,

$$\begin{aligned}
D_y^3(f(x,y)) &= 54xy^3 + 24x^2y^2 + 27(6n-14)x^2y^3 + \left(\frac{81(n-2)(n-3)}{2}\right)x^3y^3,\\
D_x^3D_y^3(f(x,y)) &= 54xy^3 + 192x^2y^2 + 216(6n-14)x^2y^3 + \left(\frac{2187(n-2)(n-3)}{2}\right)x^3y^3,\\
JD_x^3D_y^3(f(x,y)) &= 246x^4 + 216(6n-14)x^5 + \left(\frac{2187(n-2)(n-3)}{2}\right)x^6,\\
Q_{-2}JD_x^3D_y^3(f(x,y)) &= 246x^2 + 216(6n-14)x^3 + \left(\frac{2187(n-2)(n-3)}{2}\right)x^4,\\
S_x^3Q_{-2}JD_x^3D_y^3(f(x,y)) &= \frac{123x^2}{4} + 8(6n-14)x^3 + \left(\frac{729(n-2)(n-3)}{64}\right)x^4.
\end{aligned} \tag{46}$$

Now, we have

$$A(OP_n) = S_x^3Q_{-2}JD_x^3D_y^3f\big|_{x=1} = \frac{729n^2}{64} - \frac{573n}{64} - \frac{413}{32}. \tag{47}$$

□

5.2. Results for (OR_k)

Theorem 2. *Let OR_k be the swapped network; then,*

$$M(G;x,y) = nkx^ky^{k+1} + \left(\frac{n^2(k+1) - n(1+2k)}{2}\right)x^{k+1}y^{k+1}. \tag{48}$$

Proof. The OR_k network has the following two types of edges based on the degree of end vertices:

$$\begin{aligned}
E_{\{k,k+1\}}(OR_k) &= \{uv \in E(OR_k) : d_u = k, d_v = k+1\},\\
E_{\{k+1,k+1\}}(OR_k) &= \{uv \in E(OR_k) : d_u = k+1, d_v = k+1\},
\end{aligned} \tag{49}$$

such that

$$\left|E_{\{k,k+1\}}(OR_k)\right| = nk,$$
$$\left|E_{\{k+1,k+1\}}(OR_k)\right| = \frac{n^2(k+1)-n(1+2k)}{2}. \tag{50}$$

Now, from the definition of M-polynomial, we have

$$\begin{aligned} M(G;x,y) &= \sum_{\delta\le i\le j\le\Delta} m_{ij}x^i y^j \\ &= \sum_{k\le k+1} m_{k(k+1)}x^k y^{k+1} + \sum_{k+1\le k+1} m_{(k+1)(k+1)}x^{k+1}y^{k+1} \\ &= \left|E_{\{k,k+1\}}(OR_k)\right|x^k y^{k+1} + \left|E_{\{k+1,k+1\}}(OR_k)\right|x^{k+1}y^{k+1} \\ &= nkx^k y^{k+1} + \left(\frac{n^2(k+1)-n(1+2k)}{2}\right)x^{k+1}y^{k+1}. \end{aligned} \tag{51}$$

□

Corollary 10. *Let (OR_k) be the swapped network; then,*

$$M_1(OR_k) = 2nk^2 + nk + n^2(k+1)^2 - n(k+1)(1+2k). \tag{52}$$

Proof. Let

$$f(x,y) = M(G;x,y) = nkx^k y^{k+1} + \left(\frac{n^2(k+1)-n(1+2k)}{2}\right)x^{k+1}y^{k+1}. \tag{53}$$

Then,

$$\begin{aligned} D_x(f(x,y)) &= nk^2x^k y^{k+1} + (k+1)\left(\frac{n^2(k+1)-n(1+2k)}{2}\right)x^{k+1}y^{k+1}, \\ D_y(f(x,y)) &= nk(k+1)x^k y^{k+1} + (k+1)\left(\frac{n^2(k+1)-n(1+2k)}{2}\right)x^{k+1}y^{k+1}. \end{aligned} \tag{54}$$

Now, we have

$$\begin{aligned} M_1(OR_k) &= D_x f + D_y f\big|_{x=y=1} \\ &= 2nk^2 + nk + n^2(k+1)^2 - n(k+1)(1+2k). \end{aligned} \tag{55}$$

□

Corollary 11. *Let (OR_k) be the swapped network; then,*

$$M_2(OR_k) = nk^3 + nk^2 + \frac{n^2(k+1)^3}{2} - \frac{n(k+1)^2(1+2k)}{2}. \tag{56}$$

Proof. Let

$$f(x,y) = M(G;x,y) = nkx^k y^{k+1} + \left(\frac{n^2(k+1)-n(1+2k)}{2}\right)x^{k+1}y^{k+1}. \tag{57}$$

Then,

$$\begin{aligned} D_y(f(x,y)) &= nk^2x^k y^{k+1} + nkx^k y^{k+1} + (k+1)\left(\frac{n^2(k+1)-n(1+2k)}{2}\right)x^{k+1}y^{k+1}, \\ D_xD_y(f(x,y)) &= nk^3x^k y^{k+1} + nk^2x^k y^{k+1} + (k+1)^2\left(\frac{n^2(k+1)-n(1+2k)}{2}\right)x^{k+1}y^{k+1}. \end{aligned} \tag{58}$$

Now, we have

$$\begin{aligned} M_2(OR_k) &= D_xD_y f\big|_{x=y=1} \\ &= nk^3 + nk^2 + \frac{n^2(k+1)^3}{2} - \frac{n(k+1)^2(1+2k)}{2}. \end{aligned} \tag{59}$$

□

Corollary 12. *Let (OR_k) be the swapped network; then,*

$${}^mM_2(OR_k) = \frac{nk}{(k^2+k)} + \frac{n^2}{(2k+2)} - \frac{n(1+2k)}{2(k+1)^2}. \tag{60}$$

Proof. Let

$$f(x,y) = M(G;x,y) = nkx^k y^{k+1} + \left(\frac{n^2(k+1)-n(1+2k)}{2}\right)x^{k+1}y^{k+1}. \tag{61}$$

Then,

$$\begin{aligned} S_y(f(x,y)) &= \left(\frac{nk}{(k+1)}\right)x^k y^{k+1} + \left(\frac{n^2(k+1)-n(1+2k)}{2(k+1)}\right)x^{k+1}y^{k+1}, \\ S_xS_y(f(x,y)) &= \left(\frac{nk}{(k^2+k)}\right)x^k y^{k+1} + \left(\frac{n^2(k+1)-n(1+2k)}{2(k+1)^2}\right)x^{k+1}y^{k+1}. \end{aligned} \tag{62}$$

Now, we have

$$\begin{aligned} {}^{m}M_2(OR_k) &= S_xS_yf\big|_{x=y=1} \\ &= \frac{nk}{(k^2+k)} + \frac{n^2}{(2k+2)} - \frac{n(1+2k)}{2(k+1)^2}. \end{aligned} \tag{63}$$

□

Corollary 13. *Let (OR_k) be the swapped network; then,*

$$R_\alpha\Big((OR_k) = nk^{3\alpha} + nk^{2\alpha} + (k+1)^{2\alpha}\left(\frac{n^2(k+1)-n(1+2k)}{2^\alpha}\right)x^{k+1}y^{k+1}\Big). \tag{64}$$

Proof. Let

$$f(x,y) = M(G;x,y) = nkx^ky^{k+1} + \left(\frac{n^2(k+1)-n(1+2k)}{2}\right)x^{k+1}y^{k+1}. \tag{65}$$

Then,

$$\begin{aligned} D_y^\alpha(f(x,y)) &= nk^{2\alpha}x^ky^{k+1} + nk^\alpha x^ky^{k+1} + (k+1)^\alpha\left(\frac{n^2(k+1)-n(1+2k)}{2}\right)x^{k+1}y^{k+1}, \\ D_x^\alpha D_y^\alpha\Big((f(x,y)) &= nk^{3\alpha}x^ky^{k+1} + nk^{2\alpha}x^ky^{k+1} + (k+1)^{2\alpha}\left(\frac{n^2(k+1)-n(1+2k)}{2}\right)x^{k+1}y^{k+1}\Big). \end{aligned} \tag{66}$$

Now, we have

$$\begin{aligned} R_\alpha\Big((OR_k) &== D_x^\alpha D_y^\alpha f\big|_{x=y=1} \\ &= nk^{3\alpha} + nk^{2\alpha} + (k+1)^{2\alpha}\left(\frac{n^2(k+1)-n(1+2k)}{2^\alpha}\right)x^{k+1}y^{k+1}. \end{aligned} \tag{67}$$

□

Corollary 14. *Let (OR_k) be the swapped network; then,*

$$RR_\alpha\Big((OR_k) = \frac{n}{(k+1)^\alpha} + \left(\frac{n^2(k+1)-n(1+2k)}{(k+1)^{2\alpha}2^\alpha}\right)\Big). \tag{68}$$

Proof. Let

$$f(x,y) = M(G;x,y) = nkx^ky^{k+1} + \left(\frac{n^2(k+1)-n(1+2k)}{2}\right)x^{k+1}y^{k+1}. \tag{69}$$

Then,

$$\begin{aligned} S_y^\alpha(f(x,y)) &= \left(\frac{nk^\alpha}{(k+1)^\alpha}\right)x^ky^{k+1} + \left(\frac{n^2(k+1)-n(1+2k)}{2^\alpha(k+1)^\alpha}\right)x^{k+1}y^{k+1}, \\ S_x^\alpha S_y^\alpha\Big((f(x,y)) &= \left(\frac{n}{(k+1)^\alpha}\right)x^ky^{k+1} + \left(\frac{n^2(k+1)-n(1+2k)}{2^\alpha(k+1)^{2\alpha}}\right)x^{k+1}y^{k+1}\Big). \end{aligned} \tag{70}$$

Now, we have

$$\begin{aligned} RR_\alpha\Big((OR_k) &== S_x^\alpha S_y^\alpha f\big|_{x=y=1} \\ &= \frac{n}{(k+1)^\alpha} + \left(\frac{n^2(k+1)-n(1+2k)}{(k+1)^{2\alpha}2^\alpha}\right). \end{aligned} \tag{71}$$

□

Corollary 15. *Let (OR_k) be the swapped network; then,*

$$\mathrm{SSD}(OR_k) = \frac{nk^2}{(k+1)} + nk + n + n^2(k+1) - n(1+2k). \tag{72}$$

Proof. Let

$$f(x,y) = M(G;x,y) = nkx^ky^{k+1} + \left(\frac{n^2(k+1)-n(1+2k)}{2}\right)x^{k+1}y^{k+1}. \tag{73}$$

Then,

$$\begin{aligned} S_xD_y(f(x,y)) &= nkx^ky^{k+1} + nx^ky^{k+1} + \left(\frac{n^2(k+1)-n(1+2k)}{2}\right)x^{k+1}y^{k+1}, \\ D_xS_y(f(x,y)) &= \left(\frac{nk^2}{(k+1)}\right)x^ky^{k+1} + \left(\frac{n^2(k+1)-n(1+2k)}{2}\right)x^{k+1}y^{k+1}. \end{aligned} \tag{74}$$

Now, we have

$$\begin{aligned} \mathrm{SSD}(OR_k) &= \left(D_xS_yf + S_xD_yf\right)\big|_{x=y=1} \\ &= \frac{nk^2}{(k+1)} + nk + n + n^2(k+1) - n(1+2k). \end{aligned} \tag{75}$$

□

Corollary 16. *Let (OR_k) be the swapped network; then,*

$$H(OR_k) = 2\left(\frac{nk}{2k+1} + \frac{n^2(k+1)}{4k+4} - \frac{n(1+2k)}{4k+4}\right). \quad (76)$$

Proof. Let

$$f(x,y) = M(G;x,y) = nkx^k y^{k+1} + \left(\frac{n^2(k+1) - n(1+2k)}{2}\right)x^{k+1}y^{k+1}. \quad (77)$$

Then,

$$J(f(x,y)) = nkx^{2k+1} + \left(\frac{n^2(k+1) - n(1+2k)}{2}\right)x^{2k+2},$$

$$2S_xJ(f(x,y)) = 2\left(\left(\frac{nk}{2k+1}\right)x^{2k+1} + \left(\frac{n^2(k+1)}{4k+4}\right)x^{2k+2} - \left(\frac{n(1+2k)}{4k+4}\right)x^{2k+2}\right). \quad (78)$$

Now, we have

$$H(OR_k) = 2S_xJf\big|_{x=1} = 2\left(\frac{nk}{2k+1} + \frac{n^2(k+1)}{4k+4} - \frac{n(1+2k)}{4k+4}\right). \quad (79)$$

□

Corollary 17. *Let (OR_k) be the swapped network; then,*

$$I(OR_k) = \frac{nk^3}{2k+1} + \frac{nk^2}{2k+1} + (k+1)^2\left(\frac{n^2(k+1) - n(1+2k)}{4k+4}\right). \quad (80)$$

Proof. Let

$$f(x,y) = M(G;x,y) = nkx^k y^{k+1} + \left(\frac{n^2(k+1) - n(1+2k)}{2}\right)x^{k+1}y^{k+1}. \quad (81)$$

Then,

$$\begin{aligned}
D_y(f(x,y)) &= nk(k+1)x^k y^{k+1} + (k+1)\left(\frac{n^2(k+1) - n(1+2k)}{2}\right)x^{k+1}y^{k+1},\\
D_xD_y(f(x,y)) &= nk^3x^k y^{k+1} + nk^2x^k y^{k+1} + (k+1)^2\left(\frac{n^2(k+1) - n(1+2k)}{2}\right)x^{k+1}y^{k+1},\\
JD_xD_y(f(x,y)) &= nk^3x^{2k+1} + nk^2x^{2k+1} + (k+1)^2\left(\frac{n^2(k+1) - n(1+2k)}{2}\right)x^{2k+2},\\
S_xJD_xD_y(f(x,y)) &= \left(\frac{nk^3}{2k+1}\right)x^{2k+1} + \left(\frac{nk^2}{2k+1}\right)x^{2k+1} + (k+1)^2\left(\frac{n^2(k+1) - n(1+2k)}{4k+4}\right)x^{2k+2}.
\end{aligned} \quad (82)$$

Now, we have

$$I(OR_k) = S_xJD_xD_yf\big|_{x=1} = \frac{nk^3}{2k+1} + \frac{nk^2}{2k+1} + (k+1)^2\left(\frac{n^2(k+1) - n(1+2k)}{4k+4}\right). \quad (83)$$

□

Corollary 18. *Let (OR_k) be the swapped network; then,*

$$A(OR_k) = \frac{nk^5(k+1)^2}{(2k-1)^3} + \frac{nk^4(k+1)^2}{(2k-1)^3} + (k+1)^6\left(\frac{n^2(k+1) - n(1+2k)}{8k^3}\right). \quad (84)$$

Proof. Let

$$f(x,y) = M(G;x,y) = nkx^k y^{k+1} + \left(\frac{n^2(k+1) - n(1+2k)}{2}\right)x^{k+1}y^{k+1}. \quad (85)$$

Then,

$$\begin{aligned}
D_y^3(f(x,y)) &= nk^2(k+1)^2x^ky^{k+1} + nk(k+1)^2x^ky^{k+1} \\
&\quad + (k+1)^3\left(\frac{n^2(k+1)-n(1+2k)}{2}\right)x^{k+1}y^{k+1}, \\
D_y^3D_y^3(f(x,y)) &= nk^5(k+1)^2x^ky^{k+1} + nk^4(k+1)^2x^ky^{k+1} \\
&\quad + (k+1)^6\left(\frac{n^2(k+1)-n(1+2k)}{2}\right)x^{k+1}y^{k+1}, \\
JD_y^3D_y^3(f(x,y)) &= nk^5(k+1)^2x^{2k+1} + nk^4(k+1)^2x^{2k+1} \\
&\quad + (k+1)^6\left(\frac{n^2(k+1)-n(1+2k)}{2}\right)x^{2k+2}, \\
Q_{-2}JD_y^3D_y^3(f(x,y)) &= nk^5(k+1)^2x^{2k-1} + nk^4(k+1)^2x^{2k-1} \\
&\quad + (k+1)^6\left(\frac{n^2(k+1)-n(1+2k)}{2}\right)x^{2k}, \\
S_x^3Q_{-2}JD_y^3D_y^3(f(x,y)) &= \left(\frac{nk^5(k+1)^2}{(2k-1)^3}\right)x^{2k-1} + \left(\frac{nk^4(k+1)^2}{(2k-1)^3}\right)x^{2k-1} \\
&\quad + (k+1)^6\left(\frac{n^2(k+1)-n(1+2k)}{8k^3}\right)x^{2k}.
\end{aligned} \tag{86}$$

Now, we have

$$\begin{aligned}
A(OR_k) &= S_x^3Q_{-2}JD_y^3D_y^3f\big|_{x=1} \\
&= \frac{nk^5(k+1)^2}{(2k-1)^3} + \frac{nk^4(k+1)^2}{(2k-1)^3} \\
&\quad + (k+1)^6\left(\frac{n^2(k+1)-n(1+2k)}{8k^3}\right).
\end{aligned} \tag{87}$$

□

6. Conclusion

In this paper, our focus is on swapped interconnection networks that allow systematic construction of large, scalable, modular, and robust parallel architectures, while maintaining many desirable attributes of the underlying basis network comprising its clusters. We have computed several TIs of underlined networks. Firstly, we computed M-polynomials of understudy networks, and then we recovered Zagreb indices, Randić indices, and some other indices [49–52].

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they no conflicts of interest.

Authors' Contributions

All authors contributed equally to this study.

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