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Research Article

Topological Indices of Certain Transformed Chemical Structures

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Topological indices like generalized Randić index, augmented Zagreb index, geometric arithmetic index, harmonic index, product connectivity index, general sum-connectivity index, and atom-bond connectivity index are employed to calculate the bioactivity of chemicals. In this paper, we define these indices for the line graph of k-subdivided linear $[n]$ Tetracene, fullerene networks, tetracenic nanotori, and carbon nanotube networks.

1. Introduction

In chemical graph theory, we apply the concepts of graph theory to describe the mathematical model of a variety of chemical structures. The atoms of the molecules correspond to the vertices, and the chemical bond is reflected by edges. Topological indices are numerical parameters of chemical graphs associated with quantitative structure property relationship (QSPR) and quantitative structure activity relationship (QSAR). The major topological indices are distance based, degree based, and eccentricity based. Among these classes, degree-based topological indices are of great importance and are helpful tools for chemists. The concept of topological index came from the work done by Wiener, when he was working on boiling point of paraffin [1]. The Wiener index is the first and most studied topological index. The degree-based topological indices for line graph of some subdivided graphs were studied in [2]. In [3], the bounds of topological indices for some graph operations are discussed. Baĉa et al. studied some indices for families of fullerene graph in [4]. Baig et al. found the topological indices for poly oxide, poly silicate, DOX, and DSL networks in [5]. Liu et al. found the different topological indices for Eulerian graphs, fractal graphs, and generalized Sierpinski networks in [6–8]. The number of spanning trees and normalized Laplacian of linear octagonal quadrilateral networks were studied in [9]. Recently, Liu et al. in [10] calculated the generalized adjacency, Laplacian, and signless Laplacian spectra of the weighted edge corona networks. In [11], Gao et al. found the forgotten topological index on chemical structure in drugs. Imran et al. calculated the degree-based topological indices for different networks in [12–15]. In 2018, Mufti et al. [16] found the topological indices for para-line graphs of pentacene. Nadeem et al. calculated the degree-based topological indices for para-line graphs of V-Phenylenic nanostructures in [17].

Randić in 1975 introduced the Randić index [18]. Bollobas and Erdos generalized the Randić index for any real number α and named it as generalized Randić index:

$$R_\alpha(G) = \sum_{uv\in E(G)} (d_u d_v)^\alpha. \tag{1}$$

Recall that the augmented Zagreb index is [19]

$$A(G) = \sum_{uv\in E(G)} \left[\frac{d_u d_v}{d_u + d_v - 2}\right]^3. \tag{2}$$

Furtula et al. defined the geometric arithmetic index as [19]

$$GA(G) = \sum_{uv\in E(G)} \frac{2\sqrt{d_u d_v}}{d_u + d_v}. \quad (3)$$

Moreover, the harmonic index is defined as follows [20]:

$$H(G) = \sum_{uv\in E(G)} \frac{2}{d_u + d_v}. \quad (4)$$

The first degree-based connectivity index for graphs evolved by using vertex degree is product connectivity index (Randić index), proposed by the chemist Randić, as [18]

$$R(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_u d_v}}. \quad (5)$$

The general sum-connectivity index has been introduced in 2010, as [21]

$$\chi_\alpha(G) = \sum_{uv\in E(G)} (d_u + d_v)^\alpha. \quad (6)$$

One of the well-known degree-based topological indices is the atom-bond connectivity (ABC) index of a graph, proposed by Estrada et al. and defined as [22]

$$ABC(G) = \sum_{uv\in E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}. \quad (7)$$

The remaining article is characterized as follows. In Section 2, the topological indices for the line graph of subdivided graph of different nanostructures have been discussed. The conclusion has been drawn in Section 3.

2. Main Results

Let G be a finite, simple, and connected graph with order p and size q. For $k \geq 1$, a k-subdivided graph $G(k)$ of G is obtained by replacing each edge of graph G by a path P_{k+2}. A line graph $L(G)$ of graph G is a transformed graph having q vertices and two vertices have common neighbourhood in $L(G)$ if and only if their corresponding edges are adjacent in G.

2.1. Linear [n] Tetracene. We will start the debate from linear $[n]$ Tetracene, by defining its topology. It has the appearance of a pale orange powder. Tetracene is a four ringed member of the series of acnes. The original graph has order $18n$ and size $23n - 2$. The line graph of subdivided graph has $(2,2)$, $(2,3)$, and $(3,3)$ types of edges. For subdivision $k = 1$, the number of edges will be $8n + 10$, $16n - 4$, and $37n - 16$, respectively. Similarly, for subdivision $k = 2$, the number of edges will be $24n + 12$, $30n - 12$, and $30n - 12$, respectively. Further, for subdivision $k \geq 3$, the number of edges will be $24n + 12 + (k-2)(23n-2)$, $30n - 12$, and $30n - 12$, respectively. Let $G(n,k)$ be the line graph of k-subdivided linear[n] Tetracene with $k \geq 3$ vertices. Its topological indices are calculated in the next theorems.

Theorem 1. *Let $G(n,k)$ be the line graph of k-subdividing linear [n] Tetracene with $k \geq 3$. The generalized Randić index, augmented Zagreb index, geometric arithmetic index, and harmonic index of $G(n,k)$ are*

(1) $R_\alpha(G(n,k)) = 2^{2\alpha}(k-2)(23n-2) + 3.2^{2\alpha+2}(2n+1) + (2^{\alpha+1}.3^{\alpha+1} + 2.3^{2\alpha+1})(5n-2)$

(2) $A(G(n,k)) = 8(k-2)(23n-2) + (24759n/32) - (2187/16)$

(3) $GA(G(n,k)) = (k-2)(23n-2) + 6(2\sqrt{6} + 9)n - (24\sqrt{6}/5)$

(4) $H(G(n,k)) = ((k-2)/2)(23n-2) + 34n - (14/5)$

Proof. The generalized Randić index of $G(n,k)$ is computed as follows:

$$\begin{aligned} R_\alpha(G(n,k)) &= \sum_{uv\in E(G(n,k))} (d_u d_v)^\alpha \\ &= \sum_{uv\in E(2,2)} (d_u d_v)^\alpha + \sum_{uv\in E(2,3)} (d_u d_v)^\alpha \\ &\quad + \sum_{uv\in E(3,3)} (d_u d_v)^\alpha \\ &= [(k-2)(23n-2) + 24n + 12]4^\alpha \\ &\quad + (30n-12)6^\alpha + (30n-12)9^\alpha \\ &= 2^{2\alpha}(k-2)(23n-2) + 3.2^{2\alpha+2}(2n+1) \\ &\quad + \left(2^{\alpha+1}\cdot 3^{\alpha+1} + 2.3^{2\alpha+1}\right)(5n-2). \end{aligned} \quad (8)$$

The augmented Zagreb index is computed as

$$\begin{aligned} A(G(n,k)) &= \sum_{uv\in E(G(n,k))} \left[\frac{d_u d_v}{d_u + d_v - 2}\right]^3 \\ &= \sum_{uv\in E(2,2)} \left[\frac{d_u d_v}{d_u + d_v - 2}\right]^3 + \sum_{uv\in E(2,3)} \left[\frac{d_u d_v}{d_u + d_v - 2}\right]^3 \\ &\quad + \sum_{uv\in E(3,3)} \left[\frac{d_u d_v}{d_u + d_v - 2}\right]^3 \\ &= [(k-2)(23n-2) + 24n + 12]\left[\frac{4}{2+2-2}\right]^3 \\ &\quad + (30n-12)\left[\frac{6}{2+3-2}\right]^3 + (30n-12)\left[\frac{9}{3+3-2}\right]^3 \\ &= 8(k-2)(23n-2) + \frac{24759n}{32} - \frac{2187}{16}. \end{aligned} \quad (9)$$

Next, the geometric arithmetic index is computed as

$$\begin{aligned} GA(G(n,k)) &= \sum_{uv\in E(G(n,k))} \frac{2\sqrt{d_u d_v}}{d_u+d_v} \\ &= \sum_{uv\in E(2,2)} \frac{2\sqrt{d_u d_v}}{d_u+d_v} + \sum_{uv\in E(2,3)} \frac{2\sqrt{d_u d_v}}{d_u+d_v} \\ &\quad + \sum_{uv\in E(3,3)} \frac{2\sqrt{d_u d_v}}{d_u+d_v} \\ &= [(k-2)(23n-2)+24n+12]\frac{2\sqrt{4}}{2+2} \\ &\quad + (30n-12)\frac{2\sqrt{6}}{2+3} + (30n-12)\frac{2\sqrt{9}}{3+3} \\ &= (k-2)(23n-2)+6(2\sqrt{6}+9)n-\frac{24\sqrt{6}}{5}. \end{aligned} \tag{10}$$

Moreover, the harmonic index is defined as follows:

$$\begin{aligned} H(G(n,k)) &= \sum_{uv\in E(G(n,k))} \frac{2}{d_u+d_v} \\ &= \sum_{uv\in E(2,2)} \frac{2}{d_u+d_v} + \sum_{uv\in E(2,3)} \frac{2}{d_u+d_v} \\ &\quad + \sum_{uv\in E(3,3)} \frac{2}{d_u+d_v} \\ &= [(k-2)(23n-2)+24n+12]\frac{2}{2+2} \\ &\quad + (30n-12)\frac{2}{2+3} + (30n-12)\frac{2}{3+3} \\ &= \frac{(k-2)}{2}(23n-2)+34n-\frac{14}{5}, \end{aligned} \tag{11}$$

which completes the proof of the theorem. □

Theorem 2. *The product connectivity index, general sum-connectivity index, and atom-bond connectivity index of $G(n,k)$ are*

(1) $R(G(n,k)) = ((k-2)/2)(23n-2) + (22 + (10\sqrt{3}/\sqrt{2}))n - 2(\sqrt{6}+1)$

(2) $\chi_\alpha(G(n,k)) = 2^{2\alpha}(k-2)(23n-2) + (6.5^\alpha + 2^{\alpha+1}\cdot 3^{\alpha+1})(5n-2)$

(3) $ABC(G(n,k)) = ((k-2)/\sqrt{2})(23n-2) + (27\sqrt{2}+20)n - 8$

Proof. The product connectivity index of $G(n,k)$ is computed as

$$\begin{aligned} R(G(n,k)) &= \sum_{uv\in E(G(n,k))} \frac{1}{\sqrt{d_u d_v}} \\ &= \sum_{uv\in E(2,2)} \frac{1}{\sqrt{d_u d_v}} + \sum_{uv\in E(2,3)} \frac{1}{\sqrt{d_u d_v}} \\ &\quad + \sum_{uv\in E(3,3)} \frac{1}{\sqrt{d_u d_v}} \\ &= [(k-2)(23n-2)+24n+12]\frac{1}{\sqrt{4}} \\ &\quad + (30n-12)\frac{1}{\sqrt{6}} + (30n-12)\frac{1}{\sqrt{9}} \\ &= \frac{(k-2)}{2}(23n-2) + \left(22+\frac{10\sqrt{3}}{\sqrt{2}}\right)n - 2(\sqrt{6}+1). \end{aligned} \tag{12}$$

The general sum-connectivity index is computed as

$$\begin{aligned} \chi_\alpha(G(n,k)) &= \sum_{uv\in E(G(n,k))} (d_u+d_v)^\alpha \\ &= \sum_{uv\in E(2,2)} (d_u+d_v)^\alpha + \sum_{uv\in E(2,3)} (d_u+d_v)^\alpha \\ &\quad + \sum_{uv\in E(3,3)} (d_u+d_v)^\alpha \\ &= [(k-2)(23n-2)+24n+12](2+2)^\alpha \\ &\quad + (30n-12)(2+3)^\alpha + (30n-12)(3+3)^\alpha \\ &= 2^{2\alpha}(k-2)(23n-2) + \left(6.5^\alpha + 2^{\alpha+1}\cdot 3^{\alpha+1}\right) \\ &\quad \cdot (5n-2). \end{aligned} \tag{13}$$

The atom-bond connectivity (ABC) index is computed as

$$\begin{aligned} ABC(G(n,k)) &= \sum_{uv\in E(G(n,k))} \sqrt{\frac{d_u+d_v-2}{d_u d_v}} \\ &= \sum_{uv\in E(2,2)} \sqrt{\frac{d_u+d_v-2}{d_u d_v}} + \sum_{uv\in E(2,3)} \sqrt{\frac{d_u+d_v-2}{d_u d_v}} \\ &\quad + \sum_{uv\in E(2,3)} \sqrt{\frac{d_u+d_v-2}{d_u d_v}} \\ &= [(k-2)(23n-2)+24n+12]\sqrt{\frac{2+2-2}{4}} \\ &\quad + (30n-12)\sqrt{\frac{2+3-2}{6}} + (30n-12)\sqrt{\frac{3+3-2}{9}} \\ &= \frac{(k-2)}{\sqrt{2}}(23n-2) + (27\sqrt{2}+20)n - 8, \end{aligned} \tag{14}$$

which completes the proof of the theorem. □

2.2. Tetracenic Nanotori. Next, we have nanostructures F, G, K, and L. The original graph of each structure has same order $18pq$, and size of F is $27pq-4p-2q$, G is $27pq-4p$, K is $27pq-2q$, and L is $27pq$. Let r and s be the number of vertices with degree 2 and 3, respectively. The types of edges of line graph of subdivided graph for each nanostructure will be $(2,2)$, $(2,3)$, and $(3,3)$. For subdivision $k=1$, the number of edges for F will be $2q+r+4$, $16p+4q-8$, and $27pq-20p-8q+3s+4$, respectively, the number of edges for G will be r, $16p$, and $27pq-20p+3s$, respectively, the number of edges for K will be $2q+r$, $4q$, and $27pq-8q+3s$, respectively, and the number of edges for L will be r, 0, and $27pq+3s$, respectively. Similarly, for subdivision $k=2$, the number of edges for F will be $16p+8q+r$, $54pq-24p-12q$, and $3s$, respectively, the number of edges for G will be $16p+r$, $54pq-24p$, and $3s$, respectively, the number of edges for K will be $8q+r$, $54pq-12q$, and $3s$, respectively, and the number of edges for L will be r, $54pq$, and $3s$, respectively. Further, for subdivision $k\geq 3$, the number of edges for F will be $16p+8q+r+(27pq-4p-2q)(k-2)$, $54pq-24p-12q$, and $3s$, respectively, the number of edges for G will be $16p+r+(27pq-4p)(k-2)$, $54pq-24p$, and $3s$, respectively, the number of edges for K will be $8q+r+(27pq-2q)(k-2)$, $54pq-12q$, and $3s$, respectively, and the number of edges for L will be $r+27pq(k-2)$, $54pq$, and $3s$, respectively.

Theorem 3. *Let $G(n,k)$ be the line graph of subdivided nanostructures F, G, K, and L, by $k\geq 3$ vertices. Their generalized Randić index, general Zagreb index, augmented Zagreb index, geometric arithmetic index, and harmonic index are as follows.*

For F:

(1) $R_\alpha(G(n,k))=2^{2\alpha}[(k-2)(27pq-4p-2q)+16p+8q+r]+2^{\alpha+1}\cdot 3^{\alpha+1}(9pq-4p-2q)+3^{2\alpha+1}s$

(2) $A(G(n,k))=8[(k-2)(27pq-4p-2q)+54pq-8p-4q+r]+(2187s/64)$

(3) $GA(G(n,k))=(k-2)(27pq-4p-2q)+16p+8q+r+3s+(12\sqrt{6}/5)(9pq-4p-2q)$

(4) $H(G(n,k))=((k-2)/2)(27pq-4p-2q)+(108/5)pq-(8/5)p-(4/5)q+(r/2)+s$

For G:

(1) $R_\alpha(G(n,k))=2^{2\alpha}[(k-2)(27pq-4p)+16p+r]+2^{\alpha+1}\cdot 3^{\alpha+1}(9pq-4p)+3^{2\alpha+1}s$

(2) $A(G(n,k))=8[(k-2)(27pq-4p)+54pq-8p+r]+(2187s/64)$

(3) $GA(G(n,k))=(k-2)(27pq-4p)+16p+r+3s+(12\sqrt{6}/5)(9pq-4p)$

(4) $H(G(n,k))=((k-2)/2)(27pq-4p)+(108/5)pq-(8/5)p+(r/2)+s$

For K:

(1) $R_\alpha(G(n,k))=2^{2\alpha}[(k-2)(27pq-2q)+8q+r]+2^{\alpha+1}\cdot 3^{\alpha+1}(9pq-2q)+3^{2\alpha+1}s$

(2) $A(G(n,k))=8[(k-2)(27pq-2q)+54pq-4q+r]+(2187s/64)$

(3) $GA(G(n,k))=(k-2)(27pq-2q)+8q+r+3s+(12\sqrt{6}/5)(9pq-2q)$

(4) $H(G(n,k))=((k-2)/2)(27pq-2q)+(108/5)pq-(4/5)q+(r/2)+s$

For L:

(1) $R_\alpha(G(n,k))=2^{2\alpha}[(k-2)27pq+r]+2^{\alpha+1}\cdot 3^{\alpha+3}pq+3^{2\alpha+1}s$

(2) $A(G(n,k))=8[(k-2)27pq+54pq+r]+(2187s/64)$

(3) $GA(G(n,k))=(k-2)27pq+r+3s+(108\sqrt{6}/5)pq$

(4) $H(G(n,k))=((k-2)/2)27pq+(108/5)pq+(r/2)+s$

Proof. By using equations (1)–(4), we get the required results. □

Theorem 4. *Let $G(n,k)$ be the line graph obtained after subdividing nanostructures F, G, K, and L by $k\geq 3$ vertices. Their connectivity indices, i.e., product connectivity index, general sum-connectivity index, and atom-bond connectivity index, are as follows.*

For F:

(1) $R(G(n,k))=((k-2)/2)(27pq-4p-2q)+9\sqrt{6}pq+4(2-\sqrt{6})p+2(2-\sqrt{6})q+(r/2)+s$

(2) $\chi_\alpha(G(n,k))=2^{2\alpha}[(k-2)(27pq-4p-2q)+16p+8q+r]+6.5^\alpha(9pq-4p-2q)+2^\alpha\cdot 3^{\alpha+1}s$

(3) $ABC(G(n,k))=(1/\sqrt{2})[(k-2)(27pq-4p-2q)+54pq-8p-4q+r]+2s$

For G:

(1) $R(G(n,k))=((k-2)/2)(27pq-4p)+9\sqrt{6}pq+4(2-\sqrt{6})p+(r/2)+s$

(2) $\chi_\alpha(G(n,k))=2^{2\alpha}[(k-2)(27pq-4p)+16p+r]+6.5^\alpha(9pq-4p)+2^\alpha\cdot 3^{\alpha+1}s$

(3) $ABC(G(n,k))=(1/\sqrt{2})[(k-2)(27pq-4p)+54pq-8p+r]+2s$

For K:

(1) $R(G(n,k))=((k-2)/2)(27pq-2q)+9\sqrt{6}pq+2(2-\sqrt{6})q+(r/2)+s$

(2) $\chi_\alpha(G(n,k))=2^{2\alpha}[(k-2)(27pq-2q)+8q+r]+6.5^\alpha(9pq-2q)+2^\alpha\cdot 3^{\alpha+1}s$

(3) $ABC(G(n,k))=(1/\sqrt{2})[(k-2)(27pq-2q)+54pq-4q+r]+2s$

For L:

(1) $R(G(n,k))=((k-2)/2)27pq+9\sqrt{6}pq+(r/2)+s$

(2) $\chi_\alpha(G(n,k))=2^{2\alpha}[(k-2)27pq+r]+54.5^\alpha pq+2^\alpha\cdot 3^{\alpha+1}s$

(3) $ABC(G(n,k))=(1/\sqrt{2})[(k-2)27pq+54pq+r]+2s$

Proof. By using equations (5), (6), and (8), we get the required results. □

2.3. Fullerene Networks. The next model we are going to add is of fullerene. It is a regular graph of degree 3. Let KB_m^n be the Klein-bottle fullerene and H_m^n be the toroidal fullerene for $n \geq 2$ *even* and $m \geq 2$ having order $2mn$ and size $3mn$ and $KB_{(m+(1/2))}^n$ be the Klein-bottle fullerene for $n \geq 2$ *even* and $m \geq 1$ with order $2n(m+(1/2))$ and size $3n(m+(1/2))$. The line graph of subdivided graph for $k = 1$ each structure is a 3-regular graph, with KB_m^n and H_m^n having same $9mn$ edge count, whereas $KB_{(m+(1/2))}^n$ has $9n(m+(1/2))$ edges. Similarly, for subdivision $k = 2$, the types of edges are $(2,3)$ and $(3,3)$. Both KB_m^n and H_m^n have same count $6mn$ in each type of edges, and for $KB_{(m+(1/2))}^n$, each type has again same count $6n(m+(1/2))$. For subdivision $k \geq 3$, the fullerenes KB_m^n and H_m^n have (2,3), (3,3) and (2,2) types of edges with counts 6mn, 6mn and 3mn(k–2), respectively. The fullerene $KB_{(m+(1/2))}^n$ also has (2,3), (3,3) and (2,2) types of edges, and their count is 3n(2m+1), 3n(2m+1), and 3n(m+1/2)(k–2), respectively.

Theorem 5. *Let $G(n,k)$ be the line graph of k-subdivided Klein-bottle fullerenes, KB_m^n and, H_m^n for $n \geq 2$ even and $m \geq 2$, further Klein-bottle fullerene, $KB_{(m+(1/2))}^n$ for $n \geq 2$ even and $m \geq 1$, with $k \geq 3$. Their generalized Randić index, general Zagreb index, augmented Zagreb index, geometric arithmetic index, and harmonic index are*

For KB_m^n:

(1) $R_\alpha(G(n,k)) = [3.2^{2\alpha}(k-2) + 2^{\alpha+1} \cdot 3^{\alpha+1} + 2 \cdot 3^{2\alpha+1}]mn$

(2) $A(G(n,k)) = 3[8(k-2) + (1241/32)]mn$

(3) $GA(G(n,k)) = 3[(k-2) + (4\sqrt{6}/5) + 2]mn$

(4) $H(G(n,k)) = [(3(k-2)/2) + (22/5)]mn$

For $KB_{(m+(1/2))}^n$:

(1) $R_\alpha(G(n,k)) = [3.2^{2\alpha}(k-2) + 2^{\alpha+1} \cdot 3^{\alpha+1} + 2 \cdot 3^{2\alpha+1}]n(m+(1/2))$

(2) $A(G(n,k)) = 3[8(k-2) + (1241/32)]n(m+(1/2))$

(3) $GA(G(n,k)) = 3[(k-2) + (4\sqrt{6}/5) + 2]n(m+(1/2))$

(4) $H(G(n,k)) = [(3(k-2)/2) + (22/5)]n(m+(1/2))$

For H_m^n:

(1) $R_\alpha(G(n,k)) = [3.2^{2\alpha}(k-2) + 2^{\alpha+1} \cdot 3^{\alpha+1} + 2 \cdot 3^{2\alpha+1}]mn$

(2) $A(G(n,k)) = 3[8(k-2) + (1241/32)]mn$

(3) $GA(G(n,k)) = 3[(k-2) + (4\sqrt{6}/5) + 2]mn$

(4) $H(G(n,k)) = [(3(k-2)/2) + (22/5)]mn$

Proof. By using equations (1)–(4), we get the required results. □

Theorem 6. *Let $G(n,k)$ be the line graph of k-subdivided Klein-bottle fullerene, KB_m^n and toroidal fullerene, H_m^n for $n \geq 2$ even and $m \geq 2$, further Klein-bottle fullerene, $KB_{(m+1)/2}^n$ for $n \geq 2$ even and $m \geq 1$, by $k \geq 3$. Their connectivity indices, i.e., product connectivity index, general sum-connectivity index, and atom-bond connectivity index, are*

For KB_m^n:

(1) $R(G(n,k)) = [(3(k-2)/2) + \sqrt{6} + 2]mn$

(2) $\chi_\alpha(G(n,k)) = [3.2^{2\alpha}(k-2) + 6.5^\alpha + 2^{\alpha+1} \cdot 3^{\alpha+1}]mn$

(3) $ABC(G(n,k)) = 3[((k-2)/\sqrt{2}) + \sqrt{2} + 4]mn$

For $KB_{(m+(1/2))}^n$:

(1) $R(G(n,k)) = [(3(k-2)/2) + \sqrt{6} + 2]n(m+(1/2))$

(2) $\chi_\alpha(G(n,k)) = [3.2^{2\alpha}(k-2) + 6.5^\alpha + 2^{\alpha+1} \cdot 3^{\alpha+1}]n(m+(1/2))$

(3) $ABC(G(n,k)) = 3[((k-2)/\sqrt{2}) + \sqrt{2} + 4]n(m+(1/2))$

For H_m^n:

(1) $R(G(n,k)) = [(3(k-2)/2) + \sqrt{6} + 2]mn$

(2) $\chi_\alpha(G(n,k)) = [3.2^{2\alpha}(k-2) + 6.5^\alpha + 2^{\alpha+1} \cdot 3^{\alpha+1}]mn$

(3) $ABC(G(n,k)) = 3[(k-2)/\sqrt{2}) + \sqrt{2} + 4]mn$

Proof. By using equations (5)–(7), we get the required results. □

2.4. Carbon Nanotube Networks. Let NA_m^n be the nanotube, for $m, n \geq 2$ with order $2m(n+1)$ and size $m(3n+2)$. Let NB_m^n be the nanotube, for $n \geq 2$ *even* and $m \geq 1$ with order $2n(m+1)$ and size $n(3m+2)$. In the line graph of subdivided graph, both structures have $(2,2)$, $(2,3)$, and $(3,3)$ types of edges. For $k = 1$, NA_m^n has $2m$, $4m$, and $9mn - 2m$ edges, respectively, whereas NB_m^n has $3n$, $2n$, and $9mn - n$ edges, respectively. Similarly, for subdivision $k = 2$, NA_m^n has $6m$ and last two types have same $6mn$ count of edges, whereas NB_m^n has $6n$ and again last two types have same $6mn$ count of edges. Further, for subdivision $k \geq 3$, NA_m^n has $6m + (k-2)(3mn + 2m)$ and last two types have same $6mn$ count of edges, whereas NB_m^n has $6n + (k-2)(3mn + 2n)$ and last two types have same $6mn$ count of edges.

Theorem 7. *Let $G(n,k)$ be the line graph of k-subdivided NA_m^n nanotube, for $m, n \geq 2$, and NB_m^n nanotube, for $n \geq 2$ even and $m \geq 1$ with $k \geq 3$. Their generalized Randić index, general Zagreb index, augmented Zagreb index, geometric arithmetic index, and harmonic index are*

For NA_m^n

(1) $R_\alpha(G(n,k)) = 2^{2\alpha}(k-2)(3mn + 2m) + 3.2^{2\alpha+1}m + (3^{\alpha+1} \cdot 2^{\alpha+1} + 2.3^{2\alpha+1})mn$

(2) $A(G(n,k)) = 8(k-2)(3mn + 2m) + 48m + (2187/32)mn$

(3) $GA(G(n,k)) = (k-2)(3mn+2m)+6m+6((2\sqrt{6}/5)+1)mn$
(4) $H(G(n,k)) = ((k-2)/2)(3mn+2m)+3m+(17/5)mn$

For NB_m^n

(1) $R_\alpha(G(n,k)) = 2^{2\alpha}(k-2)(3mn+2n)+3.2^{2\alpha+1}n+(3^{\alpha+1}\cdot 2^{\alpha+1}+2.3^{2\alpha+1})mn$
(2) $A(G(n,k)) = 8(k-2)(3mn+2n)+48n+(2187/32)mn$
(3) $GA(G(n,k)) = (k-2)(3mn+2n)+6n+6((2\sqrt{6}/5)+1)mn$
(4) $H(G(n,k)) = ((k-2)/2)(3mn+2n)+3n+(17/5)mn$

Proof. By using equations (1)–(4), we get the required results. □

Theorem 8. *Let $G(n,k)$ be the line graph of k-subdivided NA_m^n nanotube, for $m,n\geq 2$, and NB_m^n nanotube, for $n\geq 2$ even and $m\geq 1$ with $k\geq 3$. Their connectivity indices, i.e., product connectivity index, general sum-connectivity index, and atom-bond connectivity index, are*

For NA_m^n:

(1) $R(G(n,k)) = ((k-2)/2)(3mn+2m)+3m+(\sqrt{6}+2)mn$
(2) $\chi_\alpha(G(n,k)) = 2^{2\alpha}(k-2)(3mn+2m)+3.2^{2\alpha+1}m+(6.5^\alpha+2^{\alpha+1}\cdot 3^{\alpha+1})mn$
(3) $ABC(G(n,k)) = ((k-2)/\sqrt{2})(3mn+2m)+3\sqrt{2}m+(3\sqrt{2}+4)mn$

For NB_m^n:

(1) $R(G(n,k)) = ((k-2)/2)(3mn+2n)+3n+(\sqrt{6}+2)mn$
(2) $\chi_\alpha(G(n,k)) = 2^{2\alpha}(k-2)(3mn+2n)+3.2^{2\alpha+1}n+(6.5^\alpha+2^{\alpha+1}\cdot 3^{\alpha+1})mn$
(3) $ABC(G(n,k)) = ((k-2)/\sqrt{2})(3mn+2n)+3\sqrt{2}n+(3\sqrt{2}+4)mn$

Proof. By using equations (5)–(7), we get the required results. □

3. Conclusion

All graphs are simple in this article. We have found different degree-based topological indices for the line graph of subdivided $k\geq 3$ graph of linear $[n]$ Tetracene, Klein bottle fullerene, V-tetracenic nanotube, H-tetracenic nanotube, tetracenic nanotori, toroidal fullerene, and carbon nanotubes.

4. Future Work

In future, degree-based topological indices for some additional structures can be studied. Moreover, can we study degree-based topological indices for line graph of k-subdivided graph, having any kind of edge degree sequence and type of edges?

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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