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Research Article

An Integrated Slacks-Based Measure of Super-Efficiency with Input Saving and Output Surplus Scaling Factors and its Application in Paper Chemical Mills

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Data envelopment analysis (DEA) as a nonparametric programming approach has been widely extended and applied in many areas. Conventional DEA models can well measure the efficiency of inefficient decision-making units (DMUs) but cannot further discriminate the efficient DMUs. A lot of methods are proposed to address this problem. One of the most important methods is the slacks-based measure of super-efficiency model (S-SBM model) developed by Tone in 2002. However, the projection for a DMU on the efficient frontier identified by S-SBM model may not be strongly Pareto-efficient that makes the super-efficiency score misestimated. This paper revises the usual slacks-based measure of super-efficiency by incorporating input saving and output surplus scaling factors into the objection function for measuring DMUs. We integrate SBM model and S-SBM model effectively and yield input saving and output surplus scaling factors as well as input and output slacks under only one integrated model. According to the study, the projection reference point identified by our method is strongly Pareto-efficient. Meanwhile, how each decision variable influences the efficiency score for a specific DMU is revealed and illustrated through two numerical examples and an empirical study in paper chemical mills.

1. Introduction

Over the past several decades, the data envelopment analysis (DEA) initially proposed by Charnes et al. has proved to be an effective data-oriented programming method for measuring the efficiencies of a group of homogenous decision making units (DMUs) [1]. The technique of DEA depicts a best-practice production efficient frontier formed by observed DMUs and provides a benchmark or reference point on this frontier for each DMU to compute its efficiency score. Using DEA does not assume any production function or presuppose any specific weight restriction to inputs and outputs. So in the recent years, DEA has been widely extended and applied in many fields [2]. The efficiency value obtained by the classic CCR model indicates how efficiently a DMU has performed when compared with other DMUs so as to determine its efficient level within the group of all DMUs. The CCR model works as a radial model and has both input- and output-orientation styles which permits the DMU under assessment to proportionally reduce all its inputs for producing its given outputs, or to proportionally expand all its outputs by using its given inputs.

However, when the evaluation target group includes a considerable number of DMUs, more than one unit will always get the same efficient score of unity. In this case, the CCR model cannot further differentiate the efficiency performances of these DMUs and cannot provide more recognizing information on them. For example, it does not detect whether the evaluated DMU is weakly efficient. As Chen pointed out, the "radial" efficiency model may make some DMU measured against a weakly efficient point on the efficient frontier in the production possibility set [3].

Specifically, the weakly efficient reference point for the current evaluated DMU may still have a positive amount of input excesses or output shortfalls, for it is not the strongly Pareto-efficient reference point. From the view of DEA, the efficiency score obtained by the weakly efficient reference point makes the evaluated DMU misestimated with respect to its strongly Pareto-efficient reference point on the efficient frontier.

So far, many methods have been developed and studied in order to enhance the cognition levels and discrimination abilities to distinguish DMUs, such as the cross-efficiency technique [4, 5], the benchmark ranking method, and others [6]. These newly developed methods are mainly built to solve problems resulting from the original CCR model in a certain aspect. Therein, Andersen and Petersen creatively developed the first radial super-efficiency model under the assumption of constant returns to scale (CRS) to reassess the efficient DMUs under CCR model [7]. They exclude the DMUs being evaluated from the reference set by the envelopment linear program so as to retrieve the called super-efficiency scores for those efficient DMUs, while, for the variable returns to scale (VRS) super-efficiency model, Seiford and Zhu found that the problem of infeasibility may occur [8]. Chen further ascribed the sources of super-efficiency for an efficient DMU to its achieved positive amounts of input saving and output surplus regarding its efficient reference point on the super-efficiency frontier [9]. Cook et al. derived a revised VRS super-efficiency model which could generate optimal solutions for some efficient DMUs that is infeasible in the original VRS super-efficiency model [10]. The resulting super-efficiency scores gained by their model could be described from both inputs and outputs aspects to some degree. Later, Lee et al. introduced a two-stage process to address the infeasibility issue under VRS [11]. Next, this two-stage process was merged into a single linear program by the work of Chen and Liang [12]. More recently, Lee and Zhu settled the infeasibility caused by zero input data and decompose the acquired super-efficiency score into three indices: radial efficiency index, input saving index, and output surplus index [13].

The above super-efficiency models are all of a radial type and they all have both input- and output-oriented forms. Tone built a popular nonradial model named slacks-based measure (SBM), which uses a fractional objection function depending on input and output slacks instead of a simple radial efficiency variable [14]. The SBM model only has one style, for there is no distinction between input-orientation and output-orientation under SBM. The efficiency score computed by the SBM model is also between 0 and 1. Compared to the traditional radial super-efficiency DEA models under CRS, the SBM model avoids the problem of infeasibility. Moreover, input slacks and output slacks in the SBM model can be utilized to detect the input excesses and output shortfalls of a given DMU, respectively. However, for SBM-efficient DMUs, Tone designed a super-efficiency model (S-SBM model) to examine their super-efficiency scores in order to allow SBM-efficient DMUs to be also ranked and compared [15]. Liu and Chen developed a HypoSBM model to distinguish the worst-performance DMUs from the bad ones [16]. Du et al. extended the SBM super-efficiency model to the additive slacks-based DEA model [17]. Fang et al. established a two-stage process, which was an alternative disposal treatment for the SBM method proposed by Tone [18]. They demonstrated that their two-stage approach generated the same results as Tone's models. Chen indicated that there exists a discontinuous gap between the SBM score and S-SBM score for a DMU who has a weakly reference point under S-SBM model [3]. In his study, an ambidextrous joint computation model (J-SBM model) for slacks-based measure was provided.

However, J-SBM model has two noted shortcomings. First, it fails to represent all input saving and output surplus scaling factors explicitly for each DMU. Second, it has a more complicated operational procedure and needs a three-stage computational process. The current paper further investigates this topic and presents a modified slacks-based measure of super-efficiency with input saving and output surplus scaling factors to reevaluate these DMUs. Our approach is devoted to detecting the specific scaling factors for each input and output as well as input and output slacks explicitly by one integrated model. Meanwhile, the phenomenon of discontinuousness between SBM score and S-SBM score for the same DMU can be eliminated. The results indicate that the projection obtained through our proposed model is strongly Pareto-efficient. And in this approach, we can see clearly how each decision variable influences the final efficiency score for a specific DMU.

The structure of this paper is organized as follows. Section 2 briefly reviews several kinds of slacks-based models. Section 3 presents a modified slacks-based measure of super-efficiency with input saving and output surplus scaling factors to determine DMUs' efficiency scores. In Section 4, two numerical examples are applied to compare our approach with the previous models. Section 5 applies our approach to an empirical example where the performance of 32 paper chemical mills in China is evaluated. The main conclusions and remarks are given in Section 6.

2. Preliminaries

Suppose there are n DMUs, $\{\mathrm{DMU}_k\,(k = 1, 2, \ldots, n)\}$. Let $x_k = (x_{1k}, \ldots, x_{mk})$ and $y_k = (y_{1k}, \ldots, y_{sk})$ denote the input and output vectors of the kth DMU. The ith input of the kth DMU is denoted as x_{ik} and the rth output of the kth DMU is denoted as y_{rk}, respectively. λ_j is the intensity coefficient for the kth DMU which means its contribution to forming the efficient frontier. Assume that all input and output data are positive.

2.1. Tone's SBM Measure. Tone developed the following SBM model to evaluate the relative efficiency of DMU_k [14], where input and output slacks are denoted, respectively, as $z_i^-\,(i = 1, \ldots, m)$ and $z_r^+\,(r = 1, \ldots, s)$. Unlike the CCR model, the objective function in SBM model has a fractional form which directly includes input and output slacks.

$$\rho_k^* = \min \frac{1 - (1/m)\sum_{i=1}^{m} z_i^- / x_{ik}}{1 + (1/s)\sum_{r=1}^{s} z_r^+ / y_{rk}},$$

$$\begin{aligned} \text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j x_{ij} = x_{ik} - z_i^-, \quad i = 1, \ldots, m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} = y_{rk} + z_r^+, \quad r = 1, \ldots, s, \\ & \lambda_j \geq 0, \quad j = 1, \ldots, n, \\ & z_i^- \geq 0, \quad i = 1, \ldots, m, \\ & z_r^+ \geq 0, \quad r = 1, \ldots, s, \end{aligned} \tag{1}$$

where DMU_k is called SBM-efficient if and only if $z_i^- = z_r^+ = 0$ for all i and r, that is, $\rho_k^* = 1$; otherwise, it is called SBM-inefficient. In order to further discriminate SBM-efficient DMUs with the same SBM efficiency score of 1, Tone introduced a SBM super-efficiency model which was referred to as S-SBM model in the following formula [15].

$$\delta_k^* = \min \frac{(1/m)\sum_{i=1}^{m} \overline{x}_{ik} / x_{ik}}{(1/s)\sum_{r=1}^{s} \overline{y}_{rk} / y_{rk}},$$

$$\begin{aligned} \text{s.t.} \quad & \sum_{\substack{j=1 \\ j \neq k}}^{n} \lambda_j x_{ij} \leq \overline{x}_{ik}, \quad i = 1, \ldots, m, \\ & \sum_{\substack{j=1 \\ j \neq k}}^{n} \lambda_j y_{rj} \geq \overline{y}_{rk}, \quad r = 1, \ldots, s, \\ & \overline{x}_{ik} \geq x_{ik}, \quad i = 1, \ldots, m, \\ & \overline{y}_{rk} \leq y_{rk}, \quad r = 1, \ldots, s, \\ & \lambda_j, \overline{y}_{rk} \geq 0, \quad j \neq k, j = 1, \ldots, n, i = 1, \ldots, s. \end{aligned} \tag{2}$$

It should be noticed that the S-SBM model can only be used for the DMU whose SBM efficiency score $\rho^* = 1$. Through model (2), these SBM-efficient DMUs get its super-efficiency scores. However, model (2) cannot discriminate SBM-inefficient DMUs for they will get the same efficiency score of 1.

2.2. Fang's Models. Fang et al. provided a two-stage process which brings in the same efficiency scores as those obtained by Tone's two models [18]. In the first stage, they replace $\overline{x}_{ik}$, $\overline{y}_{rk}$ with $x_{ik} + w_i^-$, $y_{rk} + w_r^+$ to form model (3) for detecting both input savings and output surpluses for all DMUs first.

$$\delta_k^* = \min \frac{1 + (1/m)\sum_{i=1}^{m} w_i^- / x_{ik}}{1 - (1/s)\sum_{r=1}^{s} w_r^+ / y_{rk}},$$

$$\begin{aligned} \text{s.t.} \quad & \sum_{\substack{j=1 \\ j \neq k}}^{n} \lambda_j x_{ij} \leq x_{ik} + w_i^-, \quad i = 1, \ldots, m, \\ & \sum_{\substack{j=1 \\ j \neq k}}^{n} \lambda_j y_{rj} \geq y_{rk} - w_r^+, \quad r = 1, \ldots, s, \\ & \lambda_j \geq 0, \quad j \neq k, j = 1, \ldots, n, \\ & w_i^- \geq 0, \quad i = 1, \ldots, m, \\ & w_r^+ \geq 0, w_r^+ \leq y_{rk}, \quad r = 1, \ldots, s. \end{aligned} \tag{3}$$

Through Model (3), the optimal w_i^{-*} and w_r^{+*} for each input and output for all DMUs can be obtained. For SBM-efficient DMUs in model (1), there will exist at least one i or r, so that $w_i^{-*} > 0$ or $w_r^{+*} > 0$ in model (3), while, for SBM-inefficient DMUs in model (1), they have no positive input saving and output surplus for each corresponding input and output at all. So these DMUs will get $w_i^- = w_r^+ = 0$ for all i and r. Then, plug w_i^{-*} and w_r^{+*} into the following model, and a slacks-based measure is reconstructed as model (4) exhibits.

$$\rho_k^* = \min \frac{1 - (1/m)\sum_{i=1}^{m} s_i^- / x_{ik}}{1 + (1/s)\sum_{r=1}^{s} s_r^+ / y_{rk}},$$

$$\begin{aligned} \text{s.t.} \quad & \sum_{\substack{j=1 \\ j \neq k}}^{n} \lambda_j x_{ij} = x_{ik} + w_i^{-*} - s_i^-, \quad i = 1, \ldots, m, \\ & \sum_{\substack{j=1 \\ j \neq k}}^{n} \lambda_j y_{rj} = y_{rk} - w_r^{+*} + s_r^+, \quad r = 1, \ldots, s, \\ & \lambda_j \geq 0, \quad j \neq k, j = 1, \ldots, n, \\ & s_i^- \geq 0, \quad i = 1, \ldots, m, \\ & s_r^+ \geq 0, \quad r = 1, \ldots, s. \end{aligned} \tag{4}$$

Obviously, model (3) is equivalent to model (2). So model (3) also determines the super-efficiency scores for SBM-efficient DMUs. The optimal SBM efficiency scores and the optimal slacks (s_i^{-*}, s_r^{+*}) for SBM-inefficient DMUs can be computed based on the optimal (w_i^{-*}, w_i^{+*}) by model (4).

At last, the final efficiency score for DMU_k based on SBM is defined as

$$\varphi_k^* = \begin{cases} \dfrac{1+(1/m)\sum_{i=1}^{m} w_i^{-*}/x_{ik}}{1-(1/s)\sum_{r=1}^{s} w_r^{+*}/y_{rk}}, & \text{if } \dfrac{1+(1/m)\sum_{i=1}^{m} w_i^{-*}/x_{ik}}{1-(1/s)\sum_{r=1}^{s} w_r^{+*}/y_{rk}} > 1, \\[2ex] \dfrac{1-(1/m)\sum_{i=1}^{m} s_i^{-*}/x_{ik}}{1+(1/s)\sum_{r=1}^{s} s_r^{+*}/y_{rk}}, & \text{otherwise.} \end{cases} \tag{5}$$

2.3. Chen's Model. Chen investigated a data set in Tone's which can be seen in Table 1 and found the reference points generated by model (2) or (3) might not be Pareto-efficient [3]. This resulted in a discontinuous gap between the SBM score and S-SBM score. The author indicated that the issue of discontinuity makes troubles to rationalize the efficiency score. So he established an ambidextrous joint computation model (6) (J-SBM model) to find the Pareto-efficient point for each DMU.

$$\begin{aligned}
\min \quad & \phi_k = \frac{\mathrm{JSBM}_k^x}{\mathrm{JSBM}_k^x} - M\left(b_1 + (1-b_1)b_2\right), \\
\text{s.t.} \quad & \mathrm{JSBM}_k^x = 1 - \frac{1}{m}\left[b_1\left(\sum_{i=1}^{m}\frac{s_i^-}{x_{ik}}\right) - (1-b_1)b_2\left(\sum_{i=1}^{m}\frac{s_i^-}{x_{ik}}\right) + (1-b_1)(1-b_2)\left(\sum_{i=1}^{m}\frac{\tilde{s}_i^-}{x_{ik}}\right)\right], \\
& \mathrm{JSBM}_k^y = 1 + \frac{1}{s}\left[b_1\left(\sum_{r=1}^{s}\frac{s_r^+}{y_{rk}}\right) - (1-b_1)b_2\left(\sum_{r=1}^{s}\frac{s_r^+}{y_{rk}}\right) + (1-b_1)(1-b_2)\left(\sum_{r=1}^{s}\frac{\tilde{s}_r^+}{y_{rk}}\right)\right], \\
& (6.1)\begin{cases} b_1\left(\sum_{j=1,j\neq k}^{n}\lambda_j x_{ij}\right) = b_1\left(x_{ik} - s_i^-\right), & i = 1,\ldots,m, \\ b_1\left(\sum_{j=1,j\neq k}^{n}\lambda_j y_{rj}\right) = b_1\left(y_{rk} + s_r^+\right), & r = 1,\ldots,s, \end{cases} \\
& (6.2)\begin{cases} (1-b_1)b_2\left(\sum_{j=1,j\neq k}^{n}\lambda_j x_{ij}\right) = (1-b_1)b_2\left(x_{ik} + s_i^-\right), & i = 1,\ldots,m, \\ (1-b_1)b_2\left(\sum_{j=1,j\neq k}^{n}\lambda_j y_{rj}\right) = (1-b_1)b_2\left(y_{rk} - s_r^+\right), & r = 1,\ldots,s, \end{cases} \\
& (6.3)\begin{cases} (1-b_1)(1-b_2)\left(\sum_{j=1,j\neq k}^{n}\lambda_j x_{ij}\right) = (1-b_1)(1-b_2)\left(x_{ik} - \tilde{s}_i^-\right), & i = 1,\ldots,m, \\ (1-b_1)(1-b_2)\left(\sum_{j=1,j\neq k}^{n}\lambda_j y_{rj}\right) = (1-b_1)(1-b_2)\left(y_{rk} + \tilde{s}_r^+\right), & r = 1,\ldots,s, \end{cases} \\
& \lambda_j \geq 0, \quad j = 1,\ldots,n, \\
& s_i^- \geq 0, \quad \tilde{s}_i^- \text{ free for } i = 1,\ldots,m, \\
& s_r^+ \geq 0, \quad \tilde{s}_r^+ \text{ free for } r = 1,\ldots,s, \\
& b_1, b_2 \in \{0,1\}, \\
& M \text{ is a large enough positive number.}
\end{aligned} \tag{6}$$

In model (6), b_1 and b_2 are two binary variables that are used to control which one of the three kinds of constraint conditions (6.1), (6.2), and (6.3) is chosen. For the SBM-inefficient DMUs, $b_1 = b_2 = 1$, constraint condition (6.1) is active; now model (6) works as the SBM model. For SBM-efficient DMUs, model (6) first acts as the S-SBM model under active constraint condition (6.2) when $b_1 = 0, b_2 = 1$. In the meantime, if the super-efficiency reference point is

Table 1: Data set 1 from Tone [15].

DMU	x_1	x_2	y_1	Tone's or Fang's approach S-SBM δ_k^*	SBM ρ_k^*	SBM φ_k^*	Chen's approach J-SBM ϕ_k^*	Our approach Integrated model (10) ψ_k^*
A	4	3	1	1	0.833	0.833	0.833	0.833
B	7	3	1	1	0.619	0.619	0.619	0.619
C	8	1	1	1.125	1	1.125	1.125	1.125
D	4	2	1	1.25	1	1.25	1.25	1.25
E	2	4	1	1.5	1	1.5	1.25	1.25
F	10	1	1	1	0.9	0.9	0.9	0.9
G	12	1	1	1	0.833	0.833	0.833	0.833

not Pareto-efficient for the evaluated DMU, model (6) will activate constraint condition (6.3), and the corresponding super-efficiency score will be corrected for the DMU. Additionally, Chen further proved that the reference points for all DMUs under model (6) are Pareto-efficient [3].

As can be seen from the above procedures in operating model (6), the constraint condition (6.3) is set intentionally to correct the misestimated efficiency score due to the less Pareto-efficient reference point caused by the S-SBM model (2) or (3). For these DMUs who have a weakly efficient reference point under model (2) or (3), however, the achievement of its super-efficiency needs a three-stage process. For instance, when DMU_E in Table 1 gets its super-efficiency of 1.25 under the constraint condition (6.3), the constraint conditions (6.1), (6.2) have been inspected before.

3. A Modified Slacks-Based Measure of Super-efficiency

In this section, we first establish an equivalent form of S-SBM model, which explicitly contains input saving and output surplus scaling factors. Note that $\overline{x}_{ik} \geq x_{ik}$ for all i and $\overline{y}_{rk} \leq y_{rk}$ for all r in model (2). Let $\overline{x}_{ik} = x_{ik} + t_i x_{ik}$, $t_i \geq 0$ and $\overline{y}_{rk} = y_{rk} - \beta_r y_{rk} \geq 0$, $0 \leq \beta_r \leq 1$, or $w_i^- = x_{ik} + t_i x_{ik}$, $t_i \geq 0$ and $w_r^+ = y_{rk} - \beta_r y_{rk} \geq 0$, $0 \leq \beta_r \leq 1$; then model (2) or model (3) will become the following model (7).

$$\begin{aligned}
\delta_k^* &= \min \frac{1 + (1/m)\sum_{i=1}^m t_i}{1 - (1/s)\sum_{r=1}^s \beta_r}, \\
\text{s.t.}\quad & \sum_{\substack{j=1\\ j\neq k}}^n \lambda_j x_{ij} \leq (1 + t_i) x_{ik}, \quad i = 1, \ldots, m, \\
& \sum_{\substack{j=1\\ j\neq k}}^n \lambda_j y_{rj} \geq (1 - \beta_r) y_{rk}, \quad r = 1, \ldots, s, \\
& \lambda_j \geq 0, \quad j \neq k, j = 1, \ldots, n, \\
& t_i \geq 0, \quad i = 1, \ldots, m, \\
& 0 \leq \beta_r \leq 1, \quad r = 1, \ldots, s.
\end{aligned} \tag{7}$$

The difference between models (7) and (3) is that model (7) can not only measure the input saving $t_i x_{ik}$ and the output surplus $\beta_r y_{rk}$, but also present the specific scaling factors t_i for x_{ik} and β_r for y_{rk} of DMU_k.

Let (t_i^*, β_r^*) be the optimal solution of model (7). Based on model (7), the standard SBM model (1) can be revised as follows:

$$\begin{aligned}
\rho_k^* &= \min \frac{1 - (1/m)\sum_{i=1}^m s_i^- / x_{ik}}{1 + (1/s)\sum_{r=1}^s s_r^+ / y_{rk}}, \\
\text{s.t.}\quad & \sum_{\substack{j=1\\ j\neq k}}^n \lambda_j x_{ij} = (1 + t_i^*) x_{ik} - s_i^-, \quad i = 1, \ldots, m, \\
& \sum_{\substack{j=1\\ j\neq k}}^n \lambda_j y_{rj} = (1 - \beta_r^*) y_{rk} + s_r^+, \quad r = 1, \ldots, s, \\
& \lambda_j \geq 0, \quad j \neq k, j = 1, \ldots, n, \\
& s_i^- \geq 0, \quad i = 1, \ldots, m, \\
& s_r^+ \geq 0, \quad r = 1, \ldots, s.
\end{aligned} \tag{8}$$

Similarly, we apply model (7) to all DMUs before the utilization of model (8) on inefficient DMUs. We define the final efficiency score for DMU_k based on SBM with the following piecewise function:

$$\varphi_k^* = \begin{cases} \dfrac{1 + (1/m)\sum_{i=1}^m t_i^*}{1 - (1/s)\sum_{r=1}^s \beta_r^*}, & \text{if } \dfrac{1 + (1/m)\sum_{i=1}^m t_i^*}{1 - (1/s)\sum_{r=1}^s \beta_r^*} > 1, \\[2ex] \dfrac{1 - (1/m)\sum_{i=1}^m s_i^{-*} / x_{ik}}{1 + (1/s)\sum_{r=1}^s s_r^{+*} / y_{rk}}, & \text{otherwise,} \end{cases} \tag{9}$$

where (s_i^{-*}, s_i^{+*}) is the optimal solution in model (8).

The above two-stage process is identical to that of Fang et al. Model (7) may also suffer from the problem that the projection reference point for a specific DMU may not be Pareto-efficient. As Chen mentioned, the SBM and S-SBM efficiency scores are calculated in two separate models with two different objection functions that have two different projecting styles for the evaluated DMU [3]. The SBM scores are achieved by their input and output slacks, while the S-SBM scores depend only on the reference point (may not be Pareto-efficient) on the super-efficiency frontier. Therefore, the whole evaluation system is not an integrated one and is divided into two styles, which results in the problem of a discontinuous gap.

In order to integrate the whole evaluation system, we maintain the basic sense of SBM model and S-SBM model into one model and ensure that the desired model has the uniform projecting way for the evaluated DMU. So, we develop the following modified SBM measure based on model (7).

$$\begin{aligned}
\mathrm{SE} = \min & \frac{1-(1/m)\sum_{i=1}^{m}\left(\left(s_i^-/x_{ik}\right)-t_i\right)}{1+(1/s)\sum_{r=1}^{s}\left(\left(s_r^+/y_{rk}\right)-\beta_r\right)} + M \times \left(\frac{1+(1/m)\sum_{i=1}^{m}t_i}{1-(1/s)\sum_{r=1}^{s}\beta_r}\right), \\
\text{s.t.} \quad & \sum_{\substack{j=1 \\ j\neq k}}^{n} \lambda_j x_{ij} = \left(1+t_i\right)x_{ik} - s_i^-, \quad i = 1,\ldots,m, \\
& \sum_{\substack{j=1 \\ j\neq k}}^{n} \lambda_j y_{rj} = \left(1-\beta_r\right)y_{rk} + s_r^+, \quad r = 1,\ldots,s, \\
& \lambda_j \geq 0, \quad j \neq k, j = 1,\ldots,n, \\
& s_i^- \geq 0, \quad i = 1,\ldots,m, \\
& s_r^+ \geq 0, \quad r = 1,\ldots,s, \\
& t_i \geq 0, \quad i = 1,\ldots,m, \\
& 0 \leq \beta_r < 1, \quad r = 1,\ldots,s.
\end{aligned} \tag{10}$$

The logic behind model (10) is based on two major attributes. First, the measure we modified as $(1-(1/m)\sum_{i=1}^{m}((s_i^-/x_{ik})-t_i))/(1+(1/s)\sum_{r=1}^{s}((s_r^+/y_{rk})-\beta_r))$, which not only includes the input and output slacks in SBM measure, but also includes input saving and output surplus scaling factors in S-SBM model. It is able to comprehensively explain the reason for the resulting efficiency scores; that is to say, when input saving scaling factors or output surplus scaling factors appear together with input or output slacks, they all contribute to the efficiency score.

Hence, the efficiency score can be defined as

$$\begin{aligned}
\psi_k^* &= \frac{1-(1/m)\sum_{i=1}^{m}\left(\left(s_i^{-*}/x_{ik}\right)-t_i^*\right)}{1+(1/s)\sum_{r=1}^{s}\left(s_r^{+*}/y_{rk}\right)-\beta_r^*} \\
&= \frac{1-(1/m)\sum_{i=1}^{m}s_i^{-*}/x_{ik}+(1/m)\sum_{i=1}^{m}t_i^*}{1+(1/s)\sum_{r=1}^{s}s_r^{+*}/y_{rk}-(1/s)\sum_{r=1}^{s}\beta_r^*},
\end{aligned} \tag{11}$$

where $(t_i^*, \beta_r^*, s_i^{-*}, s_i^{+*})$ is the optimal solution of model (10).

Second, we integrate SBM model and S-SBM model under the rule that the evaluated DMU must have the minimum super-efficiency score if it has at least one $t_i^* > 0$ or $\beta_r^* > 0$. So we add the controllable term of $M \times (1+(1/m)\sum_{i=1}^{m}t_i/1-(1/s)\sum_{r=1}^{s}\beta_r)$ to the objective function so as to make t_i and β_r as small as possible for all i and r.

Here, M (we set it 10^7 so as to make it large enough in the empirical study) is a predetermined large number used mainly for magnifying the effect of input saving and output surplus scaling factors on the objective function under the condition of minimum S-SBM score. Simultaneously, it can control the optimal values of decision variables. Note the constraints in model (10):

$$\begin{aligned}
& \sum_{\substack{j=1 \\ j\neq k}}^{n} \lambda_j x_{ij} = \left(1+t_i\right)x_{ik} - s_i^-, \quad i = 1,\ldots,m, \\
& \sum_{\substack{j=1 \\ j\neq k}}^{n} \lambda_j y_{rj} = \left(1-\beta_r\right)y_{rk} + s_r^+, \quad r = 1,\ldots,s.
\end{aligned} \tag{12}$$

A severe problem which may occur is that when $t_i > 0$ and $s_i^- > 0$ for a certain i (or $\beta_r > 0$ and $s_r^+ > 0$ for a certain r) simultaneously exist, there is an offsetting contradiction between $t_i x_{ik}$ and s_i^- (or $\beta_r y_{rk}$ and s_r^+). To avoid this kind of situation, the controllable term of $M \times ((1+(1/m)\sum_{i=1}^{m}t_i)/(1-(1/s)\sum_{r=1}^{s}\beta_r))$ added to the objective function is able to ensure that either $t_i^* > 0$ or $s_i^{-*} > 0$ for a certain i (either $\beta_r^* > 0$ or $s_r^{+*} > 0$ for a certain r) and both of them are not in existence.

Model (10) has close connections with the SBM model and S-SBM model.

Theorem 1. *For any SBM-inefficient DMU_k, $\psi_k^* = \rho_k^*$.*

Proof. For SBM-inefficient DMU_k, there must be $t_i^* = 0$ and $\beta_r^* = 0$ for all i and r in model (10). In this case, model (10) degenerates into SBM model (1). □

Theorem 2. *For SBM-efficient DMU_k, if the reference point under S-SBM model for DMU_k is Pareto-efficient, $\psi_k^* = \delta_k^*$.*

Proof. For SBM-inefficient DMU_k, suppose the reference point of DMU_k in S-SBM model is Pareto-efficient, there must be s_i^{-*} and $s_r^{+*} = 0$ for all i and r in model (10). At this moment, model (10) degenerates into model (7) which is equivalent to S-SBM model (2).

However, for other SBM-efficient DMUs, if the reference point under S-SBM model is not Pareto-efficient, model (10) will revise the super-efficiency score obtained by S-SBM model (7) but just make sure that model (10) can identify the Pareto-efficient reference for these DMUs. □

Theorem 3. *The reference point identified by model (10) is Pareto-efficient.*

Proof. For a SBM-inefficient DMU, model (10) serves as the standard SBM model (1); the reference point identified the standard SBM model as Pareto-efficient, and so does model (10).

For a SBM-efficient DMU, there at least exists one input $i \in \{1,\ldots,m\}$ or one output $r \in \{1,\ldots,s\}$ such that at least one scaling factor $t_i^* > 0$ or $\beta_r^* > 0$. Meanwhile, the existence of input and output slacks allows the evaluated DMU to decrease its inputs and increase its outputs. Thus, the reference point identified by model (10) is as follows:

$$\sum_{\substack{j=1\\ j\neq k}}^{n} \lambda_j^* x_{ij} = (1+t_i^*)x_{ik} - s_i^{-*}, \quad i=1,\ldots,m,$$
$$\sum_{\substack{j=1\\ j\neq k}}^{n} \lambda_j^* y_{rj} = (1-\beta_r^*)y_{rk} + s_r^{+*}, \quad r=1,\ldots,s. \tag{13}$$

If $((1+t_i^*)x_{ik}, (1-\beta_r^*)y_{rk})$ is not a Pareto-efficient reference point, then it moves s_i^{-*} leftward and s_r^{+*} upward to reach the Pareto-efficient reference points.

However, it should be noted that model (10) is a quite different optimization design because the objective function and constraints are quite different to the standard SBM and S-SBM models. Although the projection identified by model (10) is Pareto-efficient, for SBM-efficient DMUs, the projected reference point by model (10) may not coincide with that by S-SBM model or J-SBM model (see examples in Section 4).

Clearly, the modified measurement ψ_k has the unit-invariant property. □

Theorem 4. *ψ_k is unit-invariant.*

The fractional programming (10) can be transformed into a linear programming problem through the following Charnes-Cooper transformation by setting $w_1 = (1/(1 + (1/s)\sum_{r=1}^{s}(s_r^+/y_{rk}) - \beta_r))$, $w_2 = 1/(1-(1/s)\sum_{r=1}^{s}\beta_r)$ $(w_1 \leq w_2)$, $\lambda_j' = w_1 \cdot \lambda_j$, $s_i'^- = w_1 \cdot s_i^-$, $s_r'^+ = w_1 \cdot s_r^+$, $t_i' = w_1 \cdot t_i$, $\beta_r' = w_1\beta_r$, $t_i'' = w_2 \cdot t_i$, $\beta_r'' = w_2\beta_r$; then model (10) will become the following linear programming:

$$\begin{aligned}
\min \quad & w_1 - \frac{1}{m}\sum_{i=1}^{m}\left(\frac{s_i'^-}{x_{ik}} - t_i'\right) + M \times \left(w_2 + \frac{1}{m}\sum_{i=1}^{m} t_i''\right),\\
\text{s.t.} \quad & w_1 + \frac{1}{s}\sum_{r=1}^{s}\left(\frac{s_r'^+}{y_{rk}} - \beta_r'\right) = 1,\\
& w_2 - \frac{1}{s}\sum_{r=1}^{s}\beta_r'' = 1,\\
& \sum_{\substack{j=1\\ j\neq k}}^{n} \lambda_j' x_{ij} = w_1 x_{ik} + t_i' x_{ik} - s_i'^-, \quad i=1,\ldots,m,\\
& \sum_{\substack{j=1\\ j\neq k}}^{n} \lambda_j' y_{rj} = w_1 y_{rk} - \beta_r' y_{rk} + s_r'^+, \quad r=1,\ldots,s,\\
& \lambda_j' \geq 0, \quad j\neq k, j=1,\ldots,n,\\
& s_i'^- \geq 0, \quad i=1,\ldots,m,\\
& s_r'^+ \geq 0, \quad r=1,\ldots,s,\\
& 0 \leq t_i' \leq t_i'', \quad i=1,\ldots,m,\\
& 0 \leq \beta_r' \leq w_1, \beta_r' \leq \beta_r'', \quad r=1,\ldots,s,\\
& 0 \leq w_1 \leq w.
\end{aligned} \tag{14}$$

Suppose $(w_1^*, t_i'^*, \beta_r'^*, s_i'^{-*}, s_r'^{+})$ is the optimal solution of model (11), then $(t_i^* = t_i'^*/w_1^*, \beta_r^* = \beta_r'^*/w_1^*, s_i^{-*} = s_i'^{-*}/w_1^*, s_r^{+*} = s_r'^{+*}/w_1^*)$ is the optimal solution of model (10).

4. Illustration Examples

Two numerical examples from Tone are used to verify our approach through comparing with the SBM model, S-SBM model, and J-SBM model. In Table 1, the efficiency scores derived by the SBM model, S-SBM model, and J-SBM model are presented and those identified through our approach are listed in the last column. Table 2 shows detailed optimal solutions under J-SBM model (6) and we rewrite them in the form of input and output surplus scaling factors so as to facilitate comparison. Table 3 presents detailed optimal solutions under our model (10). As shown in Tables 2 and 3, these DMUs get the same efficiency scores but may be a different optimal solution for each decision variable. For example, DMU_D gets the super-efficiency score of 1.25 under J-SBM model (6), and the achievement of super-efficiency score seems reflected in its input saving scaling factors $t_1^* = 0.234$ and $t_2^* = 0.266$, while under our model (10) it not only owns input saving scaling factors $t_1^* = 0.137$ and $t_2^* = 0.163$, but also has the output surplus scaling factor $\beta_1^* = 0.08$. The reason is that due to the different projecting style of these two models, DMU_D gets different reference points, which are both Pareto-efficient projected points. And for DMU_E, J-SBM model (6) and our model (10) identified the same Pareto-efficient reference point. Our model also overcomes the problem of the discontinuous gap and finds its input saving scaling factor $t_1^* = 1$ and input slack $s_2^{-*} = 2$, so the rational super-efficiency score is

$$\begin{aligned}
\psi_k^* &= \frac{1-(1/m)\sum_{i=1}^{m}\left((s_i^{-*}/x_{ik}) - t_i^*\right)}{1+(1/s)\sum_{r=1}^{s}\left((s_r^{+*}/y_{rk}) - \beta_r^*\right)}\\
&= \frac{1-(1/2)\times(1/2)+(1/2)\times 1}{1} = 1.25.
\end{aligned} \tag{15}$$

In Table 4, we mainly compare the approach from Tone's or Fang's and ours using data set 2 from Tone. For the first four DMUs (1–4), two approaches gain the same efficiency scores. But for DMU_5, our approach detects that it has both input saving scaling factor $t_1^* = 0.4$ and output surplus scaling factor $\beta_2^* = 0.475$. Besides, it has input slack $s_2^{-*} = 1.9$ and output slack $s_1^{+*} = 0.4$. This situation means that S-SBM model does not project DMU_5 at a Pareto-efficient targeted point and its efficiency score should be revised. According to our approach, the efficiency score should be modified as

$$\begin{aligned}
\psi_k^* &= \frac{1-(1/m)\sum_{i=1}^{m}\left((s_i^{-*}/x_{ik}) - t_i^*\right)}{1+(1/s)\sum_{r=1}^{s}\left((s_r^{+*}/y_{rk}) - \beta_r^*\right)}\\
&= \frac{1-(1/2)\times(1.711/4)+(1/2)\times 0.526}{1+(1/2)\times 0.526-(1/2)\times 0.428} = 1.0001.
\end{aligned} \tag{16}$$

Table 2: Results from J-SBM model (6).

DMU	x_1	x_2	y_1	φ_k^*	t_1^*	t_2^*	β_1^*	s_1^{-*}	s_2^{-*}	s_1^{+*}
A	4	3	1	0.833	0	0	0	0	1	0
B	7	3	1	0.619	0	0	0	2.895	0.948	0.026
C	8	1	1	1.125	0.25	0	0	0	0	0
D	4	2	1	1.25	0.234	0.266	0	0	0	0
E	2	4	1	1.25	1	0	0	0	2	0
F	10	1	1	0.9	0	0	0	2	0	0
G	12	1	1	0.833	0	0	0	4	0	0

Table 3: Results from our model (10).

DMU	x_1	x_2	y_1	ψ_k^*	t_1^*	t_2^*	β_1^*	s_1^{-*}	s_2^{-*}	s_1^{+*}
A	4	3	1	0.833	0	0	0	0	1	0
B	7	3	1	0.619	0	0	0	1.706	0.353	0.324
C	8	1	1	1.125	0.25	0	0	0	0	0
D	4	2	1	1.25	0.137	0.163	0.08	0	0	0
E	2	4	1	1.25	1	0	0	0	2	0
F	10	1	1	0.9	0	0	0	2	0	0
G	12	1	1	0.833	0	0	0	4	0	0

Table 4: Data set 2 from Tone [14] and results from our approach.

DMU	x_1	x_2	y_1	y_2	Tone's or Fang's approach	Our approach								
					φ_k^*	ψ_k^*	t_1^*	t_2^*	β_1^*	β_2^*	s_1^{-*}	s_2^{-*}	s_1^{+*}	s_2^{+*}
1	4	3	2	3	0.7980	0.7980	0	0	0	0	0	0.357	0.714	0
2	6	3	2	3	0.5682	0.5682	0	0	0	0	0	0.643	2.286	0
3	8	1	6	2	1.3333	1.3333	0	0	0	0.5	0	0	0	0
4	8	1	6	1	0.6667	0.6667	0	0	0	0	0	0	0	1
5	2	4	1	4	1.4545	1.0001	0.526	0	0	0.428	0	1.711	0.526	0

5. An Empirical Study

After high speed development on industries and economics during the past several decades, China has accumulated a lot of air environment problems and the Chinese government is paying more and more attention to ecological and environmental assessment. Recent years have seen a lot of studies based on DEA to measure China's environmental pollution problems. For example, Zhou et al. [19, 20] construct a set of DEA models with the integral and zero-sum gain constraints for calculating air quality and a new nonradial directional distance function to scale the performance of water use and wastewater emission, respectively.

In this section, the method that we have developed is applied to examine the efficiencies and rankings of 32 paper chemical mills along the Huai River in China. In this empirical study, four input and output variables are considered to evaluate each mill's performance. The inputs of each paper mill include labor and capital, and good output as paper products as well as bad output as biochemical oxygen demand (BOD). The detailed data set of these 32 paper chemical mills is shown in Table 5.

In papermaking industry, the good products are always produced with bad products and it is impossible to increase good outputs and decrease bad outputs meanwhile. First, the attribute of "the smaller the better" for bad outputs is consistent with inputs. Second, bad outputs cannot create any new profit and dealing with them (such as sewage treatment and air purification) always comes at a price which the mill should afford [21]. Therefore, here we simply treat the bad outputs as inputs from the cost point of view.

We use model (10) to assess the performance of 32 paper chemical mills and the computed efficiency scores and ranking results are also displayed in Table 5. The last column gives their ranks according to efficiency scores obtained by our method. Of the 32 paper mills, only three mills, named mills 9, 12, and 25, are identified to obtain super-efficiency scores and they are ranked as the top three accordingly. For example, mill 9 and 12 get super-efficiency of 1.0734 and 1.4546, respectively, due to that the scaling factor of output surplus in one good output (paper) is 0.0734 and 0.4546 separately and there exist no slacks in all inputs and outputs. In contrast, 29 mills fail to get super-efficiency scores. For instance, mill 6 ranks dead last because it does not have any scaling factor of each input or output. Meanwhile, positive slacks are found to exist in two inputs and one bad output. Although mill 31 achieves the input saving scaling factor of 0.0229 in the first input (labor) and the output surplus scaling factor of 0.1077 in the good output (paper), its efficiency score is identified as 0.8538 and is ranked eighth finally. The reason for this is that there exists a slack amount of 13.7051 in the bad output (BOD), which brings more negative impact to efficiency scores than the positive effect brought by input and output scaling factors.

Table 5: The Data set and efficiency results of 32 paper mills.

Paper mills	Inputs		Bad output	Good output	ψ_k^*	Rank
	Labor (person)	Capital (¥10000)	BOD (ton)	Paper (ton)		
1	1077	2959.9	21.4290	27582	0.9005	5
2	452	3589	19.8062	29514	0.8544	7
3	319	5901.9	12.3287	14700	0.3590	26
4	1075	4892.8	9.1559	22354	0.4337	16
5	813	4079.7	11.9146	20669	0.4328	17
6	850	5239.6	5.2037	8222	0.2019	32
7	1090	3022.8	3.6054	15066	0.5286	12
8	122	3173.1	3.7278	8066	0.4937	14
9	297	2277.4	8.0765	19125	1.0734	3
10	1047	1491.9	8.9060	7601	0.3141	30
11	1010	3940.1	4.8940	11579	0.3184	28
12	262	3236.5	4.0835	23216	1.4546	1
13	551	4448.6	4.8750	21698	0.6357	10
14	671	1789.7	4.5334	8127	0.3617	25
15	577	2310.9	6.1362	11549	0.4179	20
16	208	3398.2	7.0186	10295	0.4130	21
17	667	5331.9	28.4877	29881	0.4904	15
18	878	3450.4	13.1680	19076	0.4236	19
19	640	3109.8	6.1616	12176	0.3694	24
20	927	3345.2	1.4533	5187	0.3024	31
21	167	4329.7	22.5809	24005	0.8937	6
22	903	3855.2	26.3390	23085	0.4258	18
23	720	1908.3	3.0787	6545	0.3182	29
24	629	3468.2	21.7332	27599	0.6853	9
25	152	5571.7	5.3061	23748	1.2256	2
26	1010	4647.1	9.1360	17323	0.3489	27
27	578	2513.3	5.0049	10617	0.3898	23
28	384	2247.4	4.4373	9083	0.3968	22
29	166	3968.1	10.6127	15151	0.5997	11
30	894	1368.5	4.7758	9911	0.5089	13
31	143	5350.2	17.4677	25260	0.8538	8
32	879	2873.2	26.9134	27721	0.9464	4

Source: Anhui Environmental Protection Bureau, the Fuyang Environmental Protection Bureau, and the Huainan Environmental Protection Bureau.

As can be seen from Table 5, the efficiency score or the super-efficiency score for each mill can be obtained by the integrated model (10). This course by our method prevents transformations among three submodels of the method by Chen. The final efficiency scores ψ_k^* $(k = 1, \ldots, 32)$ are affected synthetically by all decision variables that include input saving and output surplus scaling factors, input and output slacks. So we can fully differentiate the performances of all 32 paper mills and can provide a complete ranking criterion for all DMUs to be compared with regard to these efficiency scores.

6. Conclusions and Remarks

The classic SBM model proposed by Tone projects the DMU under evaluation at a Pareto-efficient reference point on the production frontier, while the S-SBM model proposed by Tone may not apply and the reference point on the super-efficiency frontier by S-SBM model may not be Pareto-efficient. Chen found an approach through transformations among three submodels to handle this issue.

In the present paper, we build a universal model to realize the integration of SBM model and S-SBM model. Our approach not only shows the specific scaling factors for each input and output of a specific DMU explicitly, but also provides input saving, output surplus, and slacks information simultaneously only in one model. And we can see clearly how the efficiency scores are obtained by the optimal value for each decision making variable. Thus, more recognizing information on DMUs is revealed via two numerical examples and an empirical study in paper chemical mills. The composite efficiency scores for 32 mills are represented and ranked by integrating the effect of all decision variables, which include input saving, output surplus scaling factors, and input and output slacks.

The current paper also overcomes the problem of a discontinuous gap between SBM score and S-SBM score found by Chen. Model (10) is proved to make sure that for each DMU under evaluation it supplies a Pareto-efficient reference point on the efficient frontier. This is achieved by incorporating input and output slacks into S-SBM model and controlling optimal values for each variable through the use of big M. For one specific input (output), input saving scaling factor (output surplus scaling factor) and input slack (output slack) are not permitted to appear at the same time. Especially, for a certain DMU, we can judge whether the reference point in S-SBM model is Pareto-efficient through our model. That is, the reference point $((1 + t_i^*)x_{ik}, (1 - \beta_r^*)y_{rk})$ for DMU_k in S-SBM model (7) is not Pareto-efficient if and only if there exists $s_{i'}^{-*} > 0$ $(i' \neq i \in (1, \ldots, m))$ or $s_{r'}^{+*} > 0$ $(r' \neq r \in (1, \ldots, m))$ under model (10).

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflicts of interest.

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