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Research Article

Grassmannian Constellation Based on Antipodal Points and Orthogonal Design and Its Simplified Detecting Algorithm

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This study presents a framework of the unitary space time modulation (USTM) constellation based on antipodal points over Grassmannian manifold. The antipodal constellation enables an intrinsic simplified ML detecting algorithm. The algebraic orthogonal USTM constellation is also an antipodal constellation which, apart from being adaptive to the antipodal simplified ML detector, also has another simplified ML detector based on its self-indexing features, and the latter is simpler because of getting rid of the matrix operation. A searching orthogonal USTM constellation based on the grid search algorithm is obtained under the presented framework and its minimum Frobenius chordal distance and simulation performance are be superior to those of the algebraic orthogonal USTM constellation.

1. Introduction

Grassmannian constellation is a set of unitary space time modulation (USTM) signal matrices defined on Gassmann manifold presented by Hochwald and Marzetta [1] and Zheng and Tse [2] for robustness against very fast fading in high speed mobile channels in which learning the channel fade coefficients becomes increasingly difficult for both transmitter and receiver. There are many methods about how to construct the USTM constellation, mainly including derivative-based optimization searching schemes [3] and algebraic structural schemes [4–8]. This study concentrated on the random and algebraic orthogonal [4] USTM constellation having the feature of antipodal point on Grassmannian manifold and their simplified maximum likelihood (ML) detecting algorithm.

The content of the paper including its main contributions is organized as follows. In Section 2, the preliminary knowledge which will be used throughout this paper is described, including the system model, the noncoherent maximum likelihood (ML) detector, and the chordal Frobenius distance measure. In Section 3, we build a framework of USTM constellation based on the antipodal points. The optimal packing method of searching the orthogonal unitary matrices over Grassmannian manifold and the corresponding searching algorithm are investigated. Under the constraint of the framework and by using the grid searching algorithm, we obtain a set of the orthogonal unitary matrices which contains many constellations of satisfying antipodal feature and orthogonality. Among them, an orthogonal USTM constellation with the optimum distribution of chordal Frobenius distance is determined by two explicit expressions. In Section 4, a simplified ML detecting algorithm based on antipodal points is derived and discussed. In Section 5, we demonstrate the antipodal feature of the algebraic orthogonal USTM constellation from [4] and derive its simplified ML detecting algorithm based on antipodal points. Furthermore, we deduce the indexing simplified ML detector of the algebraic orthogonal USTM constellation which only needs to operate several complex-values and get rid of the matrix operation. In Section 6, we show the simulation testing results between the searching and algebraic orthogonal USTM constellations which indicate that the searching constellation is superior to the algebraic that in both chordal Frobenius distance spectrum and performance with regard to symbol error probability and signal noise ratio. We conclude with some remarks in Section 7.

2. Preliminary and System Model

Consider a system with M transmit and N receive antennas. The channels between antenna pairs are Rayleigh flat fading and independent of each other. The channel fading coefficients are constant in a coherence interval T and change to a new realization in the next interval. A system model [2] is given as follows:

$$\mathbf{Y} = \sqrt{\frac{\rho}{M}}\mathbf{X} \cdot \mathbf{H} + \mathbf{W}, \quad (1)$$

where $\mathbf{X} \in \mathscr{C}^{T\times M}$ and $\mathbf{Y} \in \mathscr{C}^{T\times N}$ are, respectively the transmitted and received signal matrices, $\mathbf{H} \in \mathscr{C}^{M\times N}$ is a fading coefficient matrix and $\mathbf{W} \in \mathscr{C}^{T\times N}$ is an additive noise matrix, of which the elements of both are drawn from the i.i.d. standard complex Gaussian distribution $\mathscr{CN}(0,1)$, and ρ is the expected signal-to-noise ratio (SNR) at each receiver antenna.

The capacity-achieving space time modulation signal distribution at high SNR is modelled as a set of unitary matrices [1]: $\{\mathbf{X}\} = \sqrt{T}\{\mathbf{\Phi}_b\}_{b=1}^{B}$ in which each matrix satisfies $\mathbf{\Phi}_b^*\mathbf{\Phi}_b = \mathbf{I}_M$ and all $\mathbf{\Phi}_b$'s are points on a Stiefel manifold, or the subspace Ω_b spanned by column vectors of $T \times M$ matrix $\mathbf{\Phi}_b$ is uniformly distribution in Grassmann manifold $G_{T,M}$; that is, $\Omega_b \in G_{T,M}$ [2]. Let a set $\{\mathbf{\Phi}_b\}_{b=1}^{B}$ denote a USTM constellation which contains B $T\times M$ complex unitary matrices $\mathbf{\Phi}_b$.

As the coefficients of $\mathbf{H}$ are unknown to both receiver and transmitter, the noncoherent ML detector [1] is introduced:

$$\widehat{\mathbf{\Phi}}_{\mathrm{ML}} = \arg \max_{\mathbf{X}_i=\sqrt{T}\mathbf{\Phi}_i, \mathbf{\Phi}_i\in\{\mathbf{\Phi}_b\}_{b=1}^{B}} \operatorname{Tr}\left(\mathbf{Y}^*\mathbf{X}_i\mathbf{X}_i^*\mathbf{Y}\right), \quad (2)$$

where $\operatorname{Tr}(\cdot)$ is the trace operation of a matrix and $(\cdot)^*$ is the complex conjugate transpose.

Let vector sets $\{u_i\}$ and $\{v_i\}$ be two principal vectors corresponding to two M-planes $\mathbf{U}, \mathbf{V} \in G_{T,M}$. The principal angles $\theta_1, \theta_2, \ldots, \theta_M \in [0, \pi/2]$ between $\mathbf{U}$ and $\mathbf{V}$ are defined as $\cos\theta_i = \max_{u\in U}\max_{v\in V} u \cdot v = u_i \cdot v_i$ for $i = 1, 2, \ldots, M$, subject to $u{\cdot}u = v{\cdot}v = 1, u_i{\cdot}u_j = 0, v_i{\cdot}v_j = 0$ $(1 \le j \le i{-}1)$ [9]. The chordal Frobenius distance measure is defined as follows ([3] and references therein):

$$d(\mathbf{U}, \mathbf{V}) = \sqrt{2M - 2\operatorname{Tr}\left(\Sigma_{\mathbf{U}^*\mathbf{V}}\right)} = \sqrt{\sum_{i=1}^{M} 4\sin^2\frac{1}{2}\theta_i}, \quad (3)$$

where $\Sigma_{\mathbf{U}^*\mathbf{V}}$ denotes a diagonal matrix formed by the singular values of the matrix $\mathbf{U}^*\mathbf{V}$.

3. A Framework of Grassmannian Constellation Based on Antipodal Point

3.1. A Framework of USTM Constellation. A pair of antipodal points are defined as two points with the furthest distance on a sphere. Since the capacity-achieving USTM signal distribution at high SNR is isotropic on the Grassmannian manifold $G_{T,M}$ and each signal point is denoted as a unitary matrix $\mathbf{\Phi}$, $\overline{\mathbf{\Phi}}$ is defined as an antipodal point of $\mathbf{\Phi}$ if $\overline{\mathbf{\Phi}}$ is the orthogonal complement of $\mathbf{\Phi}$ on $G_{T,M}$. Then how can one decide whether the two matrices on $G_{T,M}$ are the antipodal matrix? This can be done with the following lemma.

Lemma 1. *Let* $\mathbf{U}, \mathbf{V}$ *be two* $T \times M$ *unitary matrix on* $G_{T,M}$. $\mathbf{U}$ *and* $\mathbf{V}$ *become a pair of antipodal matrices if and only if either* $\mathbf{U}^*\mathbf{V} = \mathbf{0}_M$ *or* $[\mathbf{U}\mathbf{V}] = \mathbf{Q}_T$ *and* $\mathbf{Q}_T^*\mathbf{Q}_T = \mathbf{I}_T$, *where* $\mathbf{I}_T$ *is a* $T\times T$ *identity matrix and* $\mathbf{0}_M$ *is a* $M \times M$ *full-zero matrix.*

Proof. $\mathbf{U}^*\mathbf{V} = \mathbf{0}_M$ implies that each of M column vectors of $\mathbf{U}$ is orthogonal to each of M column vectors of $\mathbf{V}$; that is, $\mathbf{U}$ and $\mathbf{V}$ are orthogonal and complement each other. Since $\mathbf{U}, \mathbf{V}$ are two $T \times M$ unitary matrices, they are used to construct a $T\times T$ matrix $\mathbf{Q}_T = [\mathbf{U}\mathbf{V}]$. $\mathbf{Q}_T^*\mathbf{Q}_T = \mathbf{I}_T$ indicates that T column vectors of $\mathbf{U}$ and $\mathbf{V}$ span a basis of Euclid space $\mathbb{C}^T$, so $\mathbf{U}$ and $\mathbf{V}$ are orthogonal and complement each other. □

Construction 1 (a framework of USTM). Let $C = \{\mathbf{\Phi}_b\}_{b=0}^{B-1}$ denote a constellation and $\mathbf{\Phi}_b \in C \subset G_{T,M}$. Let $\varphi_{ij} \in \mathbb{C}$ denote a complex element at the ith row and the jth column of $\mathbf{\Phi}_b$ for $i = 1, 2, \ldots, T$ and $j = 1, 2, \ldots, M$. If for any positive integer B, each code word $\mathbf{\Phi}_b$ of C has the structure

$$\mathbf{\Phi}_b = \begin{bmatrix} \varphi_{11} & \varphi_{12} & \cdots & \varphi_{1M} \\ \varphi_{21} & \varphi_{22} & \cdots & \varphi_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_{T1} & \varphi_{T2} & \cdots & \varphi_{TM} \end{bmatrix} \quad \text{for } b = 0, 1, 2, \ldots, B-1 \quad (4)$$

and satisfies the following constraints:

(1) For all $b, \mathbf{\Phi}_b^*\mathbf{\Phi}_b = \mathbf{I}_M$, where $\mathbf{I}_M$ is a $M \times M$ identity matrix;

(2) All points of $C = \{\mathbf{\Phi}_b\}_{b=0}^{B-1} = C_1 \cup C_2$ are partitioned into two parts $C_1 = \{\mathbf{\Phi}_b\}_{b=0}^{B/2-1}$ and $C_2 = \{\mathbf{\Phi}_b\}_{b=B/2}^{B-1}$ in such a way that there exist $B/2$ one-to-one antipodal points between C_1 and C_2 but there is no antipodal point in each of C_1 and C_2;

(3) The degree of freedom of elements in each $\mathbf{\Phi}_b \in C \subset G_{T,M}$ is $\dim(G_{T,M}) = M(T-M)$ [2];

(4) All $\varphi_{\alpha\beta} \neq 0$ are unknown for $\alpha = 1, 2, \ldots, T$ and $\beta = 1, 2, \ldots, M$.

Then the constellation set $C = \{\mathbf{\Phi}_b\}_{b=0}^{B-1}$ is called a framework of the full diversity USTM constellation based on antipodal points on $G_{T,M}$.

3.2. The Construction of Antipodal Constellation with Orthogonality. According to the analysis on how to choose T and

M [1, 2], the simplest case of $T = 2M = 4$ was considered. A framework of the 4×2 unitary matrix $\mathbf{\Phi}_b$ on $G_{4,2}$ was built similar to (4), and the degree of freedom of its elements was $M(T-M) = 4$. Therefore, let $\varphi_k = A_k e^{j\phi_k} \in \mathbb{C}$, $k = 1, 2, 3, 4$ be four independent complex elements of $\mathbf{\Phi}_b$, where j is an imaginary unit. Then a unitary matrix with uncertain eight values A_k, ϕ_k, $k = 1, 2, 3, 4$, is formed as follows:

$$\begin{aligned}\mathbf{\Phi}_b &= \frac{1}{\sqrt{A_1^2 + A_2^2 + A_3^2 + A_4^2}} \begin{bmatrix} A_1 e^{j\phi_1} & A_2 e^{j\phi_2} \\ -A_2 e^{-j\phi_2} & A_1 e^{-j\phi_1} \\ A_3 e^{j\phi_3} & A_4 e^{j\phi_4} \\ -A_4 e^{-j\phi_4} & A_3 e^{-j\phi_3} \end{bmatrix} \\ &= a \begin{bmatrix} \varphi_1 & \varphi_2 \\ -\varphi_2^* & \varphi_1^* \\ \varphi_3 & \varphi_4 \\ -\varphi_4^* & \varphi_3^* \end{bmatrix} = f\left(a \left\{A_k e^{j\phi_k}\right\}_{k=1}^{4}\right) \\ &= f\left(a \left\{\varphi_k\right\}_{k=1}^{4}\right).\end{aligned} \tag{5}$$

The expression (5) defines a function $f : \{\varphi_1, \varphi_2, \varphi_3, \varphi_4\} \rightarrow \mathbf{\Phi}_b$; that is, four complex numbers are mapped into a unitary matrix similar to (5) which is a point of the USTM constellation, or $f : \{A_k, \phi_k\}_{k=1}^{4} \rightarrow \mathbf{\Phi}_b$, where $\varphi_k \in \mathbb{C}$ and $A_k, \phi_k \in \mathbb{R}$.

The optimal packing method stated was used to determine $C = \{\mathbf{\Phi}_b\}_{b=0}^{B-1}$ with all points like (5). That is, for the fixed $T = 2M = 4$ and B, design a packing C in $G_{4,2}$ of cardinality $|C| = B$ so that its minimum distance similar to (3) is as large as possible. In fact, a complex number set of $\varphi_k = A_k e^{j\phi_k} \in \mathbb{C}$ needs to be obtained in order to form a constellation $C = \{\mathbf{\Phi}_b\}_{b=0}^{B-1}$ on $G_{4,2}$ so that the minimum Frobenius chordal distance $\min_{\varphi_k \in \mathbb{C}} d(\mathbf{\Phi}_u(\varphi_k), \mathbf{\Phi}_v(\varphi_k))$ of (3) is maximized. The optimal packings of B points on $G_{4,2}$ require the solution of the following optimization problem:

$$\begin{aligned} &\max_{A_k, \phi_k \in \mathbb{R}} \ \min_{u, v \in B} d\left(\mathbf{\Phi}_u\left(A_k, \phi_k\right), \mathbf{\Phi}_v\left(A_k, \phi_k\right)\right) \\ &\text{Subject to} \quad \mathbf{\Phi}_u\left(A_k, \phi_k\right), \mathbf{\Phi}_v\left(A_k, \phi_k\right) \in C = \left\{\mathbf{\Phi}_b\right\}_{b=0}^{B-1}, \\ &\qquad \mathbf{\Phi}_u\left(A_k, \phi_k\right) \neq \mathbf{\Phi}_v\left(A_k, \phi_k\right), \\ &\qquad u \neq v \in \{0, 1, \ldots, B-1\}. \end{aligned} \tag{6}$$

If the complex elements $\varphi_k \in \mathbb{C}$ ($k = 1, 2, 3, 4$) of each unitary matrix (a constellation point) are referred as to the parameter of the model for the underlying system, such as USTM, then the parameters $A_k, \phi_k \in \mathbb{R}$ can be thought of the hyperparameter of the same system. The so-called hyperparameter optimization, also called model selection, is the problem of choosing a set of hyperparameters $A_k, \phi_k \in \mathbb{R}$. Thus we need to solve the problem of hyperparameter optimization. The traditional way of performing hyperparameter optimization has been grid search algorithm, or a parameter sweep, which is simply an exhaustive searching through a manually specified subset of the hyperparameter space. From the above, we need to consider the following factors.

A grid search algorithm must be guided by some performance metric. Here the performance metric space is to maximize the minimum chordal Frobenius distance.

Since the parameter space may include real-valued or unbounded value spaces for our parameters $A_k, \phi_k \in \mathbb{R}$, our searching scheme needs to be tuned for good performance on an unknown data set; then manually set bounds and discretization may be necessary before applying grid search.

Since grid search suffers from the curse of dimensionality and doing a complete grid search may also be time-consuming, we considered using a coarse grid first. If the searched constellation cannot satisfy the some predetermined threshold of Frobenius chordal distance, we will use the fine grid.

In fact, there are several optimal methods used by [3] which can obtain the constellations with the better distribution of the minimum Frobenius chordal distance. However, there are several motivations why we prefer the simple grid search approach. One is that we want to know whether there exists the other orthogonal structural constellation whose performance is superior to the performance of the algebraic structural orthogonal constellation [4]. Hence, let $\phi_k = 0, \pi/2, \pi, 3\pi/2$ which means that an orthogonal constraint is imposed on each point of the constellation $C = \{\mathbf{\Phi}_b\}_{b=0}^{B-1}$ and which also means that the parameter ϕ_k is discretized into a coarse grid. Thus it is natural to introduce the grid searching algorithm. Another is that we expect that the value distribution of A_k has some regular pattern so that all points of the constellation can be denoted by the expression like the orthogonal design of [4] rather than by the way of enumeration.

Let $\{\mathbf{\Phi}\}$ be initialized into an empty set, $|\{\mathbf{\Phi}\}|$ be the size of $\{\mathbf{\Phi}\}$, and B be the total of the constellation points. Our searching scheme is described as follows:

(a) Select an initial point. Due to $\varphi_{\alpha\beta} = \varphi_k \neq 0$, $A_k = 1$ and $\phi_k = 0$ is stipulated for $k = 1, 2, 3, 4$. Place the initial point $\mathbf{\Phi}_{b=0} = \mathbf{\Phi}_0$ and its antipodal point $\overline{\mathbf{\Phi}}_{b=0} = \mathbf{\Phi}_{B/2}$ into $\{\mathbf{\Phi}\}$.

(b) Determine a step length. Let $\gamma = 1/m \in [0, 1]$ be a step length of increasing A_k and $\sigma = 2\pi/n \in [0, 2\pi]$ be a step length of increasing ϕ_k, where m and n are positive integers. For orthogonal scheme, let $n = 4$; then $\sigma = \pi/2$ which implies that all principal angles $\theta_1, \theta_2, \ldots, \theta_M$ between any two points $\mathbf{\Phi}_u, \mathbf{\Phi}_v$ are orthotropic each other; equivalently, $\phi_k \in \{0, \pi/2, \pi, 3\pi/2\}$. We selected the amplitude value of A_k by fixed $m = 2, 4, 8$. So two step lengths of γ and σ provide a coarse grid, which can avoid the curse of dimensionality and reduce time- consuming of the grid search.

(c) Select the distance threshold d_{th} in accordance with the practical cases. The choice of $d_{\text{th}} = 0.8$ was given by considering the Frobenius chordal distance distribution of the orthogonal design [4], such as $d_{\text{th}} = 0.7321$ shown in Figure 1.

(d) Searching method: find $\varphi_k = A_k e^{j\phi_k}$, $k = 1, 2, 3, 4$ with A_k and ϕ_k modified by γ and σ in order

Input: B, m, n, d_{th};

Output: a constellation $C = \{\boldsymbol{\Phi}_b\}_{b=0}^{B-1}$;

Point($A_k(b), \phi_k(b)$) {

$a = \dfrac{1}{\sqrt{(A_1(b))^2 + (A_2(b))^2 + (A_3(b))^2 + (A_4(b))^2}}$;

$\boldsymbol{\Phi}_b = f(a\{A_k e^{j\phi_k}\}_{k=1}^4)$ of (5)} // function of calculating points

$A_k(0) = 1$; $\phi_k(0) = 0$; // for $k = 1, 2, 3, 4$

$\boldsymbol{\Phi}_0 = $ Point($A_k(0), \phi_k(0)$);

$\boldsymbol{\Phi}_0 \rightarrow \{\boldsymbol{\Phi}\}$; $\boldsymbol{\Phi}_{B/2} \rightarrow \{\boldsymbol{\Phi}\}$ // initial the constellation set

$\gamma = \dfrac{1}{m}$; $\sigma = \dfrac{2\pi}{n}$;

For $A_k(b) = 1, A_k(b) = A_k(b) + \gamma, A_k(b) \le 2$ {{{{

  // four for-cycle to calculate A_1, A_2, A_3, A_4

  For $\phi_k(b) = 0, \phi_k(b) = \phi_k(b) + \sigma, \phi_k(b) < 2\pi$ {{{{

    // four for-cycle to calculate $\phi_1, \phi_2, \phi_3, \phi_4$

      $\boldsymbol{\Phi}_b = $ Point($A_k(b), \phi_k(b)$);

      For $l = 0, l++, l < |\{\boldsymbol{\Phi}\}|$ {

        If $d(\boldsymbol{\Phi}_b, \boldsymbol{\Phi}_l) > d_{\text{th}}$;

          $\boldsymbol{\Phi}_b \rightarrow C = \{\boldsymbol{\Phi}_b\}_{b=0}^{B-1}$; $\boldsymbol{\Phi}_{b+B/2} \rightarrow C = \{\boldsymbol{\Phi}_b\}_{b=0}^{B-1}$ }

If $|\{\boldsymbol{\Phi}\}| < B$

  $b = b + 1$ }}}}}}}}

$|\{\boldsymbol{\Phi}\}| = B$

Return $C = \{\boldsymbol{\Phi}_b\}_{b=0}^{B-1}$

Algorithm 1: Algorithm of searching the antipodal orthogonal constellation.

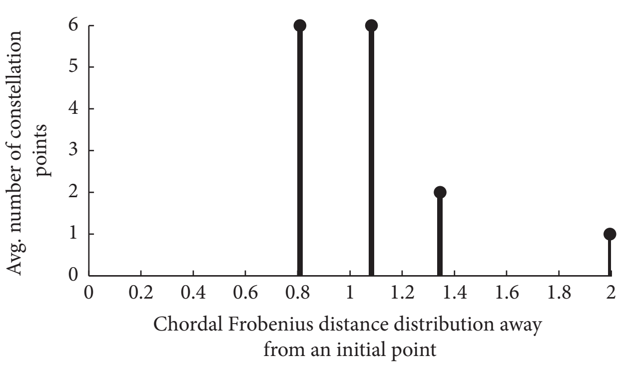

Figure 1: The distance distribution of the searching orthogonal constellation with the minimum distance $d_{\min} = 0.8$.

to generate a $\boldsymbol{\Phi}_\beta \in C$ like (5). Calculate $d = \min_{i=1,\ldots,|\{\boldsymbol{\Phi}\}|, \boldsymbol{\Phi}_i \in \{\boldsymbol{\Phi}\}} d(\boldsymbol{\Phi}_i, \boldsymbol{\Phi}_\beta)$. If $d > d_{\text{th}}$, then place $\boldsymbol{\Phi}_\beta$ into $\{\boldsymbol{\Phi}\}$; otherwise, discard $\boldsymbol{\Phi}_\beta$. If $\boldsymbol{\Phi}_\beta$ is placed into $\{\boldsymbol{\Phi}\}$, then its antipodal point $\overline{\boldsymbol{\Phi}}_\beta$ is also placed into $\{\boldsymbol{\Phi}\}$. Repeat (d) until $|\{\boldsymbol{\Phi}\}| = B$.

About the solution of the above optimization problem, there are $P_{mn} = (mn)!/(mn-4)!$ candidate matrices like (5). Hence the calculating complexity of the search algorithm is determined by m and n.

We used the grid search technique to do the following tests. For $m = 2, 4, 8$ and $n = 4$, let $d_{\text{th}} = 0.8$, we obtained the same results. There are 24 matrix points of satisfying the orthogonal condition within the range of $(mn)!/(mn-4)!$ unitary matrix points, and they constitute a set Θ of $C_{24}^{16} = 735471$ constellations in which each constellation contains 16 matrix points with the minimum distance $d_{\min} = 0.8$. By observing these 24 matrix points, we find out the following orthogonal constellation with the minimum Frobenius chordal distance $d_{\min} = 0.8376$.

$$\{\boldsymbol{\Omega}_k\} = \frac{1}{\sqrt{10}} \begin{bmatrix} 2 & 2 \\ -2 & 2 \\ e^{jk\pi/2} & e^{j(k\pm1)\pi/2} \\ -e^{-j(k\pm1)\pi/2} & e^{-jk\pi/2} \end{bmatrix} \tag{7}$$

for $k = 0, 1, 2, 3$, $\{\boldsymbol{\Omega}_k\}$ contains eight points and its antipodal points set, denoted as $\{\overline{\boldsymbol{\Omega}}_k\}$, has the following form:

$$\{\overline{\boldsymbol{\Omega}}_k\} = \frac{1}{\sqrt{10}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 2e^{jk\pi/2} & 2e^{j(k\pm1)\pi/2} \\ -2e^{-j(k\pm1)\pi/2} & 2e^{-jk\pi/2} \end{bmatrix} \tag{8}$$

for $k = 0, 1, 2, 3$; $\{\overline{\boldsymbol{\Omega}}_k\}$ also contains eight points. So the searching orthogonal constellation is

$$\{\boldsymbol{\Phi}_b\}_{b=0}^{15} = \{\boldsymbol{\Omega}_k\} \cup \{\overline{\boldsymbol{\Omega}}_k\}. \tag{9}$$

When $d_{\text{th}} \ge 0.8376$, we cannot search any constellation by using the presented algorithm. Note that when $d_{\text{th}} = 0.8376$, the grid searching algorithm cannot obtain the constellation of (9), because this constellation does not contain the initial point of searching process (see Algorithm 1).

4. Simplified Maximum Likelihood Detecting Algorithm Based on Antipodal Point

The antipodal constellation has the following feature.

Lemma 2. *Let* $C = C_1 \cup C_2 = \{\mathbf{X}_b\}_{b=1}^{B/2} \cup \{\overline{\mathbf{X}}_b\}_{b=1+B/2}^{B}$ *be an antipodal constellation defined on* $G_{T,M}$, $\mathbf{X} \in C_1 = \{\mathbf{X}_b\}_{b=1}^{B/2}$ *be a* $T \times M$ *transmitted signal matrix, and* $\overline{\mathbf{X}} \in C_2 = \{\overline{\mathbf{X}}_b\}_{b=1+B/2}^{B}$ *be an antipodal point of* $\mathbf{X}$*. If* $\mathbf{Y}$ *is a* $T \times N$ *received signal matrix, then* $\mathbf{X}$, $\overline{\mathbf{X}}$ *and* $\mathbf{Y}$ *satisfy*

$$\operatorname{Tr}\left(\mathbf{Y}^*\mathbf{X}\mathbf{X}^*\mathbf{Y}\right) + \operatorname{Tr}\left(\mathbf{Y}^*\overline{\mathbf{X}}\,\overline{\mathbf{X}}^*\mathbf{Y}\right) = \operatorname{Tr}\left(\mathbf{Y}^*\mathbf{Y}\right). \tag{10}$$

Proof. As $\mathbf{X}, \overline{\mathbf{X}} \in C \subset G_{T,M}$ are a pair of antipodal points, according to Lemma 1, there are $[\mathbf{X}\overline{\mathbf{X}}] = \mathbf{Q}_T$ and $[\mathbf{X}^*\overline{\mathbf{X}}^*]^* = \mathbf{Q}_T^*$, where $\mathbf{Q}_T$ is a $T \times T$ unitary matrix. Thus $\mathbf{X}\mathbf{X}^* + \overline{\mathbf{X}}\,\overline{\mathbf{X}}^* = \begin{bmatrix}\mathbf{X} & \overline{\mathbf{X}}\end{bmatrix}\begin{bmatrix}\mathbf{X}^* \\ \overline{\mathbf{X}}^*\end{bmatrix} = \mathbf{Q}_T\mathbf{Q}_T^* = \mathbf{I}_T$ is obtained. From here, (10) is deduced from the left to the right as follows:

$$\begin{aligned}
\text{Left} &= \operatorname{Tr}\left(\mathbf{Y}^*\mathbf{X}\mathbf{X}^*\mathbf{Y}\right) + \operatorname{Tr}\left(\mathbf{Y}^*\overline{\mathbf{X}}\,\overline{\mathbf{X}}^*\mathbf{Y}\right) \\
&= \operatorname{Tr}\left(\mathbf{Y}^*\left(\mathbf{X}\mathbf{X}^* + \overline{\mathbf{X}}\,\overline{\mathbf{X}}^*\right)\mathbf{Y}\right) \\
&= \operatorname{Tr}\left(\mathbf{Y}^*\left(\mathbf{X}\mathbf{X}^* + \overline{\mathbf{X}}\,\overline{\mathbf{X}}^*\right)\mathbf{Y}\right) = \operatorname{Tr}\left(\mathbf{Y}^*\mathbf{Q}_T\mathbf{Q}_T^*\mathbf{Y}\right) \\
&= \operatorname{Tr}\left(\mathbf{Y}^*\mathbf{I}_T\mathbf{Y}\right) = \operatorname{Tr}\left(\mathbf{Y}^*\mathbf{Y}\right) = \text{Right}
\end{aligned} \tag{11}$$

This completes the proof. □

It can be observed from (10) that the matrix $\mathbf{X}_i \in C_1$ determined by the maximum value of $\operatorname{Tr}(\mathbf{Y}^*\mathbf{X}_i\mathbf{X}_i^*\mathbf{Y})$ matches with the matrix $\overline{\mathbf{X}}_i \in C_2$ determined by the minimum value of $\operatorname{Tr}(\mathbf{Y}^*\overline{\mathbf{X}}_i\overline{\mathbf{X}}_i^*\mathbf{Y})$. Therefore, the following two lemmas are self-evident.

Lemma 3. *Let* $a = \operatorname{Tr}(\mathbf{Y}^*\mathbf{Y})$ *and* $b_i = \operatorname{Tr}(\mathbf{Y}^*\mathbf{X}_i\mathbf{X}_i^*\mathbf{Y})$ *which forms a set* $\{b_i\}$*.* $b_{max} = \max\{b_i \mid i = 1, \ldots, B/2\}$ *and* $b_{min} = \min\{b_i \mid i = 1, \ldots, B/2\}$ *is calculated. By Lemma 2,* $\overline{b}_{max} = a - b_{min}$ *and* $\overline{b}_{min} = a - b_{max}$ *are obtained.*

For the sake of obtaining the signal matrices in the subset $C_1 = \{\mathbf{X}_b\}_{b=1}^{B/2}$ corresponding to $b_{\max}$ and $b_{\min}$ of the set $\{b_i\}$, let $s_{\max}$ and $s_{\min}$ denote the indexing indicator of $\max\{b_i\}$ and $\min\{b_i\}$, respectively. ID denotes taking the index of each of all elements for the set $\{b_i\}$.

Lemma 4. $s_{max} = ID\max\{b_i\}$ *indicates that* s_{max} *is an index of the maximum value* b_{max} *in the set* $\{b_i\}$*. Similarly, we have* $s_{min} = ID\min\{b_i\}$, $\overline{s}_{max} = ID\max\{\overline{b}_i\}$*, and* $\overline{s}_{min} = ID\min\{\overline{b}_i\}$*. By Lemmas 2 and 3,* $s_{max} = \overline{s}_{min}$ *and* $s_{min} = \overline{s}_{max}$ *are obtained.*

The simplified ML detecting criterion is as follows.

Theorem 5. *Let* $C = C_1 \cup C_2 = \{\mathbf{X}_b\}_{b=1}^{B/2} \cup \{\overline{\mathbf{X}}_b\}_{b=1+B/2}^{B}$ *be the transmitted signal constellation and* $\mathbf{Y}$ *be the received signal matrix. Calculate* $a = \operatorname{Tr}(\mathbf{Y}^*\mathbf{Y})$ *and* $\{b_i\} = \{\operatorname{Tr}(\mathbf{Y}^*\mathbf{X}_i\mathbf{X}_i^*\mathbf{Y})\}$ *for* $i = 1, 2, \ldots, B/2$*. According to Lemmas 3 and 4, we obtain* b_{max}, s_{max}, b_{min}, s_{min}*, and* $\overline{b}_{max} = a - b_{min}$*. If* $b_{max} > \overline{b}_{max}$*, the detector outputs* $\widehat{\mathbf{X}} = \mathbf{X}_{s_{max}} \in C_1$ *as an estimate of the transmitted signal* $\mathbf{X} \in C$*; If* $b_{max} < \overline{b}_{max}$*, then* $\widehat{\mathbf{X}} = \mathbf{X}_{s_{min}} \in C_2$ *is given as an estimate of* $\mathbf{X} \in C$*.*

Proof. The result is obtained from Lemmas 2, 3, and 4 at once.

The complexity analysis of the simplified scheme: the ML detector of (2) requires B points (matrices) to take part in the calculation of $\operatorname{Tr}(\mathbf{Y}^*\mathbf{X}_b\mathbf{X}_b^*\mathbf{Y})$, while the antipodal simplified ML algorithm of Theorem 5 only requires $B/2$ points to take part in the calculation of $\operatorname{Tr}(\mathbf{Y}^*\mathbf{X}_b\mathbf{X}_b^*\mathbf{Y})$; in addition, looking for the maximum and minimum of $\{b_i\}$ requires the use of t comparison operations with $B/2 < t < B$; this is such that $\{b_{\max}\}$ and $\{b_{\min}\}$ are two nonempty sets initialized by the first two b_i's and if $b_i = \operatorname{Tr}(\mathbf{Y}^*\mathbf{X}_i\mathbf{X}_i^*\mathbf{Y}) > b_{\max}$, then put b_i into $\{b_{\max}\}$; else if $b_i = \operatorname{Tr}(\mathbf{Y}^*\mathbf{X}_i\mathbf{X}_i^*\mathbf{Y}) < b_{\min}$, then put b_i into $\{b_{\min}\}$; otherwise calculate the next b_i and the current b_i is discarded for $i = 1, 2, \ldots, B/2$. In one word, the calculated amount of $\operatorname{Tr}(\mathbf{Y}^*\mathbf{X}_b\mathbf{X}_b^*\mathbf{Y})$ descends by half but there is no performance loss. Refer to [10] regarding the other details of the detector. □

5. The Algebraic Orthogonal Design Constellation

In Section 3.2, the searching orthogonal USTM constellation based on the antipodal points was provided. In this Section, the feature of the antipodal point for the algebraic orthogonal USTM constellation presented [4] will be verified first, followed by the discussion of its antipodal simplified ML detector and the indexing simplified ML detector.

5.1. The Antipodal Feature of Orthogonal Design. Zhao et al. [4] presented the following Algebraic orthogonal (AO) scheme of USTM:

$$\mathbf{X}_{k,l} = \begin{bmatrix}\mathbf{E} \\ \mathbf{U}_{k,l}\end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ s_0 & s_1 \\ -s_1^* & s_0^* \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ e^{j(2\pi/Q)k} & e^{j(2\pi/Q)l} \\ -e^{-j(2\pi/Q)l} & e^{-j(2\pi/Q)k} \end{bmatrix}, \tag{12}$$

where $(k, l) \in F \times F$, $F = \{0, 1, \ldots, P-1\}$, and $B = P^2$. An AO USTM constellation is denoted as $C_{\text{AO}} = \{\mathbf{X}_{k,l} \mid (k, l) \in F \times F\}$. Let $\overline{\mathbf{X}}_{k,l} = [\mathbf{E}^*\overline{\mathbf{U}}_{k,l}^*]^* = \begin{bmatrix} 1 & -1 & s_2 & s_4 \\ 1 & 1 & s_3 & s_5 \end{bmatrix}^*$. By $[\mathbf{X}_{k,l}\overline{\mathbf{X}}_{k,l}] = \mathbf{Q}_T$ and $\mathbf{Q}_T^\dagger\mathbf{Q}_T = \mathbf{I}_T$ of Lemma 1, it is easy to verify that C_{AO} has the antipodal feature. According to the known elements of $\mathbf{X}_{k,l}$ and the antipodal relation between $\mathbf{X}_{k,l}$ and $\overline{\mathbf{X}}_{k,l}$, elements of $\overline{\mathbf{X}}_{k,l}$ are calculated as $s_2 = -s_0 = -e^{j(2\pi/P)k}$, $s_3 = -s_1 = -e^{j(2\pi/P)l}$, $s_4 = s_1^* = e^{-j(2\pi/P)l}$, and $s_5 = -s_0^* = -e^{-j(2\pi/P)k}$. For the case of $P = 4$ and $B = P^2 = 16$, we determine two distributions of antipodal points for C_{AO}, and they are given as scheme one:

$$\begin{aligned} C_{\mathrm{AO}}^1 &= \left\{\{\mathbf{X}_{k,l}\}_{k=0}^{P/2-1}\right\}_{l=0}^{P-1} \\ &= \{\mathbf{X}_{0,0}, \mathbf{X}_{0,1}, \mathbf{X}_{0,2}, \mathbf{X}_{0,3}, \mathbf{X}_{1,0}, \mathbf{X}_{1,1}, \mathbf{X}_{1,2}, \mathbf{X}_{1,3}\} \\ C_{\mathrm{AO}}^2 &= \left\{\{\mathbf{X}_{k,l}\}_{k=P/2}^{P-1}\right\}_{l=0}^{P-1} \\ &= \{\mathbf{X}_{2,2}, \mathbf{X}_{2,3}, \mathbf{X}_{2,0}, \mathbf{X}_{2,1}, \mathbf{X}_{3,2}, \mathbf{X}_{3,3}, \mathbf{X}_{3,0}, \mathbf{X}_{3,1}\} \end{aligned} \tag{13}$$

and scheme two:

$$\begin{aligned} C_{\mathrm{AO}}^1 &= \left\{\{\mathbf{X}_{k,l}\}_{k=0}^{P-1}\right\}_{l=0}^{P/2-1} \\ &= \{\mathbf{X}_{0,0}, \mathbf{X}_{1,0}, \mathbf{X}_{2,0}, \mathbf{X}_{3,0}, \mathbf{X}_{0,1}, \mathbf{X}_{1,1}, \mathbf{X}_{2,1}, \mathbf{X}_{3,1}\} \\ C_{\mathrm{AO}}^2 &= \left\{\{\mathbf{X}_{k,l}\}_{k=0}^{P-1}\right\}_{l=P/2}^{P-1} \\ &= \{\mathbf{X}_{2,2}, \mathbf{X}_{3,2}, \mathbf{X}_{0,2}, \mathbf{X}_{1,2}, \mathbf{X}_{2,3}, \mathbf{X}_{3,3}, \mathbf{X}_{0,3}, \mathbf{X}_{1,3}\}. \end{aligned} \tag{14}$$

Thus the general method of forming a pair of antipodal points in C_{AO} was derived as follows.

Theorem 6. *In the set $C_{AO} = \{\mathbf{X}_{k,l} \mid (k,l) \in F \times F\} = C_{AO}^1 \cup C_{AO}^2$, given a constellation point $\mathbf{X}_{k,l} \in C_{AO}^1$, then its antipodal point is $\mathbf{X}_{(k+P/2)(\mathrm{mod}\, P),(l+P/2)(\mathrm{mod}\, P)} \in C_{AO}^2$.*

Proof. From $d(\mathbf{U},\mathbf{V}) = \sqrt{2M - 2\,\mathrm{Tr}(\Sigma_{\mathbf{U}^*\mathbf{V}})}$ of (3), it is known that if $\mathbf{U}^*\mathbf{V} = 0$, then $d(\mathbf{U},\mathbf{V}) = \sqrt{2M}$ attains its maximum value. Thus, by calculating $\mathbf{X}_{k,l}^*\mathbf{X}_{(k+P/2)(\mathrm{mod}\, P),(l+P/2)(\mathrm{mod}\, P)} = \mathbf{0}$, we obtain $d(\mathbf{X}_{k,l}, \mathbf{X}_{(k+P/2)(\mathrm{mod}\, P),(l+P/2)(\mathrm{mod}\, P)}) = \sqrt{2M} = 2$. □

5.2. The Antipodal Simplified ML Detector of C_{AO}. The orthogonal USTM design [4] also has the relation similar to (10) in Lemma 2. Let $\mathbf{X} \in C_{\mathrm{AO}}^1 = \{\{\mathbf{X}_{k,l}\}_{k=0}^{P/2-1}\}_{l=0}^{P-1}$ (or $\mathbf{X} \in C_{\mathrm{AO}}^1 = \{\{\mathbf{X}_{k,l}\}_{l=0}^{P/2-1}\}_{k=0}^{P-1}$) and $\overline{\mathbf{X}} \in C_{\mathrm{AO}}^2 = \{\mathbf{X}_{(k+P/2)(\mathrm{mod}\, P),(l+P/2)(\mathrm{mod}\, P)}\}$; then $C_{\mathrm{AO}} = \{\mathbf{X}_{k,l} \mid (k,l) \in F \times F\} = C_{\mathrm{AO}}^1 \cup C_{\mathrm{AO}}^2$. Knowing that the shape of the transmitted signal with normalizing factor of 1/2 is a $T \times M$ unitary matrix $\mathbf{X} = [\mathbf{E}^*\mathbf{U}^*]^*$ in C_{AO} and its antipodal point is $\overline{\mathbf{X}} = [\mathbf{E}^*\overline{\mathbf{U}}^*]^*$, if $\mathbf{E}$ is invariable via transmission, then the received signal matrix is $\mathbf{Y} = [\mathbf{E}^*\mathbf{V}^*]^*$, where

$$\begin{aligned} \mathbf{E} &= \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}, \\ \mathbf{U} &= \begin{bmatrix} s_0 & s_1 \\ -s_1^* & s_0^* \end{bmatrix}, \\ \overline{\mathbf{U}} &= \begin{bmatrix} -s_0 & -s_1 \\ s_1^* & -s_0^* \end{bmatrix}, \\ \mathbf{V} &= \begin{bmatrix} y_1 & y_2 \\ y_3 & y_4 \end{bmatrix}. \end{aligned} \tag{15}$$

Lemma 7. *Let $C_{AO} = C_{AO}^1 \cup C_{AO}^2$ be a algebraic orthogonal USTM constellation with antipodal points on $G_{T,M}$, and $\mathbf{X} = [\mathbf{E}^*\mathbf{U}^*]^* \in C_{AO}^1$ and $\overline{\mathbf{X}} = [\mathbf{E}^*\overline{\mathbf{U}}^*]^* \in C_{AO}^2$. If $\mathbf{Y} = [\mathbf{E}^*\mathbf{V}^*]^*$ is a received matrix when $\mathbf{X}$ is a transmitted matrix, then $\mathbf{U}$, $\overline{\mathbf{U}}$, and $\mathbf{V}$ satisfy the constraint:*

$$\mathrm{Tr}\left(\mathbf{V}^*\mathbf{U}\mathbf{U}^*\mathbf{V}\right) + \mathrm{Tr}\left(\mathbf{V}^*\overline{\mathbf{U}}\,\overline{\mathbf{U}}^*\mathbf{V}\right) = \mathrm{Tr}\left(\mathbf{V}^*\mathbf{V}\right). \tag{16}$$

Proof. The proof is similar to Lemma 2. □

Similar to Theorem 5, the algebraic orthogonal USTM [4] has the following antipodal simplified ML detector.

Lemma 8. *Let $a = 4\,\mathrm{Tr}(\mathbf{V}^*\mathbf{V})$ and $b_i = \mathrm{Tr}(\mathbf{V}^*\mathbf{U}_i\mathbf{U}_i^*\mathbf{V})$. Calculate $b_{max} = \max\{b_i \mid i = 1,\ldots,B/2\}$ and $b_{min} = \min\{b_i \mid i = 1,\ldots,B/2\}$. By Lemma 7, $\overline{b}_{max} = a - b_{min}$ and $\overline{b}_{min} = a - b_{max}$ are obtained.*

Theorem 9. *Let $C_{AO} = C_{AD}^1 \cup C_{AO}^2$ be the transmitted signal constellation and $\mathbf{Y}$ be the received signal matrix. Calculate $a = 4\,\mathrm{Tr}(\mathbf{V}^*\mathbf{V})$ and $\{b_i\} = \{\mathrm{Tr}(\mathbf{V}^*\mathbf{U}_i\mathbf{U}_i^*\mathbf{V})\}$ for $i = 1,2,\ldots,B/2$. By Lemmas 8 and 4, obtain b_{max}, s_{max}, b_{min}, and s_{min}. If $b_{max} > a - b_{min}$, then the detector outputs $\widehat{\mathbf{X}} = \mathbf{X}_{s_{max}} \in C_{AO}^1$ as an estimate of the transmitted signal $\mathbf{X} \in C_{AO}$, else if $b_{max} < a - b_{min}$, then it outputs $\widehat{\mathbf{X}} = \mathbf{X}_{s_{min}} \in C_{AO}^2$ as an estimate of $\mathbf{X} \in C_{AO}$.*

Obviously, the calculating quantity of $\mathrm{Tr}(\mathbf{V}^*\mathbf{U}\mathbf{U}^*\mathbf{V})$ in (16) is half of that of $\mathrm{Tr}(\mathbf{Y}^*\mathbf{X}\mathbf{X}^*\mathbf{Y})$ in (10) because the sizes $T \times M$ of $\mathbf{X}$ and $\mathbf{Y}$ in $\mathrm{Tr}(\mathbf{Y}^*\mathbf{X}\mathbf{X}^*\mathbf{Y})$ for nonorthogonal design decrease by half the sizes of $M \times M$ of $\mathbf{U}$ and $\mathbf{V}$ in $\mathrm{Tr}(\mathbf{V}^*\mathbf{U}\mathbf{U}^*\mathbf{V})$ for orthogonal design.

5.3. The Indexing Simplified ML Detector of C_{AO}. Zhao et al. [4] presented the other simplified detecting algorithm for the algebraic orthogonal USTM, here called the indexing simplified detecting algorithm with k and l.

$$\begin{aligned} \left\|\mathbf{X}^*\mathbf{Y}\right\|^2 &= \mathrm{Tr}\left\{\mathbf{Y}^*\mathbf{X}\mathbf{X}^*\mathbf{Y}\right\} \\ &= 2 + \frac{1}{2}\mathrm{Tr}\left\{\mathbf{V}^*\mathbf{V}\right\} + \frac{1}{2}\mathrm{Tr}\left\{\mathbf{V}^*\mathbf{\Theta}_k + \mathbf{\Theta}_k^*\mathbf{V}\right\} \\ &\quad + \frac{1}{2}\mathrm{Tr}\left\{\mathbf{V}^*\mathbf{\Psi}_l + \mathbf{\Psi}_l^*\mathbf{V}\right\} \\ &= 2 + \frac{1}{2}\left\|\mathbf{V}\right\|^2 + \left(y_2^* - y_3\right)e^{j(2\pi/P)k} \\ &\quad + \left(y_2 - y_3^*\right)e^{-j(2\pi/P)k} + \left(y_2^* - y_3\right)e^{j(2\pi/P)l} \\ &\quad + \left(y_2 - y_3^*\right)e^{-j(2\pi/P)l}, \end{aligned} \tag{17}$$

where

$$\begin{aligned} \mathbf{\Theta}_k &= \begin{bmatrix} s_0 & 0 \\ 0 & s_0^* \end{bmatrix}, \\ \mathbf{\Psi}_l &= \begin{bmatrix} 0 & s_1 \\ -s_1^* & 0 \end{bmatrix}. \end{aligned} \tag{18}$$

Notice that $\mathrm{Tr}\{\mathbf{Y}^*\mathbf{X}\mathbf{X}^*\mathbf{Y}\}$ is decomposed into $\mathrm{Tr}\{\mathbf{V}^*\mathbf{\Theta}_k + \mathbf{\Theta}_k^*\mathbf{V}\}$ and $\mathrm{Tr}\{\mathbf{V}^*\mathbf{\Psi}_l + \mathbf{\Psi}_l^*\mathbf{V}\}$, where $\mathrm{Tr}\{\mathbf{V}^*\mathbf{\Theta}_k + \mathbf{\Theta}_k^*\mathbf{V}\}$ is only related to the first index k, while $\mathrm{Tr}\{\mathbf{V}^*\mathbf{\Psi}_l + \mathbf{\Psi}_l^*\mathbf{V}\}$ is only related to the second index l. Based on this decomposition and the exhausted expansion up to the level of elements of the matrix, the simplified ML detector [4] can further be simplified as follows:

$$\begin{aligned}\widehat{\mathbf{X}}_{\mathrm{ML}} &= \arg\max_{\mathbf{X}_{k,l}\in C_{\mathrm{AO}}} \left\|\mathbf{X}_{k,l}^*\mathbf{Y}\right\|^2 = \arg\max_{k,l=0,1,\ldots,P-1} \left\|\mathbf{U}_{k,l}^*\mathbf{V}\right\|^2 \\ &= \mathbf{X}_{\widehat{k}_{\mathrm{ML}},\widehat{l}_{\mathrm{ML}}},\end{aligned} \tag{19}$$

where $\widehat{k}_{\mathrm{ML}}$ and $\widehat{l}_{\mathrm{ML}}$ are computed as follows:

$$\begin{aligned}\widehat{k}_{\mathrm{ML}} &= \arg\max_{k=0,1,\ldots,P-1} \mathrm{Tr}\left(\mathbf{V}^*\mathbf{\Theta}_k + \mathbf{\Theta}_k{}^*\mathbf{V}\right) \\ &= \arg\max_{k=0,1,\ldots,P-1} \left[\left(y_1^* + y_4\right)e^{j(2\pi/P)k} \right. \\ &\quad \left. + \left(y_1 + y_4^*\right)e^{-j(2\pi/P)k}\right] \\ \widehat{l}_{\mathrm{ML}} &= \arg\max_{l=0,1,\ldots,P-1} \mathrm{Tr}\left(\mathbf{V}^*\mathbf{\Psi}_l + \mathbf{\Psi}_l^*\mathbf{V}\right) \\ &= \arg\max_{l=0,1,\ldots,P-1} \left[\left(y_2^* - y_3\right)e^{j(2\pi/P)l} \right. \\ &\quad \left. + \left(y_2 - y_3^*\right)e^{-j(2\pi/P)l}\right].\end{aligned} \tag{20}$$

The simplified approach of Zhao et al. [4] is required to calculate $\mathrm{Tr}\{\mathbf{Y}\mathbf{Y}^*\mathbf{A}_k\}$, P times and $\mathrm{Tr}\{\mathbf{Y}\mathbf{Y}^*\mathbf{B}_l\}$, P times. The aforementioned simplified approach only needs to calculate the $\widehat{k}_{\mathrm{ML}}$ and $\widehat{l}_{\mathrm{ML}}$ expressions formed by two complex multiplications and three complex additions, P times, respectively, ignoring matrix operations.

6. Numerical Results

In order to compare the quality between the searching orthogonal constellations and the algebraic orthogonal constellation, we plot their distance spectrums about average number of constellation points versus chordal Frobenius distance distribution away from an initial point. Figure 1 shows the distance spectrum of an searching orthogonal constellation with the minimum chordal Frobenius distance $d_{\min} = 0.8$ which consists of the first 16 points obtained by the grid search algorithm via setting the threshold $d_{\mathrm{th}} = 0.8$. This fist constellation belongs to the set Θ of C_{24}^{16} constellations (see the last in Section 3.2). In the set Θ, the best constellation is of (9) with the minimum distance $d_{\min} = 0.8376$ and its distance spectrum is shown in Figure 2. The algebraic orthogonal constellation like (12) does not belong to the set Θ because its minimum Frobenius chordal distance is $d_{\min} = 0.7321$ and its distance spectrum is shown in Figure 3.

In Figure 4, we demonstrate the corresponding performances of three constellations when used in noncoherent communication and operated on the additive white Gaussian noise (AWGN) channel, by plotting the curves between the symbol error probability versus the signal noise ratio.

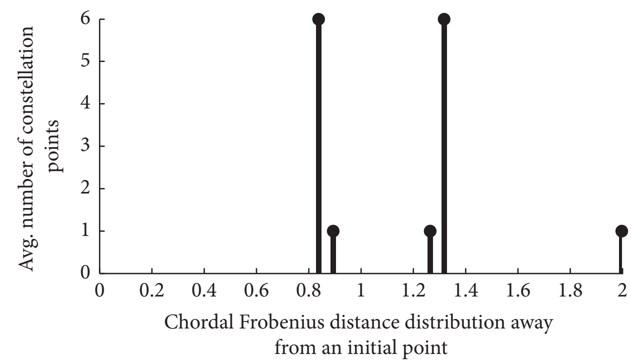

Figure 2: The distance distribution of the searching orthogonal constellation with the minimum distance $d_{\min} = 0.8376$.

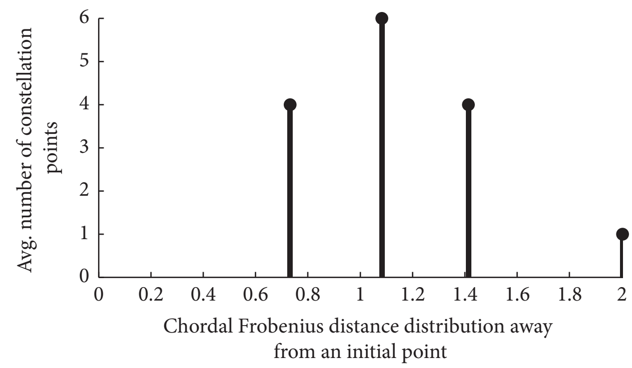

Figure 3: The distance distribution of the algebraic orthogonal constellation with the minimum distance $d_{\min} = 0.7321$.

At a symbol error probability of 10^{-5}, the best searching orthogonal constellation with the minimum distance $d_{\min} = 0.8376$ yields a SNR gain of about 2 dB over the algebraic orthogonal constellation of [4] when the number of receive antennas $N = 2$. In order to compare our testing results with those from the algebraic orthogonal constellation of [4], Figure 4 also shows the case that the number of receive antennas is $N = 1$. At a symbol error probability of 10^{-3}, the orthogonal constellation system for two receive antennas yields an SNR gain of about 8 dB over the system for one receive antenna.

7. Conclusion

We build a framework of generating a general USTM constellation based on full diversity and antipodal feature. Under the constraint of this framework, we search a set of the orthogonal constellations all of which are superior to the algebraic orthogonal constellation of [4] in both the distance spectrum and the performance of symbol error probability versus signal noise ratio. But the algebraic orthogonal constellation has a simpler ML detecting algorithm which is only the linear combination of complex elements of the received matrix without dependence on matrix operations.

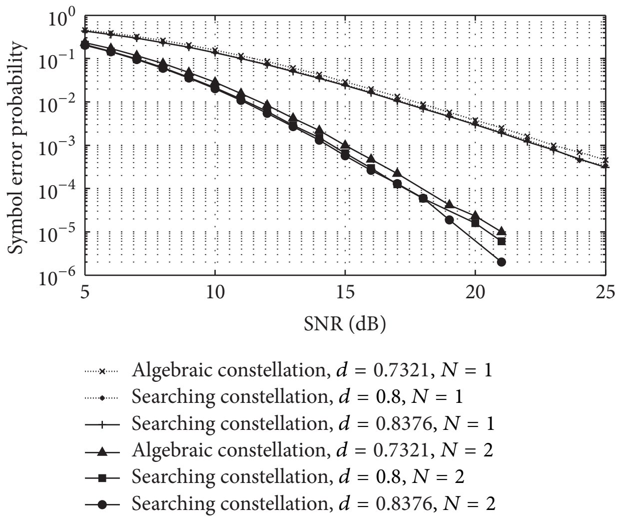

Figure 4: The performance comparisons between two searching orthogonal constellations and an algebraic orthogonal constellation for two cases with one and two receive antennas.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] B. M. Hochwald and T. L. Marzetta, "Unitary space-time modulation for multiple-antenna communications in Rayleigh flat fading," *IEEE Transactions on Information Theory*, vol. 46, no. 2, pp. 543–564, 2000.

[2] L. Zheng and D. N. Tse, "Communication on the Grassmann manifold: a geometric approach to the noncoherent multiple-antenna channel," *IEEE Transactions on Information Theory*, vol. 48, no. 2, pp. 359–383, 2002.

[3] R. H. Gohary and T. N. Davidson, "Noncoherent MIMO communication: Grassmannian constellations and efficient detection," *IEEE Transactions on Information Theory*, vol. 55, no. 3, pp. 1176–1205, 2009.

[4] W. Zhao, G. Leus, and G. B. Giannakis, "Orthogonal design of unitary constellations for uncoded and trellis-coded noncoherent space-time systems," *IEEE Transactions on Information Theory*, vol. 50, no. 6, pp. 1319–1327, 2004.

[5] W. Zhao, G. Leus, and G. B. Giannakis, "Algebraic design of unitary space-time constellations," in *Proceedings of the International Conference on Communications (ICC '03)*, pp. 3180–3184, May 2003.

[6] J. Kim, K. Cheun, and S. Choi, "Unitary space-time constellations based on quasi-orthogonal sequences," *IEEE Transactions on Communications*, vol. 58, no. 1, pp. 35–39, 2010.

[7] V. Tarokh and I.-M. Kim, "Existence and construction of noncoherent unitary space-time codes," *IEEE Transactions on Information Theory*, vol. 48, no. 12, pp. 3112–3117, 2002.

[8] Y. Jing and B. Hassibi, "Unitary space-time modulation via Cayley transform," *IEEE Transactions on Signal Processing*, vol. 51, no. 11, pp. 2891–2904, 2003.

[9] J. H. Conway, R. H. Hardin, and N. J. Sloane, "Packing lines, planes, etc.: packings in Grassmannian spaces," *Experimental Mathematics*, vol. 5, no. 2, pp. 139–159, 1996.

[10] L. Peng and D. Fu, "Antipodal demodulation method and antipodal demodulator for non-coherent unitary space-time modulation in MIMO wireless communication," Patent Number US 8, 879, 660 B1, November 2014.