Research Article

Robust Preamble-Based Timing Synchronization for OFDM Systems

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This study presents a novel preamble-based timing offset estimation method for orthogonal frequency division multiplexing (OFDM) systems. The proposed method is robust, immune to the carrier frequency offset (CFO), and independent of the structure of the preamble. The performance of the new method is demonstrated in terms of mean square error (MSE) obtained by simulation in multipath fading channels. The results indicate that the new method significantly improves timing performance in comparison with existing methods.

1. Introduction

Orthogonal frequency division multiplexing (OFDM) has become an increasingly popular technology in high bit-rate wireless communication systems such as WiFi, WiMax, and 3GPP LTE. Due to the sensitivity of OFDM systems to synchronization errors, reliable synchronization methods must be designed for these systems [1]. Many techniques have been proposed in the literature for timing and frequency offset estimation either jointly or individually in OFDM. In general, timing methods in OFDM systems fall into two categories: blind methods and those that use one or more preamble at the start of the frame. Considering that the preamble-based methods are always more efficient in computation than the blind methods, we focus on preamble-aided methods in this paper.

In [2], Schmidl and Cox proposed a timing and frequency estimation method using a preamble containing two identical halves. Schmidl’s method is simple and robust. However, owing to a quite wide plateau existing in the timing-metric function, the timing offset estimation in [2] suffers from large mean square error (MSE). To promote the timing accuracy, Minn et al. in [3] tried to remove this plateau by convoluting Schmidl's timing metric with a rectangular widow, whose length is equivalent to the cyclic prefix. In [4], Shi and Serpedin extended Minn’s method and exploited all self-correlation products to get smaller estimation MSE. However, the estimation accuracy of the methods in [3, 4] is still limited due to the side lobes in the timing-metric function. In [5], Park et al. proposed a preamble composed of two symmetrical parts and get a quite sharp timing metric without side-lobe by calculating the correlation between the symmetrical parts. All the proposed timing methods in [2–5] are based on self-correlation operation, immune to the CFO, and simple in realization, but they generally have much bigger estimation MSEs compared to the cross-correlation based methods.

Kang et al. [6], Hamed and Mahrokh (HM) [7], and Yang and Zhang [8] proposed impulse-like timing-metric functions using the cross-correlation between the received signal and the known preamble. Kang’s method is highly vulnerable to the CFO. Furthermore, since the number of the cross-correlation terms used in the timing metric is limited to N, the length of the preamble, Kang’s method performs poorly when N and the signal-noise ratio (SNR) are both small. HM’s method in [7] improves the timing performance by using much more correlation terms but it is also sensitive to the CFO. Yang’s estimator does not work when the absolute value of the CFO is larger than the frequency space between adjacent subcarriers.
In [9–11], there are timing methods in which self-correlation and cross-correlation are used in combination. Ren et al. [9] presented a pseudo-noise-sequence weighted preamble, whose specific structure is then exploited in the timing estimator. Ren’s method improves the accuracy of the timing offset estimator significantly. In [10, 11], HM and Liu proposed preamble-independent timing methods, which perform better than all the previous ones since the number of the available correlation terms is far more than that of other methods. It is worth mentioning that the method proposed in [11] has the same performance but it is less computationally complex than that in [10].

To further improve the performance of the timing synchronization, we investigate the maximum-likelihood (ML) timing algorithm in this paper. In the literature, several ML timing algorithms have been presented [12–14]. The ML estimators in [12, 13] exploit the redundant information contained within the cyclic prefix (CP); thus they are not fit for OFDM systems without CP, such as zero-prefix OFDM. The ML estimator in [14] is based on a preamble composed of more than 2 repeated parts. Compared to the existing ML timing methods, our method is independent of the type of the prefix or the structure of the preamble.

The remainder of this paper is organized as follows. Section 2 describes the signal model of OFDM system, and Section 3 presents our new timing estimation method and its characteristics. Section 4 gives the simulation results. The paper concludes with remarks in Section 5.

2. System Description

2.1. Signal Model. In an OFDM system, the samples of complex-valued baseband signal at the transmitter output can be expressed by

\[ x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi kn/N}, \quad 0 \leq n \leq N - 1, \quad (1) \]

where \( n \) is the time domain sample index, \( N \) is the total number of subcarriers with \( N_{\text{sub}} \) subcarriers being inactive, and \( X_k \) represents the modulated data symbol on the \( k \)th subcarrier. In practical applications, \( x(n) \) is computed by inverse fast Fourier transformation (IFFT). In order to avoid intersymbol interference (ISI) and intercarrier interference (ICI) in multipath channels, a cyclic prefix of length \( G \) is appended in front of \( x(n) \) as follows:

\[ x(n) = \begin{cases} x(n+N), & -G \leq n \leq -1 \\ x(n), & 0 \leq n \leq N - 1. \end{cases} \quad (2) \]

The length of the CP is longer than the possible length of the channel impulse response (CIR).

At the beginning of our study, we consider a flat fading channel to derive the ML estimator in an intuitive fashion as commonly assumed in [12, 14]. Later it will be shown by computer simulations that the proposed estimator works well for frequency-selective channels too. Now, the received signal is expressed as

\[ r[n] = x[n - \tau] e^{j2\pi \epsilon n/N + \theta} + \omega[n], \quad (3) \]

where \( \tau \) is the timing offset, \( \epsilon \) is the CFO normalized by the subcarrier spacing \( 1/T \) (\( T \) is the OFDM symbol interval), \( \theta \) is the phase offset, and \( \omega(n) \) is the sample of additive white Gaussian noise with zero mean and variance \( \sigma^2 \).

We assume a packet-based OFDM system in which each frame is composed of a preamble and \( M - 1 \) data OFDM symbols as shown in Figure 1. The preamble in the time domain is denoted by a vector \( S = [s_0, s_1, \ldots, s_{N-1}] \), where \( s_k \) is the \( k \)th element of \( S \). In addition, we assume that the frames are sent consecutively, so each frame is preceded by data samples of the last frame. It is worth mentioning that the proposed estimator, derived in Section 3, also performs well in the case of burst communication, where each frame is preceded by null samples at the transmitter.

2.2. Timing Synchronization. In [2], Schmidl and Cox proposed a preamble of the form \( [A \quad A] \) where \( A \) is a random complex vector of length \( N/2 \). The preamble can be generated by modulating a pseudo-noise-sequence on the even subcarriers and a zero sequence on the odd ones. According to Schmidl’s estimation method, the timing point maximizing the timing metric of \( M_{\text{Sch}}(n) \) in (4) shows the start of the frame,

\[ M_{\text{Sch}}(d) = \frac{|P_{\text{Sch}}(d)|^2}{(E_{\text{Sch}}(d))^2}, \quad (4) \]

where

\[ P_{\text{Sch}}(d) = \sum_{n=0}^{N/2-1} r^*(d+n) r(d+n+N/2), \quad (5) \]

\[ E_{\text{Sch}}(d) = \sum_{n=0}^{N/2-1} |r(d+n)|^2. \quad (6) \]

\( P_{\text{Sch}}(d) \) is defined to examine the correlation of the transmitted preamble. \( E_{\text{Sch}}(n) \) represents the energy of the received signal. In (5), \((\cdot)^*\) denotes the complex conjugation operation. Based on (4), the timing offset is estimated from

\[ \hat{\tau} = \arg \max_{d} (M_{\text{Sch}}(d)). \quad (7) \]

Due to the cyclic prefix of the preamble, Schmidl's timing metric has a plateau of \( G \) samples in length, which results in large uncertainty of the timing estimation.

To exploit the advantage of the constant envelope preamble in the transmission, Ren et al. in [9] proposed a synchronization method in which time and frequency offset are estimated.
jointly using one constant envelop preamble of length $N$. Ren's preamble can be defined as

$$x_n' = s_n x_n, \quad n = 0, 1, \ldots, N-1,$$

where $x_n'$ is a constant envelop preamble consisting of two repetitive sequences similar to Schmidl's, $s_n$ is a pseudorandom sequence, the correlation function $P_{\text{Ren}}(d)$ and normalization function $E_{\text{Ren}}(d)$ defined by

$$P_{\text{Ren}}(d) = \sum_{n=0}^{N/2-1} s_n s_{n+N/2} r(d) r(d+n) + \sum_{n=N/2}^{N-1} s_n s_{n-N/2} r(d) r(d+n), \quad n \in I_p,$$

$$E_{\text{Ren}}(d) = \frac{1}{2} \sum_{n=0}^{N-1} |r(d+n)|^2.$$

(9)

Since $s_n$ is a pseudorandom sequence, the correlation function $P_{\text{Ren}}(d)$ has an impulsive value only at the exact timing point $d$.

The number of the correlation products used in (9) is only $N/2$. However, for a given checkpoint $d$, there are $N(N-1)/2$ correlation products made by samples $[r(d), r(d+1), \ldots, r(d+N-1)]$. And all the available product terms can be represented by the following set:

$$C_d^d = \bigcup_{i=1}^{N-1} C_d^i,$$

(10)

where

$$C_d^i = \{r(d) r^*(d+i), r(d+1) r^*(d+1+i), \ldots, r(d+N-1-i) r^*(d+N-1)\}$$

is the subset of $C_d^d$. In [10], HM proposed a robust timing method which can exploit all the available product terms defined by (11). HM's estimator gets a significant improvement in timing accuracy compared to Ren's method but performs still unsatisfactorily when the SNR of the received signal is low.

3. Proposed Method

3.1. ML Synchronization. Without loss of generality, we denote the received samples with the length of a frame as

$$\mathcal{F} = \{r[n] \mid n = 0, 1, \ldots, \tau, \ldots, MN-1\},$$

(13)

where $r(0)$ and $r(\tau)$ denote the first sample of the received signal and the start point of the preamble, respectively. Here, we neglect the effect of the CP in the analysis since the CP little affects the timing synchronization performance but makes the analysis less intuitive. Therefore, those received signals are composed of $(M-1)N$ data samples and $N$ preamble samples, with indices defined by $I_p = \{n \mid \tau \leq n < \tau + N\}$ and $I_d = \{n \mid 0 \leq n < \tau \} \cup \{n \mid \tau + N \leq n < MN\}$, respectively. When $N$ is large, the data samples can be regarded as a Gaussian process [12, 14], and these samples follow independent complex Gaussian distribution with zero mean and variance $\sigma_r^2$ ($\sigma_d^2 = \sigma_r^2 + \sigma_c^2$, where $\sigma_c^2$ is the variance of $x(n)$). Thus, the conditional probability density function (PDF) of $r(n)$ for given $\tau$, $\epsilon$, and $\theta$ can be represented as

$$f(r(n) \mid \tau, \epsilon, \theta) = \begin{cases} \frac{1}{\pi \sigma_r^2} e^{-||r(n) - s_\epsilon||^2/\sigma_r^2}, & n \in I_p \\ \frac{1}{\pi \sigma_d^2} e^{-||r(n)||^2/\sigma_d^2}, & n \in I_d. \end{cases}$$

(14)

It is obvious that elements in $\mathcal{F}$ are independent of each other and their conditional PDF can be obtained as

$$f(\mathcal{F} \mid \tau, \epsilon, \theta) = \prod_{n \in I_p} f(r(n) \mid \tau, \epsilon, \theta).$$

(15)

The optimal joint ML estimation of the delays $\tau$, $\epsilon$, and $\theta$ can be obtained by maximizing $\log f(\mathcal{F} \mid \tau, \epsilon, \theta)$, that is, using the following log-likelihood function:

$$\Lambda(\tau, \epsilon, \theta) = \log \left( \prod_{n \in I_p} f(r(n) \mid \tau, \epsilon, \theta) \right).$$

After substituting (14) and (15) into (16), by omitting the constant terms $-N \log(\sigma_r^2) - (M-1)N \log(\sigma_d^2)$, and multiplying a constant number $\sigma_r^2$, we get a reduced likelihood function described as

$$\Lambda'(\tau, \epsilon, \theta) = -\rho_0 \sum_{n \in I_p} |r(n)|^2 - \sum_{n \in I_p} |r(n)|$$

$$- s_{n-\epsilon} e^{i2\pi n (\epsilon + \theta)} \left| r(n) \right| - \rho_0 \sum_{n \in I_p} |r(n)|^2 - \sum_{n \in I_p} |r(n)|$$

$$- s_{n-\epsilon} e^{i2\pi n (\epsilon + \theta)} \left| r(n) \right| - \rho_1 \sum_{i=0}^{N-1} \left| r(\tau+i) s_\epsilon e^{-i2\pi i/2} - c \right|,$$

(17)

where $\rho_0 = \sigma_r^2/(\sigma_d^2 + \sigma_r^2)$. Then, the ML estimates of $(\tau, \epsilon, \theta)$ can be represented as (18) at the top of the next page, where $\rho_1 = \sigma_r^2/(\sigma_d^2 + \sigma_r^2)$, and $c$ is a constant number, independent of $(\tau, \epsilon, \theta)$, described as

$$c = \rho_0 \sum_{i=0}^{MN-1} |r(i)|^2 + \sum_{i=0}^{N-1} |s_\epsilon|^2.$$

(19)

It is observed that, for any given $\tau$ and $\epsilon$, the right-hand side of (18) can be maximized to

$$2 \sum_{i=0}^{N-1} |r(\tau+i) s_\epsilon e^{-i2\pi i/2}| - \rho_1 \sum_{i=0}^{N-1} |r(\tau+i)|^2 - c$$

(20)
when
\[
\theta = -\frac{2\pi \tau}{N} + \angle \left\{ \sum_{i=0}^{N-1} r(\tau+i)s_i^* e^{-j2\pi i/N} \right\},
\]  
(21)
where \( \angle \) denotes the argument of a complex number. As a result, we can obtain the ML estimation of \((\tau, \epsilon)\) at first using (20) as the likelihood function, with the constant \(c\) omitted. And the ML estimator is written as
\[
(\tau_o, \epsilon_o) = \arg \max_{(\tau, \epsilon)} \left\{ \left| \sum_{i=0}^{N-1} r(\tau+i)s_i^* e^{-j2\pi i/N} \right| \right\}.
\]  
(22)

Note that, \(\rho_1\) in (22) is a constant number dependent on the SNR of the received signal. Furthermore, when \(N\) is large, \(\sum_{i=0}^{N-1} |r(\tau+i)|^2\) in (22) can also be regarded as a constant number \(N\sigma_1^2\) (where \(\sigma_1^2\) is the average power of \(r(x)\)). As a result, by omitting the multiplication of the two aforementioned constant numbers, we get the reduced ML estimator of \((\tau_o, \epsilon_o)\) written as
\[
(\tau_o, \epsilon_o) = \arg \max_{d} \left\{ \left| \sum_{i=0}^{N-1} r(d+i)s_i^* e^{-j(2\pi/\epsilon) i} \right| \right\}.
\]  
(23)

In some applications, for example, in wired OFDM systems including baseband digital subscriber line (DSL), the CFO error may be negligible. In this case, the joint estimation of \((\tau_o, \epsilon_o)\) in (23) reduces to ML timing offset estimation, which can be easily obtained by searching \(\tau\) maximizing the inner product of \([r(\tau), r(\tau+1), \ldots, r(\tau+N-1)]\) and the known preamble \(S\). When the CFO is uncertain, the optimum joint ML estimation of \((\tau, \epsilon)\), given in (23), needs to be obtained by numerical method, requiring high computational complexity because of the continuous variable \(\epsilon\).

### 3.2. Simplified Synchronization

It is observed that, on the right-hand side of (23), the operations in the absolute value bars are by nature a discrete-time Fourier transformation (DTFT) of the signal \(r(\tau+i)s_i^*\), with the frequency variable \((2\pi/\epsilon)\). Considering the truth that discrete Fourier transform (DFT) is just the equally spaced samples of the DTFT, and, for a time domain sequence of length \(N\), the \(N\) points DFT hold the same quantity of information as that of DTFT, we propose a near ML timing estimator written as
\[
\hat{\tau} = \arg \max_d \left\{ M(d) \right\},
\]  
(24)
where
\[
M(d) = \max \left\{ \left| \sum_{i=0}^{N-1} r(d+i)s_i^* e^{-j(2\pi/\epsilon) i} \right|, \ 0 \leq k < N \right\}.
\]  
(25)

is the timing metric of the proposed timing offset estimator. Similar to the timing metric proposed by [9, 10], the envelop of the proposed metric is impulse-like, because \(M(d)\) has its peak value at the starting point of the preamble, while the values at all other points are comparatively smaller. In Figures 2(a) and 2(b), we have depicted the metric of the proposed method in two cases of ideal condition (no noise and no channel distortion) and in a frequency-selective channel of 5 paths, respectively. It is obvious that, under both channels, the proposed timing metric has a sharp envelope, which leads to a much smaller mean square error of timing offset estimation as shown in later simulation results.
After the timing offset is acquired by the proposed method, suboptimal estimation of the CFO can be realized by methods in existing work, such as [15, 16]. The CFO normalized by the subcarrier spacing in a system can be expressed as

\[ \epsilon = \epsilon_i + \epsilon_f, \]

where \(\epsilon_i\) and \(\epsilon_f\) denote the integer part and the fractional part of the frequency offset, respectively. In [15], firstly, the fractional part is achieved by using the correlation between the fractional-part-corrected preamble (chosen from the received signal after timing synchronization) and its pure transmitted version. When the preamble does not have repeated identical parts, the integer part and fractional part can be achieved by the estimator, in [16], which is independent of the preamble structure.

### 3.3. Computational Complexity

Besides the performance issues, computational complexity is equally important in a practical sense. We evaluated the complexity of the proposed timing method by comparing the number of multiplications and additions needed by the timing-metric function with that of other timing offset estimators. In a OFDM system, \(N\) is always set as \(2^m\) where \(m\) is an integer, so that the FFT/IFFT operations can be used. Thus, the DFT in (25) can also be performed by FFT, which greatly reduces the computational complexity.

To make the analysis more intuitive, we rewrite the timing metric in (25) as

\[ M(d) = \max \left\{ \left| \left( R_d \ast S^* \right) W_k \right|^2, \ 0 \leq k < N \right\}, \]

where

\[ R_d = \left[ r(d), r(d+1), \ldots, r(d+N-1) \right], \]
\[ W_k = \left[ 1, e^{j(2\pi/N)k}, e^{j(2\pi/N)2k}, \ldots, e^{j(2\pi/N)(N-1)k} \right]. \]

The operator \(\ast\) denotes the Hadamard product, which is defined as element-by-element multiplications of two vectors, and \((\cdot)^H\) is the conjugate transpose operation.

Now, we calculate the metric \(M(d)\) in (27) for a given checkpoint indexed by \(d\). At first, a Hadamard product, which needs \(N\) complex multiplications, is used to calculate the cross-correlation of received signal vector \(R_d\) and local preamble vector \(S\). Then the resulting vector transformed to the frequency-domain vector by FFT operations, which require \(0.5N \log_2 N\) complex multiplications and \(N \log_2 N\) additions. At last, \(N\) complex multiplications and \(N-1\) complex additions are used to find the maximum element from the resulting frequency-domain vector. In total, we need \(N(0.5 \log_2 N + 2)\) complex multiplications and \((\log_2 N + 1)N - 1\) additions to calculate \(M(d)\) for the checkpoint \(d\).

In Table 1, we have demonstrated the complexity of the proposed method in comparison with the previous ones. Note that although the proposed method is more complex than methods of Schmidl, Ren, and Kang, it achieves a significantly better performance than those methods, as shown in the next section. As to HM's method, the computational complexity is adjustable by setting reasonable index vectors \(F = \left[ F_1, F_2, \ldots, F_L \right]\) and \(G = \left[ G_1, G_2, \ldots, G_L, \ldots, G_L \right]\), in which \(F_i\) or \(G_i\) is a subvector. The number of elements in \(F\) or \(G\) is noted as \(L\), which can be extended up to \((N-1)(N/2)\) if necessary. Naturally, higher \(L\) means higher timing performance. However, from the simulation results given in Section 4, we will see that the performance of the proposed method is better than that of HM's estimator even when all the available products are used in the latter method.

### 4. Simulation Results

The performance of the proposed method has been investigated in comparison with other methods using Monte Carlo simulations. The parameters used in the OFDM system are 64 subcarriers, 64-point IFFT/FFT, 1/8 guard interval (\(G = 8\)), and 100,000 simulation runs in each examination. The normalized CFO is set to \(\epsilon = 3.1\). Considering that the OFDM systems always work under multipath channel, we adopt two different multipath channels for the simulation. The first channel is based on Stanford university interim (SUI) channel modeling [17] with the sampling rate of 5 MHz. The second one is a multipath Rayleigh fading channel of 5 paths. We evaluate the performance of the proposed timing methods by mean square error (MSE) since MSE reflects both the bias and the variance of the estimation.

In Figure 3, we have depicted the MSE of the proposed method in comparison with the MSEs of five previous methods under the SUI-1 channel. We can see that the proposed method has a significantly better performance than the previous methods. It is worth mentioning that all the possible product terms have been used in HM's method in the simulation. Thus the proposed estimator performs better than HM's method even though the latter involves much higher computational complexity. Furthermore, we can see that Kang's and Yang's methods suffer great performance degradation from the CFO in the simulation.

Figure 4 demonstrates the MSEs of different estimators in another multipath Rayleigh fading channel with 5 paths, path delays \(\ell = 0, 1, \ldots, 4\), and the channel delay profile \(\epsilon_{\ell}(N)\). Here, we have \(N = 64\) and \(G = 12\). For HM's method [10], to get the best performance it could achieve, we use

<table>
<thead>
<tr>
<th>Method</th>
<th>Complex multiplication</th>
<th>Complex addition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Schmidl [2]</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Ren [9]</td>
<td>1</td>
<td>(N-1)</td>
</tr>
<tr>
<td>Kang [6]</td>
<td>2</td>
<td>(N-1)</td>
</tr>
<tr>
<td>HM [10]</td>
<td>(I_i + L + 1), where (0 &lt; I_i &lt; N, L \leq (N^2 - N)/2)</td>
<td>(L)</td>
</tr>
<tr>
<td>Proposed</td>
<td>(N(0.5 \log_2 N + 2))</td>
<td>(N(\log_2 N + 1) - 1)</td>
</tr>
</tbody>
</table>
plified timing synchronization method has been proposed based on the ML estimator. The proposed timing method is independent of the structure of the preamble and immune to the normalized CFO (by subcarrier space) in the range of $[-N/2 N/2]$. As shown in the simulation result, the proposed estimator performs better than the existing methods. For any given preamble, the joint timing and frequency synchronization for OFDM systems can be achieved by using our proposed timing method and the CFO estimator in [16].

**Competing Interests**

The authors declare that there are no competing interests regarding the publication of this paper.

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