Research Article

Comparative Study of Some New Hybrid Fuzzy Algorithms for Manipulator Control

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The robot manipulator is a highly complex system, which is multi-input, multi-output, nonlinear, and time variant. Controlling such a system is a tedious and challenging task. In this paper, some new hybrid fuzzy control algorithms have been proposed for manipulator control. These hybrid fuzzy controllers consist of two parts: a fuzzy controller and a conventional or adaptive controller. The outputs of these controllers are superimposed to produce the final actuation signal based on current position and velocity errors. Simulation is used to test these controllers for different trajectories and for varying manipulator parameters. Various performance indices like the RMS error, steady state error, and maximum error are used for comparison. It is observed that the hybrid controllers perform better than only fuzzy or only conventional/adaptive controllers.

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1. INTRODUCTION

Many different control strategies have been proposed in literature for control of a robot arm. These range from conventional [1] to adaptive [2, 3] to more recent fuzzy [4–6] and adaptive fuzzy [7, 8] control strategies.

Use of fuzzy logic for control of robotic manipulators has found vast interest in the control literature. Fuzzy logic deals with problems of vagueness, uncertainty, or imprecision, unlike Boolean logic, which deals with crisp values. It provides control designers liberty to exploit their understanding of the problem and to construct intelligent control strategies. Nonlinear controllers can be devised easily by using fuzzy logic principles [9]. Fuzzy controllers are thus powerful tools to deal with nonlinear systems.

In this paper, an attempt has been made to do a comparative study of some hybrid fuzzy control schemes. We have tried to combine some well-established conventional and adaptive controllers with a lookup table-based fuzzy controller and have done a comparative analysis of their simulated performance. This paper follows a similar pattern as our earlier paper [10], where we compared some adaptive fuzzy algorithms used for manipulator control.

2. FUZZY CONTROL

The fuzzy control strategy consists of situation and action pairs. This is very similar to how a human operator uses his experience about the plant to control it. A block diagram for the fuzzy controller is shown in Figure 1. The fuzzy controller here defines error (e) as

$$e = \theta_d - \theta \quad (1)$$

and rate of change of error ($\dot{e}$) as

$$\dot{e} = \dot{\theta}_d - \dot{\theta}, \quad (2)$$

where θ and $\dot{\theta}$ are actual position and velocity of joints, θ_d and $\dot{\theta}_d$ are desired position and velocity of joints and τ is the output of fuzzy controller applied as control input to the robot system.

A detailed view of internal of the Fuzzy controller block shown in Figure 1 is shown in Figure 2.

For the simulation study carried out in this paper, the input variables to the fuzzy controller ($e, \dot{e}$) are quantized into thirteen levels represented by $-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6$, and a set of linguistic variables such as negative big (NB), negative medium (NM), negative small (NS), zero

FIGURE 1: Block diagram of fuzzy controller.

FIGURE 2: Details of fuzzy controller block.

(ZE), positive small (PS), positive medium (PM), positive big (PB) are assigned.

The membership functions chosen for the linguistic variables, for simulation study, are triangular in shape and are shown in Figure 3. Some of the control rules framed are the following.

- (R1) If e is ZE and $\dot{e}$ is ZE, then u is ZE.
- (R2) If e is ZE and $\dot{e}$ is NS, then u is NS.
- (R3) If e is NM and $\dot{e}$ is ZE, then u is NM.
- (R4) If e is NM and $\dot{e}$ is NB, then u is NB.

As stated earlier, the rules formulated are based on operator's experience regarding the plant. The rule strength of the individual rule is evaluated using the intersection operation defined as

$$\mu_{NB}(u) = \min(\mu_{NM}(e^*), \mu_{NB}(\dot{e}^*)), \quad (3)$$

where $\mu_{NB}(u)$ is the rule strength of the rule R4, $\mu_{NM}(e^*)$ is the membership of the crisp input e^* in the fuzzy set NM, and $\mu_{NB}(\dot{e}^*)$ is the membership of $\dot{e}^*$ in the fuzzy set NB. For each possible pair of e^* and $\dot{e}^*$, the rules are fired individually to give the degree to which the rule antecedent has been matched by the crisp value. The clipped values for the individual rules thus obtained are summed forming the overall control values. The output value is then defuzzified by using the center of gravity method, which is given by

$$u^* = \frac{\sum_{Ri} \mu_{Ri}(u_{Ri}) \cdot u_{Ri}}{\sum_{Ri} \mu_{Ri}(u_{Ri})}. \quad (4)$$

The output values thus obtained for all the $(e^*, \dot{e}^*)$ pairs are stored in the form of a lookup table (LUT) as shown in Table 1.

The array implementation improves execution speed, as the run-time inference is reduced to a table lookup, which is

FIGURE 3: Membership functions of the linguistic variables.

a lot faster. This is true at least when the correct entry can be found without too much searching.

The controller output values shown in Table 1 were obtained after some manual adjustment through trial and error to give best possible results. This was required because the manipulator control problem is highly nonlinear and the rules formulated through user experience are not always correct under different situations. To do the manual adjustment, we tracked the entries of lookup table used in a typical simulation run. These entries were then matched against the error profiles. The table entries, which were in use when the errors were high, were then modified accordingly.

The control strategies were tested for a two-link manipulator. Figure 4 shows the manipulator with frames assigned to the links.

The inverse dynamics of the manipulator can be derived using the Lagrange or Newton-Euler method. The joints were assumed to have only viscous friction. This model was used for all simulations. The details of the manipulator model used for simulations are provided in the appendices.

3. HYBRID FUZZY CONTROL

Many hybrid fuzzy controllers have been proposed in the past [11, 12]. In this section, we investigate some new hybrid fuzzy control schemes. The primary characteristic of these controllers is that in these schemes the final control output applied to the plant is summation of individual output of two controllers. One of them is the fuzzy controller while the other could be a conventional or adaptive controller. The general block diagram of the controller is shown in Figure 5. As both the controllers are individually BIBO stable, the combination is also BIBO stable. The proof for this statement is provided in the appendices. We first discuss the results of combining fuzzy and conventional controllers and then fuzzy and adaptive controllers.

All the controllers discussed in the following subsections were tested for the case when the parameters of the manipulator change during motion because of it picking up a load sometime during its motion.

3.1. Fuzzy plus computed torque control

The most common and perhaps one of the oldest nonlinear control techniques for manipulator control is the computed torque control (CTC) [13, 14]. Recently, there has been

TABLE 1: Lookup table for the fuzzy controller.

e	$\dot{e}$													
	Membership function													
	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	
-6	-5.6	-5.4	-5.0	-4.8	-4.8	-4.7	-4.7	-4.6	-4.5	-4.4	-4.3	-4.3	-4.2	
-5	-4.7	-4.5	-4.4	-4.3	-4.2	-4.1	-4.0	-3.9	-3.8	-3.8	-3.7	-3.6	-3.5	
-4	-3.7	-3.6	-3.5	-3.2	-3.0	-3.0	-3.0	-2.9	-2.9	-2.8	-2.8	-2.7	-2.7	
-3	-2.0	-2.0	-1.9	-1.9	-1.8	-1.8	-1.7	-1.7	-1.6	-1.5	-1.4	-1.3	-1.3	
-2	0.0	0.0	-0.8	-1.0	-1.2	-1.7	-2.3	-2.2	-2.2	-2.0	-2.0	-1.0	-1.0	
-1	1.0	1.0	0.0	0.0	-0.5	-0.5	-0.5	-1.0	-1.2	-1.5	-1.7	-1.0	-1.0	
0	1.3	1.2	1.0	0.8	0.6	0.0	-0.2	-0.4	-0.6	-0.8	-1.0	-1.0	-1.0	
1	2.0	2.0	1.9	1.8	1.8	1.8	1.8	1.8	1.5	0.0	-0.3	-1.0	-0.8	
2	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	1.2	0.8	0.0	0.0	
3	2.0	2.1	2.3	2.5	2.5	2.5	2.6	2.7	2.8	2.8	2.9	2.9	3.0	
4	2.7	2.7	2.8	3.1	3.2	3.3	3.5	3.6	3.6	3.8	3.8	3.9	3.9	
5	3.6	3.3	3.7	4.0	4.1	4.3	4.3	4.4	4.4	4.5	4.5	4.6	4.7	
6	4.4	4.4	4.3	4.8	5.0	5.0	5.1	5.2	5.3	5.4	5.6	5.6	5.6

FIGURE 4: Manipulator used for simulations.

renewed interest in this controller being used along with fuzzy controllers [15].

Here the computed torque τ_{ctc} is given by

$$\tau_{ctc} = M(\theta)[\ddot{\theta}_d + K_D\dot{e} + K_P e] + V_M(\theta, \dot{\theta})\dot{\theta} + F_M(\theta, \dot{\theta})\dot{\theta} + G(\theta), \quad (5a)$$

where $M(\theta)$ is manipulator mass matrix, $V_M(\theta, \dot{\theta})$ is matrix containing centrifugal and coriolis terms, $F_M(\theta, \dot{\theta})$ is the friction matrix, and $G(\theta)$ is the gravity matrix. Further, K_P and K_D are controller gain matrices. The values of K_P and K_D chosen for simulation were 100 and 50, respectively.

If the manipulator model is known exactly, then this scheme results in asymptotically stable linear time invariant error dynamics and provides asymptotically exact tracking.

With the additional fuzzy error compensation, the control law becomes

$$\tau_{ctcf} = M(\theta)[\ddot{\theta}_d + K_D\dot{e} + K_P e] + V_M(\theta, \dot{\theta})\dot{\theta} + G(\theta) + \tau_f. \quad (5b)$$

FIGURE 5: Hybrid fuzzy controller.

3.2. Fuzzy plus feed forward inverse dynamics control

A slightly different approach that is more suitable to adaptation is sometimes used instead of computed torque scheme [16, 17]. This scheme uses the inverse dynamics in feed forward mode and is thus known as feed forward inverse dynamics control (FFIDC). In this strategy, the torque is calculated as

$$\begin{aligned} \tau_{ffidc} &= M(\theta)\ddot{\theta}_d + V_M(\theta, \dot{\theta})\dot{\theta}_d + F_M(\theta, \dot{\theta})\dot{\theta}_d + G(\theta) + K_D\dot{e} + K_P e \\ &= W(\theta, \dot{\theta}, \ddot{\theta}_d, \dot{\theta}_d)P + K_D\dot{e} + K_P e. \end{aligned} \quad (6a)$$

Equation (6a) uses the inverse dynamics model with $W(\theta, \dot{\theta}, \ddot{\theta}_d, \dot{\theta}_d)$ as the regressor matrix and P as the vector of manipulators parameters. The regressor matrix is dependent both on actual and the desired values of acceleration and velocity instead of the actual values alone. The error system resulting from this controller can be shown to be globally asymptotically stable when K_P and K_D are diagonal and all the scalar values are positive. The values of K_P and K_D chosen for simulation were 100 and 50, respectively.

With the additional fuzzy error compensation, the control law becomes

$$\tau_{\text{fddcf}} = W(\theta, \dot{\theta}, \ddot{\theta}_d)P + K_D \dot{e} + K_P e + \tau_f. \quad (6b)$$

3.3. Fuzzy plus critically damped inverse dynamics control

The critically damped inverse dynamics control (CDIDC) strategy is almost same as feed forward inverse dynamics except that the regressor matrix is calculated using reference velocity and reference acceleration instead of the desired values. These reference values are defined as

$$\begin{aligned} \dot{\theta}_R &= \dot{\theta}_d + \Lambda(\theta_d - \theta), \\ \ddot{\theta}_R &= \ddot{\theta}_d + \Lambda(\dot{\theta}_d - \dot{\theta}), \end{aligned} \quad (7)$$

where Λ is a positive definite gain matrix.

If we define $W(\theta, \dot{\theta}, \ddot{\theta}_R)$ as the manipulator regressor matrix and the error, $\dot{e}$, as

$$\dot{e} = \dot{\theta} - \dot{\theta}_R = \dot{\theta} - \dot{\theta}_d - \Lambda(\theta_d - \theta), \quad (8)$$

then the torque is calculated as

$$\tau_{\text{cdidc}} = W(\theta, \dot{\theta}, \ddot{\theta}_R) - K_D \dot{e}. \quad (9a)$$

This control law results in a system of stable first-order subspace. An exponentially stable system forced by an input that decays to zero has an output that decays to zero. Then $\lim_{t \rightarrow \infty} e(t) \rightarrow 0$. This result was given by Sadeh and Horowitz [18] and is used to prove the stability of the controller.

As can be seen from (8) and (9a), there are two main differences between this controller and the previous FFIDC. First, in this controller, the manipulator regressor matrix is calculated as a function of actual positions and velocities and also as a function of reference velocities and accelerations, while in the FFIDC, the regressor matrix was calculated as a function of only actual positions, velocities, and accelerations. Second, the effective proportional gain of the CDIDC becomes ΛK_D , while in the FFIDC, it is equal to K_P . The Λ matrix in the CDIDC effectively has the same role as the K_P matrix has in the FFIDC. As a result, the effective proportional gain constant of the CDIDC increases by a factor of K_D , if we choose $\Lambda = K_P$. For our simulation study, we chose Λ and K_D as diagonal matrices with elements equal to 100 and 50, respectively.

With the additional fuzzy error compensation, the control law becomes

$$\tau_{\text{cdidcf}} = W(\theta, \dot{\theta}, \ddot{\theta}_R) - K_D \dot{e} + \tau_f. \quad (9b)$$

3.4. Fuzzy plus adaptive critically damped inverse dynamics control

The first adaptive controller investigated is the adaptive critically damped inverse dynamics controller (ACDIDC) proposed in [19]. This is a direct adaptive controller in the sense that the parameter values are adapted directly from the information about position and velocity errors of the different joints. The adaptation law is derived starting from the manipulator dynamic equation written in a linear form. If we define

$$\tilde{P} = \hat{P} - P \quad (10)$$

as the parameter estimation error vector, with P as the true parameter values vector and $\hat{P}$ as the vector of parameter estimates, then the linearity property of the robot dynamics enables us to write

$$\tilde{M}(\theta)\ddot{\theta}_R + \tilde{V}_M(\theta, \dot{\theta})\dot{\theta}_R + \tilde{F}_M(\theta, \dot{\theta})\dot{\theta}_R + \tilde{G}(\theta) = W(\theta, \dot{\theta}, \ddot{\theta}_R)\tilde{P}, \quad (11)$$

where

$$\begin{aligned} \tilde{M} &= \hat{M} - M, \\ \tilde{V}_M &= \hat{V}_M - V_M, \\ \tilde{F}_M &= \hat{F}_M - F_M, \\ \tilde{G} &= \hat{G} - G, \end{aligned} \quad (12)$$

and $\dot{\theta}_R$ and $\ddot{\theta}_R$ are reference trajectories as defined in (7).

The control law used can be written as

$$\tau_{\text{acdide}} = \hat{M}(\theta)\ddot{\theta}_R + \hat{V}_M(\theta, \dot{\theta})\dot{\theta}_R + \hat{F}_M(\theta, \dot{\theta})\dot{\theta}_R + \hat{G}(\theta) - K_D \dot{e}, \quad (13a)$$

where $\dot{e}$ is as defined in (8) and K_D is uniformly positive definite controller gain matrix. For our simulation study, we chose Λ and K_D as diagonal matrices with elements equal to 100 and 50, respectively.

With the additional Fuzzy error compensation, the control law becomes

$$\begin{aligned} \tau_{\text{acdidecf}} &= \hat{M}(\theta)\ddot{\theta}_R + \hat{V}_M(\theta, \dot{\theta})\dot{\theta}_R + \hat{F}_M(\theta, \dot{\theta})\dot{\theta}_R + \hat{G}(\theta) - K_D \dot{e} + \tau_f. \end{aligned} \quad (13b)$$

The stability of the adaptive controller can be proved by considering the Lyapunov function candidate,

$$V(t) = \frac{1}{2}[\dot{e}^T M(\theta)\dot{e} + \tilde{P}^T \Gamma^{-1} \tilde{P}], \quad (14)$$

where Γ is a constant positive definite matrix.

This leads to an adaptation law as

$$\dot{\hat{P}} = -\Gamma W^T \dot{e}. \quad (15)$$

For our simulation study, we chose Γ as a diagonal matrix with elements equal to 10.

3.5. Fuzzy plus direct model reference adaptive control

This model reference adaptive controller (MRAC), proposed in [20], has two stages. First, the known dynamics are separated out and used to perform a global linearization on the nonlinear system. Second, a model reference adaptive control, based on the Lyapunov stability criterion, is designed for the remaining unknown portion of the plant.

The torque for this controller is calculated as follows:

$$\tau_{\text{mrac}} = (I + \Delta_v)v - (K_p + \Delta_1)\theta - (K_v + \Delta_2)\dot{\theta} + \tau_k, \quad (16a)$$

where τ_k is the torque due to known dynamics, I is the identity matrix, Δ 's are the controller gains which are adapted according to the present errors, and $v = \ddot{\theta}_d + K_p\theta + K_v\dot{\theta}$.

For our simulation study, we chose K_p and K_v as diagonal matrices with elements equal to 500 and 100, respectively. The initial values of all Δ 's were taken as zero.

With the additional fuzzy error compensation, the control law becomes

$$\tau_{\text{mracf}} = (I + \Delta_v)v - (K_p + \Delta_1)\theta - (K_v + \Delta_2)\dot{\theta} + \tau_k + \tau_f. \quad (16b)$$

3.6. Fuzzy plus decentralized adaptive control

The decentralized adaptive controller (DAC) does not make use of the centralized mathematical model of the robot manipulator [21]. It thus results in much fewer calculations to be performed in the control loop. The basic structure of the controller is that of a simple PID controller. The torque is calculated as

$$\tau_{\text{dac}} = K_f\ddot{\theta}_d + K_p e + K_d \dot{e}, \quad (17a)$$

where K_f , K_p , and K_d are adaptable controller gains.

For our simulation study, we started with zero values of the controller gains and these were adapted depending on the adaptation laws and the present errors.

With the additional fuzzy error compensation the control law becomes

$$\tau_{\text{dacf}} = K_f\ddot{\theta}_d + K_p e + K_d \dot{e} + \tau_f. \quad (17b)$$

4. REFERENCE TRAJECTORIES AND MANIPULATOR PARAMETER VALUES

The above controllers were tested for two different trajectories. In the first trajectory, the first joint was required to move

FIGURE 6: Desired trajectory 1.

from its initial home position (0°) to a final position of $+90^\circ$ in 5 seconds. On reaching the final position, the manipulator picks up a load and returns back to its home position in another 5 seconds. On reaching the home position, the manipulator was required to stay there with the load for another 5 seconds. Thus, the desired position of first joint remains constant at 90° for the last 5 seconds of its motion. This kind of trajectory enables us to test the steady state performance of the controller. The desired motion for the second joint is exactly the same as for the first one except that it is required to move from 0° to -90° and then back to 0° in a total time of 15 seconds. Figure 6 shows the desired joint position profiles for this trajectory.

The second test trajectory was chosen to simulate the motion of manipulator during a typical pick and place operation. Here, the manipulator's first joint was required to move from its home position of 0° to a final position of $+45^\circ$ in 2 seconds. At this point, the manipulator picks up a load and returns back to its home position in the next 2 seconds. On reaching home, the manipulator releases the load and this cycle is repeated all over again. The second joint of the manipulator has a motion similar to the first one except that it moves to a final position of -45° . The errors for this trajectory were traced for two cycles, that is, 8 seconds. The RMS and the maximum values of the errors were used for quantitative performance comparisons of various controllers for this trajectory. Figure 7 shows the joint motion profiles for this trajectory.

The controllers were tested using the above trajectories. We assumed that there is some initial estimate available of the various manipulator parameter values. This estimate is a rough approximation of the real actual values and can be arrived at by some elementary measurements. The real and the estimated values of the parameters were taken as below.

Real parameters	Estimated parameters
$m_1 = 2.0 \text{ kg}$	$m_1 = 1.0 \text{ kg}$
$m_2 = 2.0 \text{ kg}$	$m_2 = 1.0 \text{ kg}$
$l_1 = 0.26 \text{ m}$	$l_1 = 0.26 \text{ m}$
$x_1 = 0.13 \text{ m}$	$x_1 = 0.11 \text{ m}$
$x_2 = 0.14 \text{ m}$	$x_2 = 0.12 \text{ m}$
$I_{zz1} = 0.09 \text{ kg-m}^2$	$I_{zz1} = 0.05 \text{ kg-m}^2$
$I_{zz2} = 0.09 \text{ kg-m}^2$	$I_{zz2} = 0.05 \text{ kg-m}^2$
$F_1 = 2.5 \text{ N-m/rad/s}$	$F_1 = 2.0 \text{ N-m/rad/s}$
$F_2 = 2.5 \text{ N-m/rad/s}$	$F_2 = 2.0 \text{ N-m/rad/s}$

where

m_i = Mass of the i th link (kg),

l_i = Length of the i th link (m),

x_i = Location of the center of mass of the i th link along the respective x -axis (m),

I_{zzi} = Moment of inertia of the i th link about Z_i axis (kg-m^2),

F_i = Coefficient of viscous friction for i th joint.

The adaptive algorithms in all the cases start with this apriori estimate. We thus say that the manipulator makes a *warm start*. The parameters of the manipulator were further assumed to have changed to new values whenever it picked up a load. These new values of the parameters of manipulator with load were taken as below.

Parameters with load
$m_1 = 3.0 \text{ kg}$
$m_2 = 3.0 \text{ kg}$
$l_1 = 0.26 \text{ m}$
$x_1 = 0.15 \text{ m}$
$x_2 = 0.16 \text{ m}$
$I_{zz1} = 1.5 \text{ kg-m}^2$
$I_{zz2} = 0.09 \text{ kg-m}^2$
$F_1 = 2.5 \text{ N-m/rad/s}$
$F_2 = 2.5 \text{ N-m/rad/s}$

5. SIMULATION RESULTS AND OBSERVATIONS

The values of different errors for various control strategies are tabulated in Table 2.

Following observations are made based on our simulation studies.

(1) The FFIDC performs appreciably better than the CTC. This clearly indicates the merits of using the inverse dynamics in the feed forward mode and further illustrates the advantage of using the desired velocity and acceleration instead of the actual ones for composing the manipulator regressor matrix. The use of desired acceleration instead of the actual value has further advantage in terms of real world implementation. Measuring the actual acceleration of the manipulator joints is a difficult task, as acceleration sensors are bulky and difficult to mount at the joints. Further, if the acceleration is found by differentiating the velocity information given by a tachogenerator or by double differentiating the position information given by an optical encoder, there

FIGURE 7: Desired trajectory 2.

are always possibilities of error creeping in due to even a very low amplitude noise signal.

(2) The CDIDC is the best performer in the category of conventional controllers. It is observed that both the RMS and steady state values of errors, for both the links for both the trajectories are considerably reduced in magnitude over the corresponding values for FFIDC. As explained earlier, the greater value of the proportional gain constant results in a better steady state performance, while, the use of reference velocities and accelerations for calculation of the manipulator regressor matrix instead of the actual values results in an improved transient performance of the arm. This can be mainly attributed to the fact that the reference values are "cleaner" compared to the actual values, which are sensor derived and, hence, always ridden with noise. Moreover if only an optical encoder is used for feedback (as is usual), the values of velocity and acceleration of the joints has to be derived by differentiating the position information provided by the encoders. This numerical differentiation can further reduce the validity of data and the problem becomes more severe with any increase in the noise in the environment where the manipulator is working. The error profiles for the two links are shown in Figure 8 for the first trajectory, and Figure 9 for the second trajectory.

(3) Adaptive controllers outperform their fixed model counterparts.

(4) Among the adaptive controllers, the ACDIDC gives the best performance because it has the same advantages as its nonadaptive counterpart (CDIDC). The error profiles for the two links are shown in Figure 10 for the first trajectory and Figure 11 for the second trajectory.

(5) The pure fuzzy controller does not give very encouraging performance. In fact, its performance is poorer than the conventional CDIDC. But the advantage it offers is

TABLE 2: Errors (in degrees) for different controllers.

S.no	Control strategy	Trajectory no.1				Trajectory no.2			
		link1		link2		link1		link2	
		0° → 90° → 0° →		0° → -90° → 0° →		0° → 45° → 0° → 45° → 0°		0° → -45° → 0° → -45° → 0°	
		RMS	SS	RMS	SS	RMS	MAX	RMS	MAX
(1)	CTC	6.1340	8.5617	4.6260	-4.7008	4.9007	12.2731	4.4288	5.0733
(2)	FFIDC	2.5940	3.6395	0.8689	1.1186	2.0083	4.1268	0.6431	1.0846
(3)	CDIDC	0.0616	0.0695	0.0211	0.0214	0.0707	0.1244	0.0265	0.0509
(4)	ACDIDC	0.0247	0.0001	0.0238	-0.0002	0.0391	0.0772	0.0344	0.0642
(5)	MRAC	0.9283	1.3691	0.2827	0.4039	0.6935	1.4323	0.2628	0.5390
(6)	DAC	0.0613	0.0043	0.0259	-0.0007	0.0674	0.1387	0.0284	0.0600
(7)	Pure Fuzzy	1.3926	2.0000	0.2289	0.1670	1.2304	2.6153	0.4585	1.1183
(8)	CTC + Fuzzy	0.1761	0.1670	0.2361	0.1667	0.6324	1.8571	0.2895	-1.0030
(9)	FFIDC + Fuzzy	0.1534	0.1667	0.2716	0.1663	0.1471	0.2104	0.3702	-0.9186
(10)	CDIDC + Fuzzy	0.0216	0.0	0.0212	0.0	0.0343	-0.0610	0.0325	0.0507
(11)	ACDIDC + Fuzzy	0.0130	0.0	0.0119	-0.0001	0.0291	0.0599	0.0226	-0.0564
(12)	MRAC + Fuzzy	0.2472	0.3565	0.1261	0.1673	0.2350	0.3648	0.1423	-0.2403
(13)	DAC + Fuzzy	0.0657	0.0101	0.0351	0.0001	0.0567	0.1103	0.0323	0.0791

FIGURE 8: CDIDC errors for trajectory 1.

FIGURE 9: CDIDC errors for trajectory 2.

in terms of much lesser number of calculations to be performed. This is mainly due to use of lookup table. Moreover, if the lookup table entries and the scaling factors can be further fine tuned, the performance of the controller may improve.

(6) Fine tuning the lookup table manually is very tedious task. The lookup table can actually be built up automatically like in self-organizing fuzzy controllers.

(7) The Hybrid fuzzy/conventional controllers show significant performance improvement over their conventional counterparts. In fact, the performance of even the CDIDC

is improved to a large extent by addition of fuzzy controller. The error profiles for the two links are shown in Figure 12 for the first trajectory and Figure 13 for the second trajectory.

(8) The Hybrid fuzzy/adaptive controllers also show performance improvement over their adaptive counterparts. However, the improvement is not that significant as in case of hybrid fuzzy/conventional controllers. This is mainly because of the fact that adaptive controllers by themselves give very good performance leaving little scope for further improvements.

FIGURE 10: Errors for ACDIDC, trajectory 1.

FIGURE 12: Errors for CDIDC + fuzzy control, trajectory 1.

FIGURE 11: Errors for ACDIDC, trajectory 1.

FIGURE 13: Errors for CDIDC + fuzzy control, trajectory 2.

(9) The best performance is that of hybrid Fuzzy + ACDIDC. The error profiles for the two links are shown in Figure 14 for the first trajectory and Figure 15 for the second trajectory.

6. CONCLUSION

This paper presents an investigation into some hybrid fuzzy control algorithms for manipulator control. It is seen that the performance of hybrid fuzzy plus conventional controllers improves appreciably compared to their respective fuzzy only or conventional only counterparts. The perfor-

mance improvement noticed for hybrid fuzzy plus adaptive controllers on the other hand is not very appreciable. The Fuzzy controller used in these algorithms was a lookup table-based controller. Building up of lookup table is a tedious and cumbersome task. It will be better if the lookup table could be built up automatically like in case of self-organizing fuzzy controllers. It will be worthwhile investigating the above hybrid fuzzy algorithms with the present fuzzy controller replaced by a self-organizing fuzzy controller. Also, experimental implementation of these algorithms needs to be carried out to verify the simulation results.

FIGURE 14: Errors for ACDIDC + fuzzy control (trajectory 1).

FIGURE 15: Errors for ACDIDC + fuzzy control (trajectory 2).

FIGURE 16: Hybrid fuzzy controller.

APPENDICES

A. MANIPULATOR MODEL USED FOR SIMULATIONS

The mathematical model of the two-link manipulator shown in Figure 4 is given as follows:

$$\begin{aligned}
 & \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \\
 &= \begin{bmatrix} (m_2 x_2^2 + 2m_2 x_2 c_2 l_1 + m_1 x_1^2 + m_2 l_1^2) + [I_{zz1} + I_{zz2}] & (m_2 x_2^2 + m_2 x_2 c_2 l_1 + I_{zz2}) \\ (m_2 l_1 x_2 c_2 + m_2 x_2^2 + I_{zz2}) & m_2 x_2^2 + I_{zz2} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} \\
 &+ \begin{bmatrix} -2m_2 x_2 s_2 l_1 \dot{\theta}_2 & -m_2 x_2 s_2 l_1 \dot{\theta}_2 \\ m_2 x_2 l_1 s_2 \dot{\theta}_1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \\
 &+ \begin{bmatrix} m_2 x_2 g c_{12} + (m_1 x_1 + m_2 l_1) g c_1 \\ m_2 x_2 g c_{12} \end{bmatrix} \\
 &= M(\theta) \ddot{\theta} + V_M(\theta, \dot{\theta}) \dot{\theta} + F_M \dot{\theta} + G(\theta), \tag{A.1}
 \end{aligned}$$

where

θ_i , $\dot{\theta}_i$, and $\ddot{\theta}_i$ are actual position, velocity, and accelerations of joint i ,

τ_i is the torque applied to the joint i ,

m_i , l_i , x_i , I_{zzi} , and F_i are as defined in Section 4,

g is the gravity acceleration, taken as 9.8 m/s^2 ,

c_2 is same as $\cos(\theta_2)$,

s_2 is same as $\sin(\theta_2)$, and

c_{12} is same as $\cos(\theta_1 + \theta_2)$.

B. STABILITY PROOF

The argument for stability proof is as follows.

Consider the controller structure as shown in Figure 16.

Assume that at some time $t = t_o$, switch S is open and hence only controller $C1$ is in control loop.

Since $C1$ is stable by definition, then for bounded inputs $\dot{\theta}_d$ and θ_d , the outputs $\dot{\theta}$ and θ are bounded. Also the error and its derivative (e and $\dot{e}$) are bounded.

At this point of time (t_o), the output of second controller τ_{C2} is also bounded because all inputs to $C2$ are bounded and $C2$ is stable by definition.

Now assume that switch S is closed and we can treat τ_{C2} as a bounded disturbance input to the first controller $C1$.

Since the disturbance is bounded, and $C1$ is stable, the system outputs $\dot{\theta}$ and θ will still be bounded and the overall system will be stable.

In the actual system, the switch S is closed at time $t_o = 0$, and at that time all inputs and outputs of the system are bounded as the system is at rest.

Hence, the controller shown in Figure 16 is BIBO stable.

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