

## Research Article

# 2-Norm-Based Iterative Design of Filterbank Transceivers: A Control Perspective

Yang Shi<sup>1</sup> and Tongwen Chen<sup>2</sup>

<sup>1</sup>Department of Mechanical Engineering, University of Saskatchewan, Saskatoon, Saskatchewan, Canada S7N 5A9

<sup>2</sup>Department of Electrical and Computer Engineering, University of Alberta, Edmonton, Alberta, Canada T6G 2V4

Correspondence should be addressed to Yang Shi, yang.shi@usask.ca

Received 17 April 2007; Revised 30 November 2007; Accepted 3 March 2008

Recommended by Brett Ninness

This paper considers design of filterbank-based transceivers. A composite error criterion is proposed to capture all the three traditional distortions. Incorporating noise attenuation and filter bandlimiting properties into this error criterion, an optimal design procedure is developed and applied to a transceiver design example, yielding an FIR transceiver that has good frequency-selective properties and is close to perfect reconstruction. As a least-squares solution is given in closed form in each iteration, the algorithm is easy to implement.

Copyright © 2008 Y. Shi and T. Chen. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

## 1. INTRODUCTION

Multirate systems, that is, digital systems with signals of different sampling rates, have wide applications in control [1], signal processing [2], communications, econometrics and numerical mathematics. There are several reasons for this [3]: (1) in large scale multivariable digital systems, often it is unrealistic, or sometimes impossible, to sample all physical signals uniformly at one single rate; in such situations, one is forced to use multirate sampling; (2) multirate systems can often achieve objectives that cannot be achieved by single-rate systems. The study of multirate systems goes back to late 1950s [4]. A renaissance of research in multirate systems has occurred since 1980 in the control, signal processing, and communications communities.

- (i) In the control community, two directions of research stand out: first, using multirate control to achieve what single rate control cannot as well as the limitations of doing this, and second, the optimal design of multirate controllers [1].
- (ii) The driving force for studying multirate systems in signal processing comes from the need of sampling rate conversion, subband coding, and their ability to generate wavelets [2].

- (iii) In the communications community, multirate sampling is used for blind system identification and equalization.

Intersymbol interference (ISI) is a common problem in telecommunication systems, such as terrestrial television broadcasting, digital data communication systems, and cellular mobile communication systems. Usually the channel distortion results in ISI, which, if left uncompensated, causes high error rates. The solution to the ISI problem is to design a receiver that employs a means for compensating or reducing the ISI in the received signal. The compensator for the ISI is the so-called equalizer. For these problems, the filterbank approach [5–10] has gained practical interest recently.

The multirate filterbank-based transceiver model was proposed in [7] as a unifying framework able to encompass existing modulations and equalization schemes. Further in [9], the multirate filterbank-based transceiver model, as shown in Figure 1, was studied in detail. The filterbank-based transceiver introduces transmitter redundancy using filterbank precoders and generalizes existing modulations including OFDM, DMT, TDMA, and CDMA schemes encountered with single- and multiuser communications [9]. Recently in [8], based on the filterbank approach, the optimal channel equalization is studied: an optimal receiver filterbank, which is a modified Kalman filter, is obtained.

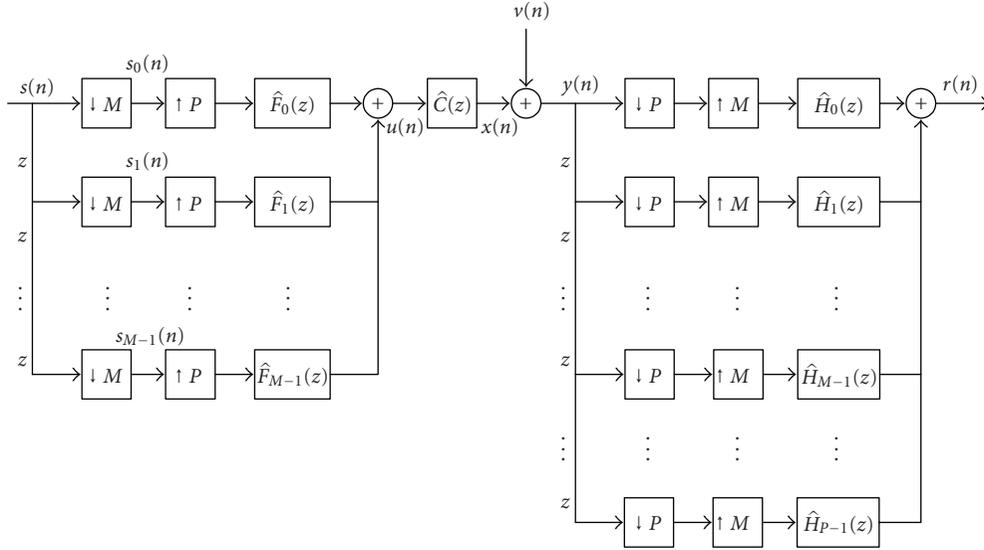


FIGURE 1: Multirate discrete-time baseband equivalent transmitter/channel/receiver model.

Motivated by the generality and importance of filterbank-based transceivers, this paper focuses on frequency-domain analysis and optimal design issues for filterbank-based transceivers. The contributions of this work are as follows.

- (i) Contrary to a time-domain study [9], our work is developed along the line of frequency-domain analysis. Frequency-domain models are obtained using the blocking technique. Based on such models, an composite error criterion is proposed to quantify the degree to perfect reconstruction. This frequency-domain criterion captures the traditional distortions.
- (ii) In the transceiver design, we apply and incorporate the model matching methodology [11]: instead of designing for perfect reconstruction, we design for close to perfect reconstruction with effective band separation. This idea was explored in filterbanks [11] and transmultiplexers [12]. Under some mild condition, one can always get arbitrarily close to perfect reconstruction by trading off filter complexity and time delay in reconstruction.
- (iii) Control of filter stopband energy is incorporated in our design. This is important, since narrowband noise could induce serious impairment due to poor stopbands of filters involved [13]: if the receiving filters have poor stopband attenuation, all the neighboring bands would be affected when there is a strong narrowband noise. The resulting ISI can seriously degrade the system performance [14].

The rest of the paper is organized as follows. In Section 2 the blocked model of the filterbank-based transceiver and the perfect reconstruction are briefly reviewed. In Section 3 the transceiver system is analyzed in several aspects based on the blocked model. Section 4 formulates the optimal design

problem as a least squares one and develops an iterative design procedure for the filterbank-based transceiver. The proposed design method is illustrated in detail with an example in Section 5. Finally, Section 6 offers some concluding remarks.

We conclude this section by introducing some notation. The signals are denoted by small letters, for example,  $r$ .  $\underline{r}$  (underlining denotes blocking) is the blocked signal.  $\hat{r}(z)$  denotes the  $z$ -transform of  $r$ . The systems are represented as time-domain operators, denoted by capital letters, for example,  $T$ . If a system  $T$  is linear time invariant (LTI), its transfer function (matrix) is written as  $\hat{T}(z)$ . The notation  $\|\cdot\|_2$  stands for the 2-norm for transceiver matrices.

## 2. BLOCKED MODELS AND PERFECT RECONSTRUCTION

Figure 1 shows the discrete-time multirate filterbank model for the baseband communication system [9]. It consists of the transmitter filterbank  $\hat{F}_m(z)$  ( $m = 0, \dots, M-1$ ), the receiver filterbank  $\hat{H}_p(z)$  ( $p = 0, \dots, P-1$ ), and the communication channel modeled by the transfer function  $C(z)$ , which is assumed to be causal and stable. The input serial data stream  $s(n)$  and its successively time-advanced versions are downsampled by a factor  $M$  to get  $M$  parallel substreams  $s_0(n), s_1(n), \dots, s_{M-1}(n)$  as shown in Figure 1. These substreams are then upsampled by a factor  $P$  and processed by filters  $\hat{F}_m(z)$ ; the combined output  $u(n)$  is then transmitted over the channel  $\hat{C}(z)$ , which is corrupted at the output by an additive noise  $v(n)$ , assumed to be stationary and white. At the receiver end, the received signal  $y(n)$  and its successively shifted versions are then downsampled by a factor  $P$ , upsampled by a factor  $M$ , and processed by filters  $\hat{H}_p(z)$ ; the combined output forms the reconstructed signal  $r(n)$ .

By blocking the transmitter, channel, and receiver, respectively, we can obtain the blocked model [6, 8] for the multirate system in Figure 1:

$$\hat{r}(z) = \hat{H}(z)\hat{C}(z)\hat{E}(z)\hat{s}(z) + \hat{H}(z)\hat{v}(z), \quad (1)$$

where  $\hat{E}(z)$ ,  $\hat{C}(z)$ , and  $\hat{H}(z)$  are the transfer matrices of the blocked transmitter filterbank, the blocked channel, and the receiver filterbank, respectively.

The blocked general multirate system (in the absence of noise) is LTI with  $\hat{T}(z) = \hat{H}(z)\hat{C}(z)\hat{E}(z)$ . The transceiver achieves perfect reconstruction if in Figure 1  $r$  is a delayed version of  $s$ , that is, if there exists nonnegative integer  $d$  such that  $T = T_d$ , where  $T_d$  is the time-delay system with transfer function  $\hat{T}_d(z) = z^{-d}$ . Blocking  $T_d$  the same way as we blocked  $T$ , the perfect reconstruction condition is equivalent to [8]

$$\hat{H}(z)\hat{C}(z)\hat{E}(z) = z^{-d} \begin{bmatrix} 0 & z^{-1}I_l \\ I_{M-l} & 0 \end{bmatrix}, \quad (2)$$

where integers  $d$  and  $l$  satisfy  $k \geq 0$  and  $0 \leq l \leq M - 1$ , and  $I_l$  and  $I_{M-l}$  are the  $l \times l$  and  $(M - l) \times (M - l)$  identity matrices, respectively. Moreover, if this condition is satisfied,  $\hat{T}(z) = z^{-(kM+l)}$ .

### 3. ANALYSIS

In this section, we study the distortion analysis, the effect of noise, and the frequency selectivity.

#### 3.1. Distortion analysis

Perfect reconstruction synthesis filterbanks at the transmitter and analysis filterbanks at the receiver allow perfect recovery of communication symbols, but the challenges arise with ISI-inducing channels and noise, either of which destroying the perfect reconstruction property. Many practical transceivers do not achieve perfect reconstruction but get close to perfect reconstruction. In order to measure the degree of closeness to perfect reconstruction, we will introduce three traditional quantities to measure sources of distortions: aliasing distortion, magnitude and phase distortions [2]. These distortion measures are based on the blocked model  $\hat{T}(z)$ .

**Lemma 1** (see [15]). *An LPTV (linear periodic time variant) system  $G$  with period  $t$  can be uniquely decomposed into*

$$G = G^{\text{ti}} + G^{\text{tv}} \quad (3)$$

satisfying the two properties

- (i)  $G^{\text{ti}}$  is the optimal LTI approximation of  $G$  in the sense that it minimizes  $\|\hat{G}(z) - \hat{Q}(z)\|_2$  over the class of LTI  $Q$ 's ( $\hat{G}$  denotes the blocked system  $L_t G L_t^{-1}$ , similarly for  $\hat{Q}$ . Here  $L_t$  is the lifting operator, and  $L_t^{-1}$  the inverse lifting operator [1]).
- (ii)  $\|\hat{G}(z)\|_2^2 = t\|\hat{G}^{\text{ti}}(z)\|_2^2 + \|\hat{G}^{\text{tv}}(z)\|_2^2$ .

Back to our transceiver problem, the system from  $s$  to  $r$  is LPTV with period  $M$ ; decompose this into  $G^{\text{ti}} + G^{\text{tv}}$ , where  $G^{\text{ti}}$  is the LTI component and  $G^{\text{tv}}$  the time-varying component. Thus we have

$$\hat{T}(z) = \hat{G}^{\text{ti}}(z) + \hat{G}^{\text{tv}}(z). \quad (4)$$

Aliasing distortion in the system is defined by

$$\text{AD} = \|\hat{G}^{\text{ti}}(z)\|_2 \quad (5)$$

Even if AD is zero, and then the LPTV system reduces to an LTI system  $G^{\text{ti}}$ , it may still have errors in magnitude and phase compared with the ideal time delay  $z^{-d}$ ; define the following quantities:

$$\begin{aligned} \text{MD} &= \left[ \frac{1}{2\pi} \int_0^{2\pi} (|\hat{G}^{\text{ti}}(e^{j\omega})| - 1)^2 d\omega \right]^{1/2}, \\ \text{PD} &= \left\{ \frac{1}{2\pi} \int_0^{2\pi} \sin^2[\angle \hat{G}^{\text{ti}}(e^{j\omega}) + d \times \omega] d\omega \right\}^{1/2}. \end{aligned} \quad (6)$$

Note that MD and PD are defined across all frequencies:  $(\text{MD})^2$  is the energy of the magnitude distortion and PD the energy of sine of the phase distortion  $\phi(\omega) = \angle \hat{G}^{\text{ti}}(e^{j\omega}) + d\omega$ . It is worth noting that there are two reasons why we apply sine to characterize the phase distortion PD: (1) if  $\phi(\omega)$  is within  $\pm(\pi/2)$ , which is usually the case,  $\sin^2(\phi(\omega))$  is a good indicator of the size of  $\phi(\omega)$ ; (2) a connection with the 2-norm-based distortion measure  $J$ , to be introduced, could be conveniently established.

Next we propose a composite distortion measure which captures all the three types of distortions and is relatively easy to use in design. The new distortion measure is the 2-norm of the blocked error transfer matrix:

$$J = \|\hat{T}(z) - \hat{T}_d(z)\|_2. \quad (7)$$

Such a measure is appropriate because in the next theorem we establish connections between  $J$  and the three types of distortions discussed earlier.

**Lemma 2** (see [16]). *Let  $G$  be a stable LTI system. Comparing  $\hat{G}(z)$  with the time delay  $z^{-d}$ , one has the following inequalities:*

$$\begin{aligned} |\hat{G}(e^{j\omega}) - e^{-jd\omega}| &\geq ||\hat{G}(e^{j\omega})| - 1|, \\ |\hat{G}(e^{j\omega}) - e^{-jd\omega}|^2 &\geq \sin^2[\angle \hat{G}(e^{j\omega}) + d\omega]. \end{aligned} \quad (8)$$

**Theorem 1.** *AD and MD relate to  $J$  via*

$$\text{AD}^2 + M(\text{MD})^2 \leq J^2, \quad (9)$$

whereas AD and PD relate to  $J$  via

$$\text{AD}^2 + M(\text{PD})^2 \leq J^2. \quad (10)$$

*Proof 1.* From (4) and Lemma 1 we get

$$\begin{aligned} J^2 &= \|\hat{T}(z) - \hat{T}_d(z)\|_2^2 \\ &= \|\hat{G}^{\text{ti}}(z) + \hat{G}^{\text{tv}}(z) - \hat{T}_d(z)\|_2^2 \\ &= M\|\hat{G}^{\text{ti}}(z) - z^{-d}\|_2^2 + \|\hat{G}^{\text{tv}}(z)\|_2^2. \end{aligned} \quad (11)$$

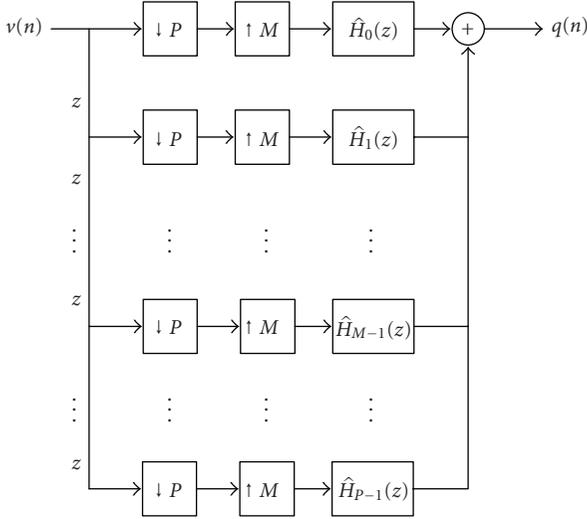


FIGURE 2: A model of the noise effect at the receiver end.

Note (5) to get

$$J^2 = AD^2 + M \|\hat{G}^{\text{ti}}(z) - z^{-d}\|_2^2. \quad (12)$$

Now by definition,

$$\|\hat{G}^{\text{ti}}(z) - z^{-d}\|_2^2 = \frac{1}{2\pi} \int_0^{2\pi} |\hat{G}^{\text{ti}}(e^{j\omega}) - e^{-jd\omega}|^2 d\omega. \quad (13)$$

Invoke Lemma 2 to get

$$\begin{aligned} |\hat{G}^{\text{ti}}(e^{j\omega}) - e^{-jd\omega}| &\geq |\hat{G}^{\text{ti}}(e^{j\omega})| - 1, \\ |\hat{G}^{\text{ti}}(e^{j\omega}) - e^{-jd\omega}|^2 &\geq \sin^2[\angle \hat{G}^{\text{ti}}(e^{j\omega}) + d\omega]. \end{aligned} \quad (14)$$

Combining these two equalities with (13) and noting the definitions of MD and PD in (6), one has

$$\begin{aligned} \|\hat{G}^{\text{ti}}(z) - z^{-d}\|_2 &\geq \text{MD}, \\ \|\hat{G}^{\text{ti}}(z) - z^{-d}\|_2 &\geq \text{PD}. \end{aligned} \quad (15)$$

The proof is complete by noting (12) and the above two inequalities.  $\square$

Then it is clear from Theorem 1 that all distortions (AD, MD, and PD) are bounded above by  $J$ . Therefore, it makes sense to minimize  $J$  in transceiver design because this suboptimizes the three distortions simultaneously.

### 3.2. Noise suppression

In Figure 2,  $q(n)$  represents the noise effect at the receiver end. If we want to minimize the root-mean-square value of  $q$ , it is equivalent to minimize the 2-norm of the blocked transfer matrix from  $\underline{v}(n)$  (standard white noise) to  $\underline{q}(n)$ . Therefore, the objective function for attenuating the output noise effect can be

$$J_N = \|\hat{H}(z)\|_2. \quad (16)$$

### 3.3. Frequency selectivity of filters

Filters with frequency selectivity are of particular importance in communications systems. In addition, frequency responses of designed filters would be deteriorated in the iterative design procedure, if no constraints are imposed on the filters. In order to obtain better bandlimiting property, we minimize the stopband energy of filters involved. To see this, take the objective function(s) for  $H_i$  in the receiver filterbank as an example; we have

$$J_{H_i} = \int_{\Omega_{H_i}} |\hat{H}_i(w)|^2 dw, \quad i = 0, 1, \dots, P-1, \quad (17)$$

where  $\Omega_{H_i}$  defines the stopband frequency interval(s) for  $H_i$ . Similarly,  $J_{F_i}$  can be computed.

In this paper, we will consider only FIR filters. The frequency response of a real  $N$ -tap FIR filter  $H_i$  is given by

$$\hat{H}_i(\omega) = \sum_{n=0}^{N-1} h_i(n) e^{-jn\omega} = h_i^T \phi_i(\omega), \quad (18)$$

where

$$\begin{aligned} h_i^T &= [h_i(0) \ h_i(1) \ h_i(2) \ \dots \ h_i(N-1)], \\ \phi_i^H(\omega) &= [1 \ e^{j\omega} \ e^{j2\omega} \ \dots \ e^{j(N-1)\omega}]. \end{aligned} \quad (19)$$

(The superscript  $H$  indicates the complex conjugate transpose.) The objective function in (17) can then be written as

$$J_{H_i} = h_i^T Q_i h_i, \quad (20)$$

where the fixed  $N \times N$  matrix  $Q_i$  is defined by

$$Q_i = \int_{\Omega_{H_i}} \phi_i(w) \phi_i^H(w) dw. \quad (21)$$

The elements  $q_{mn}$  for  $Q_i$  can be calculated easily if  $\Omega_{H_i}$  is given; for example, assume the filters  $H_i$  have the passband  $[\omega_{pi1}, \omega_{pi2}]$  over  $[0, \pi]$ , then

$$Q_i = \int_0^{\omega_{pi1}} \phi_i(w) \phi_i^H(w) dw + \int_{\omega_{pi2}}^{\pi} \phi_i(w) \phi_i^H(w) dw, \quad (22)$$

and thus the elements  $q_{mn}$  for  $Q_i$  are

$$q_{mn} = \begin{cases} \frac{\sin(\omega_{pi1}(m-n)) - \sin(\omega_{pi2}(m-n))}{(m-n)}, & m \neq n, \\ \pi + \omega_{pi1} - \omega_{pi2}, & m = n. \end{cases} \quad (23)$$

## 4. PROBLEM FORMULATION AND DESIGN

In view of the new distortion measure  $J$  discussed in the preceding section, we wish to design transmitter and receiver subsystems to minimize  $J$ . Thus our optimal filterbank-based transceiver design problem using FIR subsystems can be stated as follows: given the FIR channel and desired reconstruction time delay  $d$ , design FIR transmitter and receiver

subsystems of some given lengths to minimize  $J$  subject to some constraint on  $J_N$  in (16) and the stopband energy constraints on  $J_{H_i}$  in (20).

In order to incorporate both the noise attenuation and filter bandlimiting constraints, such an optimal design problem can be recast by including penalties on  $J_N$ ,  $J_{H_i}$ , and  $J_{F_i}$ . Because both  $\hat{F}(z)$  and  $\hat{H}(z)$  are designable, this optimization problem is in general nonlinear and difficult to solve. Thus we take the following iterative design procedure which turns out to be very effective in the design example to follow.

*Step 1.* Design transmitter subsystems to satisfy desired frequency limiting properties (without considering reconstruction performance); these are used to initiate the iteration.

*Step 2.* Fixing the transmitter subsystems, design FIR receiver subsystems by minimizing the following objective function (using the blocked models)

$$\min_{\hat{H}(z)} \left( \|\hat{H}(z)\hat{C}(z)\hat{F}(z) - \hat{T}_d(z)\|_2^2 + \alpha_N \|\hat{H}(z)\|_2^2 + \sum_{i=1}^P \alpha_{H_i} J_{H_i} \right) := \min_{\hat{H}(z)} (J_1). \quad (24)$$

*Step 3.* Fixing the receiver subsystems just designed, now redesign FIR transmitter subsystems by minimizing the following objective function (using the blocked models)

$$\min_{\hat{F}(z)} \left( \|\hat{H}(z)\hat{C}(z)\hat{F}(z) - \hat{T}_d(z)\|_2^2 + \sum_{j=1}^M \alpha_{F_j} J_{F_j} \right) := \min_{\hat{F}(z)} (J_2). \quad (25)$$

*Step 4.* Repeat Steps 2 and 3 until the corresponding objective function is sufficiently small.

We note that the idea of iteratively designing transmitter and receiver filters was used effectively in transmultiplexers design in [12]. The advantage of this procedure is evident: by fixing either  $\hat{F}(z)$  or  $\hat{H}(z)$  in Steps 2 and 3, the optimization problems become mathematically tractable; in fact, they are finite-dimensional, convex optimization with a quadratic cost function, whose global optimal solution can be always computed. Even analytical solutions can be obtained.

For example, looking at the optimal design problem in Step 2, we define

$$\hat{P}(z) = \hat{H}(z)\hat{C}(z)\hat{F}(z) - \hat{T}_d(z). \quad (26)$$

This system is FIR and hence can be represented by its finitely many coefficient matrices  $P_i$ . By Parseval's equality,

$$J_1^2 = \|\hat{P}(z)\|_2^2 = \left[ \sum_i \text{trace}(P_i P_i') \right]. \quad (27)$$

Since  $\hat{F}(z)$  and  $\hat{T}_d(z)$  are given and thus  $P(z)$  depends on  $\hat{H}(z)$  in an affine manner, it follows that  $P_i$  relates to the coefficients of  $\hat{H}(z)$  (to be designed) too in an affine manner.

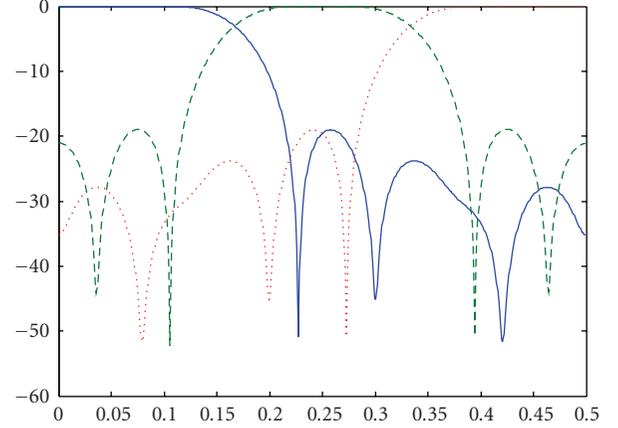


FIGURE 3: The magnitude Bode plots for the initial transmitter filters  $F_0$  (solid),  $F_1$  (dash-dot), and  $F_2$  (dotted): dB versus  $\omega/2\pi$ .

It is obvious that both  $J_N$  and  $J_{H_i}$  are of quadratic forms, therefore we can rewrite the quantity in (24) in the following way:

$$J_1^2 = (M_F x - b)'(M_F x - b) + x' M_N x + x' M_H x. \quad (28)$$

Here  $x$  is a column vector containing all the parameters in  $\hat{H}(z)$ , to be designed,  $b$  is a column vector depending on only  $\hat{T}_d(z)$ ,  $M_F$  is a matrix depending on  $\hat{F}(z)$  and the way  $x$  is formed,  $M_N$  depends on  $\alpha_N$  and  $\hat{H}(z)$ , and finally  $M_H$  depends on  $\alpha_{H_i}$  and  $\hat{H}_i(z)$ . The matrices  $M$ ,  $M_N$ ,  $M_H$ , and  $b$  can be computed and are independent of the design parameters ( $x$ ). Now the optimal design problem in Step 2 becomes a least squares problem:

$$\min_x [(M_F x - b)'(M_F x - b) + x'(M_N + M_H)x]. \quad (29)$$

If  $M_F' M_F + M_N + M_H$  is invertible, the optimal solution can be obtained to be

$$x_{\text{opt}} = (M_F' M_F + M_N + M_H)^{-1} M_F' b. \quad (30)$$

From here we can recover the optimal receiver subsystems. The optimal design problem in Step 3 can be solved similarly.

## 5. DESIGN EXAMPLE

The filterbank-based transceiver with  $M = 3$  and  $P = 8$  is designed. The channel to be used in this example is

$$\begin{aligned} \hat{C}(z) = & 1 - 0.3z^{-1} + 0.5z^{-2} - 0.4z^{-3} + 0.1z^{-4} \\ & - 0.02z^{-5} + 0.3z^{-6} - 0.1z^{-7}. \end{aligned} \quad (31)$$

The transmitter and receiver filters involved in design are all FIR and causal with a fixed order of 12. The magnitude Bode plots of the initial transmitter filters are given in Figure 3, and the reconstruction time delay is taken as  $d = 8$ . The constants  $\alpha_N$ ,  $\alpha_{H_i}$ , and  $\alpha_{F_j}$ , reflecting relative weightings among multiple objectives, are tuned in the design process.

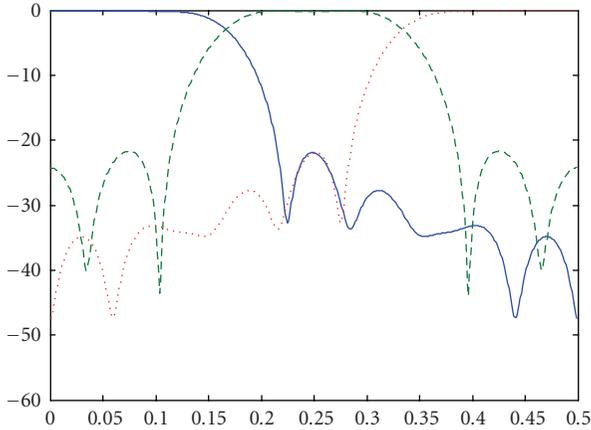


FIGURE 4: The magnitude Bode plots for the designed transmitter filters  $F_0$  (solid),  $F_1$  (dash-dot), and  $F_2$  (dotted): dB versus  $\omega/2\pi$ .

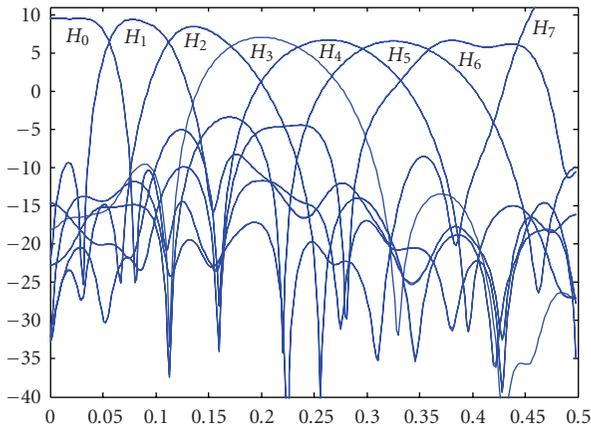


FIGURE 5: The magnitude Bode plots for the designed receiver filters  $H_0$ – $H_7$ : dB versus  $\omega/2\pi$ .

In our design, these are taken to be  $\alpha_N = 0.02$ ,  $\alpha_{H_i} = 0.03$  ( $i = 0, 1, \dots, 7$ ),  $\alpha_{F_j} = 0.01$  ( $j = 0, 1, 2$ ).

Next we apply the iterative design procedure to the example. In the first iteration,  $J_1 = 5.806$ , and the resulting  $\alpha_N \|\hat{H}(z)\| = 0.02 \|\hat{H}(z)\|_2^2 = 1.553$ . After some iterations, the value of the objective function gradually decreases as the number of iterations increases, and finally converges to the value  $J_1 = 0.156$ ,  $0.02 \|\hat{H}(z)\|_2^2 = 0.0872$ , and finally  $J = 0.048$ . The above comparison on  $J_1$  clearly illustrates the improvement obtained by using the proposed iterative algorithm. The designed transmitter and receiver filters are shown in Figures 4 and 5, respectively.

## 6. CONCLUSION

In this paper, we investigated the problem of optimal design of filterbank-based transceivers. We proposed quantities to measure various distortions. We also introduced a composite distortion ( $J$ ) that captures all distortions. Finally, by incorporating two important practical issues, the noise

suppression and filter bandlimiting property, we developed an iterative design procedure based on minimizing the objective function and successfully applied this procedure to design of a filterbank-based transceiver. At each iteration, the least squares solution should be found, thus the final transmitter and receiver filters can be obtained with relative ease.

## ACKNOWLEDGMENTS

The authors wish to thank the associate editor and anonymous reviewers for providing many constructive suggestions which have improved the presentation of the paper. This research was supported by the Natural Sciences and Engineering Research Council of Canada and the Canada Foundation of Innovation.

## REFERENCES

- [1] T. Chen and B. A. Francis, *Optimal Sampled-Data Control Systems*, Springer, London, UK, 1995.
- [2] P. P. Vaidyanathan, *Multirate Systems and Filter Banks*, Prentice-Hall, Englewood Cliffs, NJ, USA, 1993.
- [3] L. Chai and L. Qiu, "Model validation of multirate systems from time-domain experimental data," *IEEE Transactions on Automatic Control*, vol. 47, no. 2, pp. 346–351, 2002.
- [4] G. Kranc, "Input-output analysis of multirate feedback systems," *IEEE Transactions on Automatic Control*, vol. 3, no. 1, pp. 21–28, 1957.
- [5] G. Cherubini, E. Eleftheriou, S. Oker, and J. M. Cioffi, "Filter bank modulation techniques for very high-speed digital subscriber lines," *IEEE Communications Magazine*, vol. 38, no. 5, pp. 98–104, 2000.
- [6] Y.-P. Lin and S.-M. Phoong, "ISI-free FIR filterbank transceivers for frequency-selective channels," *IEEE Transactions on Signal Processing*, vol. 49, no. 11, pp. 2648–2658, 2001.
- [7] G. B. Giannakis, "Filterbanks for blind channel identification and equalization," *IEEE Signal Processing Letters*, vol. 4, no. 6, pp. 184–187, 1997.
- [8] G. Gu and E. F. Badran, "Optimal design for channel equalization via the filterbank approach," *IEEE Transactions on Signal Processing*, vol. 52, no. 2, pp. 536–545, 2004.
- [9] A. Scaglione, G. B. Giannakis, and S. Barbarossa, "Redundant filterbank precoders and equalizers—I: unification and optimal designs," *IEEE Transactions on Signal Processing*, vol. 47, no. 7, pp. 1983–2006, 1999.
- [10] H.-T. Chiang, S.-M. Phoong, and Y.-P. Lin, "Design of nonuniform filter bank transceivers for frequency selective channels," *EURASIP Journal on Advances in Signal Processing*, vol. 2007, Article ID 61396, 12 pages, 2007.
- [11] T. Chen, "Nonuniform multirate filter banks: analysis and design with an  $H_\infty$  performance measure," *IEEE Transactions on Signal Processing*, vol. 45, no. 3, pp. 572–582, 1997.
- [12] Y. Shi and T. Chen, "Optimal design of multi-channel transmultiplexers with stopband energy and passband magnitude constraints," *IEEE Transactions Circuits Systems II*, vol. 50, no. 9, pp. 659–662, 2003.
- [13] G. W. Wornell, "Emerging applications of multirate signal processing and wavelets in digital communications," *Proceedings of the IEEE*, vol. 84, no. 4, pp. 586–603, 1996.
- [14] J. G. Proakis, *Digital Communications*, McGraw-Hill, New York, NY, USA, 1995.

- 
- [15] T. Chen and L. Qiu, "Linear periodically time-varying discrete-time systems: aliasing and LTI approximations," *Systems & Control Letters*, vol. 30, no. 5, pp. 225–235, 1997.
- [16] T. Liu and T. Chen, "Design of multi-channel nonuniform transmultiplexers using general building blocks," *IEEE Transactions on Signal Processing*, vol. 49, no. 1, pp. 91–99, 2001.



**Hindawi**

Submit your manuscripts at  
<http://www.hindawi.com>

