Research Article

Intelligent Hybrid Control Strategy for Trajectory Tracking of Robot Manipulators

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We address the problem of robust tracking control using a PD-plus-feedforward controller and an intelligent adaptive robust compensator for a rigid robotic manipulator with uncertain dynamics and external disturbances. A key feature of this scheme is that soft computer methods are used to learn the upper bound of system uncertainties and adjust the width of the boundary layer base. In this way, the prior knowledge of the upper bound of the system uncertainties does need not to be required. Moreover, chattering can be effectively eliminated, and asymptotic error convergence can be guaranteed. Numerical simulations and experiments of two-DOF rigid robots are presented to show effectiveness of the proposed scheme.

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1. INTRODUCTION

Robot control has been an active research field during the past decade. A main concern of the control task is to find an effective controller to achieve accurate tracking of desired motions. However, robotic manipulators have to face many uncertainties in their dynamics, such as payload parameter, friction, and disturbance. Yet, the work using PD-plus-feedforward on robust control of robot [1–6] shows great attraction to researches. In [1], a PD-plus-feedforward controller is firstly proposed for robot manipulators which guarantees exponential convergence. The adaptive version of this controller is addressed later [2] considering the robustness of velocity measurement. However, the main drawback of them is that they require high controller gains in order to overcome the uncertainty in the initial parameter errors. Combining the ideas of [1, 3], Berghuis [4] put forward a modified plan ensuring global exponential convergence. Wen [5] treats the analysis of a robot controller based on energy-like Lyapunov function, which leads to a new class of global stability control design, but is still a high-gain method. Kelly [6] designs a class of PD-plus-feedforward controller on the assumption that parameters are precisely known, nevertheless obtaining local but not global stability

results. Unfortunately, all above works do not consider the external disturbances and unmodeled dynamics.

Recently, considerable research efforts have been directed towards applying intelligent control methods to robot control [7–14]. In [9–11], intelligent controllers are proposed for the compensation of effects yielded by nonlinearities and uncertainties. Some theses [8, 12, 13] use soft computer methods to approximate the whole robot dynamics rather than its nonlinear components as done by static intelligent means. Man [7] and Qi [15] make use of intelligent systems to replace the constant switching control gain and the width of the boundary layer. However, [7] has the shortcoming that the inverse of inertia matrix needs to be calculated.

In this note, motivated by the PD-plus-feedforward control idea and Man's work [7], a new intelligent adaptive robust control scheme is proposed for robot manipulators with model uncertainties and external disturbance. The novel control scheme proposed in this paper contains a PD-plus-feedforward part and an intelligent robust compensational part. As compared with afore-mentioned results, not only does we suppose that the mathematical model of robot system is totally unknown, but we do not require calculating the inverse of inertia matrix. Moreover, the parameter of the proposed dynamic compensator is adaptively updated by

using the output of intelligent system that is used to learn the unknown upper bound of system uncertainties, and a fuzzy logic is used to adjust the width of the boundary based on the state. In this way, the vibrancy of the controller will be effectively reduced. Application to a two-link robotic manipulator is presented. Numerical simulations and experiments are proposed to show the excellent tracking performance of our controller.

2. PRELIMINARIES

In this section, we firstly give some descriptions of intelligent systems, and after that, present the robot manipulator model and some dynamic properties which are crucial to our work.

2.1. Description of fuzzy-logic systems

The basic configuration of the fuzzy system is constructed from the fuzzy if-then rules using some specific inference, fuzzification, and defuzzification strategies [16]. Let $x = (x_1, \dots, x_N)^T \in U \subset R^N$ and $y \in V \subset R$ be the input and output of the fuzzy system. The fuzzifier maps a crisp point in U into a fuzzy set in U . The fuzzy rule base consists of a collection of fuzzy if-then rules.

$R^{(l)}$: If x_1 is F_1^l and $\dots$ and x_N is F_N^l , then y is G^l , in which F_i^l and G^l are fuzzy sets, and $l = 1, \dots, M$, where M denotes the number of fuzzy if-then rules. The fuzzy inference engine performs a mapping from fuzzy sets in U to fuzzy sets in V . The defuzzifier maps a fuzzy set in V to a crisp point in V . A more detailed description of these systems can be found in [16].

The fuzzy systems with center-average defuzzifier, product inference, and singleton fuzzifier can be expressed as in the following form:

$$y(x) = \frac{\sum_{l=1}^M \theta_l (\prod_{i=1}^N \mu_{F_i^l}(x_i))}{\sum_{l=1}^M (\prod_{i=1}^N \mu_{F_i^l}(x_i))}, \quad (1)$$

where $\mu_{F_i^l}(\cdot)$ is the numbership function of the fuzzy set F_i^l and θ_l is the point at which μ_{G^l} achieves its maximum value (it is assumed here that $\mu_{G^l}(\theta_l) = 1$). By the universal approximation theorem [16], for any given real continuous function $f(x)$ on a compact subset $U \subset R^N$ and arbitrary $\epsilon > 0$, there exists a fuzzy system $y(x)$ in the form of (1) such that $\max_{x \in U} \|f(x) - y(x)\| < \epsilon$.

2.2. Description of neural-network systems

The basic configuration of the neural network system is implemented by using massive connections among processing units. Let $x \in U \subset R^N$ be the input of the neural network system. A two-layer network has a network output given by [17, 18]

$$y(x) := \sum_{l=1}^M \theta_l H_l \left(\sum_{j=1}^M \omega_{jl} x_j + m_l \right), \quad (2)$$

where M is the number of hidden neurons, $H_l(\cdot)$ denotes the activation function which must be a nonconstant, bounded,

and monotonically increasing continuous function (e.g., hyperbolic tangent function), ω_{ji} is the first-to-second layer interconnection weights, m_l is the biases, and θ_l is the second-to-output interconnection weights.

The neural network system is also a popular class of function approximators that can be used in a similar setting as the fuzzy logic system described above. That is, for any given real continuous function $f(x)$ and arbitrary $\epsilon > 0$, there exists a neural network system $y(x)$ in the form of (2) such that $\max_{x \in U} \|f(x) - y(x)\| < \epsilon$.

2.3. Description of robot manipulator model

The basics of robot dynamics and control are sufficiently well known by now that we will be brief in our derivation of the control algorithm. Thus, given the Euler-Lagrange dynamic equations for an n -link robot [19]

$$\begin{aligned} M(q)\ddot{q} + N(q, \dot{q}) &= \tau, \\ N(q, \dot{q}) &= C(q, \dot{q})\dot{q} + G(q) + F_d(\dot{q}) + F_s(\dot{q}) + \tau_d(t, q, \dot{q}), \end{aligned} \quad (3)$$

where $q, \dot{q}, \ddot{q} \in R^n$ are the joint position, velocity, and acceleration vectors, respectively; $M(q)$ denotes the inertia matrix; $C(q, \dot{q})$ expresses the coriolis, centripetal matrix; $G(q)$ is the gravity vector; $F_d, F_s(\dot{q}) \in R^{n \times n}$ represent the dynamic friction coefficient matrix and static friction vector, respectively; $\tau_d(t, q, \dot{q})$ is the vector of disturbances and unmodeled dynamics; τ is the control vector representing the torque exerting on joints. For our purposes, we assume (3) to have the following properties.

Property 1. $M(q)$ is a positive symmetric matrix defined by $M_m \leq \|M(q)\| \leq M_M$ with $M_m, M_M > 0$ being known constants.

Property 2. $C(q, \dot{q})$ defined by using the Christoffel symbols satisfies that

- (a) $\dot{M}(q) - 2C(q, \dot{q})$ is Skew symmetry;
- (b) $\|C(q, \dot{q})\| \leq C_M \|\dot{q}\|$ and $C(q, x)y = C(q, y)x$, $x, y \in R^n$, $C_M > 0$.

Property 3. Linear parameterization of the local dynamics of each subsystem:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = Y(q, \dot{q}, \ddot{q})\theta, \quad (4)$$

where θ is a constant p -dimensional vector of inertia parameters and Y is an $n \times p$ matrix called regressor, which contains known functions and is uniformly continuous in its arguments.

Moreover, without losing generality, the following assumptions on model (3) are made.

Assumption 1. The bounding function of the external disturbances and unmodeled dynamics $\tau_d(t, q, \dot{q})$ is assumed to be

$$\|\tau_d(t, q, \dot{q})\| \leq c_0 + c_1 \|q\| + c_2 \|q\|^2 + c_3 \|\dot{q}\| + c_4 \|\dot{q}\|^2, \quad (5)$$

where c_i ($i = 0, \dots, 4$) are positive constants.

FIGURE 1: Schematic diagram of proposed PD plus feedforward robust control.

Assumption 2. The desired trajectory q_d , $\dot{q}_d$, and $\ddot{q}_d$ are uniformly bounded.

3. INTELLIGENT ADAPTIVE ROBUST CONTROLLER DESIGNS

We now come to the question of trajectory tracking of robot dynamic system (2). Setting $\tilde{q} = \hat{q} - q$, $\dot{\tilde{q}} = \dot{\hat{q}} - \dot{q}$, $s = \dot{\tilde{q}} + \Lambda\tilde{q}$, $\dot{q}_r = \dot{q}_d - \Lambda\tilde{q}$, $\dot{q}_0 = \dot{q} - \Lambda\tilde{q}$, with Λ a positive definite matrix. Let us consider the following robust controller:

$$\begin{aligned} \tau &= Y(f_1, \dots, f_5)\hat{\theta} - K_v\dot{\tilde{q}} - K_p\tilde{q} + u(t), \\ Y(f_1, \dots, f_5)\hat{\theta} &= \hat{M}(f_1)\ddot{q} + \hat{C}(f_2, f_3)\dot{q} + \hat{G}(f_5), \\ \hat{\theta} &= \text{Proj}[-\Gamma^{-1}Y^T(f_1, \dots, f_5)s], \end{aligned} \quad (6)$$

where $(\hat{\cdot})$ denotes the estimate of $(\cdot)$; $f_1, f_2, f_5 \in \{q, q_d\}$, $f_3, f_4 \in \{\dot{q}, \dot{q}_d, \dot{q}_r, q_0\}$; Γ is a positive definite matrix; $\text{Proj}[\cdot]$ expresses a discontinuous projection mapping [20], and particularly, in our paper, it is defined as (7); $u(t)$ represents the nonlinear control component used to stabilize the closed-loop system and enhance the system robustness against the external disturbances and unmodeled dynamics. The schematic diagram of overall control system is shown in Figure 1 (set $f_4 \stackrel{\text{def}}{=} \dot{q}_d$ for ease of illustration). In the following part, we consider how to design the robust compensator $u(t)$ so that the controller (6) can ensure the asymptotic convergence of tracking error:

$$\text{Proj}[\omega_i] = \begin{cases} \omega_i, & \begin{cases} \underline{\theta}_i < \hat{\theta}_i < \bar{\theta}_i, \\ \hat{\theta}_i \geq \bar{\theta}_i, \omega_i \leq 0, \\ \hat{\theta}_i \leq \underline{\theta}_i, \omega_i \geq 0, \end{cases} \\ 0, & \begin{cases} \hat{\theta}_i \geq \bar{\theta}_i, \omega_i > 0, \\ \hat{\theta}_i \leq \underline{\theta}_i, \omega_i < 0, \end{cases} \end{cases} \quad (7)$$

where $\omega_i = -\Gamma_i^{-1} \sum_{j=1}^r Y_{ji}s_j$, $\underline{\theta}_i < \hat{\theta}_i(0) < \bar{\theta}_i$, $i = 1, 2, \dots, p$.

Substituting (6) into (3) yields the error dynamic system

$$\dot{E} = AE + B(u(t) + Y(f_1, \dots, f_5)\tilde{\theta} + N(t, f_1, \dots, f_5)), \quad (8)$$

where $E = [\tilde{q}, \dot{\tilde{q}}]^T$, $B = [0, M^{-1}(q)]^T$, $\tilde{\theta} = \hat{\theta} - \theta$, and

$$\begin{aligned} A &= \begin{pmatrix} 0 & I \\ -M^{-1}(q)(K_p + \lambda C(q, \dot{q})) & -M^{-1}(q)(K_v + C(q, \dot{q}) + \lambda M(q)) \end{pmatrix}, \\ N(t, f_1, \dots, f_5) &= Y(f_1, \dots, f_5)\theta - [M(q)(\ddot{q}_d - \lambda\ddot{\tilde{q}}) + C(q, \dot{q})(\dot{q}_d - \lambda\dot{\tilde{q}}) \\ &\quad + G(q) + F_s(\dot{q}) + \tau_d(t, q, \dot{q})]. \end{aligned} \quad (9)$$

Due to the model uncertainty and external disturbance, $N(t, f_1, \dots, f_5)$ in (8) is an uncertain unknown nonlinear function. In order to obtain the upper bound of $N(t, f_1, \dots, f_5)$, inspired by the idea of [21], we give the following lemma.

Lemma 1. Using Properties 1–3 and Assumptions 1–2, one obtains the upper bound of $N(t, f_1, \dots, f_5)$ as follows:

$$N(t, f_1, \dots, f_5) \leq \bar{N}(t) = d_1 + d_2\|E\| + d_3\|E\|^2 + d_4\|E\|^3 \quad (10)$$

for some $d_i > 0$, $i = 1, 2, 3, 4$.

Proof. Using Properties 1, 2 and Assumption 1, we have

$$\begin{aligned} N(t, f_1, \dots, f_5) &\leq N(t) \\ &= \xi_1 + \xi_2\|f_1 - q\| + \xi_3\|\dot{q} - f_3\| \cdot \|f_4\| \\ &\quad + \xi_4\|q - f_2\| \cdot \|\dot{q}\| \cdot \|f_4\| + \xi_5\|f_4\| \cdot \|\dot{q}\| \\ &\quad + \xi_6\|\dot{q}\| \cdot \|f_4 - \dot{q}_d\| + \xi_7\|f_5 - q\| + \xi_8\|\dot{\tilde{q}}\| \\ &\quad + \xi_9\|\dot{q}\| \cdot \|\tilde{q}\| + \|\tau_d(t, q, \dot{q})\|, \end{aligned} \quad (11)$$

where ξ_i ($i = 1, \dots, 9$) are known constants.

Due to the definition of f_i ($i = 1, \dots, 5$) and Assumption 2, the following inequalities can be obtained immediately:

$$\begin{aligned} \|f_i - q\| &\leq \|\tilde{q}\| \leq \|E\|, \quad i = 1, 2, 5, \\ \|f_i - q\| &\leq \|\tilde{q}\| + \|\Lambda\tilde{q}\| \leq (1 + \bar{\Lambda})\|E\|, \\ \bar{\Lambda} &= \lambda_{\max}(\Lambda), \quad i = 3, 4, \\ \|f_4\| &\leq b_1 + b_2\|E\|, \\ \|\dot{q}\| &\leq \|\dot{\tilde{q}}\| + \|\dot{q}_d\| \leq b_3 + \|E\|, \\ \|q\| &\leq \|\tilde{q}\| + \|q_d\| \leq b_4 + \|E\|, \end{aligned} \quad (12)$$

where b_i ($i = 1, 2, 3, 4$) are known constants, $\lambda_{\max}$ denotes the maximum eigenvalue. Substituting (12) into (11) yields (10). $\square$

Remark 2. There are a lot of literature [22, 23], even when a small perturbation exists in system, pointing out that parameter drift occurs and devastates the total control scheme. Therefore, a nonlinear projection mapping $\text{Proj}[\bullet]$ is led into our controller for purpose of guaranteeing globally asymptotical converges and distinct transient performance.

Remark 3. According to the differential choice of f_i , $i = 1, \dots, 5$, as many as $2^3 \times 4^2 = 128$ kinds of global robust stability controllers could be obtained. It can be seen that our control scheme encompasses a wide range of circumstances.

Remark 4. From error (2), it is known that the key to synthesis controller (3) is how to design a suitable adaptive control compensator $u(t)$ so that the effect of uncertain function $N(t, f_1, \dots, f_5)$ can be effectively eliminated.

3.1. Routine robust compensator

When knowing the upper bound function $\bar{N}(t)$ of concentrated uncertainty, the following continued dynamic compensator can be designed as follows:

$$u(t) = -\frac{\bar{N}(t)s}{\|s\|\bar{N}(t) + \alpha\|E\|^2}, \quad (13)$$

where constant $\alpha > 0$ satisfies $\beta := \lambda_{\min}(Q) - \alpha > 0$, $\lambda_{\min}(\cdot)$ denotes the minimum eigenvalue of matrix, respectively, Q is a symmetrical positive matrix chosen as

$$Q = \begin{pmatrix} \lambda K_p & 0 \\ 0 & K_v \end{pmatrix}. \quad (14)$$

Apparently, $\lambda_{\min}(Q) = \min(\lambda_{\min}(\lambda K_p), \lambda_{\min}(K_v))$.

The following theorem gives the dynamic performance of closed loop system based on controller (6) with compensator (13).

Theorem 1. Consider the error dynamic model (8) of the robot system (3) with Assumptions 1-2. If the dynamic compensator of controller (6) is given by expression (13), then, the close-loop system is exponentially stable.

Proof. Choose the Lyapunov function as

$$V = \frac{1}{2}E^T P E + \frac{1}{2}\tilde{q}^T P \tilde{q} + \frac{1}{2}\tilde{\theta}^T \Gamma \tilde{\theta}, \quad (15)$$

with P the unique symmetrical positive matrix solution to Lyapunov function

$$A^T P + P A = -Q, \quad (16)$$

where Q is defined as (14).

Differentiating V with respect to time along solution trajectory of (8), and using Property 3, we get

$$\begin{aligned} \dot{V} &= E^T P \dot{E} + \frac{1}{2}E^T \dot{P} E + \tilde{q}^T P \dot{\tilde{q}} + \tilde{\theta}^T \Gamma \dot{\tilde{\theta}} \\ &= E^T P (A E + B(u(t) + Y(f_1, \dots, f_5)\tilde{\theta} + N(t, f_1, \dots, f_5))) \\ &\quad + \frac{1}{2}E^T \dot{P} E + \tilde{q}^T P \dot{\tilde{q}} + \tilde{\theta}^T \Gamma \dot{\tilde{\theta}} \\ &= -E^T Q E + E^T \begin{pmatrix} 0 & 0 \\ -K_p & 0 \end{pmatrix} E + \tilde{q}^T P \dot{\tilde{q}} + \tilde{\theta}^T \Gamma \dot{\tilde{\theta}} \\ &\quad + E^T \begin{pmatrix} \lambda I \\ I \end{pmatrix} (u(t) + Y(f_1, \dots, f_5)\tilde{\theta} + N(t, f_1, \dots, f_5)) \\ &= -E^T Q E + s^T (u(t) + N(t, f_1, \dots, f_5)) \\ &\leq -\lambda_{\min}(Q)\|E\|^2 + s^T u(t) + \|s\|\bar{N}(t) \\ &= -\lambda_{\min}(Q)\|E\|^2 - \frac{\bar{N}(t)^2 \|s\|^2}{\|s\|\bar{N}(t) + \alpha\|E\|^2} + \|s\|\bar{N}(t) \\ &\leq -(\lambda_{\min}(Q) - \alpha)\|E\|^2, \end{aligned} \quad (17)$$

that is,

$$\dot{V} \leq -\beta\|E\|^2. \quad (18)$$

From expression (15), the following inequality can be obtained:

$$\begin{aligned} (\lambda_{\min}(P) + \lambda_{\min}(K_p))\|E\|^2 \\ \leq 2V \leq (\lambda_{\max}(P) + \lambda_{\max}(K_p))\|E\|^2. \end{aligned} \quad (19)$$

Setting

$$\lambda_1 = \lambda_{\min}(P) + \lambda_{\min}(K_p), \quad \lambda_2 = \lambda_{\max}(P) + \lambda_{\max}(K_p), \quad (20)$$

and then from expressions (18) and (19), we get

$$\|E\| \leq \left(\frac{\lambda_2}{\lambda_1}\right)^{1/2} \|E(t_0)\| \exp\left(-\frac{\beta}{\lambda_2} t\right). \quad (21)$$

Hence, the close loop system is globally exponentially stable. $\square$

Remark 5. In dynamic compensator (13), the prior knowledge on the upper bound $\bar{N}(t)$ of system uncertainty is required. However, in general, precise bound on system uncertainty is difficult to compute; and moreover, the conventional estimating method may lead to a conservative design. Therefore, we introduce a new dynamic compensator based on intelligent systems due to the excellent nonlinear mapping.

3.2. Intelligent adaptive robust compensator

Consider the upper bound $\bar{N}(t)$ in (10) that cannot be directly available in the control design. An adaptive universal

FIGURE 2: Response using proposed conventional robust control law ($\alpha = 10$).

approximation system $\hat{N}(x_E, \hat{\Theta}) \in R^m$ with input vector $x_E \in \Omega$ for some compact set $\Omega \in R^\kappa$, $\kappa > 0$, is proposed here to approximate the behavior of $\bar{N}(t)$ where x_E is a function of E and t , and Θ is a vector containing the tunable approximation parameters. Let $\hat{N}_i(x_E, \hat{\Theta}_i)$ denote the i th component of $\hat{N}(x_E, \hat{\Theta})$. As in many previous studies [16–18], the linearly parameterized fuzzy model and neural network model are adopted in the approximation procedure, and so $\hat{N}_i(x_E, \hat{\Theta}_i)$ can be expressed as

$$\hat{N}_i(x_E, \hat{\Theta}_i) = h_i^T(x_E) \hat{\Theta}_i, \quad i = 1, \dots, m, \quad (22)$$

where $\hat{\Theta}_i = [\hat{\Theta}_{i1}, \dots, \hat{\Theta}_{im_i}]^T$ for some $m_i > 0$ is a parameter vector and $h_i(x_E) = [h_{i1}(x_E), \dots, h_{im_i}(x_E)]^T$ is a regressive vector with the regressor defined for the adaptive fuzzy

approximator as follows:

$$h_{il}(x_E) = \frac{\prod_{j=1}^{\kappa} \mu_{\bar{N}_j^l}(x_{Ej})}{\sum_{l=1}^M \left(\prod_{j=1}^{\kappa} \mu_{\bar{N}_j^l}(x_{Ej}) \right)}, \quad l = 1, \dots, m_i, \quad (23)$$

and for the adaptive neural network approximator

$$h_{il}(x_E) = \frac{e^{\sigma_{il}(x_E)} - e^{-\sigma_{il}(x_E)}}{e^{\sigma_{il}(x_E)} + e^{-\sigma_{il}(x_E)}}, \quad l = 1, \dots, m_i. \quad (24)$$

The membership functions $\mu_{H_j^l}(x_{Ej})$ in the fuzzy system and the activation functions $\sigma_{il}(x_E)$ in the neural network system are specified beforehand.

Consequently, the totally adaptive approximation system $\hat{N}(x_E, \hat{\Theta})$ can be expressed as

$$\hat{N}(x_E, \hat{\Theta}) = W(x_E) \hat{\Theta}, \quad (25)$$

FIGURE 3: Response using proposed conventional robust control law ($\alpha = 100$).

where $W(x_E) = \text{diag}[h_1^T(x_E), \dots, h_m^T(x_E)]$ denotes a basis matrix and $\hat{\Theta} = [\hat{\Theta}_1^T, \dots, \hat{\Theta}_m^T]$. According to the universal approximation theorem [16–18], there is an optimal approximation parameter Θ such that $\hat{N}(x_E, \hat{\Theta})$ can approximate $\bar{N}(t)$ as best as possible. Therefore, the upper bound $\bar{N}(t)$ can be expressed as

$$\bar{N}(t) = W(x_E)\Theta + \Delta\bar{N}(t), \quad (26)$$

where $\Delta\bar{N}(t)$ denotes the optimal approximation error.

Remark 6. Since both adaptive fuzzy system and adaptive neural network system possess the property of universal approximation, the structure of upper bound $\bar{N}(t)$ can be allowed to be entirely unknown. On the other hand, suppose that the structure of $\bar{N}(t)$ is known, then according to the design philosophy of classical adaptive control, $\bar{N}(t)$ can be expressed as $\bar{N}(t) = W(E, t)\Theta + \Delta\bar{N}(t)$, that is, as in

the form of (10), in which $\bar{N}(t)$ is a regressor matrix of known functions, Θ is a constant vector with components depending on the plant parameters, and $\Delta\bar{N}(t)$ denotes the residual part that cannot be linearly parameterized.

For further analysis, the following assumptions are needed.

Assumption 3. Given an arbitrary small positive constant ω , there exists an optimal weight vector Θ^* so that the approximation error $\Delta\bar{N}(t)$ of intelligent systems satisfies

$$|\Delta\bar{N}(t)| = |\Theta^{*T}W(x_E) - \bar{N}(t)| < \omega. \quad (27)$$

Assumption 4. For ω in expression (27), the upper bound $\bar{N}(t)$ of system uncertainty $N(t, f_1, \dots, f_5)$ satisfies

$$\bar{N}(t) - \|N(t, f_1, \dots, f_5)\| > \omega. \quad (28)$$

FIGURE 4: Response using proposed intelligent robust control law ($\alpha = 10, \eta = 0.5$).

Now, we use intelligent systems (25) to design the adaptive robust compensator $u(t)$, and get the following theorem.

Theorem 2. Consider the error equation (8) for the robot manipulator system (3) with Assumptions 1–4. If the compensator $u(t)$ in controller (6) is designed as

$$u(t) = -\frac{(W(x_E)\hat{\Theta})^2 s}{\|s\| (W(x_E)\hat{\Theta}) + \alpha \|E\|^2}, \quad (29)$$

and the weight vector $\hat{\Theta}$ is updated by the following adaptive mechanism:

$$\dot{\hat{\Theta}} = \eta W(x_E) \|s\|, \quad (30)$$

with $\eta > 0$ the adaptive rate, then controllers (6), (29) can ensure that the tracking error of robot system asymptotically converges to zero.

Proof. Define the following Lyapunov function

$$V = \frac{1}{2} E^T P E + \frac{1}{2} \tilde{q}^T P \tilde{q} + \frac{1}{2} \tilde{\theta}^T \Gamma \tilde{\theta} + \frac{1}{2} \tilde{\Theta}^T \eta^{-1} \tilde{\Theta}, \quad (31)$$

where $\tilde{\Theta} = \Theta^* - \hat{\Theta}$, $\dot{\tilde{\Theta}} = -\dot{\hat{\Theta}}$, matrix P is defined by (15).

Differentiating V with respect to time along trajectory of error equation (8), similar to the proof of Theorem 1, we get

$$\begin{aligned} \dot{V} &= -E^T Q E + s^T (u(t) + N(t, f_1, \dots, f_5)) + \tilde{\Theta}^T \eta^{-1} \dot{\tilde{\Theta}} \\ &\leq -E^T Q E + s^T u(t) + \|s\| \cdot \|N(t, f_1, \dots, f_5)\| + \tilde{\Theta}^T \eta^{-1} \dot{\tilde{\Theta}} \\ &= -E^T Q E + s^T u(t) + \|s\| \cdot (\|N(t, f_1, \dots, f_5)\| - \bar{N}(t)) \\ &\quad + \|s\| \cdot \bar{N}(t) + \tilde{\Theta}^T \eta^{-1} \dot{\tilde{\Theta}}. \end{aligned} \quad (32)$$

FIGURE 5: Response using proposed intelligent robust control law ($\alpha = 100, \eta = 0.5$).

Using both expressions (27) and (30), and considering Assumption 4, we have

$$\begin{aligned}
 \dot{V} &\leq -E^T Q E + s^T u(t) + \|s\| \cdot (\|N(t, f_1, \dots, f_5)\| - \bar{N}(t)) \\
 &\quad + \|s\| \cdot (W(x_E) \Theta^{*T} + \Delta \bar{N}(t)) + (\Theta^* - \hat{\Theta})^T \eta^{-1} \tilde{\Theta} \\
 &\leq -E^T Q E + s^T u(t) + \|s\| \cdot (-\omega + \Delta \bar{N}(t)) + \hat{\Theta}^T W(x_E) \|s\| \\
 &\leq -\lambda_{\min}(Q) \|E\|^2 - \frac{(W(x_E) \hat{\Theta})^2 \|s\|^2}{\|s\| (W(x_E) \hat{\Theta}) + \alpha \|E\|^2} \\
 &\quad + \hat{\Theta}^T W(x_E) \|s\| \\
 &\leq -(\lambda_{\min}(Q) - \alpha) \|E\|^2 \\
 &= -\beta \|E\|^2.
 \end{aligned} \tag{33}$$

Hence, it is easily concluded that $E \in L_2 \cap L_\infty$, $\tilde{\Theta} \in L_\infty$, and then $s \in L_\infty$. Furthermore, from expression (8), we obtain that $\dot{E} \in L_\infty$. Using Barabarat lemma, it follows that $\lim_{t \rightarrow \infty} \tilde{q} = 0$. $\square$

In the dynamic compensator (29), $\alpha \|E\|^2$ represents the width of the boundary layer which is chosen to be a constant. The constant-width boundary layer design can reduce chattering of the control signals, but it decreases the control accuracy. The compromise between the smoothness of control signals and the accuracy of the control results is dictated by the choice of the boundary layer width. Through simulation we know that, for the control with a small boundary layer width, chattering occurs only during the transient stage when the system state is far from the origin. When the state gets closer and closer to the origin, the chattering phenomenon gradually disappears even though the boundary layer width is enough small. Based on the

FIGURE 6: High-performance robot used as testbed.

observations, this note proposes a method, which makes use of fuzzy boundary layer control [24].

We selected $\|E\|$ and $\|dE/dt\|$ as the control input, and the boundary layer width h as the control output. The fuzzy control rules are set as follows.

If $\|E\|$ is PB AND $\|dE/dt\|$ is PB, then h is PM.

If $\|E\|$ is PB AND $\|dE/dt\|$ is PS, then h is PB.

If $\|E\|$ is PS AND $\|dE/dt\|$ is PB, then h is PM.

If $\|E\|$ is PS AND $\|dE/dt\|$ is PS, then h is PS.

Where “PB, PM, PS” stand for positive big, positive middle, and positive small. “AND” is defined by “ μ_A AND $\mu_B = \mu_A \times \mu_B$ ” for any two membership values μ_A and μ_B over the fuzzy subsets A and B , respectively. Product-sum-gravity method of the inference technique is introduced for fuzzy logic control system.

Now the control law is proposed to be

$$u(t) = -\frac{(W(x_E)\hat{\Theta})^2 s}{\|s\| (W(x_E)\hat{\Theta}) + h}. \quad (34)$$

Theorem 3. Consider the error equation (8) for the robot manipulator system (3) with Assumptions 1–4. If the compensator $u(t)$ in controller (6) is designed as

$$u(t) = -\frac{(W(x_E)\hat{\Theta})^2 s}{\|s\| (W(x_E)\hat{\Theta}) + h}, \quad (35)$$

where constant h denotes the fuzzy boundary layer width, and the weight vector $\hat{\Theta}$ is updated by the following adaptive mechanism:

$$\dot{\hat{\Theta}} = \eta W(x_E) \|s\|, \quad (36)$$

with $\eta > 0$ the adaptive rate, then controllers (6), (34) can ensure that the tracking error of robot system asymptotically converges to zero.

Proof. The proof is similar to that of Theorem 1 above, thus we choose to omit the details. $\square$

Remark 7. In compensator (13), α is a small positive constant. Moreover, it can be seen from the above proof that feedback gain matrix K_p and K_v are only required to satisfy $\lambda_{\min}(Q) - \alpha > 0$. Hence, the proposed control scheme does not require the higher feedback gain matrix, whose

TABLE 1

m_1	m_2	l_1	l_2	I_{c1}	I_{c2}	I_1	I_2
10	5	1	1	0.5	0.5	10/12	5/12

robustness is guaranteed by intelligent adaptive dynamic compensator that effectively eliminated the effects of system uncertainties.

Remark 8. It has been shown that the proposed control scheme does need to calculate the inverse of inertia matrix as it was in [7].

4. COMPUTER SIMULATION

Consider (3) of a two-DOF planar robot arm, in which $G(q) = 0$ (no gravity torque) for simplicity. The entries of $M(q)$, $C(q, \dot{q})$, F_d , $F_s(\dot{q})$, and $\tau_d(t, q, \dot{q})$ are

$$M(q) = \begin{bmatrix} \theta_1 + \theta_2 + 2\theta_3 \cos(q_2) & \theta_2 + \theta_3 \cos(q_2) \\ \theta_2 + \theta_3 \cos(q_2) & \theta_2 \end{bmatrix}, \quad (37)$$

$$C(q, \dot{q}) = \begin{bmatrix} -\theta_3 \sin(q_2) \dot{q}_2 & -\theta_3 \sin(q_2) (\dot{q}_1 + \dot{q}_2) \\ \theta_3 \sin(q_2) \dot{q}_1 & 0 \end{bmatrix},$$

$F_d = \text{diag}[2, 2]$, $F_s(\dot{q}) = \text{sgn}(\dot{q})$, $\tau_d(t, q, \dot{q}) = [\dot{q}_1 \sin t, \dot{q}_2 \cos t]^T$, and one parameterization of this robot is given by

$$Y(q, \dot{q}, \ddot{q})\theta = \tau, \quad \theta \in R^3,$$

$$\theta_1 = m_1 l_{c1}^2 + m_2 l_1^2 + I_1,$$

$$\theta_2 = m_2 l_{c2}^2 + I_2,$$

$$\theta_3 = m_2 l_1 l_{c2},$$

$$y_{11} = \ddot{q}_1,$$

$$y_{21} = 0,$$

$$y_{12} = \ddot{q}_1 + \ddot{q}_2,$$

$$y_{22} = \ddot{q}_1 + \ddot{q}_2,$$

$$y_{13} = \cos(q_2)(2\ddot{q}_1) - \sin(q_2)(\dot{q}_2^2 + 2\dot{q}_1\dot{q}_2),$$

$$y_{23} = \cos(q_2)\ddot{q}_1 + \sin(q_2)\dot{q}_1^2.$$

In the equations above, physical parameters and their numerical values are shown in Table 1. The reference trajectories specified are

$$q_{d1} = \sin t + 0.1 \sin 3t - 0.2 \sin 4t, \quad (39)$$

$$q_{d2} = -0.1 \sin 2t - 0.2 \sin 3t + 0.1 \sin 4t,$$

with selection of parameters $T = 1$ ms, $K_v = \text{diag}[20, 20]$, $K_p = \text{diag}[100, 100]$, $\Lambda = \text{diag}[5, 5]$, $\Gamma = \text{diag}[50, 10, 10]$. In simulation, the nominal robot parameter is $\theta = [2.9, 1.0, 0.9]^T$, with choice of its bound $0 \leq \theta_1 \leq 4$, $0 \leq \theta_2 \leq 2$, $0 \leq \theta_3 \leq 2$. For ease of illustration, let $f_1 = f_2 = f_3 = q_d$, $f_3 = f_4 = \dot{q}_d$.

For the case of adopting conventional robust controller, namely, the upper bound on the parametric uncertainty is

FIGURE 7: Comparative plots for tracking errors of both joints between the controller [12] (--) and our proposed controller (—).

FIGURE 8: Comparative plots for control inputs of both joints between the controller [12] (--) and our proposed controller (—).

FIGURE 9: Comparative plots for tracking errors of both joints between the controller [14] (–) and our proposed controller (—).

known, we have the following simulation results as shown in Figure 2 ($\alpha = 10$) and Figure 3 ($\alpha = 100$), by the choice of the upper bound function as $\bar{N}(t) = 20 + 7.5\|E\| + 5.0\|E\|^2$.

The results of the intelligent hybrid robust control algorithm, while the upper bound is assumed to be unknown a priori, are also presented here in Figure 4 ($\alpha = 10$) and Figure 5 ($\alpha = 100$) as a comparison, with selection of the adaptive rate $\eta = 0.5$.

It can be observed from the above simulation results that both the conventional robust control method and the proposed intelligent hybrid robust control algorithm have satisfying robust results, and the latter achieves more excellent tracking performance than the former, for the reason that chattering of the control signals is obviously weakened by the fuzzy boundary layer, and the control precision is improved significantly. In addition, for the conventional robust control method, simulation results indicate that larger α (e.g., $\alpha = 100$) may yield better performance in attenuating the chattering of control torque than the case of smaller α (e.g., $\alpha = 10$); to the proposed intelligent hybrid robust

control algorithm, yet the system behavior in the case of larger α (e.g., $\alpha = 100$) is almost the same as the case of smaller α (e.g., $\alpha = 10$). This implies that our proposed intelligent hybrid robust control algorithm has better robust performance in its insensitivity to parameter variation than the conventional robust control method.

5. EXPERIMENTS

In this section, the experimental setup and system realization are described. Then experimental data is discussed and analyzed.

5.1. Hardware

A 2-DOF direct-drive ShinMaywa R3C robot arm is used as testbed, see Figure 6. It is a rigid-link, low-friction, planar robot with two joints.

The dimensions of the robot are given in Table 2, where subindexes 1 and 2 stand for the first and second links,

FIGURE 10: Comparative plots for control inputs of both joints between the controller [14] (.-) and our proposed controller (—).

TABLE 2

Par	Mass	Length	L_c	I	Δ
Link ₁	21.20	0.25	0.15	0.21	± 160
Link ₂	15.22	0.45	0.19	0.18	± 145
Units	Kg	m	m	Kgm ²	deg

respectively, and L_c stands for the center of mass, Par means parameters, I stands for inertia, and Δ defines joint limits. Joint angles q are measured with optical encoders at a high resolution of 120 000 pulse/rad, and joint-velocity signal $\dot{q}$ is estimated from position signals using a first-order filter with a constant $\tau = 10.0$.

5.2. Firmware

A Digital Signal Processor Loughborough Sound Images Ltd. DPC/C40B board control system was integrated on a 16-

bit expansion bus slot of a DX—486 personal computer. TMS320 floating point DSP C v.1.0.1 compiler provided the programming environment. The control input is transmitted to the servomotors through the Shinmaywa servosystem, which powers the pulse-wide modulators motor drives at each joint.

The performance of the proposed controller is shown in comparison with the neurofuzzy adaptive controller [12] and fuzzy supervisory sliding-mode controller [14]. Experiments are carried out at high velocities in order to show the performance of the system at inertial dominated dynamics.

Tuning the parameters is not obvious, and special attention is paid to avoid misleading conclusions. Since we present a comparison among similar but structurally different controllers, we set common gains to the same value. All feedback gains, including those of [12, 14], are given as

$$\begin{aligned}
 r &= \begin{pmatrix} 12 & 0 \\ 0 & 12 \end{pmatrix}, & K &= \begin{pmatrix} 100 & 0 \\ 0 & 80 \end{pmatrix}, \\
 I &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, & \Lambda &= \begin{pmatrix} 1 & 0 \\ 0 & 5 \end{pmatrix}, \\
 C &= (r, I), & r_0 &= \begin{pmatrix} 0.12 & 0 \\ 0 & 0.12 \end{pmatrix}.
 \end{aligned} \tag{40}$$

Comparative experiments are difficult to obtain because any comparative result can be dangerously biased. Besides, it is difficult to qualitatively compare controllers that are structurally different. Therefore, we show experimental results obtained “at first run,” that is, when reasonable performance, as dictated by the theory, is obtained without perhaps obtaining the best plots.

5.3. Experimental results

For better visualization of the plots, some figures are shown in two windows. Window 1 shows the time interval $[0, 1]$ s, while Window 2 shows the time interval $[1, 5]$ s.

Figure 7 shows a comparative plot of position tracking errors at high velocities of the neurofuzzy adaptive controller [12] and our intelligent hybrid control. Desired paths at each joint are $q_{d1} = 0.5 \sin(6\pi t)$ and $q_{d2} = 0.2 \sin(\pi t + \pi/2)$. Initial conditions are set to -5.0° with zero initial velocity. After a transient of 1 second, our controller yields tracking errors threefold and fourteenfold smaller at each joint, respectively. In this figure, we can observe that our controller yields better tracking accuracy with smooth control input, even when [3] is started with smaller position tracking errors, see controllers in Figure 8.

Figure 9 shows a comparative plot of position tracking errors at high velocities of the fuzzy supervisory sliding-mode controller [14] and our Intelligent hybrid control. Desired paths at each joint are $q_{d1} = 1.25 - (7/5)e^{-t} + (7/20)e^{-4t}$ and $q_{d2} = 1.25 + e^{-t} - (1/4)e^{-4t}$. Initial conditions are set to 1.0 with zero initial velocity. In this figure, we can observe that our controller yields better tracking accuracy with smooth control input, even when [14] is started with smaller position tracking errors, see controllers in Figure 10.

6. CONCLUSION

A new adaptive intelligent hybrid robust controller based on sliding mode is proposed in this paper for the trajectory tracking of robot manipulators with unknown nonlinear dynamics. We use intelligent systems to approximate the upper bound on the parametric uncertainty, and propose the idea of fuzzy bound later used to weaken the chattering effect. The system global stability and the tracking error convergence are proved by Lyapunov techniques. Simulation results and experimental data show that the intelligent hybrid controller make a significant improvement in the tracking performance.

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REFERENCES

- [1] J. T. Wen and D. S. Bayard, "New class of control laws for robotic manipulators—part 1: non-adaptive case," *International Journal of Control*, vol. 47, no. 5, pp. 1361–1385, 1988.
- [2] D. S. Bayard and J. T. Wen, "New class of control laws for robotic manipulators—part 2: adaptive case," *International Journal of Control*, vol. 47, no. 5, pp. 1387–1406, 1988.
- [3] L. L. Whitcomb, A. A. Rizzi, and D. E. Koditschek, "Comparative experiments with a new adaptive controller for robot arms," *IEEE Transactions on Robotics and Automation*, vol. 9, no. 1, pp. 59–70, 1993.
- [4] H. Berghuis, "A robust adaptive robot controller," *IEEE Transactions on Robotics and Automation*, vol. 9, no. 6, pp. 825–830, 1993.
- [5] J. T. Wen, K. Kreutz-Delgado, and D. S. Bayard, "Lyapunov function-based control laws for revolute robot arms: tracking control, robustness, and adaptive control," *IEEE Transactions on Automatic Control*, vol. 37, no. 2, pp. 231–237, 1992.
- [6] R. Kelly and R. Carelli, "A class of nonlinear PD-type controllers for robot manipulators," *Journal of Robotic Systems*, vol. 13, no. 12, pp. 793–802, 1996.
- [7] Z. Man, X. H. Yu, K. Eshraghian, and M. Palaniswami, "A robust adaptive sliding mode tracking control using an RBF neural network for robotic manipulators," in *Proceedings of the IEEE International Conference on Neural Networks*, vol. 5, pp. 2403–2408, Perth, Australia, November–December 1995.
- [8] S. S. Ge and C. C. Hang, "Direct adaptive neural network control of robots," *International Journal of Systems Science*, vol. 27, no. 6, pp. 533–542, 1996.
- [9] J. B. Mbede, X. Huang, and M. Wang, "Robust neuro-fuzzy sensor-based motion control among dynamic obstacles for robot manipulators," *IEEE Transactions on Fuzzy Systems*, vol. 11, no. 2, pp. 249–261, 2003.
- [10] S. Purwar, I. N. Kar, and A. N. Jha, "Adaptive control of robot manipulators using fuzzy logic systems under actuator constraints," *Fuzzy Sets and Systems*, vol. 152, no. 3, pp. 651–664, 2005.
- [11] Z. Song, J. Yi, D. Zhao, and X. Li, "A computed torque controller for uncertain robotic manipulator systems: fuzzy approach," *Fuzzy Sets and Systems*, vol. 154, no. 2, pp. 208–226, 2005.
- [12] F. Sun, Z. Sun, L. Li, and H.-X. Li, "Neuro-fuzzy adaptive control based on dynamic inversion for robotic manipulators," *Fuzzy Sets and Systems*, vol. 134, no. 1, pp. 117–133, 2003.
- [13] B. Daachi and A. Benallegue, "A neural network adaptive controller for end-effector tracking of redundant robot manipulators," *Journal of Intelligent and Robotic Systems*, vol. 46, no. 3, pp. 245–262, 2006.
- [14] H. Hu and P.-Y. Woo, "Fuzzy supervisory sliding-mode and neural-network control for robotic manipulators," *IEEE Transactions on Industrial Electronics*, vol. 53, no. 3, pp. 929–940, 2006.
- [15] Z. Qi, J. E. McInroy, and F. Jafari, "Trajectory tracking with parallel robots using low chattering, fuzzy sliding mode controller," *Journal of Intelligent and Robotic Systems*, vol. 48, no. 3, pp. 333–356, 2007.
- [16] L. Wang, *Adaptive Fuzzy Systems and Control: Design and Stability Analysis*, Prentice-Hall, Englewood Cliff, NJ, USA, 1994.
- [17] R. M. Sanner and J.-J. E. Slotine, "Gaussian networks for direct adaptive control," *IEEE Transactions on Neural Networks*, vol. 3, no. 6, pp. 837–863, 1992.
- [18] J. T. Spooner and K. M. Passino, "Stable adaptive control using fuzzy systems and neural networks," *IEEE Transactions on Fuzzy Systems*, vol. 4, no. 3, pp. 339–359, 1996.
- [19] J. Craig, *Introduction to Robotics*, Addison-Wesley, Boston, Mass, USA, 1986.
- [20] S. Sastry and M. Bodson, *Adaptive Control: Stability, Convergence and Robustness*, Prentice Hall, Englewood Cliffs, NJ, USA, 1989.
- [21] M. Liu, "Decentralized control of robot manipulators: nonlinear and adaptive approaches," *IEEE Transactions on Automatic Control*, vol. 44, no. 2, pp. 357–363, 1999.
- [22] J. Craig, *Adaptive Control of Mechanical Manipulators*, Addison-Wesley, New York, NY, USA, 1988.
- [23] J. Slotine and W. Li, *Applied Nonlinear Control*, Prentice-Hall, Upper Saddle River, NJ, USA, 1991.
- [24] Y.-G. Leu, T.-T. Lee, and W.-Y. Wang, "Observer-based adaptive fuzzy-neural control for unknown nonlinear dynamical systems," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 29, no. 5, pp. 583–591, 1999.