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Research Article

Passivity-Based Synchronization of Unified Chaotic System

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This letter further improves and extends the work of Kemih et al. In detail, feedback passivity synchronization with only one controller for a unified chaotic system is discussed here. It is noticed that the unified system contains the noted Lorenz, Lu, and Chen systems. Numerical simulations are given to show the effectiveness of these methods.

1. INTRODUCTION

Chaotic systems are characterized by being extremely sensitive to initial conditions, deterministically random, and hence ultimately unpredictable. Chaotic systems have numerous potential applications in mechanics, laser and chemical technologies, communications, biology and medicine, economics, ecology, and so forth [1, 2].

During the last two decades, synchronization in chaotic dynamic systems has received a great deal of interest among scientists from various research fields, since Pecora and Carroll [3] introduced a method to synchronize two identical chaotic systems with different initial conditions. The idea of synchronization is to use the output of the master system to control the slave system so that the output of the response system follows the output of the master system asymptotically. A wide variety of approaches have been proposed for the synchronization of various chaotic systems which include PC method, OGY method, active control approach, impulsive control method, generalized predictive control, adaptive control method, time-delay feedback approach, and backstepping design technique [5–16]. Many people have begun to give their attention to passive network theory [17–20]. The character of passive system is one of the network theory concepts, which show characteristics of dissipative network. Passive network theory can be used to analyze the dynamical character of system, such as stabilization and dynamic characteristics.

In this paper, our aim is to synchronize two unified chaotic systems. To achieve this goal, passive control theory is used to study passive synchronization of two unified chaotic systems. We first show that the passive synchronization problem is a passive control problem. In the stable control region, the error dynamical system is transformed to a passive system, and then using the state feedback, the error dynamical system can be globally asymptotically stabilized at zero. Finally, some simulation results are provided for illustration and verification.

2. PROPRETIES OF PASSIVE SYSTEM

Some preliminaries of passivity theory used in this paper will be shortly reviewed for the consistency of the presentation. Passivity is applied to nonlinear systems which are modelled by ordinary differential equations with input vector $u(t)$ and output vector $y(t)$ [17]:

$$\begin{aligned} \dot{x}(t) &= f(x(t), u(t)), \\ y(t) &= h(x(t)). \end{aligned} \tag{1}$$

The system (1) is dissipative with the supply rate $W(u(t), y(t))$, if it is not able to generate power by itself, that is, the energy stored in the system is less than or equal to the supplied power:

$$V(x(t)) \geq 0, \qquad V(x(T)) - V(x(0)) \leq \int_0^T W(u(t), y(t))\,dt. \tag{2}$$

Furthermore, the storage function $V(x(t))$ must satisfy the requirements for a Lyapunov function. If there exists a positive semidefinite Lyapunov function, such that

$$\int u^T(\tau)y(\tau)d\tau \geq \int \left[\frac{\partial V(x(\tau))}{\partial x(\tau)} f(x(\tau),u(\tau)) + eu^T + \delta y^T(\tau)y(\tau) + \rho\phi(x(\tau))\right]d\tau \tag{3}$$

then the system (1) is passive. A passive system implies that any increase in storage energy is due solely to an external power supply.

Then the equilibrium point of the system:

$$\dot{x}(t) = f(t,x(t),0) \tag{4}$$

is asymptotically stable in either of the two cases:

(i) $\rho > 0$,
(ii) $e + \delta > 0$ and the system is zeros-state observation.

The system (1) can be represented as the normal form:

$$\begin{aligned} \dot{x} &= f(z) + g(z,y)y, \\ \dot{y} &= l(z,y) + k(z,y)u. \end{aligned} \tag{5}$$

The nonlinear system (5) may be rendered bay a state feedback of the form [17]

$$u = \gamma_{12} + \beta(x)v. \tag{6}$$

3. PASSIVE SYNCHRONIZATION OF UNIFIED CHAOTIC SYSTEM

We consider unified chaotic system [21]:

$$\begin{aligned} \dot{x}_1 &= (25\alpha + 10)(x_2 - x_1), \\ \dot{x}_2 &= -x_1x_3 + (29\alpha - 1)x_2 + (28 - 35\alpha)x_1, \\ \dot{x}_3 &= x_1x_2 - \frac{8+\alpha}{3}x_3, \end{aligned} \tag{7}$$

where $\alpha \in [0,1]$, which contains the canonical Lorenz system [21, 23] and Chen system [24] as two extremes and Lü system [25] as a special case. Obviously, when $\alpha = 0$, $\alpha = 0.8$, and $\alpha = 1$, it is the Lorenz chaotic attractor, Lü chaotic attractor and Chen chaotic attractor, respectively. What is interesting is that as the parameter α changes continuously from 0 to 1, the resulting system remains continuously to be chaotic.

We will study the passive synchronization of two chaotic systems. Let system (7) be the drive system, and the response system is modelled by the following equation:

$$\begin{aligned} \dot{y}_1 &= (25\alpha + 10)(y_2 - y_1), \\ \dot{y}_2 &= -y_1y_3 + (29\alpha - 1)y_2 + (28 - 35\alpha)y_1, \\ \dot{y}_3 &= y_1y_2 - \frac{8+\alpha}{3}y_3. \end{aligned} \tag{8}$$

Let $e = (e_1,e_2,e_3)^T = (y_1 - x_1, y_2 - x_2, y_3 - x_3)^T$ be the synchronization error, and then the controlled model of error synchronization system is given by

$$\begin{aligned} \dot{e}_1 &= -(25\alpha + 10)e_1 + (25\alpha + 10)e_2, \\ \dot{e}_2 &= (28 - 35\alpha - x_3)e_1 + (29\alpha - 1)e_2 - x_1e_3 - e_1e_3 + u, \\ \dot{e}_3 &= x_2e_1 + x_1e_2 - \frac{8+\alpha}{3}e_3 + e_1e_2. \end{aligned} \tag{9}$$

The error synchronization system (9) is already in the normal form of (5), where $z_1 = e_1$, $z_2 = e_3$, $y = e_2$, $z = [z_1, z_2]^T$:

$$\begin{aligned} f(z) &= \left[-(25\alpha + 10)z_1, x_2z_1 - \frac{8+\alpha}{3}z_2\right], \\ g(z,y) &= [(25\alpha + 10), x_1 + z_1]^T, \\ l(z,y) &= (28 - 35\alpha - x_3)z_1 + (29\alpha - 1)y - x_1z_2 - z_1z_2, \\ k(z,y) &= 1. \end{aligned} \tag{10}$$

Our object is to design a smooth control (6) for the error synchronization system to make the closed-loop system passive. Choose a storage function candidate:

$$V(z,y) = W(z) + \frac{1}{2}y^2, \tag{11}$$

where $W(z)$ is Lyapunov function, with $W(0) = 0$,

$$W(z) = \frac{1}{2}(z_1^2 + z_2^2). \tag{12}$$

The zero dynamics of the system (5) describes those internal dynamics which are consistent with external constraint y = 0, that is,

$$\dot{z} = f(z). \tag{13}$$

Considering (13) and because $\alpha > 0$ is positive constant,

$$\begin{aligned} \frac{d}{dt}W(z) &= -(25\alpha + 10)z_1^2 + x_2z_1z_2 - \frac{8+\alpha}{3}z_2^2, \\ \frac{d}{dt}W(z) &= -(25\alpha + 10)\left(z_1 - \frac{x_2z_2}{2(25\alpha + 10)}\right)^2 \\ &\quad - \left(\frac{8+\alpha}{3} - \frac{x_2^2}{4(25\alpha + 10)}\right)z_2^2. \end{aligned} \tag{14}$$

In addition, we have

$$\frac{8+\alpha}{3} - \frac{x_2^2}{4(25\alpha + 10)} = \frac{(100\alpha^2 + 840\alpha + 320) - 3x_1^2}{12(25\alpha + 10)} > 0. \tag{15}$$

Then

$$\frac{d}{dt}W(z) \leq 0. \tag{16}$$

The zero dynamics of error synchronization system is Lyapunov stable. The derivative of $V(z, y)$ along the trajectory of the error synchronization system (9) is

$$\begin{aligned}\frac{d}{dt}V(z,y) &= \frac{\partial}{\partial z}W(z)\dot{z} + y\dot{y} \\ &= \frac{\partial}{\partial z}W(z)f(z) + \frac{\partial}{\partial z}W(z)g(z,y)y \\ &\quad + l(z,y)y + k(z,y)y.\end{aligned} \tag{17}$$

The error synchronization system is minimum phase:

$$\frac{d}{dt}W(z)f(z) \le 0. \tag{18}$$

Equation (17) becomes

$$\frac{d}{dt}V(z,y) \le \frac{\partial}{\partial z}W(z)g(z,y)y + \left(l(z,y) + k(z,y)u\right)y. \tag{19}$$

If we select the feedback control (6) of the following form and consider (10):

$$\begin{aligned}u &= k^{-1}(z,y)\left[-l^T(z,y) - \frac{\partial W}{\partial z}g(z,y) - \gamma_{12}y + v\right] \\ &= -(38 - 10\alpha - x_3)z_1 - (29\alpha - 1 + \gamma_{12})y + v,\end{aligned} \tag{20}$$

where γ_{11} is positive constant, and v is an external signal which is connected with the reference input, the above inequality can be rewritten as

$$\frac{d}{dt}V(z,y) \le -(\gamma_{12} + 29\alpha - 1)y^2 + vy. \tag{21}$$

Then by integrating both sides of (21), we have

$$\begin{aligned}&V(z,y) - V(z_0,y_0) \\ &\quad \le \int_0^t -(\gamma_{12} + 29\alpha - 1)y(\tau)^2 d\tau + \int_0^t v(\tau)y(\tau)d\tau,\end{aligned} \tag{22}$$

$$V(z,y) \ge 0, \text{ and } \rho = V(z_0,y_0),$$

$$\begin{aligned}V(z,y) - V(z_0,y_0) &\le \int_0^t -(\gamma_{12} + 29\alpha - 1)y(\tau)^2 d\tau + V(z,y) \\ &\ge \int_0^t -(\gamma_{12} + 29\alpha - 1)y(\tau)^2 d\tau.\end{aligned} \tag{23}$$

It satisfies the passive definition (3). The error synchronization system (9) is rendered to be output strict passive (OSP) under the feedback control. If external signal $v = 0$, then we want to steer state of error synchronization system to the origin.

4. SIMULATION RESULTS

Unified chaotic system has attractor for $\alpha = 0.8$. The typical unified chaotic attractor is shown in Figure 1.

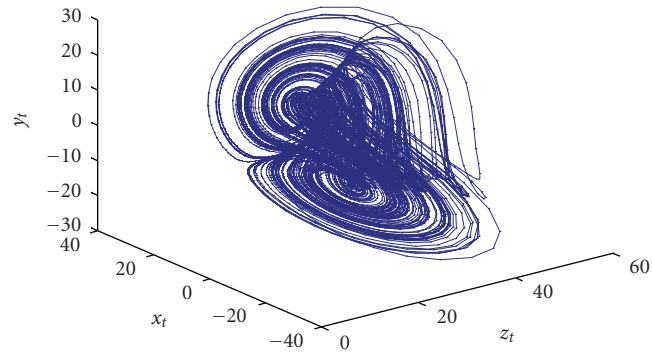

Figure 1: The typical unified chaotic attractors.

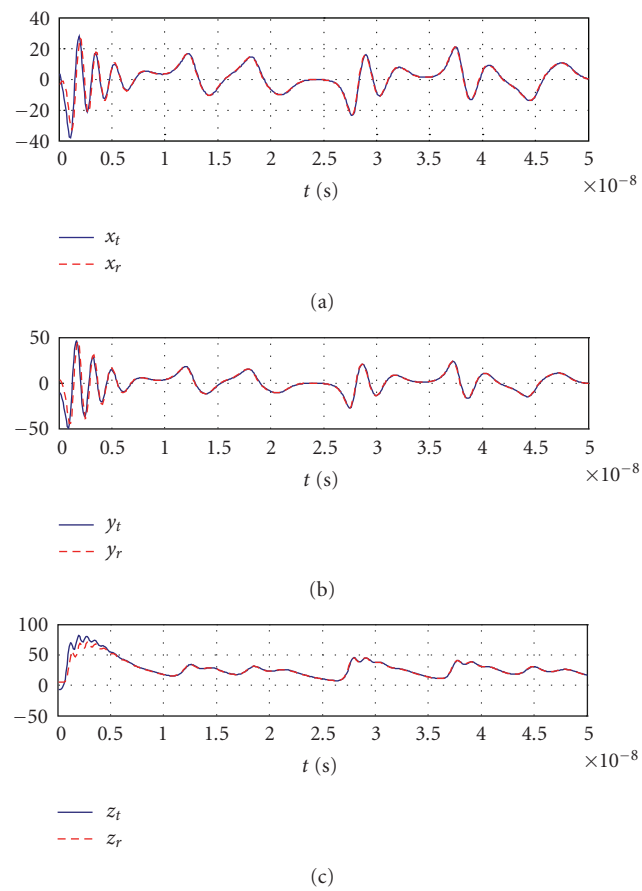

Figure 2: Synchronization of two unified chaotic systems.

The goal is to force the two systems to synchronize under control, while they have different initial conditions. The drive system starts from $[8\ -9\ -7]$ and the response system from $[-6\ 5\ 6]$. We select $\gamma_{12} = 11$ and use the controller as in (20): $u = -(30 - x_3)e_1 - 33.2e_2$.

In Figures 2 and 3, we note that the system is quickly and perfectly synchronized, also the bigger γ_{12} gives the best performance.

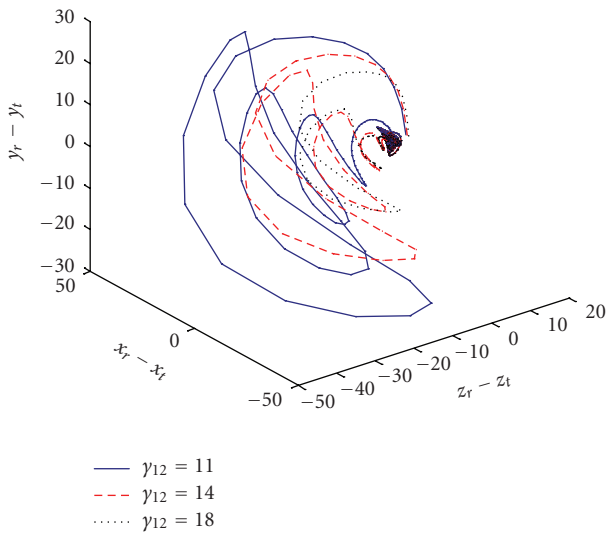

Figure 3: Phase-space trajectory.

5. CONCLUSION

In this paper, we have shown that the passivity-based synchronization can be used to synchronize unified chaotic system. The controller is very simple and has linear feedback form for the synchronization error system. It is noticed that the unified system contains the noted Lorenz, Lu, and Chen systems. Numerical simulations given show the effectiveness of these methods.

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