Research Article

Synchronization and Antisynchronization of a Planar Oscillation of Satellite in an Elliptic Orbit via Active Control

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We have investigated the synchronization and antisynchronization behaviour of two identical planar oscillation of a satellite in elliptic orbit evolving from different initial conditions using the active control technique based on the Lyapunov stability theory and the Routh-Hurwitz criteria. The designed controller, with our own choice of the coefficient matrix of the error dynamics that satisfy the Lyapunov stability theory and the Routh-Hurwitz criteria, is found to be effective in the stabilization of the error states at the origin, thereby, achieving synchronization and antisynchronization between the states variables of two nonlinear dynamical systems under consideration. The results are validated by numerical simulations using *mathematica*.

1. Introduction

In the last two decades, considerable research has been done in nonlinear dynamical systems and their various properties. One of the most important property of nonlinear dynamical systems is synchronization, which classically represents the entrainment of frequencies of oscillators due to weak interactions [1–3]. Studies in this field are partly motivated by experimental realization in lasers, electronic circuits, plasma discharge, and chemical reactions [2–4]. Synchronization techniques have been improved in recent years, and many different methods are applied theoretically as well as experimentally to synchronize the chaotic systems. Some of them are nonlinear feedback method [5], adaptive control method [6], antisynchronization method [7], sliding mode control method [8], and chaos synchronization using active control, which is introduced in [9] is one of these methods. Notable among these methods, chaos synchronization using active control scheme has recently been widely accepted as an efficient technique to synchronize the chaotic systems. The reason is because it can be used to synchronize identical as well as nonidentical systems; a feature that gives it an advantage over other synchronizing methods. This method based on the Lyapunov stability theory and the Routh-Hurwitz criteria to active control in order to achieve stable synchronization

has been applied to many practical systems such as the electronic circuits, which model a third-order “jerk” equation [10], Lorenz, Chen, and Lü system [11], geophysical systems [12], nonlinear equations of acoustic gravity waves (Lorenz-Stenflo system) [13], Van-der Pol-Duffing oscillator [14], forced damped pendulum [15], nuclear magnetic resonance modeled by the Bloch equations [16], parametrically excited oscillators [17], permanent magnet reluctance machine [18], inertial ratchets [19], RCL-shunted Josephson junction [20], and modified projective synchronization [21].

In this article, we have applied the active control technique based on the Lyapunov stability theory and the Routh-Hurwitz criteria to study the synchronization and antisynchronization behavior of two identical planar oscillation of a satellite in elliptic orbit evolving from different initial conditions. The system under consideration is chaotic for some values of parameter involved in the system. In synchronization, the two systems (master and slave) are synchronized and start with different initial conditions. The same problem may be treated as the design of control laws for full chaotic slave system using the known information of the master system so as to ensure that the controlled receiver synchronizes with the master system. Hence, the slave chaotic system completely traces the dynamics of the master system in the course of time. The aim of this study is to trace the chaotic

dynamics of the planar oscillation of a satellite in elliptic orbit based on synchronization and antisynchronization. To the best of my knowledge nobody studied this before.

2. Description of the Model

Elliptically orbiting planar oscillations of satellites in the solar system make an interesting study, and significant contributions to this end can be found [22–30], all of which have studied the influence of certain perturbative forces, such as solar radiation pressure, tidal force, and air resistance. In the present work, we consider the planar oscillation of a satellite in elliptic orbit with the spin axis fixed perpendicular to the orbital plane. Let the long axis of the satellite makes an angle θ with a reference axis that is fixed in inertial space, the long axis of the satellite makes an angle ψ with satellites planet centre line [31], and the satellite to be a triaxial ellipsoid with principal moments of inertia $A < B < C$, where C is the moment about the spin axis. The orbit is taken to be a fixed ellipse with semimajor axis a , eccentricity e , true anomaly f , and instantaneous radius r . The equation of motion of satellite planar oscillation in an elliptic orbit around the earth is

$$2C\ddot{\theta} - 3(B - A)\frac{Gm_p}{r^3} \sin 2\psi = 0. \quad (1)$$

Taking $3(B - A)/C = \omega_0^2$, $Gm_p = \mu$, $\psi = f - \theta$ and $l/r = 1 + e \cos f$, (1) can be written as

$$\frac{d^2\theta}{dt^2} = \frac{\mu\omega_0^2}{l^3} (1 + e \cos f)^3 \sin 2(f - \theta). \quad (2)$$

In order to reduce three variables ψ , θ , and f we are taking f as independent variable.

Now,

$$\begin{aligned} \frac{d\theta}{dt} &= \frac{d\theta}{df} \frac{df}{dt} = \dot{f} \frac{d\theta}{df} = \frac{h}{r^2} \frac{d\theta}{df}, \quad (\text{using } r^2 \dot{f} = h), \\ \frac{d^2\theta}{dt^2} &= \frac{-2h\dot{r}}{r^3} \frac{d\theta}{df} + \frac{h}{r^2} \frac{d^2\theta}{df^2} \frac{df}{dt}, \\ \frac{d^2\theta}{dt^2} &= \frac{-2h\dot{r}}{r^3} \frac{d\theta}{df} + \frac{h^2}{r^4} \frac{d^2\theta}{df^2} \quad (\text{using } r^2 \dot{f} = h). \end{aligned} \quad (3)$$

Now, from $l/r = 1 + e \cos f$, we have

$$\begin{aligned} \dot{r} &= \frac{-l(-e \sin f) \dot{f}}{(1 + e \cos f)^2}, \\ \dot{r} &= \frac{el \sin f}{(1 + e \cos f)^2} \frac{h}{r^2}. \end{aligned} \quad (4)$$

Using (3) and (4) in (2), we have

$$\begin{aligned} \frac{h^2}{r^4} \frac{d^2\theta}{df^2} - \frac{2elh^2 \sin f}{r^5 (1 + e \cos f)^2} \frac{d\theta}{df} &= \frac{\mu\omega_0^2}{r^3} \sin 2(f - \theta) \\ \frac{h^2}{r^4} \frac{d^2\theta}{df^2} - \frac{2eh^2 \sin f}{r^4 (1 + e \cos f)} \frac{d\theta}{df} &= \frac{h^2\omega_0^2}{lr^3} \sin 2(f - \theta) \quad (\text{using } h^2 = \mu l), \\ \frac{d^2\theta}{df^2} &= \frac{2e \sin f}{(1 + e \cos f)} \frac{d\theta}{df} + \omega_0^2 \frac{\sin 2(f - \theta)}{(1 + e \cos f)}. \end{aligned} \quad (5)$$

Using the binomial expansion for $(1 + e \cos f)^{-1}$ and for very small value of e ignoring the higher order terms in e , we have

$$\frac{d^2\theta}{df^2} = 2e \sin f \frac{d\theta}{df} + \frac{\omega_0^2}{2} (1 - e \cos f) \sin 2(f - \theta). \quad (6)$$

3. Synchronization via Active Control

For a system of two coupled chaotic oscillators, the master system ($\dot{x} = f(x, y)$) and the slave system ($\dot{y} = g(x, y)$), where $x(t)$ and $y(t)$ are the phase space (state variables), and $f(x, y)$ and $g(x, y)$ are the corresponding nonlinear functions, synchronization in a direct sense implies $\lim_{t \rightarrow \infty} |x(t) - y(t)| \rightarrow 0$. When this occurs the coupled systems are said to be completely synchronized. Chaos synchronization is related to the observer problem in control theory [32]. The problem may be treated as the design of control laws for full chaotic slave system using the known information of the master system so as to ensure that the controlled receiver synchronizes with the master system. Hence, the slave chaotic system completely traces the dynamics of the master in the course of time.

In order to formulate the active controllers, we write the system (6) in two first-order differential equations as shown below.

Let $d\theta/df = x'_1$ and $d^2\theta/df^2 = x'_2$, then we have

$$\begin{aligned} x'_1 &= x_2, \\ x'_2 &= 2ex'_1 \sin f + \frac{\omega_0^2}{2} (1 - e \cos f) \sin 2(f - x_1). \end{aligned} \quad (7)$$

Let us define another system

$$\begin{aligned} y'_1 &= y_2 + u_1(f), \\ y'_2 &= 2ey'_1 \sin f + \frac{\omega_0^2}{2} (1 - e \cos f) \sin 2(f - y_1) + u_2(f), \end{aligned} \quad (8)$$

where (7) and (8) are called the master system and the slave system, respectively, and in slave system, $u_1(f)$ and $u_2(f)$ are control functions to be determined. Let $e_1(f) = y_1(f) - x_1(f)$ and $e_2(f) = y_2(f) - x_2(f)$ be the synchronization

errors such that $\lim_{f \rightarrow \infty} e_i(f) \rightarrow 0$ for $i = 1, 2$. From (7) and (8), we have

$$\begin{aligned} e'_1(f) &= e_2(f) + u_1(f), \\ e'_2(f) &= 2ee'_1(f) \sin f + \frac{\omega_0^2}{2}(1 - e \cos f) \\ &\quad \times [\sin 2(f - y_1) - \sin 2(f - x_1)] + u_2(f). \end{aligned} \quad (9)$$

In order to express (9) as only linear terms in $e_1(f)$ and $e_2(f)$, we redefine the control functions as follows:

$$\begin{aligned} u_1(f) &= v_1(f), \\ u_2(f) &= -2ee'_1(f) \sin f - \frac{\omega_0^2}{2}(1 - e \cos f) \\ &\quad \times [\sin 2(f - y_1) - \sin 2(f - x_1)] + v_2(f). \end{aligned} \quad (10)$$

From (9) and (10), we have

$$\begin{aligned} e'_1(f) &= e_2(f) + v_1(f), \\ e'_2(f) &= v_2(f). \end{aligned} \quad (11)$$

Equation (11) is the error dynamics, which can be interpreted as a control problem where the system to be controlled is a linear system with control inputs $v_1(f) = v_1(e_1(f), e_2(f))$ and $v_2(f) = v_2(e_1(f), e_2(f))$. As long as these feedbacks stabilize the system, $\lim_{f \rightarrow \infty} e_i(f) \rightarrow 0$ for $i = 1, 2$. This simply implies that the two systems (7) and (8) evolving from different initial conditions are synchronized. As functions of $e_1(f)$ and $e_2(f)$, we choose $v_1(f)$ and $v_2(f)$ as follows:

$$\begin{pmatrix} v_1(f) \\ v_2(f) \end{pmatrix} = D \begin{pmatrix} e_1(f) \\ e_2(f) \end{pmatrix}, \quad (12)$$

where $D = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is a 2×2 constant feedback matrix to be determined. Hence the error system (11) can be written as

$$\begin{pmatrix} e'_1(f) \\ e'_2(f) \end{pmatrix} = C \begin{pmatrix} e_1(f) \\ e_2(f) \end{pmatrix}, \quad (13)$$

where $C = \begin{pmatrix} a & 1+b \\ c & d \end{pmatrix}$ is the coefficient matrix.

According to the Lyapunov stability theory and the Routh-Hurwitz criteria, if

$$\begin{aligned} a + d &< 0, \\ c(1 + b) - ad &< 0, \end{aligned} \quad (14)$$

then the eigen values of the coefficient matrix of error system (11) must be real or complex with negative real parts and, hence, stable synchronized dynamics between systems (7) and (8) is guaranteed. Let

$$a + d = c(1 + b) - ad = -E, \quad (15)$$

where $E > 0$ is a real number which is usually set equal to 1. There are several ways of choosing the constant elements a, b, c, d of matrix D in order to satisfy the Lyapunov stability theory and the Routh-Hurwitz criteria (14).

4. Numerical Simulation for Synchronization

For the constant elements of feedback matrix, choosing $a = d = -0.5$ and for the parameters involved in system under investigation, $e = 0.15$, $\omega_0 = 0.3$ with the initial conditions $[x_1(0), y_1(0)] = [0, 0]$, and $[x_2(0), y_2(0)] = [1, 1.5]$, we have simulated the system under consideration using *mathematica*. The results obtained show that the system under consideration achieved synchronization. Phase portraits, time series analysis, and error diagrams are the witness of achieving synchronization between master and slave system. Further, it also has been confirmed by the convergence of the synchronization quality defined by

$$e(f) = \sqrt{e_1^2(f) + e_2^2(f)}. \quad (16)$$

Figures 1–6 confirms the convergence of the synchronization quality defined by (16).

5. Antisynchronization via Active Control

Antisynchronization of two coupled systems $\dot{x} = f(x, y)$ (master system) and $\dot{y} = g(x, y)$ (slave system) means $\lim_{t \rightarrow \infty} |x(t) + y(t)| \rightarrow 0$. This phenomenon has been investigated both experimentally and theoretically in many physical systems [18, 19, 32–37]. A recent study of the antisynchronization phenomenon in nonequilibrium systems suggests that it could be used as a technique for particle separation in a mixture of interacting particles [19].

In order to formulate the active controllers for Antisynchronization, we need to redefine the error functions as $e_1(f) = y_1(f) + x_1(f)$ and $e_2(f) = y_2(f) + x_2(f)$, where $e_1(f)$ and $e_2(f)$ are called the antisynchronization errors such that $\lim_{f \rightarrow \infty} e_i(f) (i = 1, 2) \rightarrow 0$. From (7) and (8), error dynamics can be written as

$$\begin{aligned} e'_1(f) &= e_2(f) + u_1(f), \\ e'_2(f) &= 2ee'_1(f) \sin f + \frac{\omega_0^2}{2}(1 - e \cos f) \\ &\quad \times [\sin 2(f - y_1) + \sin 2(f - x_1)] + u_2(f). \end{aligned} \quad (17)$$

In order to express (17) as only linear terms in $e_1(f)$ and $e_2(f)$, we redefine the control functions as follows:

$$\begin{aligned} u_1(f) &= v_1(f), \\ u_2(f) &= -2ee'_1(f) \sin f - \frac{\omega_0^2}{2}(1 - e \cos f) \\ &\quad \times [\sin 2(f - y_1) + \sin 2(f - x_1)] + v_2(f). \end{aligned} \quad (18)$$

From (17) and (18), we have

$$\begin{aligned} e'_1(f) &= e_2(f) + v_1(f), \\ e'_2(f) &= v_2(f). \end{aligned} \quad (19)$$

FIGURE 1: Phase plot of master system.

FIGURE 4: Time series analysis of slave system.

FIGURE 2: Phase plot of slave system.

FIGURE 5: Time series analysis of $e_1(f)$.

FIGURE 3: Time series analysis of master system.

FIGURE 6: Time series analysis of $e_2(f)$.

Equation (19) is the error dynamics, which can be interpreted as a control problem where the system to be controlled is a linear system with control inputs $v_1(f) = v_1(e_1(f), e_2(f))$, and $v_2(f) = v_2(e_1(f), e_2(f))$. As long as these feedbacks stabilize the system, $\lim_{f \rightarrow \infty} e_i(f) \rightarrow 0$ for $i = 1, 2$. This simply implies that the two systems (7) and (8) evolving from different initial conditions are synchronized. As functions of $e_1(f)$ and $e_2(f)$, we choose $v_1(f)$ and $v_2(f)$ as follows:

$$\begin{pmatrix} v_1(f) \\ v_2(f) \end{pmatrix} = D \begin{pmatrix} e_1(f) \\ e_2(f) \end{pmatrix}, \quad (20)$$

where $D = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is a 2×2 constant feedback matrix to be determined. Hence the error system (19) can be written as

$$\begin{pmatrix} e'_1(f) \\ e'_2(f) \end{pmatrix} = C \begin{pmatrix} e_1(f) \\ e_2(f) \end{pmatrix}, \quad (21)$$

where $C = \begin{pmatrix} a & 1+b \\ c & d \end{pmatrix}$, is the coefficient matrix.

According to the Lyapunov stability theory and the Routh-Hurwitz criteria, if

$$\begin{aligned} a + d &< 0, \\ c(1 + b) - ad &< 0, \end{aligned} \quad (22)$$

FIGURE 7: Phase plot of slave system.

FIGURE 10: Time series analysis of $e_2(f)$.

FIGURE 8: Time series analysis of slave system.

FIGURE 11: Convergence of errors in synchronization.

FIGURE 9: Time series analysis of $e_1(f)$.

FIGURE 12: Convergence of errors in antisynchronization.

then the eigen values of the coefficient matrix of error system (19) must be real or complex with negative real parts, and, hence, stable synchronized dynamics between systems (7) and (8) is guaranteed. Let

$$a + d = c(1 + b) - ad = -E, \quad (23)$$

where $E > 0$ is a real number which is usually set equal to 1. There are several ways of choosing the constant elements a , b , c , d of matrix D in order to satisfy the Lyapunov stability theory and the Routh-Hurwitz criteria (19).

6. Numerical Simulation for Antisynchronization

For the constant elements of feedback matrix, choosing $a = d = -0.5$ and for the parameters involved in system under investigation, $e = 0.15$, $\omega_0 = 0.3$ with the initial conditions $[x_1(0), y_1(0)] = [0, 0]$ and $[x_2(0), y_2(0)] = [1, 1.5]$, we have simulated the system under consideration using *mathematica*. The results obtained show that the system under consideration achieved synchronization. Phase portraits, time series analysis, and error diagrams are the witness of achieving synchronization between master and slave system.

Further, it also has been confirmed by the convergence of the synchronization quality defined by

$$e(f) = \sqrt{e_1^2(f) + e_2^2(f)}. \quad (24)$$

Figures 7–12 confirms the convergence of the synchronization quality defined by (24).

7. Conclusion

In this paper, we have investigated the synchronization and antisynchronization behaviour of the two identical planar oscillation of a satellite in elliptic orbit evolving from different initial conditions via the active control technique based on the Lyapunov stability theory and the Routh-Hurwitz criteria. The results were validated by numerical simulations using *mathematica*. For the errors in synchronization and antisynchronization behavior of the system under study, we have observed that the rate of convergence of errors is faster in antisynchronization. Figures 1–12 are proof of it.

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