Research Article

H_∞ Filtering over Networks for a Class of Discrete-Time Stochastic System with Randomly Occurred Sensor Nonlinearity

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Received 2 December 2011; Accepted 24 February 2012

Academic Editor: Ya-Jun Pan

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The H_∞ filtering problem for a class of discrete-time stochastic system with randomly occurred nonlinearity (RON) suffering from network packet loss is considered. Based on the Lyapunov-Krasovskii functional method, the asymptotical mean-square stability condition of the filtering system with a prescribed H_∞ level is derived. Besides, the filter parameters can be obtained simultaneously by solving the matrix inequalities we achieve. It is worth noting that no slack variable is introduced in the proposed conditions. The effectiveness of the theory developed is verified through a numerical example.

1. Introduction

In the past few decades, the H_∞ control problem has been extensively developed in many applications, for example, [1] in the state-space version [2], on output feedback H_∞ control for nonlinear systems, with [3, 4] for uncertain linear systems, since the H_∞ filtering was introduced in [5]. The objective of the H_∞ filtering is to design an estimator for a given system, such that the l_2 gain from the exogenous disturbance to the estimation error is less than a given level $\gamma > 0$, where the noise signals are assumed to be arbitrary but with bounded energy or bounded average power rather than just Gaussian. Without the assumption of the statistical properties in Kalman filtering, the H_∞ filtering has been widely applied; see [6–12] and the references therein.

The H_∞ filtering issue has been extended to a variety of complex dynamical systems, such as Markovian jumping systems [13], time-varying systems [14], fuzzy systems [15], and nonlinear systems [16]. Recently, stochastic H_∞ control and filtering problems for the systems expressed by Itô-type stochastic differential or difference equations have become a popular research area and have gained a lot of attention. In [17], an asymptotically stable (in some sense) observer was constructed which leads to a stable estimation error process whose l_2 gain with respect to uncertain disturbance signal is

less than a prescribed level. The stochastic analysis presented in [18] was conducted to enforce the performance for the newly formulated NCS system under the usual stability requirement. The work of [19] formulated the filters for both continuous- and discrete-time Itô-type stochastic systems, with the nonlinear sensor and all admissible uncertainties under a prescribed H_∞ disturbance performance, respectively.

With the rapid development in the networked control technology, in practice, more and more systems are taking wired or wireless networked control system (NCS) as a solution, which has many advantages, such as low cost and easy maintenance and installation. However, due to the limited transmission capacity of the wired cable or wireless channel, issues like quantization [20–22], transmission delay [23], and packet dropouts [24] inevitably emerged. On the other hand, nonlinearity is a main source that complexes the control algorithms; hence, the H_∞ filtering problem has attracted a lot of research attention in the previous two areas. In [25], the nonlinearity was assumed to satisfy the sector bounded conditions, and for general stochastic systems, the nonlinear H_∞ filtering problem was investigated in [26]. Nevertheless, it is worth mentioning that one interesting problem that has been persistently overlooked is the so-called RON in the sensor parts or the transmission channel

from sensor to controller. As is well known, the sensor parts of a wide class of practical systems are influenced by RON disturbances that are caused by environmental circumstances such as random failure stochastic fault on the linearization part of the sensor. Unfortunately, to the best of the authors' knowledge, the H_∞ filtering problem for discrete-time stochastic systems with RON on sensor parts has not been fully studied, which motivates the work of this paper.

In this paper, the H_∞ filtering problem against randomly occurred sensor failures for a class of discrete time stochastic systems with norm-bounded noises suffering from network packet loss is considered. First of all, the RON model was used to describe the binary switches between the linear and nonlinear sensor by a Bernoli distribution with a known probability. Such a novel idea was first illustrated in [27, 28] to investigate the synchronization problem of stochastic delayed complex networks. The RON, also named as stochastic nonlinearity, has recently attracted many researchers' interests. Readers interested in this area are suggested to refer to [27–29] and the references therein. Besides, the lossy network is also taken into consideration which is also modeled as a Bernoli process [29]. Next, the asymptotically mean-square stability condition of the filtering error dynamics with a prescribed H_∞ performance level is derived by using Lyapunov-Krasovskii functional technique. Finally, a simulation example is utilized to illustrate the effectiveness of the approach developed.

Notation 1. The notations used in the paper is fairly standard. The superscript “ T ” stands for matrix transposition; $\mathbb{R}^n$ denotes the n -dimensional Euclidean space; $\mathbb{R}^{m \times n}$ is the set of all real matrices of dimension $m \times n$; I and 0 represent the identity matrix and zero matrix, respectively. The notation $P > 0$ means that P is a real symmetric and positive definite matrix; the notation $\|A\|$ refers to the norm of matrix A , defined by $\|A\| = \sqrt{\text{tr}(A^T A)}$, and $\|\cdot\|_2$ stands for the usual l_2 norm. In symmetric block matrices or complex matrix expressions, an asterisk ($*$) is used to represent a term that is induced by symmetry. Besides, $\mathbb{E}\{x\}$ and $\mathbb{E}\{x \mid y\}$, respectively, represent the expectation of x and the expectation of x conditional on y . The set of all nonnegative integers is denoted by $\mathbb{I}^+$ and the set of all nonnegative real numbers is represented by $\mathbb{R}^+$. If A is a matrix, then $\lambda_{\max}(A)$ (resp., $\lambda_{\min}(A)$) means the largest (resp., smallest) eigenvalue of A . Matrices in this paper, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem Formulation

In this section, a class of discrete-time stochastic systems (Σ) with sensor nonlinearity occurred randomly is considered, which is represented by the following equations:

$$\begin{aligned} (\Sigma) : x(k+1) &= Ax(k) + Bv(k) + [Ex(k) + Gv(k)]w(k), \\ y(k) &= \alpha(k)\phi(Cx(k)) + (1 - \alpha(k))Cx(k) + Dv(k), \\ z(k) &= Lx(k), \end{aligned} \quad (1)$$

where $x(k) \in \mathbb{R}^n$ is the system state; $y(k) \in \mathbb{R}^q$ is the system output; $z(k) \in \mathbb{R}^r$ is the combination of the state to be estimated; $\alpha(k)$ is a Bernoli process, taking different values to indicate that the output of the plant $y(k)$ is linear or not [29], with $\Pr\{\alpha(k) = 1\} = \alpha_e$, and $\Pr\{\alpha(k) = 0\} = 1 - \alpha_e$. $w(k)$ is a real scalar random process on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ related to an increasing family $(\mathcal{F}_k)_{k \in \mathbb{N}}$ of σ -algebra $\mathcal{F}_k \in \mathcal{F}$ generated by $(w(k))_{k \in \mathbb{N}}$ with $\mathbb{N}$ being the set of natural numbers and is assumed to satisfy $\mathbb{E}\{w(k)\} = 0$, $\mathbb{E}\{w(k)^2\} = 1$ and $\mathbb{E}\{w(i)w(j)\} = 0$, for all $i \neq j$. $v(k)$ is the exogenous disturbance signal, which belongs to $l_2([0, \infty); \mathbb{R}^p)$, and is adapted and $\mathcal{F}_{k-1}$ is measurable for all $k \in \mathbb{N}$, where $l_2([0, \infty); \mathbb{R}^k)$ denotes the space of k -dimensional nonanticipatory square-summable stochastic process $f(\cdot) = (f(k))_{k \in \mathbb{N}}$ with respect to $(\mathcal{F}_k)_{k \in \mathbb{N}}$ satisfying

$$\|f\|_{e_2}^2 = \mathbb{E} \left\{ \sum_{k=0}^{\infty} |f(k)|^2 \right\} < \infty. \quad (2)$$

Additionally, in (1), A, B, E, G, C, D , and L are real constant matrices with appropriate dimensions; the function $\phi \in [K_1, K_2]$ for some given diagonal matrices $K_1 \geq 0$ and $K_2 \geq 0$ with $K_2 \geq K_1$ satisfies the following sector condition:

$$(\phi(u) - K_1 u)^T (\phi(u) - K_2 u) \leq 0 \quad \forall u \in \mathbb{R}^q. \quad (3)$$

Remark 1. It is assumed here that when $\alpha(k) = 1$, the output signal is the nonlinear function $\phi(Cx(k))$ other than the linear combination of the system states $Cx(k)$. Additionally, for the convenience of further discussion, we introduce the following two symbols: $\phi_s(u) = \phi(u) - K_1 u$ and $K = K_2 - K_1$.

Definition 2 (see [30]). The discrete-time stochastic system in the form of (1) with $v(k) = 0$ is said to be stochastic stable if there is a scalar $\mu > 0$ such that for all admissible uncertainties,

$$\mathbb{E} \left\{ \sum_{k=0}^{\infty} |x(k)|^2 \right\} \leq \mu \mathbb{E} \left\{ \|x(0)\|^2 \right\}, \quad (4)$$

where $x(k)$ denotes the solution of stochastic systems with initial state $x(0)$.

For discrete-time stochastic system (Σ), the estimation of $z(k)$ from measured output $y(k)$ is considered here. To accomplish this, we construct the following filter:

$$\begin{aligned} (\mathcal{F}) : \hat{x}(k+1) &= A_f \hat{x}(k) + B_f \hat{y}(k), \\ \hat{z}(k) &= L \hat{x}(k), \end{aligned} \quad (5)$$

where $\hat{y}(k) = \beta(k)y(k)$, $\hat{x}(k) \in \mathbb{R}^n$, $\hat{z}(k) \in \mathbb{R}^r$. A_f and B_f are the matrices with appropriate dimension to be determined. The process $\{\beta(k), k \in \mathbb{I}^+\}$ is a Bernoli process, indicating that the network packet is successfully received or not [29], with the probabilities $\Pr\{\beta(k) = 1\} = \beta_e$, $\Pr\{\beta(k) = 0\} = 1 - \beta_e$ relatively. It is also assumed here that the random processes $\alpha(k)$, $\beta(k)$, $w(k)$ and $v(k)$, for all $k \in \mathbb{I}^+$ are mutually independent. Hence, the filtering problem can be stated as follows.

Discrete-Time Filtering. Given a disturbance attenuation level $\gamma > 0$, the parameters A_f and B_f of filter (5) are designed such that the resultant filtering error system is stochastically stable for $v(k) = 0$ and any $\phi \in [K_1, K_2]$, satisfying $\|z - \hat{z}\|_{e_2} < \gamma\|v\|_{e_2}$ under zero initial conditions for all $v(k) \in l_{e_2}([0, \infty); \mathbb{R}^p)$.

Let $\tilde{x}(k) = x(k) - \hat{x}(k)$, $\tilde{z} = z(k) - \hat{z}(k)$, and $e(k) = [x(k)^T \tilde{x}(k)^T]^T$. By means of the system (Σ) and filter ($\mathcal{F}$), we obtain the filtering error dynamics as

$$\begin{aligned} (\Sigma_e) : e(k+1) &= [\bar{A} + (\beta_e - \beta(k))\tilde{A} + (\beta_e\alpha_e - \beta(k)\alpha(k))\hat{A}] \\ &\quad \times e(k) - \beta(k)\alpha(k)\hat{B}\phi_s(u(k)) \\ &\quad + [\bar{B} + (\beta_e - \beta(k))\tilde{B}]v(k) \\ &\quad + [\bar{E}e(k) + \bar{G}v(k)]w(k), \\ \tilde{z}(k) &= \bar{L}e(k), \end{aligned} \quad (6)$$

where $u(k) = Cx(k)$, and

$$\begin{aligned} \bar{A} &= \begin{bmatrix} A & 0 \\ A - A_f - \beta_e(B_f C + \alpha_e B_f K_1 C - \alpha_e B_f C) & A_f \end{bmatrix}, \\ \tilde{B} &= \begin{bmatrix} 0 \\ B_f D \end{bmatrix}, \quad \bar{G} = \begin{bmatrix} G \\ G \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} 0 \\ B_f \end{bmatrix}, \\ \hat{A} &= \begin{bmatrix} 0 & 0 \\ B_f K_1 C - B_f C & 0 \end{bmatrix}, \quad \tilde{A} = \begin{bmatrix} 0 & 0 \\ B_f C & 0 \end{bmatrix}, \\ \bar{L} &= [0 \ L], \quad \bar{B} = \begin{bmatrix} B \\ B - \beta_e B_f D \end{bmatrix}, \quad \bar{E} = \begin{bmatrix} E & 0 \\ E & 0 \end{bmatrix}. \end{aligned} \quad (7)$$

3. Filter Design

The following theorem provides a sufficient condition for the solvability of discrete-time filtering problem for the system (Σ_e).

Theorem 3. Consider the discrete-time stochastic systems (Σ_e), for a given disturbance attenuation level $\gamma > 0$, if there exist matrices $X = X^T > 0$, $Y = Y^T > 0$ satisfying the following matrix inequalities:

$$\begin{bmatrix} \begin{bmatrix} \Theta_1 & 0 \\ * & \Theta_6 \end{bmatrix} & \begin{bmatrix} \Theta_2 \\ 0 \end{bmatrix} & \begin{bmatrix} \Theta_3 \\ 0 \end{bmatrix} & \bar{A}^T & \sqrt{\beta_e(1-\beta_e)}\tilde{A}^T & \Theta_4\hat{A}^T & \begin{bmatrix} \Theta_5 \\ 0 \end{bmatrix} & 0 \\ * & \Theta_7 & 0 & \bar{B}^T & \sqrt{\beta_e(1-\beta_e)}\tilde{B}^T & 0 & 0 & 0 \\ * & * & -2\alpha_e I & -\beta_e\alpha_e\hat{B}^T & 0 & \Theta_4\hat{B}^T & 0 & \Theta_8 \\ * & * & * & -\bar{P} & 0 & 0 & 0 & 0 \\ * & * & * & * & -\bar{P} & 0 & 0 & 0 \\ * & * & * & * & * & -\bar{P} & 0 & 0 \\ * & * & * & * & * & * & -\bar{Y} & 0 \\ * & * & * & * & * & * & * & -\bar{Y} \end{bmatrix} < 0, \quad (8)$$

where

$$\begin{aligned} \Theta_1 &= E^T X E + E^T Y E - X, & \Theta_2 &= E^T X G + E^T Y G, \\ \Theta_3 &= \alpha_e C^T K^T, & \Theta_4 &= \sqrt{\beta_e\alpha_e(1-\beta_e\alpha_e)}, \\ \Theta_5 &= \sqrt{\beta_e\alpha_e(1-\beta_e)}(B_f C + B_f K_1 C)^T, \\ \Theta_6 &= L^T L - Y, & \Theta_7 &= G^T X G + G^T Y G - \gamma^2 I, \\ \Theta_8 &= \alpha_e \sqrt{\beta_e(1-\beta_e)}B_f^T, & \bar{P} &= (\text{diag}\{X, Y\})^{-1}, \\ & & \bar{Y} &= Y^{-1}. \end{aligned} \quad (9)$$

Then the systems (Σ_e) is globally asymptotically stable with given disturbance attenuation level $\gamma > 0$.

Proof. First, we would like to establish the stochastic stability of the filtering error system (Σ_e), by choosing a stochastic Lyapunov functional candidate as

$$V(e(k)) = e(k)^T P e(k) \quad (10)$$

for the filtering error system (6). For $v(k) = 0$ we have

$$\begin{aligned} &\mathbb{E}\{V(e(k+1)e(k))\} - V(e(k)) \\ &\leq \mathbb{E}\{V(e(k+1) \mid e(k))\} - V(e(k)) \\ &\quad - 2\phi_s^T(\mathbb{E}\{\alpha(k)\}\phi_s - \mathbb{E}\{\alpha(k)\}K C x(k)) \end{aligned}$$

$$\begin{aligned}
&= \mathbb{E} \left\{ \left[\left(\bar{A} + (\beta_e - \beta(k)) \tilde{A} + (\beta_e \alpha_e - \beta(k) \alpha(k)) \hat{A} \right) e(k) \right. \right. \\
&\quad \left. \left. - \beta(k) \alpha(k) \hat{B}_f \phi_s(u(k)) \right]^T \right. \\
&\quad \times P \left[\left(\bar{A} + (\beta_e - \beta(k)) \tilde{A} \right. \right. \\
&\quad \left. \left. + (\beta_e \alpha_e - \beta(k) \alpha(k)) \hat{A} \right) e(k) \right. \\
&\quad \left. \left. - \beta(k) \alpha(k) \hat{B}_f \phi_s(u(k)) \right] \right\} \\
&\quad + e^T(k) \bar{E}^T P \bar{E} e(k) \\
&\quad - 2 \phi_s^T(\mathbb{E}\{\alpha(k)\}) \phi_s - \mathbb{E}\{\alpha(k)\} K C x(k) - V(x(k)) \\
&= [e(k)^T \phi_s^T] \Omega_s [e(k)^T \phi_s^T]^T,
\end{aligned} \tag{11}$$

where

$$\Omega_s = \begin{bmatrix} \bar{\Omega}_1 & \bar{\Omega}_2 \\ * & \beta_e \alpha_e \hat{B}^T P \hat{B} - 2 \alpha_e I \end{bmatrix}, \tag{12}$$

with,

$$\begin{aligned}
\bar{\Omega}_1 &= \bar{A}^T P \bar{A} + \beta_e (1 - \beta_e) \tilde{A}^T P \tilde{A} - P \\
&\quad + \beta_e \alpha_e (1 - \beta_e \alpha_e) \hat{A}^T P \hat{A} \\
&\quad + 2 \beta_e \alpha_e (1 - \beta_e) \tilde{A}^T P \hat{A} + \bar{E}^T P \bar{E}, \\
\bar{\Omega}_2 &= \beta_e \alpha_e (1 - \beta_e) \tilde{A}^T P \hat{B} - \beta_e \alpha_e \bar{A}^T P \hat{B} \\
&\quad + \beta_e \alpha_e (1 - \beta_e \alpha_e) \hat{A}^T P \hat{B} + \alpha_e \bar{C}^T K^T, \\
\bar{C} &= [C \ 0].
\end{aligned} \tag{13}$$

As (8) holds, one may have $\Omega_s < 0$, which means that there is a scalar $\varepsilon > 0$ small enough, such that

$$\Omega_s < \begin{bmatrix} \varepsilon I & 0 \\ 0 & 0 \end{bmatrix}. \tag{14}$$

It follows from (14) that

$$\mathbb{E}\{V(x(k+1)|x(k))\} - V(x(k)) < -\varepsilon \|e(k)\|^2. \tag{15}$$

Taking expectations with respect to both sides of (15), we have

$$\mathbb{E}\{V(x(k+1))\} - \mathbb{E}\{V(x(k))\} < -\varepsilon \mathbb{E}\{\|e(k)\|^2\}. \tag{16}$$

Hence, by summing up both sides of (16), from 0 to N for any integer $N > 1$, we can get

$$\mathbb{E}\{V(x(N+1))\} - \mathbb{E}\{V(0)\} < -\varepsilon \mathbb{E}\left\{\sum_{k=0}^N \|e(k)\|^2\right\}, \tag{17}$$

which leads to

$$\begin{aligned}
\mathbb{E}\left\{\sum_{k=0}^N \|e(k)\|^2\right\} &< \frac{1}{\varepsilon} [\mathbb{E}\{V(0)\} - \mathbb{E}\{V(x(N+1))\}] \\
&\leq \frac{1}{\varepsilon} \mathbb{E}\{V(0)\} \\
&\leq \mu \mathbb{E}\{\|x(0)\|^2\},
\end{aligned} \tag{18}$$

where $\mu = 1/\varepsilon \lambda_{\max}\{P\}$. Taking $N \rightarrow \infty$, it is shown from (18) and Definition 2 that the filtering error system (7) is stochastically stable for $v(k) = 0$.

Next, we will present that the filtering error system (6) satisfies

$$\|\tilde{z}(k)\|_{e_2} < \gamma \|v(k)\|_{e_2} \tag{19}$$

for all $v(k) \in l_{e_2}([0, \infty); \mathbb{R}^p)$. To accomplish this, first of all, we would introduce the filtering error index:

$$J(N) = \mathbb{E}\left\{\sum_{k=1}^N [\|\tilde{z}(k)\|^2 - \gamma^2 \|v(k)\|^2]\right\} \tag{20}$$

for any $N \geq 1$. Thus, under zero initial conditions, for all $v(k) \in l_{e_2}([0, \infty); \mathbb{R}^p)$, there is

$$\begin{aligned}
J(N) &= \mathbb{E}\left\{\sum_{k=1}^N [|\tilde{z}(k)|^2 - \gamma^2 |v(k)|^2] \right. \\
&\quad \left. + \mathbb{E}\{V(x(k+1)|x(k))\} - V(x(k))\right\} \\
&\quad - \mathbb{E}\{V(x(N+1))\} \\
&\leq \mathbb{E}\left\{\sum_{k=1}^N [|\tilde{z}(k)|^2 - \gamma^2 |v(k)|^2] \right. \\
&\quad \left. + \mathbb{E}\{V(x(k+1)|x(k))\} - V(x(k))\right\} \\
&\leq \mathbb{E}\left\{\left[e(k)^T v(k)^T \phi_s(Cx(k))^T\right] \bar{\Omega}_f \right. \\
&\quad \left. \times \left[e(k)^T v(k)^T \phi_s(Cx(k))^T\right]^T\right\},
\end{aligned} \tag{21}$$

where

$$\bar{\Omega}_f = \begin{bmatrix} \bar{\Omega}_1 + \bar{L}^T \bar{L} & \bar{\Omega}_3 & \bar{\Omega}_2 \\ * & \left\{ \begin{array}{c} \beta_e (1 - \beta_e) \tilde{B}^T P \tilde{B} - \gamma^2 I \\ + \bar{B}^T P \bar{B} + \bar{G}^T P \bar{G} \end{array} \right\} & \beta_e \alpha_e (1 - \beta_e) \tilde{B}^T P \hat{B} - \beta_e \alpha_e \bar{B}^T P \hat{B} \\ * & * & \beta_e \alpha_e \hat{B}^T P \hat{B} - 2 \alpha_e I \end{bmatrix} \tag{22}$$

with $\bar{\Omega}_3 = \bar{A}^T P \bar{B} + \beta_e(1 - \beta_e)\tilde{A}^T P \tilde{B} + \beta_e \alpha_e(1 - \beta_e)\hat{A}^T P \tilde{B} + \bar{E}^T P \bar{G}$. As $\bar{\Omega}_f < 0$ means $\Omega_s < 0$ which can be deduced from (8), one may conclude that if there are matrices $X > 0$ and $Y > 0$, with appropriate dimensions, $\|\tilde{z}(k)\|_{e_2} < \gamma\|v(k)\|_{e_2}$ holds for all $v(k) \in l_{e_2}([0, \infty); \mathbb{R}^p)$. $\square$

Remark 4. When $\alpha_e = 1$, $\beta_e = 1$, for all $k \in \mathbb{N}$, that is, the output of the sensor is always nonlinear, the problem formulated previously equals to the case that discussed in [19] without considering the uncertainty and network packets loss.

4. Numerical Example

In this section, a numerical example is presented to illustrate the approach proposed in this work. The discrete-time stochastic system parameters are described as follows:

$$\begin{aligned} A &= \begin{bmatrix} 0.15 & 0.2 & 0.3 \\ 0.1 & 0.5 & 0.33 \\ 0.3 & -0.1 & 0.23 \end{bmatrix}, & B &= \begin{bmatrix} 0.2 & -0.12 \\ 0.08 & 0.07 \\ -0.06 & -0.01 \end{bmatrix}, \\ E &= \begin{bmatrix} 0.02 & 0.01 & 0.012 \\ 0.04 & 0.03 & 0.05 \\ 0.05 & 0.02 & 0.01 \end{bmatrix}, & G &= \begin{bmatrix} 0.2 & -0.1 \\ 0.2 & -0.3 \\ 0.2 & -0.3 \end{bmatrix}, \\ D &= \begin{bmatrix} 0.1 & 0.25 \\ 0.3 & 0.6 \end{bmatrix}, & C &= \begin{bmatrix} 0.03 & -0.01 & -0.04 \\ 0.06 & 0.01 & -0.03 \end{bmatrix}, \\ L &= \begin{bmatrix} 0.0367 & -0.0293 & -0.0450 \\ 0.0119 & 0.0133 & 0.0031 \end{bmatrix}, & K_1 &= \begin{bmatrix} 0.6 & 0 \\ 0 & 0.5 \end{bmatrix}, \\ K_2 &= \begin{bmatrix} 1 & 0 \\ 0 & 0.8 \end{bmatrix}, & \phi(u) &= \frac{K_1 + K_2}{2}u + \frac{K_2 - K_1}{2}\sin(u). \end{aligned} \quad (23)$$

With the initial conditions being $x(0) = [1.5 \ 0.4 \ 0.1]^T$ and $\hat{x}(0) = [2 \ 0 \ 1]^T$ for the system and the filter, respectively.

By Theorem 3 and the techniques presented in [31] one can obtain the following matrices through solving LMI (8) for $\gamma = 0.8$, $\alpha_e = 0.8$, and $\beta_e = 0.95$; the filter parameters in (5) can be obtained as

$$\begin{aligned} A_f &= \begin{bmatrix} -0.0083 & 0.0344 & 0.1658 \\ -0.0626 & 0.1099 & 0.1631 \\ 0.0620 & -0.0689 & 0.0550 \end{bmatrix}, \\ B_f &= \begin{bmatrix} -0.1941 & -0.1172 \\ -0.1212 & -0.0262 \\ -0.0352 & -0.0189 \end{bmatrix}. \end{aligned} \quad (24)$$

Under the condition that the external disturbance is taken as $v(k) = [1/(1 + k^2) \ 1/(1 + k^2) \ 1/(1 + k^2)]^T$, the trajectories of the system state $x(k)$, filtering system state $\hat{x}(k)$, and the difference between them $x(k) - \hat{x}(k)$ are shown in Figures 1, 2, and 3, respectively.

FIGURE 1: The trajectories of state $x(k)$.

FIGURE 2: The trajectories of filtering state $\hat{x}(k)$.

FIGURE 3: The trajectories of $x(k) - \hat{x}(k)$.

5. Conclusion

The filtering problem for a class of discrete-time stochastic systems with RON on the sensor parts over communication channels is studied in this paper. Both the theoretical analysis and numerical simulations are introduced to verify the effectiveness of the proposed method in this work. These conditions are obtained by introducing a simple Lyapunov function, so the computation process is greatly simplified, and the filter parameters can be obtained simultaneously by solving the sufficient conclusion. Moreover, H_∞ performance analysis has been satisfied.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (NSFC: 61134007), National Natural Science Foundation of China (NSFC: 60736021), The National High Technology Research and Development Program of China (863 Program 2008AA042902), Open Project of the State Key Laboratory of Industrial Control Technology under Grant no. ICT1002, National Natural Science Foundation of China (61104102), and China Postdoctoral Science Foundation (20100480086).

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