Research Article

Stabilization of a Wireless Networked Control System with Packet Loss and Time Delay: An ADS Approach

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The stabilization problem of a wireless networked control system is considered in this paper. Both time delay and packet loss exist simultaneously in the wireless network. The system is modeled as an asynchronous dynamic system (ADS) with unstable subsystems. A sufficient condition for the system to be stable is presented. A numerical example is given to demonstrate the effectiveness of the proposed approach.

1. Introduction

Networked control system (NCS) has received increasing attention in recent years due to its advantages, such as low cost, easy maintenance, and flexibility [1], and it has been applied in many areas, such as automobiles, building supervision, and manufacturing plants [2]. As an alternative for the wired network, the wireless network also has a wide range of applications due to its mobility; see [3–5] and the references therein.

As the network is introduced into the control system, many new problems arise due to the limited transmission capacity of the network, such as network-induced delay and packet loss. Many studies have been done in these areas, and some results are available in the literature, for example, the delay issue was studied in [6–12] and packet loss was studied in [13–18]. However, in all the results above, the time delay and packet loss are considered separately, while in practice they exist simultaneously in the network. Recently, some papers also deal with this problem [19–22]. In [19], a continuous system is considered and the effect of packet loss is considered to be equal to the time delay. A new model is proposed in [20] while the delay is assumed to be less than one sampling period. However, as to the discrete-time

system, if the plant to be controlled is unstable intuitively, no method has been addressed, which will be considered here. In this paper, a switched system with time-varying delay model is firstly proposed to describe the wireless NCS, and a sufficient condition is given to stabilize the closed-loop system, and the controller is designed to be dependent on both the packet loss and time delay. A numerical example shows the effectiveness of the proposed approach.

The organization of this paper is as follows. The structure of the wireless network control system is given in Section 2 and a switched system with time-varying delay model is established. New criteria for the switched time-varying delay system with unstable subsystem to be stable are obtained in Section 3. The controller design problem is solved in Section 4, while Section 5 gives an example to illustrate the effectiveness of the proposed approach. Finally, the conclusion is given in Section 6.

2. Modeling of the Wireless NCS

In practice, the sensors may be very far from the controller and actuators, for example [16], where the sensors and

FIGURE 1: System configuration.

FIGURE 2: System graph.

the controller are connected by a wireless network. The system structure is as in Figure 1.

As the wireless network is introduced into the system, the data, from the sensor to the controller, will be lost or delayed due to the disturbance in the environment and the limited transmission capacity of the wireless network. The graph of the system can be described as in Figure 2, where the system to be controlled is

$$x(k+1) = Ax(k) + Bu(k), \quad (1)$$

$x(k) \in \mathbb{R}^n$ is the system state, $u(k) \in \mathbb{R}^m$ is the control input d_k is the time-varying delay of the network, and $K_{\text{delay, loss}}$ is the controller to be designed, which depends on the time delay and packet loss, denoted as $K_{d,s}$ in the graph for simplicity. The data loss in the network can be viewed as the behaviour results of a pair of switches [14]. If the switches are closed, the data is transmitted successfully (3), otherwise, the previous value will be used in the controller (2). Thus, the dynamics of the switches can be modeled as

$$S : w(k) = w(k-1), \quad (2)$$

$$\bar{S} : \begin{cases} x(k) & \text{Not delayed,} \\ x(k-d_k) & \text{Delayed.} \end{cases} \quad (3)$$

Without loss of generality, the following assumptions are made for the description of the system model.

- (A1) The sensors and controller are time driven.
- (A2) The data transmitted by the sensors are time stamped, so the controller can obtain the delay value of each data packet.

- (A3) The maximum delay in the network is d_M , where d_M is a known integer.

Let

$$z(k) = \begin{bmatrix} x^T(k) & w^T(k-1) \end{bmatrix}^T. \quad (4)$$

Then the dynamics of the closed-loop system can be described by the following subsystems.

Case 1. When the data is lost, the corresponding controller is denoted as $K_{0,0}$, and the closed-loop system is described as

$$\Sigma_1 : z(k+1) = A_1 z(k), \quad A_1 = \begin{bmatrix} A & BK_{0,0} \\ 0 & I \end{bmatrix}. \quad (5)$$

Case 2. When the data is transmitted successfully without delay, the corresponding controller is denoted as $K_{0,1}$, and the closed-loop system is described as

$$\Sigma_2 : z(k+1) = A_2 z(k), \quad A_2 = \begin{bmatrix} A + BK_{0,1} & 0 \\ I & 0 \end{bmatrix}. \quad (6)$$

Case 3. When the data is transmitted successfully with d_k -step delay, the corresponding controller is denoted as $K_{d_k,1}$, and the closed-loop system is described as

$$\Sigma_3 : z(k+1) = A_3 z(k) + A_{d3} z(k-d_k), \quad (7)$$

$$A_3 = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}, \quad A_{d3} = \begin{bmatrix} BK_{d_k,1} & 0 \\ I & 0 \end{bmatrix}.$$

So, the closed-loop system can be described as a system switched within three subsystems: Σ_1 , Σ_2 , and Σ_3 .

If $d_k = 0$ in Σ_3 , then Σ_3 becomes Σ_2 , and the difference of Σ_2 and Σ_1 is only the first element value of its system matrix. So, Case 3 includes Cases 1 and 2 by appropriately changing the value of the matrices. So the wireless networked control system can be represented by the following discrete-time switched system with time-varying delay:

$$z(k+1) = A_i z(k) + A_{di} z(k-d_k), \quad i = 1, 2, 3, \quad (8)$$

$$d_k = 1, \dots, d_M,$$

where $A_{d1} = A_{d2} = 0$.

From above, it is easy to see that if the open system is unstable, when the data loss happens, the system will be uncontrollable, and the corresponding subsystem Σ_1 is unstable.

The following lemma plays a crucial role in the proof of our main results.

Lemma 1 (see [23]). For any appropriately dimensioned matrices $R = R^T > 0$, Y , X , and two positive integer time-varying d_{k_1} , d_{k_2} satisfying $d_{k_1} + 1 \leq d_{k_2} < d_M$, the following equality holds:

$$\begin{aligned} - \sum_{l=k-d_{k_2}}^{k-d_{k_1}-1} \eta^T(l) R \eta(l) &= 2\xi^T(k) Y (x(k-d_{k_1}) - x(k-d_{k_2})) \\ &+ (d_{k_2} - d_{k_1}) \xi^T(k) X \xi(k) \\ &- \sum_{l=k-d_{k_2}}^{k-d_{k_1}-1} \begin{bmatrix} \xi(k) \\ \eta(l) \end{bmatrix}^T \begin{bmatrix} X & Y \\ * & R \end{bmatrix} \begin{bmatrix} \xi(k) \\ \eta(l) \end{bmatrix}. \end{aligned} \quad (9)$$

3. Stability of the Switched System with Time-Varying Delay

As to the discrete-time switched system with time-varying delay, some results are obtained in the literature to test its stability [24–30]. However, all the results above are obtained under the assumption that all the subsystems are stable and they cannot be applied here. So we will give criteria to test the stability of the discrete-time switched time-varying delay system with unstable subsystem in this section.

Consider the following discrete-time switched system with time-varying delay,

$$\begin{aligned} z(k+1) &= A_i z(k) + A_{di} z(k-d_k), \quad i = 1, 2, \dots, N, \\ d_k &= 1, \dots, d_M, \end{aligned} \quad (10)$$

where N is the number of the subsystems.

Choose the following Lyapunov function:

$$\begin{aligned} V(k) &= V_1(k) + V_2(k) + V_3(k) + V_4(k), \\ V_1(k) &\triangleq z^T(k) P z(k), \\ V_2(k) &\triangleq \sum_{l=k-d_k}^{k-1} \lambda^{-2(l-k)} z^T(l) Q_1 z(l) \\ &+ \sum_{m=-d_M+2}^0 \sum_{l=k+m-1}^{k-1} \lambda^{-2(l-k)} z^T(l) Q_1 z(l), \quad (11) \\ V_3(k) &\triangleq \sum_{l=k-d_M}^{k-1} \lambda^{-2(l-k)} z^T(l) Q_2 z(l), \\ V_4(k) &\triangleq \sum_{m=-d_M}^{-1} \sum_{l=k+m}^{k-1} \lambda^{-2(l-k)} \eta^T(l) R \eta(l), \end{aligned}$$

where $\eta(l) = \lambda^{-1} z(l+1) - z(l)$.

In the following lemma, a criterion is given to guarantee that the Lyapunov function $V(k)$ exponentially decays or increases along any state trajectory of system (10).

Lemma 2. For a given scalar $\lambda > 0$ and any delay satisfying $0 \leq d \leq d_M$, if there exist symmetric positive definite matrices

P , Q_1 , Q_2 , R , symmetric matrix $X = \begin{bmatrix} X_1 & X_2 \\ * & X_3 \end{bmatrix} \geq 0$ and appropriate matrices G , $N = [N_1^T \ N_2^T]^T$, $M = [M_1^T \ M_2^T]^T$, such that the following LMIs hold:

$$\begin{aligned} \Psi(d_k) &= \begin{bmatrix} \Psi_{11} & \Psi_{12} & -N_1 & P + (\bar{A}_i - I)G^T - G \\ * & \Psi_{22} & -N_2 & \bar{A}_{di}G^T \\ * & * & -Q_2 & 0 \\ * & * & * & P + d_M R - G - G^T \end{bmatrix} < 0, \\ d_k &= 0, \dots, d_M, \\ \begin{bmatrix} X & N \\ * & R \end{bmatrix} &> 0, \quad \begin{bmatrix} X & M \\ * & R \end{bmatrix} > 0, \end{aligned} \quad (12)$$

where

$$\begin{aligned} \Psi_{11} &= d_M Q_1 + Q_2 + d_M X_1 + \text{Sym}(M_1 + G(\bar{A}_i^T - I)), \\ \Psi_{12} &= d_M X_2 + N_1 - M_1 + M_2^T + G \bar{A}_{di}^T, \\ \Psi_{22} &= -Q_1 + d_M X_3 + \text{Sym}(N_2 - M_2), \end{aligned} \quad (13)$$

and $\bar{A}_i = \lambda^{-1} A_i$, $\bar{A}_{di} = \lambda^{-1-d_k} A_{di}$, then

$$V(k) < \lambda^{2(k-k_0)} V(k_0). \quad (14)$$

Proof. See the appendix. $\square$

Remark 3. The last two terms of ΔV_4 are abandoned to obtain our result, which may introduce some conservatism. In order to reduce the conservatism, a free matrix G is introduced.

Suppose that the rates of the corresponding subsystems are $r_1, r_2, \dots, r_N$, which represent the fraction of time that each subsystem occurs, thus $\sum_{i=1}^N r_i = 1$. The following theorem gives a sufficient condition for the system (10) to be stable.

Theorem 4. The switched system with time-varying delay (10) is stable if there exist a Lyapunov function $V(k)$ and scalars $\lambda_1, \dots, \lambda_N$ corresponding to each subsystem such that

$$\begin{aligned} V(k) &< \lambda_i^{2(k-k_0)} V(k_0), \quad i = 1, \dots, N, \\ \prod_{i=1}^N \lambda_i^{r_i} &< 1. \end{aligned} \quad (15)$$

Proof. Suppose that the transition time of the subsystems is $t_1 = 0, t_2, \dots, t_k = k$, then

$$\begin{aligned} V(k) &< \lambda_{t_k}^{t_k - t_{k-1}} V(t_{k-1}) < \lambda_{t_k}^{t_k - t_{k-1}} \lambda_{t_{k-1}}^{t_{k-1} - t_{k-2}} V(t_{k-2}) \\ &< \dots < \prod_{i=1}^N \lambda_i^{T_i} V(0), \end{aligned} \quad (16)$$

where T_i is the total time that one subsystem occurs and as $k \rightarrow \infty$, $r_i k = T_i$. Thus,

$$V(k) < \prod_{i=1}^N \lambda_i^{T_i} V(0) = \left(\prod_{i=1}^N \lambda_i^{r_i} \right)^k V(0). \quad (17)$$

It is obvious that the system is stable if $\prod_{i=1}^N \lambda_i^{r_i} < 1$, which completes the proof. $\square$

4. Controller Design for the Wireless NCS

In this section, the controller of the wireless networked control system is given using the criteria in Section 3. As the data packets are time stamped, and the packet loss and time delay are known to the controller, the controller is designed to depend on both the packet loss and time delay.

Theorem 5. *The system (8) is stable if there exist scalars λ_i , $i = 1, 2, 3$, satisfying $\prod_{i=1}^3 \lambda_i^{r_i} < 1$ and there exist $2n \times 2n$ symmetric positive definite matrices $\mathcal{P}$, $\mathcal{Q}_1$, $\mathcal{Q}_2$, $\mathcal{R}$, $\mathcal{X}_1$, $\mathcal{X}_3$, matrices $\mathcal{M}_1$, $\mathcal{M}_2$, $\mathcal{N}_1$, $\mathcal{N}_2$, $\mathcal{X}_2$ and $n \times n$ matrices G_1 , G_2 , $m \times n$ matrices $K_{g_{0,0}}$, $K_{g_{0,1}}$, $K_{g_{d_k,1}}$, $d_k = 1, \dots, d_M$ such that the following LMIs hold:*

$$\Phi_0 + \Phi_1 < 0,$$

$$\Phi_0 + \Phi_2 < 0,$$

$$\Phi_0 + \Phi_3(d_k) < 0, \quad d_k = 1, \dots, d_M, \quad (18)$$

$$\begin{bmatrix} \mathcal{X}_1 & \mathcal{X}_2 & \mathcal{N}_1 \\ * & \mathcal{X}_3 & \mathcal{N}_2 \\ * & * & \mathcal{R} \end{bmatrix} > 0, \quad \begin{bmatrix} \mathcal{X}_1 & \mathcal{X}_2 & \mathcal{M}_1 \\ * & \mathcal{X}_3 & \mathcal{M}_2 \\ * & * & \mathcal{R} \end{bmatrix} > 0,$$

where

$$\Phi_0 = \begin{bmatrix} \Theta_{11} & d_M \mathcal{X}_2 + \mathcal{N}_1 - \mathcal{M}_1 + \mathcal{M}_2^T & -\mathcal{N}_1 & \mathcal{P} - \mathcal{G}^T - \mathcal{G} \\ * & -\mathcal{Q}_1 + d_M \mathcal{X}_3 + \text{Sym}(\mathcal{N}_2 - \mathcal{M}_2) & -\mathcal{N}_2 & 0 \\ * & * & -\mathcal{Q}_2 & 0 \\ * & * & * & \mathcal{P} + d_M \mathcal{R} - \mathcal{G} - \mathcal{G}^T \end{bmatrix}, \quad (19)$$

$$\Theta_{11} = d_M \mathcal{Q}_1 + \mathcal{Q}_2 + d_M \mathcal{X}_1 + \text{Sym}(\mathcal{M}_1 + \mathcal{G}),$$

$$\mathcal{G} = \begin{bmatrix} G_1 & 0 \\ 0 & G_2 \end{bmatrix},$$

$$\Phi_1 = \begin{bmatrix} \Lambda_{11} & 0 & 0 & \Lambda_{14} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Lambda_{11} = \begin{bmatrix} \lambda_1^{-1} A G_1^T + G_1 \lambda_1^{-1} A^T & B K_{g_{0,0}} \\ * & 0 \end{bmatrix},$$

$$\Lambda_{14} = \begin{bmatrix} \lambda_1^{-1} A G_1^T & B K_{g_{0,0}} \\ 0 & 0 \end{bmatrix},$$

$$\Phi_2 = \begin{bmatrix} \Omega_{11} & 0 & 0 & \Omega_{14} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Omega_{11} = \begin{bmatrix} \text{Sym}(\lambda_2^{-1} A G_1^T + B K_{g_{0,1}}) & 0 \\ * & 0 \end{bmatrix},$$

$$\Omega_{14} = \begin{bmatrix} \lambda_2^{-1} A G_1^T + B K_{g_{0,1}} & 0 \\ 0 & 0 \end{bmatrix},$$

$$\Phi_3 = \begin{bmatrix} Y_{11} & Y_{12} & 0 & Y_{14} \\ 0 & 0 & 0 & Y_{24} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$Y_{11} = \begin{bmatrix} \lambda_3^{-1} A G_1^T & 0 \\ 0 & 0 \end{bmatrix},$$

$$Y_{12} = \begin{bmatrix} K_{g_{d_k,1}} B^T & G_1 \\ 0 & 0 \end{bmatrix},$$

$$Y_{14} = \begin{bmatrix} \lambda_3^{-1} A G_1^T & 0 \\ 0 & 0 \end{bmatrix}, \quad Y_{24} = \begin{bmatrix} B K_{g_{d_k,1}} & 0 \\ G_1^T & 0 \end{bmatrix}, \quad (20)$$

and the controllers are given by

$$K_{0,0} = K_{g_{0,0}} G_2^{-T}, \quad K_{0,1} = K_{g_{0,1}} G_1^{-T}, \quad (21)$$

$$K_{d_k,1} = K_{g_{d_k,1}} G_1^{-T}, \quad d_k = 1, \dots, d_M.$$

Proof. Substituting the system matrices of (8) into (12) and letting $K_{g_{0,0}} = K_{0,0} G_2^T$, $K_{g_{0,1}} = K_{0,1} G_1^T$, and $K_{g_{d_k,1}} = K_{d_k,1} G_1^T$, it is easy to get the results according to Lemma 2 and Theorem 4. $\square$

Remark 6. The computing complexity of Theorem 5: the line number of the LMIs is $(36n + 8d_M n)$ and the number of the scalar variables is $(32n^2 + 5n + 3mn)$.

5. Numerical Example

In this section, an example is given to show the effectiveness of the proposed approach.

Consider the following discrete-time system:

$$A = \begin{bmatrix} 0.8862 & 0.2586 & 0.5038 \\ 0.9311 & 0.8979 & 0.6128 \\ 0.1908 & 0.5934 & 0.8194 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}. \quad (22)$$

The eigenvalues of A are 1.8734 , $0.3650 + 0.2369i$, and $0.3650 - 0.2369i$ and the system will diverge if there is no control. Suppose the event rates of the packet loss and time delay are $r_1 = 0.4$, and $r_3 = 0.5$ respectively, and the maximum delay is $d_M = 3$, and choose $\lambda_1 = 1.9$, $\lambda_2 = 0.4$, and $\lambda_3 = 0.7$. As

$$\lambda_1^{r_1} \lambda_2^{r_2} \lambda_3^{r_3} = 1.9^{0.4} \times 0.4^{0.1} \times 0.7^{0.5} = 0.9869 < 1, \quad (23)$$

using Theorem 5, a feasible solution is given and the corresponding controllers are

$$\begin{aligned} K_{0,0} &= [-0.0009 \quad 0.3295 \quad -0.2700] \times 10^{-3}, \\ K_{0,1} &= [-0.7707 \quad -0.6772 \quad -0.7465], \\ K_{1,1} &= [-0.5713 \quad -0.5048 \quad -0.5528], \\ K_{2,1} &= [-0.3999 \quad -0.3533 \quad -0.3870], \\ K_{3,1} &= [-0.2799 \quad -0.2473 \quad -0.2709]. \end{aligned} \quad (24)$$

The simulation is shown in Figure 3. The first graph depicts the time delay and packet loss of the wireless network, if the delay value is -1 , it means the packet is lost. The second graph is the trajectories of the closed-loop system. We can see that the trajectories of the system diverge at the instance when the packets are lost, while they converge to zeros at last, which proves the effectiveness of the proposed approach.

6. Conclusion

The stabilization problem for a wireless networked control system is studied in this paper. A new model is proposed to describe the system as a discrete-time switched system with time-varying delays. New criteria for the system to be stable with unstable subsystem are obtained, based on which, delay- and packet-loss-dependent controller is given to stabilize the system. A numerical example illustrates the effectiveness of the proposed approach.

Appendix

Proof of Lemma 2.

FIGURE 3: System simulation.

Proof. Applying the transformation $z(k) = \lambda^{(k-k_0)} \xi(k)$, $\lambda > 0$, we obtain the following system from (10):

$$\begin{aligned} \xi(k+1) &= \bar{A}_i \xi(k) + \bar{A}_{di} \xi(k-d_k), \quad i = 1, 2, \dots, N, \\ d_k &= 1, \dots, d_M, \end{aligned} \quad (A.1)$$

where $\bar{A}_i = \lambda^{-1} A_i$ and $\bar{A}_{di} = \lambda^{-1-d_k} A_{di}$.

Choose the following Lyapunov function for system (A.1):

$$\begin{aligned} W(k) &= W_1(k) + W_2(k) + W_3(k) + W_4(k), \\ W_1(k) &\triangleq \xi^T(k) P \xi(k), \\ W_2(k) &\triangleq \sum_{l=k-d_k}^{k-1} \xi^T(l) Q_1 \xi(l) + \sum_{m=-d_M+2}^0 \sum_{l=k+m-1}^{k-1} \xi^T(l) Q_1 \xi(l), \\ W_3(k) &\triangleq \sum_{l=k-d_M}^{k-1} \xi^T(l) Q_2 \xi(l), \\ W_4(k) &\triangleq \sum_{m=-d_M}^{-1} \sum_{l=k+m}^{k-1} \delta^T(l) R \delta(l), \end{aligned} \quad (A.2)$$

where $\delta(l) = \xi(l+1) - \xi(l)$. Using Lemma 1, it is easy to obtain that

$$\begin{aligned} \Delta W(k) &= \xi^T(k+1) P \xi(k+1) - \xi^T(k) P \xi(k) \\ &\quad + \xi^T(k) Q_1 \xi(k) + \sum_{l=k-d_{k+1}+1}^{k-1} \xi^T(l) Q_1 \xi(l) \end{aligned}$$

$$\begin{aligned}
& - \sum_{l=k-d_k+1}^{k-1} \xi^T(l) Q_1 \xi(l) + \xi^T(k) Q_2 \xi(k) \\
& - \xi^T(k-d_M) Q_2 \xi(k-d_M) \\
& + d_M \delta^T(k) R \delta(k) + d_M \begin{bmatrix} \xi(k) \\ \xi(k-d_k) \end{bmatrix}^T X \begin{bmatrix} \xi(k) \\ \xi(k-d_k) \end{bmatrix} \\
& + 2 \begin{bmatrix} \xi(k) \\ \xi(k-d_k) \end{bmatrix}^T \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} (\xi(k-d_k) - \xi(k-d_M)) \\
& + 2 \begin{bmatrix} \xi(k) \\ \xi(k-d_k) \end{bmatrix}^T \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} (\xi(k) - \xi(k-d_k)) \\
& - \sum_{l=k-d_M}^{k-d_k-1} \begin{bmatrix} \xi(k) \\ \xi(k-d_k) \\ \delta(l) \end{bmatrix}^T \begin{bmatrix} X & N \\ * & R \end{bmatrix} \begin{bmatrix} \xi(k) \\ \xi(k-d_k) \\ \delta(l) \end{bmatrix} \\
& - \sum_{l=k-d_k}^{k-1} \begin{bmatrix} \xi(k) \\ \xi(k-d_k) \\ \delta(l) \end{bmatrix}^T \begin{bmatrix} X & M \\ * & R \end{bmatrix} \begin{bmatrix} \xi(k) \\ \xi(k-d_k) \\ \delta(l) \end{bmatrix} \\
& + 2(x^T(k) + \eta^T(k)) G \left[(A_i^T - I)x(k) \right. \\
& \quad \left. + A_{di}^T x(k-d_k) - \eta(k) \right] \\
& < \zeta^T(k) \Psi(d_k) \zeta(k) < 0,
\end{aligned} \tag{A.3}$$

where $\zeta^T(k) = [\xi^T(k) \ \xi^T(k-d_k) \ \xi^T(k-d_M) \ \delta^T(k)]$, which implies that $W(k) < W(k_0)$ for any $k \geq k_0$. Furthermore,

$$\begin{aligned}
V_1(k) &= z^T(k) P z(k) = \lambda^{2(k-k_0)} \xi^T(k) P \xi(k) = \lambda^{2(k-k_0)} W_1(k), \\
V_2(k) &= \sum_{l=k-d_k}^{k-1} \lambda^{-2(l-k)} \lambda^{2(l-k_0)} \xi^T(l) Q_1 \xi(l) \\
&+ \sum_{m=-d_M+2}^0 \sum_{l=k+m-1}^{k-1} \lambda^{-2(l-k)} \lambda^{2(l-k_0)} \xi^T(l) Q_1 \xi(l) \\
&= \lambda^{2(k-k_0)} W_2(k), \\
V_3(k) &= \sum_{l=k-d_M}^{k-1} \lambda^{-2(l-k)} \lambda^{2(l-k_0)} \xi^T(l) Q_2 \xi(l) \\
&= \lambda^{2(k-k_0)} W_3(k), \\
V_4(k) &\triangleq \sum_{m=-d_M}^{-1} \sum_{l=k+m}^{k-1} \lambda^{-2(l-k)} \lambda^{2(l-k_0)} \delta^T(l) R \delta(l) = \lambda^{2(k-k_0)} W_4(k),
\end{aligned} \tag{A.4}$$

which means that $V(k) = \lambda^{2(k-k_0)} W(k)$.

Noting that $W(k_0) = V(k_0)$, it is easy to verify that $V(k) < \lambda^{2(k-k_0)} V(k_0)$. This completes the proof. $\square$

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