Research Article

Robust Fault-Tolerant Control for Uncertain Networked Control Systems with State-Delay and Random Data Packet Dropout

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A robust fault-tolerant controller design problem for networked control system (NCS) with random packet dropout in both sensor-to-controller link and controller-to-actuator link is investigated. A novel stochastic NCS model with state-delay, model uncertainty, disturbance, probabilistic sensor failure, and actuator failure is proposed. The random packet dropout, sensor failures, and actuator failures are characterized by a binary random variable. The sufficient condition for asymptotical mean-square stability of NCS is derived and the closed-loop NCS satisfies H_∞ performance constraints caused by the random packet dropout and disturbance. The fault-tolerant controller is designed by solving a linear matrix inequality. A numerical example is presented to illustrate the effectiveness of the proposed method.

1. Introduction

Networked control systems (NCSs) are one type of distributed control systems, in which the information of control system components (sensors, controllers, actuators, etc.) is exchanged via communication networks (see Figure 1). Compared with the conventional point-to-point control systems, NCSs have many advantages, such as low cost of installation and maintenance, ease of diagnosis, and flexible architectures. However, the network in the control systems also bring many problems, such as network-induced delay, packet dropout, multiple channel transmission (as in [1–4]). Recent researches have a deep look into the controller and filter design for NCSs without faults (see [5–10]) and the references therein.

Actually, NCSs are more vulnerable to faults than conventional control systems, due to the complexity introduced by the network. It is very significant to guarantee security and reliability of NCSs, because modern technological systems rely on sophisticated control systems to meet increased performance and safety requirement. Therefore, research on fault-tolerant control (FTC) for NCSs has attracted more and more attention from both industry and academia.

However, research on FTC for NCSs is quite different from the ones for conventional control systems in many aspects (see [11–15]). A suitable architecture for FTC of NCSs must take into consideration the dynamical behaviour of network. In most research on FTC for NCSs, the fault model is described in a static way (as in [16–20]). Actually, faults often happen in a random way, so it is suitable to be studied in a dynamic way (as in [21–25]). In [21], the probabilistic sensor reductions are modeled by using a random variable that obeys a specific distribution in a known interval. In [22], the entire sensor failures or missing measurements have been described as a Bernoulli distributed variable. The reliable control design is considered for NCSs against probabilistic actuator fault with different failure rates in [23]. But, only actuator failures or sensor failures are considered in [16–23].

As we know, many engineering control systems such as conventional oil-chemical industrial processes, nuclear reactors, long transmission lines in pneumatic, hydraulic, and rolling mill systems, NCSs contain some time-delay effects, model uncertainties (as in [26, 27]), and external disturbances. However, most of the aforementioned researches discuss FTC for NCSs without model uncertainty and disturbance (as in [16–20, 23, 24]).

FIGURE 1: Schematic description of a distributed NCS.

Therefore, from the above description, considering random packet dropout, the robust FTC for state-delay uncertain NCSs with both probabilistic sensors failures and actuators failures is still a challenging problem.

In this paper, we study the robust FTC problem for NCSs with random packet dropout in both sensor-to-controller (S-C) link and controller-to-actuator (C-A) link. A new stochastic NCS model with fault is proposed, which includes the state-delay, model uncertainty, disturbance, random packet dropout, probabilistic sensors failures, and actuators failures. The random packet dropout, the sensor failure and the actuator failure are described as a binary random variable. The aim of this paper is to design a dynamic fault-tolerant controller for the NCS including packet dropouts, both sensor failures and actuator failures. The closed-loop NCS can be asymptotical mean-square stability and satisfies the performance constraint.

The rest of the paper is organized as follows. The problem is formulated in Section 2, a new stochastic NCS with probabilistic sensors failures and actuators failures is modelled. Section 3 presents the integral analysis of asymptotical mean-square stability for stochastic NCS with sensor and actuator faults. Section 4 designs a dynamic fault-tolerant controller. Section 5 gives a numerical example to demonstrate the effectiveness of the proposed method. Concluding remarks are made in Section 6.

2. Problem Formulation

Consider the following uncertain linear state-delay system:

$$\begin{aligned} x_{k+1} &= (A + \Delta A)x_k + A_d x_{k-d} + B u_k + D_0 \omega_k, \\ y_k &= C_0 x_k, \\ z_k &= C x_k + C_d x_{k-d} + D_1 \omega_k, \end{aligned} \quad (1)$$

where $x_k \in \mathbb{R}^n$, $y_k \in \mathbb{R}^m$, $u_k \in \mathbb{R}^p$, $z_k \in \mathbb{R}^q$ denote the state, the sensor measurement, the control input, and the controlled output, respectively. $\omega_k \in \mathbb{R}^r$ is the disturbance input belonging to $l_2[0, \infty)$. $A, A_d, B, C_0, C, C_d, D_0, D_1$, are known real constant matrices with appropriate dimensions. $d > 0$ is a known delay. ΔA denotes the model uncertainty, which satisfies $\Delta A = H F_k N$, $F_k^T F_k < I$, F_k represents an unknown real-valued time-varying matrix.

Figure 1 shows a typical feedback loop of NCS. Due to network congestion, traffic load balancing, or other unpredictable network behavior, the network-induced delay, data packet dropout, disorder may occur at the same time. In this paper, we focus on the data packet dropout phenomenon. Some assumptions in this paper are as follows.

- (1) The sensor is clock-driven, the controller and the actuator are event-driven.
- (2) Data packet dropouts occur in both S-C link and C-A link.
- (3) Data are single-packet transmission with timestamp.
- (4) Ignore the effects of quantization and asynchronous error in this paper.

Remark 1. A clock-driven sensor can send measurements to network periodically and is often used in real-time computing. The advantage of event-driven controller/actuator is that the controller/actuator will be updated as soon as the new data packet comes.

Remark 2. Taking the Internet as an illustration, the Transmission Control Protocol (TCP) is one of the core protocols of the Internet Protocol Suite. TCP is responsible for verifying the correct delivery of data from client to server. Data can be lost in the intermediate network. TCP adds support to detect errors or lost data and to trigger retransmission until the data is correctly and completely received. Thus, TCP is optimized for accurate delivery rather than timely delivery. Furthermore, for real-time feedback control, it is appropriate to discard the old data and transmit a new packet if it is available.

Therefore, we assume if the total network-induced delay τ_k is larger than a sampling period, the output terminal will actively discard this packet, which means the network-induced delay problem can be considered as a packet-dropout problem. And the receiver with a buffer can rearrange the packets by reading the information of timestamp; in this way, the data disorder problem will be solved. Hence, we only focus on the data packet dropout issue in this paper.

The binary random variable θ_k is an identically distributed (i.i.d.) process. $\theta_k = 1$ means that there is no packet dropout, and the sensors and the actuators are reliable; $\theta_k = 0$ means packet is lost, and the sensors and the actuators have failures. The probability distribution of θ_k is $P\{\theta_k = 0\} = p$, $P\{\theta_k = 1\} = 1 - p$, where $p \in (0, 1)$ indicates the sensor/actuator failure rate and the packet dropout rate.

Considering the channel from the sensor to the controller, the sensor measurement y_k will be updated to $\bar{y}_k$ as follows:

$$\bar{y}_k = \theta_k y_k + (1 - \theta_k) \bar{y}_{k-1}. \quad (2)$$

Considering the channel from the controller to the actuator, the control output u_k will be updated as follows:

$$u_k = \theta_k u_k + (1 - \theta_k) u_{k-1}. \quad (3)$$

Remark 3. From expressions (2) and (3), when $\theta_k = 0$ at time k , the sensor measurements and control information at time k are missing. The last available measurement $\bar{y}_{k-1}$ and controller output u_{k-1} stored in a buffer are utilized to substitute the missing data, which means at least one packet can be transmitted successfully in a sampling period.

Since the random variable θ_k also represents sensor failures and actuator failures, the dynamic fault-tolerant controller is designed

$$\begin{aligned}\hat{x}_{k+1} &= A_f \hat{x}_k + B_f \bar{y}_k, \\ u_k &= C_f \hat{x}_k.\end{aligned}\quad (4)$$

Define $\eta_k = [x_k^T, \hat{x}_k^T]^T$, $d_k = [u_{k-1}^T, \bar{y}_{k-1}^T]^T$, the stochastic NCS with probabilistic sensor failures and actuator failures

$$\begin{aligned}\eta_{k+1} &= \tilde{A}\eta_k + (\theta_k - p)\tilde{A}_2\eta_k + \tilde{A}_d Z \eta_{k-d} \\ &\quad + (\theta_k - p)\tilde{B}_1 d_k + \tilde{B}_2 d_k + \tilde{D}_1 \omega_k, \\ z_k &= \tilde{C}\eta_k + C_d Z \eta_{k-d} + D_1 \omega_k,\end{aligned}\quad (5)$$

where

$$\begin{aligned}\tilde{A} &= \tilde{A}_1 + \tilde{H}F_k\tilde{N} = \begin{bmatrix} A & pBC_f \\ pB_f C_0 & A_f \end{bmatrix} + \begin{bmatrix} H \\ 0 \end{bmatrix} F_k \begin{bmatrix} N & 0 \end{bmatrix}, \\ \tilde{A}_2 &= \begin{bmatrix} 0 & BC_f \\ B_f C_0 & 0 \end{bmatrix}, \quad \tilde{A}_d = \begin{bmatrix} A_d \\ 0 \end{bmatrix}, \\ \tilde{B}_1 &= \begin{bmatrix} -B & 0 \\ 0 & -B_f \end{bmatrix}, \quad \tilde{B}_2 = \begin{bmatrix} (1-p)B & 0 \\ 0 & (1-p)B_f \end{bmatrix}, \\ \tilde{C} &= \begin{bmatrix} C & 0 \end{bmatrix}, \quad \tilde{D}_1 = \begin{bmatrix} D_0 \\ 0 \end{bmatrix}, \quad Z = \begin{bmatrix} I & 0 \end{bmatrix}.\end{aligned}\quad (6)$$

The aim of this paper is to design a dynamic fault-tolerant controller for the NCS (5), such that for all the possible data packet dropout and failures, the system (5) satisfies the following requirements.

(Q1) The closed-loop NCS (5) is asymptotically mean-square stable.

(Q2) Under the zero-initial condition, the output z_k satisfies

$$\sum_{k=0}^{\infty} \mathbb{E}\{\|z_k\|^2\} < \gamma^2 \sum_{k=0}^{\infty} \mathbb{E}\{\|\omega_k\|^2\} + \delta^2 \sum_{k=0}^{\infty} \mathbb{E}\{\|d_k\|^2\} \quad (7)$$

for all nonzero d_k, ω_k , where $\gamma, \delta > 0$ are the scalars we will design.

3. The Stability Analysis of NCS

In this section, the stability analysis for the NCS (5) is discussed.

Lemma 4 (as in [28]). Let $W = W^T < 0$, H, N be matrices of appropriate dimensions, with F satisfying $F^T F \leq I$, then $W + HFN + N^T F^T H^T < 0$ holds, if and only if there exists a $\varepsilon > 0$ such that $W + \varepsilon HH^T + \varepsilon^{-1} N^T N < 0$.

Definition 5. The NCS with faults given by (5) with $\omega_k = 0$, $d_k = 0$, is asymptotically mean-square stable, if for any initial state, $\lim_{k \rightarrow \infty} \mathbb{E}\{\|z_k\|^2\} = 0$ holds.

Theorem 6. Given $\gamma > 0, \delta > 0$, the system (5) is asymptotically mean-square stable, and the output z_k satisfies (7), if there exist matrices $P = P^T > 0$, and $Q = Q^T > 0$ satisfying

$$\begin{bmatrix} -P & 0 & P\tilde{A} & P\tilde{A}_d & P\tilde{D}_1 & 0 & \sigma P\tilde{B}_1 & P\tilde{B}_2 & 0 \\ * & -I & \tilde{C} & C_d & D_1 & 0 & 0 & 0 & 0 \\ * & * & -P & 0 & 0 & \sigma\tilde{A}_2^T P & 0 & 0 & Z^T \\ * & * & * & -Q & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -P & 0 & 0 & 0 \\ * & * & * & * & * & * & -\delta^2 I & 0 & 0 \\ * & * & * & * & * & * & * & -\delta^2 I & 0 \\ * & * & * & * & * & * & * & * & -Q^{-1} \end{bmatrix} < 0. \quad (8)$$

Proof. Let $\Theta_k = [\eta_k^T, \eta_{k-1}^T, \dots, \eta_{k-d}^T]^T$, for all nonzero η_k , consider the Lyapunov function $V_k(\Theta_k) = V_{1k} + V_{2k}$ with

$$V_{1k} = \eta_k^T P \eta_k, \quad V_{2k} = \sum_{i=k-d}^{k-1} \eta_i^T Z^T Q Z \eta_i. \quad (9)$$

The difference of V_k is

$$\begin{aligned}\Delta V_{1k} &= \mathbb{E}\{V_{1k}(\Theta_{k+1})\} - V_{1k}(\Theta_k) \\ &= \eta_k^T \tilde{A}^T P \tilde{A} \eta_k + \eta_k^T \tilde{A}^T P \tilde{A}_d Z \eta_{k-d} + \eta_k^T \tilde{A}^T P \tilde{B}_2 d_k \\ &\quad + \eta_k^T \tilde{A}^T P \tilde{D}_1 \omega_k + \mathbb{E}\{(\theta_k - p)^2\} \eta_k^T \tilde{A}_2^T P \tilde{A}_2 \eta_k \\ &\quad + \mathbb{E}\{(\theta_k - p)^2\} \eta_k^T \tilde{A}_2^T P \tilde{B}_1 d_k + \eta_{k-d}^T Z^T \tilde{A}_d^T P \tilde{A} \eta_k \\ &\quad + \eta_{k-d}^T Z^T \tilde{A}_d^T P \tilde{A}_d Z \eta_{k-d} + \eta_{k-d}^T Z^T \tilde{A}_d^T P \tilde{B}_2 d_k \\ &\quad + \eta_{k-d}^T Z^T \tilde{A}_d^T P \tilde{D}_1 \omega_k + \mathbb{E}\{(\theta_k - p)^2\} d_k^T \tilde{B}_1^T P \tilde{A}_2 \eta_k \\ &\quad + \mathbb{E}\{(\theta_k - p)^2\} d_k^T \tilde{B}_1^T P \tilde{B}_1 d_k + d_k^T \tilde{B}_2^T P \tilde{A} \eta_k \\ &\quad + d_k^T \tilde{B}_2^T P \tilde{A}_d Z \eta_{k-d} + d_k^T \tilde{B}_2^T P \tilde{B}_2 d_k \\ &\quad + d_k^T \tilde{B}_2^T P \tilde{D}_1 \omega_k + \omega_k^T \tilde{D}_1^T P \tilde{A} \eta_k + \omega_k^T \tilde{D}_1^T P \tilde{A}_d Z \eta_{k-d} \\ &\quad + \omega_k^T \tilde{D}_1^T P \tilde{B}_2 d_k + \omega_k^T \tilde{D}_1^T P \tilde{D}_1 \omega_k - \eta_k^T P \eta_k \\ \Delta V_{2k} &= \mathbb{E}\{V_{2k}(\Theta_{k+1})\} - V_{2k}(\Theta_k) \\ &= \eta_k^T (Z^T Q Z) \eta_k - \eta_{k-d}^T Z^T Q Z \eta_{k-d}.\end{aligned}\quad (10)$$

For $\mathbb{E}\{\theta_k - p\} = 0$, and denote $\mathbb{E}\{(\theta_k - p)^2\} = \sigma^2$, we have

$$\begin{aligned}\Delta V_k &= \begin{bmatrix} \eta_k \\ Z \eta_{k-d} \\ d_k \\ \omega_k \end{bmatrix}^T \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} \\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} \\ * & * & \Phi_{33} & \Phi_{34} \\ * & * & * & \Phi_{44} \end{bmatrix} \begin{bmatrix} \eta_k \\ Z \eta_{k-d} \\ d_k \\ \omega_k \end{bmatrix} \\ &< \zeta_k^T \Phi_{ij} \zeta_k \quad (i, j = 1, \dots, 4),\end{aligned}\quad (11)$$

where $\zeta_k = [\eta_k^T, \eta_{k-d}^T Z^T, d_k^T, \omega_k^T]^T$, $\Phi_{11} = \tilde{A}^T P \tilde{A} + Z^T Q Z + \sigma^2 \tilde{A}_2^T P \tilde{A}_2 - P$, $\Phi_{12} = \tilde{A}^T P \tilde{A}_d$, $\Phi_{13} = \tilde{A}^T P \tilde{B}_2 + \sigma^2 \tilde{A}_2^T P \tilde{B}_1$, $\Phi_{14} = \tilde{A}^T P \tilde{D}_1$, $\Phi_{22} = \tilde{A}_d^T P \tilde{A}_d - Q$, $\Phi_{23} =$

$\tilde{A}_d^T P \tilde{B}_2$, $\Phi_{24} = \tilde{A}_d^T P \tilde{D}_1$, $\Phi_{33} = \tilde{B}_2^T P \tilde{B}_2 + \sigma^2 \tilde{B}_1^T P \tilde{B}_1$, $\Phi_{34} = \tilde{B}_2^T P \tilde{D}_1$, $\Phi_{44} = \tilde{D}_1^T P \tilde{D}_1$, $*$ is implicitly defined by the fact that the matrix is symmetric.

When $\omega_k = 0, d_k = 0$, (11) is rewritten as

$$\begin{aligned} \Delta V_{k, \omega_k = d_k = 0} &= \begin{bmatrix} \tilde{A}^T P \tilde{A} + Z^T Q Z + \sigma^2 \tilde{A}_2^T P \tilde{A}_2 - P & \tilde{A}^T P \tilde{A}_d \\ \tilde{A}_d^T P \tilde{A} & \tilde{A}_d^T P \tilde{A}_d - Q \end{bmatrix} \quad (12) \\ &= \Pi. \end{aligned}$$

From (8) and the Schur complement theorem, $\Pi < 0$ is arrived. Therefore, for all nonzero η_k , we have $\Delta V_k < 0$, then the NCS (5) with sensor and actuator fault is asymptotically mean-square stable.

Next, For any nonzero ω_k, d_k , it follows from (5), (8), and (11) that

$$\begin{aligned} &\mathbb{E}\{V(\Theta_{k+1})\} - \mathbb{E}\{V(\Theta_k)\} + \mathbb{E}\{z_k^T z_k\} \\ &- \gamma^2 \mathbb{E}\{\omega_k^T \omega_k\} - \delta^2 \mathbb{E}\{d_k^T d_k\} \\ &\leq \mathbb{E}\{\zeta_k^T \Phi_{ij} \zeta_k + \eta_k^T \tilde{C}^T \tilde{C} \eta_k \\ &\quad + \eta_k^T \tilde{C}^T C_d Z \eta_{k-d} + \eta_k^T \tilde{C}^T D_1 \omega_k \\ &\quad + \eta_{k-d}^T Z^T C_d^T \tilde{C} \eta_k + \eta_{k-d}^T Z^T C_d^T C_d Z \eta_{k-d} \\ &\quad + \eta_{k-d}^T Z^T C_d^T D_1 \omega_k + \omega_k^T D_1^T \tilde{C} \eta_k \\ &\quad + \omega_k^T D_1^T C_d Z \eta_{k-d} + \omega_k^T D_1^T D_1 \omega_k \\ &\quad - \gamma^2 \omega_k^T \omega_k - \delta^2 d_k^T d_k\}. \end{aligned} \quad (13)$$

Then, we have

$$\begin{aligned} &\mathbb{E}\{V(\Theta_{k+1})\} - \mathbb{E}\{V(\Theta_k)\} + \mathbb{E}\{z_k^T z_k\} \\ &- \gamma^2 \mathbb{E}\{\omega_k^T \omega_k\} - \delta^2 \mathbb{E}\{d_k^T d_k\} < 0. \end{aligned} \quad (14)$$

Now, summing (14) from 0 to ∞ with respect to k yields

$$\begin{aligned} \sum_{k=0}^{\infty} \mathbb{E}\{z_k^T z_k\} &< \gamma^2 \sum_{k=0}^{\infty} \mathbb{E}\{\omega_k^T \omega_k\} + \delta^2 \sum_{k=0}^{\infty} \mathbb{E}\{d_k^T d_k\} \\ &+ \mathbb{E}\{V_0\} - \mathbb{E}\{V_{\infty}\}. \end{aligned} \quad (15)$$

Since the system (5) is asymptotically mean-square stable, we can get that the following inequality:

$$\sum_{k=0}^{\infty} \mathbb{E}\{z_k^T z_k\} < \gamma^2 \sum_{k=0}^{\infty} \mathbb{E}\{\omega_k^T \omega_k\} + \delta^2 \sum_{k=0}^{\infty} \mathbb{E}\{d_k^T d_k\} \quad (16)$$

holds under the zero initial condition. The proof is thus, complete. $\square$

4. Robust H_{∞} Controller Design

In this section, a theorem will be proposed to solve the controller design problem for stochastic state-delay NCS (5).

Theorem 7. Given a scalar $\gamma > 0, \delta > 0$, the system (5) is asymptotically mean-square stable, and the controlled output z_k satisfies the H_{∞} constraints (7), if there exist real scalars $\varepsilon > 0$, and matrices $S = S^T > 0$, $Q = Q^T > 0$, and $R = R^T > 0$, and real matrices W_1, W_2 , and W_3 , such that the following inequality holds:

$$[\Omega_{ij}] < 0, \quad i, j = 1, \dots, 16, \quad (17)$$

where $\Omega_{11} = -S^{-1}$, $\Omega_{12} = -I$, $\Omega_{14} = A + pBW_3$, $\Omega_{15} = A$, $\Omega_{16} = A_d$, $\Omega_{17} = D_0$, $\Omega_{1,10} = -\sigma B$, $\Omega_{1,12} = (1 - p)B$, $\Omega_{1,15} = H$, $\Omega_{21} = -I$, $\Omega_{22} = -R$, $\Omega_{24} = RA + pW_2C_0 + W_1$, $\Omega_{25} = RA + pW_2C_0$, $\Omega_{26} = RA_d$, $\Omega_{27} = RD_0$, $\Omega_{2,10} = -\sigma RB$, $\Omega_{2,11} = -\sigma W_2$, $\Omega_{2,12} = (1 - p)RB$, $\Omega_{2,13} = (1 - p)W_2$, $\Omega_{2,15} = RH$, $\Omega_{33} = -I$, $\Omega_{34} = C$, $\Omega_{35} = C$, $\Omega_{36} = C_d$, $\Omega_{37} = D_1$, $\Omega_{41} = \Omega_{14}^T$, $\Omega_{42} = \Omega_{15}^T$, $\Omega_{43} = C^T$, $\Omega_{44} = -S$, $\Omega_{45} = -S$, $\Omega_{48} = \sigma W_3^T B^T$, $\Omega_{49} = \sigma W_3^T B^T R + \sigma C_0^T W_2^T$, $\Omega_{4,14} = I$, $\Omega_{4,16} = \varepsilon N^T$, $\Omega_{51} = A^T$, $\Omega_{52} = \Omega_{25}^T$, $\Omega_{53} = C^T$, $\Omega_{54} = -S$, $\Omega_{55} = -R$, $\Omega_{59} = \sigma C_0^T W_2^T$, $\Omega_{5,14} = Q$, $\Omega_{5,16} = \varepsilon N^T$, $\Omega_{61} = A_d^T$, $\Omega_{61} = A_d^T R$, $\Omega_{63} = C_d^T$, $\Omega_{66} = -Q$, $\Omega_{71} = D_0^T$, $\Omega_{72} = D_0^T R$, $\Omega_{73} = D_1^T$, $\Omega_{77} = -\gamma^2 I$, $\Omega_{84} = \Omega_{48}^T$, $\Omega_{88} = -S$, $\Omega_{89} = -S$, $\Omega_{94} = \Omega_{49}^T$, $\Omega_{95} = \Omega_{59}^T$, $\Omega_{99} = -R$, $\Omega_{10,1} = -\sigma B^T$, $\Omega_{10,2} = -\sigma B^T R$, $\Omega_{10,10} = \Omega_{11,11} = \Omega_{12,12} = \Omega_{13,13} = -\delta^2 I$, $\Omega_{11,2} = \Omega_{2,11}^T$, $\Omega_{12,1} = \Omega_{1,12}^T$, $\Omega_{12,2} = \Omega_{2,12}^T$, $\Omega_{13,2} = \Omega_{2,13}^T$, $\Omega_{14,4} = I$, $\Omega_{14,5} = Q^T$, $\Omega_{14,14} = -I$, $\Omega_{15,1} = H^T$, $\Omega_{15,2} = H^T R$, $\Omega_{15,15} = \Omega_{16,16} = -\varepsilon I$, $\Omega_{16,4} = \Omega_{16,5} = \varepsilon N$, the rest of matrix entries are zero.

The fault-tolerant controller parameters are

$$\begin{aligned} A_f &= X_{12}^{-1} \left[(W_1 - pRBW_3)S^{-1} (Y_{12}^T)^{-1} \right], \\ B_f &= X_{12}^{-1} W_2, \quad C_f = W_3 S^{-1} (Y_{12}^T)^{-1}. \end{aligned} \quad (18)$$

Proof. The system (5) is a parameter-dependent system. By Lemma 4, (8) is rewritten as

$$\begin{bmatrix} -P & 0 & P\tilde{A}_1 & P\tilde{A}_d & P\tilde{D}_1 & 0 & \sigma P\tilde{B}_1 & P\tilde{B}_2 & 0 & P\tilde{H} & 0 \\ * & -I & \tilde{C} & C_d & D_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -P & 0 & 0 & \sigma \tilde{A}_2^T P & 0 & 0 & Z^T & 0 & \varepsilon \tilde{N}^T \\ * & * & * & -Q & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -P & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\delta^2 I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\delta^2 I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -Q^{-1} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0. \quad (19)$$

Next, partition P and P^{-1} as

$$P = \begin{bmatrix} R & X_{12} \\ X_{12}^T & X_{22} \end{bmatrix}, \quad P^{-1} = \begin{bmatrix} S^{-1} & Y_{12} \\ Y_{12}^T & Y_{22} \end{bmatrix}. \quad (20)$$

Define

$$T_1 = \begin{bmatrix} S^{-1} & I \\ Y_{12}^T & 0 \end{bmatrix}, \quad T_2 = \begin{bmatrix} I & R \\ 0 & X_{12}^T \end{bmatrix}. \quad (21)$$

Obviously, we have $PT_1 = T_2$, $T_1^T PT_1 = T_1^T T_2$. Performing the congruence transformation $\text{diag}\{T_1^T, I, T_1^T, I, T_1^T I, I, T_1^T, I, I\}$ to (19), we obtain the following:

$$[\Lambda_{i,j}] < 0, \quad i, j = 1, \dots, 16, \quad (22)$$

where $\Lambda_{11} = -S^{-1}$, $\Lambda_{12} = -I$, $\Lambda_{14} = AS^{-1} + pBC_f Y_{12}^T$, $\Lambda_{15} = A$, $\Lambda_{16} = A_d$, $\Lambda_{17} = D_0$, $\Lambda_{1,10} = -\sigma B$, $\Lambda_{1,12} = (1-p)B$, $\Lambda_{1,15} = H$, $\Lambda_{21} = -I$, $\Lambda_{22} = -R$, $\Lambda_{24} = (RA + pX_{12}B_f C_0)S^{-1} + (pRBC_f + X_{12}A_f)Y_{12}^T$, $\Lambda_{25} = RA + pX_{12}B_f C_0$, $\Lambda_{26} = RA_d$, $\Lambda_{27} = RD_0$, $\Lambda_{2,10} = -\sigma RB$, $\Lambda_{2,11} = -\sigma X_{12}B_f$, $\Lambda_{2,12} = (1-p)RB$, $\Lambda_{2,13} = (1-p)X_{12}B_f$, $\Lambda_{2,15} = RH$, $\Lambda_{33} = -I$, $\Lambda_{34} = CS^{-1}$, $\Lambda_{35} = C$, $\Lambda_{36} = C_d$, $\Lambda_{37} = D_1$, $\Lambda_{41} = \Lambda_{41}^T$, $\Lambda_{42} = \Lambda_{42}^T$, $\Lambda_{43} = S^{-1}C^T$, $\Lambda_{44} = -S^{-1}$, $\Lambda_{45} = -I$, $\Lambda_{48} = \sigma Y_{12}C_f^T B^T$, $\Lambda_{49} = \sigma Y_{12}C_f^T B^T R + \sigma S^{-1}C_0^T B_f^T X_{12}^T$, $\Lambda_{4,14} = S^{-1}$, $\Lambda_{4,16} = \epsilon S^{-1}N^T$, $\Lambda_{51} = A^T$, $\Lambda_{52} = \Lambda_{25}^T$, $\Lambda_{53} = C^T$, $\Lambda_{54} = -S$, $\Lambda_{55} = -R$, $\Lambda_{59} = \sigma C_0^T B_f^T X_{12}^T$, $\Lambda_{5,12} = I$, $\Lambda_{5,16} = \epsilon N^T$, $\Lambda_{61} = A_d^T$, $\Lambda_{62} = A_d^T R$, $\Lambda_{63} = C_d^T$, $\Lambda_{66} = -Q$, $\Lambda_{71} = D_0^T$, $\Lambda_{72} = D_0^T R$, $\Lambda_{73} = D_1^T$, $\Lambda_{77} = -\gamma^2 I$, $\Lambda_{84} = \Lambda_{48}^T$, $\Lambda_{88} = -S^{-1}$, $\Lambda_{89} = -I$, $\Lambda_{94} = \Lambda_{49}^T$, $\Lambda_{95} = \Lambda_{59}^T$, $\Lambda_{99} = -R$, $\Lambda_{10,1} = -\sigma B^T$, $\Lambda_{10,2} = -\sigma B^T R$, $\Lambda_{11,2} = -\sigma X_{12}B_f$, $\Lambda_{12,1} = (1-p)B^T$, $\Lambda_{12,2} = (1-p)B^T R$, $\Lambda_{13,2} = (1-p)X_{12}B_f$, $\Lambda_{14,4} = S^{-1}$, $\Lambda_{14,5} = I$, $\Lambda_{10,10} = \Lambda_{11,11} = \Lambda_{12,12} = \Lambda_{13,13} = -\delta^2 I$, $\Lambda_{14,14} = -Q^{-1}$, $\Lambda_{15,15} = \Lambda_{16,16} = -\epsilon I$, $\Lambda_{15,1} = H^T$, $\Lambda_{15,2} = H^T R$, $\Lambda_{16,4} = \epsilon NS^{-1}$, $\Lambda_{16,5} = \epsilon N$, the rest of matrix entries are zero.

Applying the congruence transformation $\text{diag}\{I, I, I, S, I, I, I, S, I, I, I, I, Q, I, I\}$ to (22) again, then (17) is achieved. Therefore, by Theorem 6, the desired result follows immediately.

Next, we will design the scalars γ and δ by solving the optimization problem,

$$\min_{\epsilon, S, R, Q > 0, W_1, W_2, W_3} \gamma, \delta, \quad \text{subject to (17)}. \quad (23)$$

Using the robust control toolbox, we will get the optimal value of γ and δ , and the proof is thus, complete. $\square$

Remark 8. From Theorems 6 and 7, we know if (17) is feasible, $I - RS^{-1} = X_{12}Y_{12}^T < 0$, hence, the square and non-singular matrices X_{12} and Y_{12} can be always found (as in [29]). Then, the fault-tolerant controller parameters (18) are obtained.

FIGURE 2: State responses under the fault-tolerant controller.

5. Simulation Example

Consider the system (1) with parameters (as in [22]) as follows:

$$A = \begin{bmatrix} -0.3 & 0 & -0.3 \\ 0 & 0.6 & 0.2 \\ 0.5 & 0 & 0.7 \end{bmatrix}, \quad A_d = \begin{bmatrix} -0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.2 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \quad D_0 = \begin{bmatrix} 0.5 \\ 0 \\ 0.2 \end{bmatrix}, \quad H = \begin{bmatrix} 0.5 \\ 0.5 \\ 1 \end{bmatrix},$$

$$C_0 = \begin{bmatrix} 1 & 1 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 0.1 & 0 & 0 \end{bmatrix},$$

$$C_d = \begin{bmatrix} 0.1 & 0 & 0 \end{bmatrix}, \quad N = \begin{bmatrix} 0.1 & 0 & 0 \end{bmatrix}, \quad D_1 = 0.1. \quad (24)$$

Choose the same parameters as [22], packet loss rate is $p = 0.1$, delay constant is $d = 3$. With the method in [22], the optimal performance γ is 3.3339. Using the proposed method in this paper, the optimal performance γ is 1.3614, which means the smaller performance has been obtained.

Next, let the packet loss rate and fault probability $p = 0.3$, delay constant is $d = 2$, under the initial condition $x(0) = [-0.2 \ 0 \ 0.5]^T$, $\gamma = 1.6845$, and $\delta = 2.0753$, the fault-tolerant controller parameters are designed by Theorem 7 as follows.

$$A_f = \begin{bmatrix} 0.1357 & 2.4132 & -0.9861 \\ -2.1036 & -1.0003 & -0.0584 \\ -1.5372 & -0.4739 & -2.9271 \end{bmatrix},$$

$$B_f = \begin{bmatrix} -0.0697 \\ 0.7264 \\ -0.0983 \end{bmatrix}, \quad C_f = \begin{bmatrix} -0.1016 & -0.0712 & 0.2598 \end{bmatrix}. \quad (25)$$

From Figure 2, it can be seen that state responses under the designed controller can stabilize the NCS with data packet loss, probabilistic actuator failures, and/or sensor

failures, which can illustrate the effectiveness of the proposed method.

6. Conclusion

Motivated by robust FTC problem over networks, a new stochastic NCS model with fault is addressed, which includes the state-delay, model uncertainty, disturbance, random packet dropout, probabilistic sensors failures, and actuators failures. The random packet dropout in both S-C link and C-A link, the sensor failure and the actuator failure are described as a binary random variable. The sufficient condition for asymptotical mean-square stability of the NCS has been derived and the closed-loop NCS satisfies H_∞ performance constraints. Finally, by solving a linear matrix inequality, the fault tolerant controller is designed.

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