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Research Article

Partial Pole Placement in LMI Region

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A new approach for pole placement of single-input system is proposed in this paper. Noncritical closed loop poles can be placed arbitrarily in a specified convex region when dominant poles are fixed in anticipant locations. The convex region is expressed in the form of linear matrix inequality (LMI), with which the partial pole placement problem can be solved via convex optimization tools. The validity and applicability of this approach are illustrated by two examples.

1. Introduction

In classic control theory and application, pole placement (PP) of linear system is a well-known method to reach some desired transient performances [1] in terms of settling time, overshooting, and damping ratio. Indeed, the shape of the transient response strongly depends on the locations of the closed loop poles in complex plane. A strict PP is always achievable by a state feedback control law if the system is controllable. PP can be performed in a transfer function or state-space context, through a classical eigenvalue assignment based on characteristic polynomial of the closed loop system.

However, when the system suffers uncertainties, strict PP in desirable locations is no longer suitable. For this reason, nonstrict placement in a subregion of the complex plane, such as a sector or a disc, is developed. A slight migration of the closed loop poles around desirable location may not induce a strong modification of the transient response, so the robust performance of the system can be assured. Chilali et al. proposed a linear matrix inequality (LMI) region [2, 3], which is convenient to depict typical convex subregion symmetric about real axis. With it, the robust controller is easy to design via solving some LMI [4] problems. The LMI region was expanded to quadratic matric inequality (QMI) region by Peaucelle et al. [5], and controller design for system suffering to different uncertainties was studied [6, 7]. Henrion et al. researched PP in QMI region with respect to polynomial system [8–10]. Maamri et al. [11–13] proposed a novel strategy with respect to PP in nonconnected QMI regions, while Yang placed poles in union of disjointed circular regions [14].

Partial pole placement by full state feedback is a new strategy for single-input linear system proposed by Datta et al. [15, 16]. $n-m$ critical closed loop poles of an nth order single-input linear system are placed at prespecified locations in the complex plane, while the remaining m noncritical poles can be placed arbitrarily inside a QMI region defined by a 2×2 real symmetric matrix. These noncritical poles are optimized with minimum norm of feedback gain k as the object. The QMI region constraint is reduced to an LMI with respect to $\mathbf{k}$. However, it is worth noting that the derivation of the LMI conditions involves the inner approximation of the nonconvex polynomial stability region [8–10]. This may introduce conservatism more or less.

In order to reduce the conservatism, we derive a new sufficient condition of region constraint at first. Then an iterative strategy is proposed to solve the nonlinear optimal problem of partial pole placement based on the new sufficient condition. This can produce a better result than the method in [16].

This paper is organized as follows. In the next section, the partial pole placement problem is described. Then the new method is given in Section 3. In Section 4, two systems' poles are placed with the new method. Finally, Section 5 is conclusion.

Notation is standard. The transpose and complex conjugate transpose of matrix $\mathbf{A}$ are, respectively, denoted by $\mathbf{A}'$ and $\mathbf{A}^*$. For symmetric matrices $\mathbf{A}$ and $\mathbf{B}$, $\mathbf{A} > (\geq)\mathbf{B}$ denotes

that $\mathbf{A}-\mathbf{B}$ is positive (semi)definite. $\otimes$ is the Kronecker product of two matrices, and its operation involves

$$\begin{aligned}(\mathbf{A}\otimes\mathbf{B})(\mathbf{C}\otimes\mathbf{D}) &= (\mathbf{AC})\otimes(\mathbf{BD}),\\ (\mathbf{A}+\mathbf{B})\otimes\mathbf{C} &= \mathbf{A}\otimes\mathbf{C}+\mathbf{B}\otimes\mathbf{C},\\ (\mathbf{A}\otimes\mathbf{B})' &= \mathbf{A}'\otimes\mathbf{B}'.\end{aligned} \tag{1}$$

2. Problem Formulation

A linear single-input system with full state feedback control can be written as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}+\mathbf{b}u; \quad u = -\mathbf{k}\mathbf{x}, \tag{2}$$

where $\mathbf{x}\in\mathbb{R}^{n\times 1}$, $u\in\mathbb{R}$, $\mathbf{A}\in\mathbb{R}^{n\times n}$, $\mathbf{b}\in\mathbb{R}^{n\times 1}$, and $\mathbf{k} := [k_1\ \ k_2\ \ \cdots\ \ k_n]\in\mathbb{R}^{1\times n}$. Provided that the pair $(\mathbf{A},\mathbf{b})$ is controllable, all the poles $\{\mu_1,\mu_2,\ldots,\mu_n\}$ of the closed loop system

$$\dot{\mathbf{x}} = (\mathbf{A}-\mathbf{b}\mathbf{k})\,\mathbf{x} \tag{3}$$

can be placed at any arbitrary locations of the complex plane via a unique choice of $\mathbf{k}$.

Different from strict PP, partial pole placement only assigns $n-m$ critical poles $\{\mu_{m+1},\mu_{m+2},\ldots,\mu_n\}$ at $\{\lambda_1,\lambda_2,\ldots,\lambda_{n-m}\}$ as the dominant poles to obtain desirable transient response. And the rest m poles $\{\mu_1,\mu_2,\ldots,\mu_m\}$ can be placed in some LMI region $\mathfrak{R}_{\mathrm{LMI}}$ of the complex plane.

Definition 1 (see [2]). $\mathfrak{R}_{\mathrm{LMI}}$ is defined as

$$\mathfrak{R}_{\mathrm{LMI}} = \left\{z\in\mathbb{C} : \mathbf{R}_{11}+\mathbf{R}_{12}z+\mathbf{R}_{12}'z^{*} < 0\right\}, \tag{4}$$

where $\mathbf{R}_{11} = \mathbf{R}_{11}'\in\mathbb{R}^{d\times d}$ and $\mathbf{R}_{12}\in\mathbb{R}^{d\times d}$, and they can be written in the form of partitioned matrix:

$$\mathbf{R} = \begin{bmatrix}\mathbf{R}_{11} & \mathbf{R}_{12}\\ \mathbf{R}_{12}' & \mathbf{0}\end{bmatrix}. \tag{5}$$

Many convex regions symmetric about the real axis can be depicted according to definition (4). For example, when $\mathbf{R}$ are chosen as

$$\mathbf{R} = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}, \quad \text{or} \quad \mathbf{R} = \left[\begin{array}{cc|cc}-1 & 0 & 0 & 1\\ 0 & -1 & 0 & 0\\ \hline 0 & 0 & 0 & 0\\ 1 & 0 & 0 & 0\end{array}\right], \tag{6}$$

the LMI regions are, respectively, the left-hand side of complex plane (stable region for continuous systems) and the unitary disk with centre at the origin (stable region for discrete systems).

The characteristic equation of system (3) can be written as

$$\begin{aligned}\sigma(s) &= s^n+\sigma_{n-1}s^{n-1}+\cdots\sigma_1 s+\sigma_0\\ &= \det\left(s\mathbf{I}_n-\mathbf{A}+\mathbf{b}\mathbf{k}\right)\\ &= \prod_{i=1}^{n}(s-\mu_i)\\ &= \underbrace{\left[\prod_{i=1}^{m}(s-\mu_i)\right]}_{\alpha(s)}\underbrace{\left[\prod_{i=1}^{n-m}(s-\lambda_i)\right]}_{\beta(s)},\end{aligned} \tag{7}$$

where

$$\begin{aligned}\alpha(s) &= s^m+\alpha_{m-1}s^{m-1}+\cdots\alpha_1 s+\alpha_0,\\ \beta(s) &= s^{n-m}+\beta_{n-m-1}s^{n-m-1}+\cdots\beta_1 s+\beta_0.\end{aligned} \tag{8}$$

$\alpha(s)$ is a monic polynomial of unknown coefficients, while $\beta(s)$ is a monic polynomial of known coefficients that are determined by poles $\{\lambda_1,\lambda_2,\ldots,\lambda_{n-m}\}$.

Let $a(s)$ be the open loop characteristic polynomial of system (2):

$$\begin{aligned}a(s) &= \det\left(s\mathbf{I}_n-\mathbf{A}\right)\\ &= s^n+a_{n-1}s^{n-1}+\cdots a_1 s+a_0,\end{aligned} \tag{9}$$

and then we can define

$$\begin{aligned}\widehat{\mathbf{a}} &:= [a_0\ \ a_1\ \ \cdots\ \ a_{n-1}],\\ \widehat{\boldsymbol{\sigma}} &:= [\sigma_0\ \ \sigma_1\ \ \cdots\ \ \sigma_{n-1}],\\ \widehat{\boldsymbol{\alpha}} &:= [\alpha_0\ \ \alpha_1\ \ \cdots\ \ \alpha_{m-1}],\\ \widehat{\boldsymbol{\beta}} &:= [\beta_0\ \ \beta_1\ \ \cdots\ \ \beta_{n-m-1}].\end{aligned} \tag{10}$$

As we know [16], when $\beta(s)$ is known,

$$\mathbf{k} = \left[\operatorname{conv}\left([\widehat{\boldsymbol{\alpha}}\ \ 1],[\widehat{\boldsymbol{\beta}}\ \ 1]\right)-[\widehat{\mathbf{a}}\ \ 1]\right]\begin{bmatrix}\left(\mathscr{C}\mathscr{A}'\right)^{-1}\\ \mathbf{0}\end{bmatrix}, \tag{11}$$

where

$$\begin{aligned}\mathscr{A} &= \begin{bmatrix}a_1 & a_2 & \cdots & a_{n-1} & 1\\ a_2 & a_3 & \cdots & 1 & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ a_{n-1} & 1 & \cdots & 0 & 0\\ 1 & 0 & \cdots & 0 & 0\end{bmatrix},\\ \mathscr{C} &= [\mathbf{b}\ \ \mathbf{A}\mathbf{b}\ \ \mathbf{A}^2\mathbf{b}\ \ \cdots\ \ \mathbf{A}^{n-1}\mathbf{b}].\end{aligned} \tag{12}$$

Our main object can be summarized as follows.

Question 1. Find a $\mathbf{k}$ that satisfies

$$\begin{aligned}&\min\ \mathbf{k}\mathbf{k}'\\ &\text{s.t.}\quad (1)\ \mathbf{k}\ \text{satisfies}\ (11);\\ &\qquad (2)\ \mu_i\in\mathfrak{R}_{\mathrm{LMI}},\quad i=1,2,\ldots,m.\end{aligned} \tag{13}$$

The optimal objective of Question 1 is to minimize the norm of the feedback gain vector, which means lowest control efforts.

3. Main Result

Before stating the main result, we first recall an important lemma.

Definition 2 (see [2]). A matrix $\mathbf{A} \in \mathbb{R}^{n\times n}$ is $\mathfrak{R}_{\mathrm{LMI}}$-stable if and only if all its eigenvalues lie in the $\mathfrak{R}_{\mathrm{LMI}}$ region defined by (4).

Lemma 3 (see [5]). *$\mathbf{A} \in \mathbb{R}^{n\times n}$ is $\mathfrak{R}_{LMI}$-stable if and only if there exists a symmetric positive definite matrix $\mathbf{P} \in \mathbb{R}^{n\times n}$ such that*

$$\mathbf{R}_{11} \otimes \mathbf{P} + \mathbf{R}_{12} \otimes (\mathbf{PA}) + \mathbf{R}_{12}' \otimes \left(\mathbf{A}'\mathbf{P}\right) < 0. \tag{14}$$

In the same way as Definition 2, we can define $\mathfrak{R}_{\mathrm{LMI}}$-stability with respect to polynomial $\alpha(s)$ and then derive corresponding theorem.

Definition 4. $\alpha(s)$ (or $\widehat{\boldsymbol{\alpha}}$) is $\mathfrak{R}_{\mathrm{LMI}}$-stable if and only if all roots of $\alpha(s) = 0$ lie in the $\mathfrak{R}_{\mathrm{LMI}}$ region defined by (4).

Theorem 5. *$\alpha(s)$ (or $\widehat{\boldsymbol{\alpha}}$) is $\mathfrak{R}_{LMI}$-stable if and only if there exists a symmetric positive definite matrix $\mathbf{P} \in \mathbb{R}^{n\times n}$ such that*

$$\mathbf{R}_{11} \otimes \mathbf{P} + \mathbf{R}_{12} \otimes \left(\mathbf{PE}'\right) + \mathbf{R}_{12}' \otimes (\mathbf{EP}) < 0, \tag{15}$$

where

$$\mathbf{E} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{m-1} \end{bmatrix}. \tag{16}$$

Proof. Because $\det(s\mathbf{I}_m - \mathbf{E}') = \alpha(s)$, all eigenvalues of $\mathbf{E}'$ are also roots of $\alpha(s) = 0$. The conclusion can be easily obtained. □

Theorem 6. *If there exists $\mathbf{Y} \in \mathbb{R}^{1\times m}$ and a symmetric positive definite matrix $\mathbf{X} \in \mathbb{R}^{m\times m}$ such that*

$$\mathbf{R}_{11} \otimes \mathbf{X} + \mathbf{R}_{12} \otimes \left(\mathbf{XF} + \mathbf{Y}'\mathbf{b}_0'\right) + \mathbf{R}_{12}' \otimes \left(\mathbf{FX} + \mathbf{b}_0\mathbf{Y}\right) < 0, \tag{17}$$

where

$$\mathbf{F} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \qquad \mathbf{b}_0 = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ -1 \end{bmatrix}, \tag{18}$$

then $\boldsymbol{\xi} = \mathbf{YX}^{-1}$ is $\mathfrak{R}_{LMI}$-stable.

Proof. Taking $\mathbf{Y} = \boldsymbol{\xi}\mathbf{X}$, condition (17) is reduced to Theorem 6. □

Considering Theorem 6, Question 1 can be written as follows.

Question 2. Find a symmetric positive definite matrix $\mathbf{X}$ and a matrix $\mathbf{Y}$ that satisfies

$$\begin{aligned} &\min \ \mathbf{kk}' \\ &\text{s.t.} \quad (1)\ \mathbf{k} \text{ satisfies } (11); \\ &\qquad (2)\ \widehat{\boldsymbol{\alpha}} = \mathbf{YX}^{-1}; \\ &\qquad (3)\ \mathbf{X}, \mathbf{Y} \text{ satisfy LMI } (17). \end{aligned} \tag{19}$$

However, Question 2 is a nonlinear matrix equality problem. We need to translate it into two LMI problems and solve them iteratively.

Question 3. When $\widehat{\boldsymbol{\alpha}}$ is known, find a feasible symmetric positive definite matrix $\mathbf{X}_{\widehat{\alpha}}$ that satisfies (15).

Question 4. When $\mathbf{X}$ is fixed, find a $\mathbf{Y}_X$ that minimizes $\mathbf{kk}'$ when $\mathbf{Y}_X$ is subject to (17).

The solving steps are as follows.

(1) Choose the $\mathfrak{R}_{\mathrm{LMI}}$-stable $\widehat{\boldsymbol{\alpha}}_0$ that is generated by Datta's method as the initial value and calculate $\gamma_0 = \mathbf{k}_0\mathbf{k}_0'$.

(2) Solve Question 3 and get a feasible solution $\mathbf{X}_0 = \mathbf{X}_{\widehat{\alpha}0}$.

(3) Solve Question 4 and get the optimal $\mathbf{Y}^* = \mathbf{Y}_{X_0}$.

(4) Calculate $\gamma^* = \mathbf{k}^*(\mathbf{k}^*)'$; if $|\gamma^* - \gamma_0| < \varepsilon$, stop ($\varepsilon$ is the permitted tolerance).

(5) If $\gamma^* < \gamma_0$, let $\widehat{\boldsymbol{\alpha}}_0 = \mathbf{Y}^*\mathbf{X}_0^{-1}$ and repeat step (2) to step (4); else, stop.

Because Datta's method involves replacing a nonconvex set with its inner convex approximation, necessarily there exists some conservatism. With the iterative optimization above, we can reduce $\mathbf{kk}'$ more or less.

4. Examples

In this section, the proposed method is applied to those two examples given by Datta et al. [16]. The results show that the new method produces better results than Datta's.

Example 1. Consider a single-input system with

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 51 & -10 & -30 & -10 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}. \tag{20}$$

A critical closed loop pole is placed at -1 and the remaining 3 poles are allowed to be placed arbitrarily on the left side of a

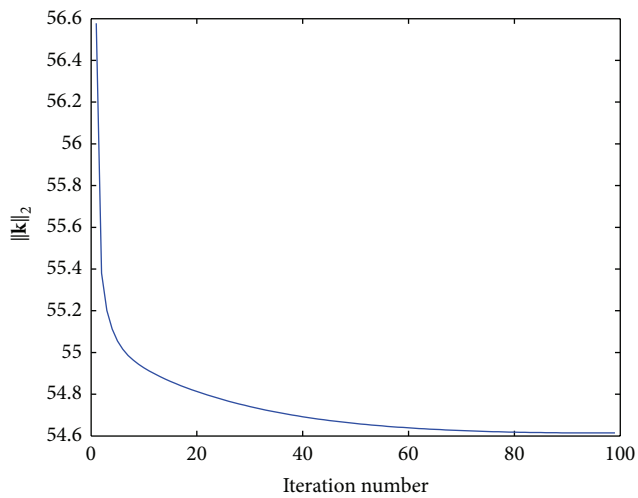

FIGURE 1: Iterative process of Example 1.

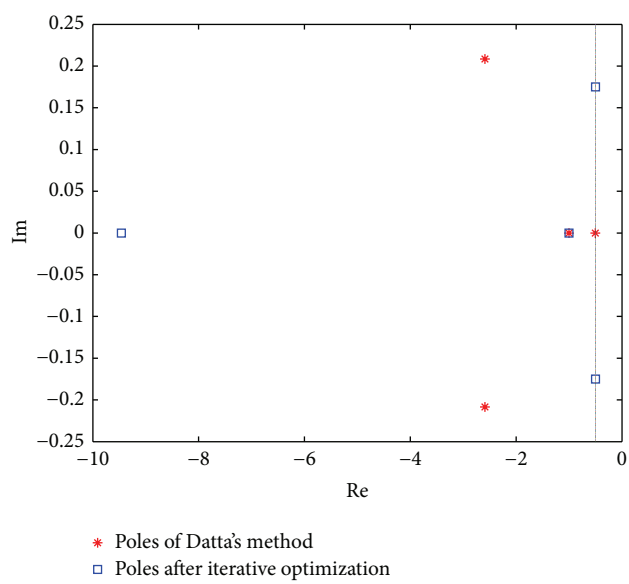

FIGURE 2: Closed loop poles of Example 1.

vertical line at −0.5 in the complex plane. So the $\mathfrak{R}_{\mathrm{LMI}}$ region is chosen as

$$\mathbf{R} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}. \tag{21}$$

Placing poles via the proposed method, we can get the feedback gain $\mathbf{k} = [54.0866 \;\; 2.8465 \;\; -12.5669 \;\; -1.3268]$. Then the closed loop poles are at −6.1732, −1, −1, and −0.5, and the 2-norm of $\mathbf{k}$ is $\|\mathbf{k}\|_2 = 55.6161$, while Datta's result is 56.5775. The iterative process and closed loop poles are, respectively, depicted in Figures 1 and 2.

Example 2. In this example, a 10th order single-input LTI system provided by Datta is considered, where

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix},$$

$$\mathbf{A}_{11} = \begin{bmatrix} -0.0489 & -3.9341 & 0.0165 & 0.2312 & -0.0085 \\ 3.9341 & -0.0447 & 0.0150 & 0.2268 & -0.0082 \\ -0.0165 & 0.0150 & -0.0444 & -1.6702 & 0.0434 \\ 0.2312 & -0.2268 & 1.6702 & -41.9647 & 5.2157 \\ 0.0085 & -0.0082 & 0.0434 & -5.2157 & -0.1365 \end{bmatrix},$$

$$\mathbf{A}_{12} = \begin{bmatrix} -0.0196 & 0.0028 & 0.0256 & 0.0115 & -0.0115 \\ -0.0191 & 0.0027 & 0.0248 & 0.0111 & -0.0111 \\ 0.1193 & -0.0149 & -0.1456 & -0.0643 & 0.0641 \\ 5.1763 & -1.2345 & -8.4006 & -4.0808 & 4.1262 \\ -6.1878 & 0.0616 & 1.0803 & 0.3854 & -0.3742 \end{bmatrix},$$

$$\mathbf{A}_{21} = \begin{bmatrix} -0.0196 & 0.0191 & -0.1193 & 5.1763 & 6.1878 \\ -0.0028 & 0.0027 & -0.0149 & 1.2345 & 0.0616 \\ 0.0256 & -0.0248 & 0.1456 & -8.4006 & -1.0803 \\ 0.0115 & -0.0111 & 0.0643 & -4.0808 & -0.3854 \\ -0.0115 & 0.0111 & -0.0641 & 4.1262 & 0.3742 \end{bmatrix},$$

$$\mathbf{A}_{22} = \begin{bmatrix} -0.8103 & 0.4617 & 1.6651 & 0.9067 & -0.9343 \\ -0.4617 & -0.0325 & -1.2753 & -0.2957 & 0.2765 \\ 1.6651 & 1.2753 & -5.1393 & -3.9349 & 4.3699 \\ 0.9067 & 0.2957 & -3.9349 & -5.9469 & 8.9488 \\ -0.9343 & -0.2765 & 4.3699 & 8.9488 & -18.0850 \end{bmatrix},$$

$$\mathbf{b} = [-0.3176 \;\; 0.3001 \;\; -0.0519 \;\; 0.7565 \;\; 0.0277 \;\; -0.0639 \;\; -0.0091 \;\; 0.0832 \;\; 0.0374 \;\; -0.0374]'. \tag{22}$$

Four critical poles are placed at $-2 \pm 6.9261i$ and -2 ± 3.9352, and the remaining six noncritical poles are placed in a disc with center at $(-8, 0)$ and radius 7.6. The matrix $\mathbf{R}$ is chosen as

$$\mathbf{R} = \begin{bmatrix} -7.6 & 8 & 0 & 1 \\ 8 & -7.6 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}. \tag{23}$$

The feedback gain produced by the new method is

$$\mathbf{k} = [-5.0729 \;\; 1.6630 \;\; -6.7656 \;\; -32.1531 \;\; -11.7833 \;\; -3.0735 \;\; -1.8311 \;\; -16.1811 \;\; -11.8403 \;\; 10.1572] \tag{24}$$

with its 2-norm being $\|\mathbf{k}\|_2 = 42.0400$, which is better than Datta's 42.1279. The iterative solving process is as in Figure 3, and closed loop poles are as in Figure 4.

5. Conclusion

A new sufficient condition for $\mathfrak{R}_{\mathrm{LMI}}$-stability is derived first. Then, based on it, partial pole placement in LMI region

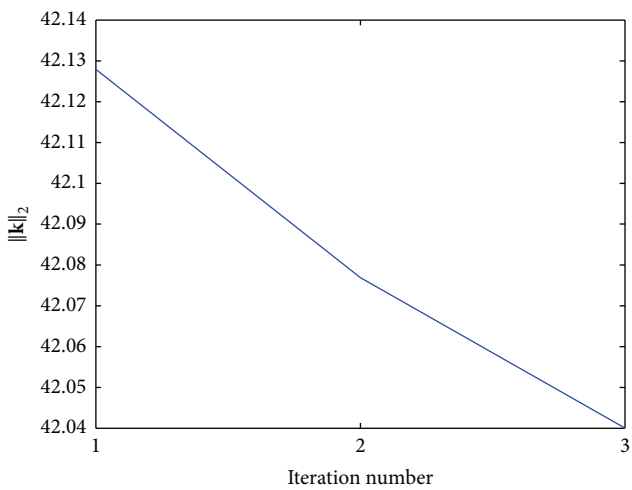

Figure 3: Iterative process of Example 2.

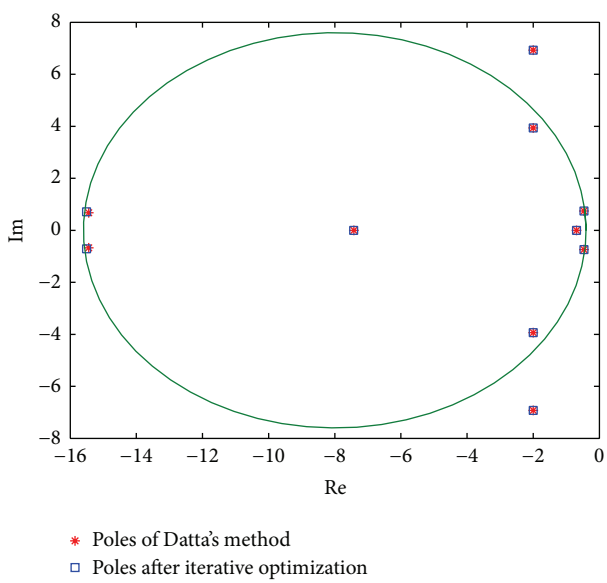

Figure 4: Closed loop poles of Example 2.

with minimum norm controller is established as a nonlinear matrix inequality problem. An iterative strategy is proposed to deal with this problem. The new method is shown to produce better results than Datta's. The future work is to extend the partial pole placement from single-input system to multi-input system.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References

[1] T. Kailath, *Linear Systems*, Prentice-Hall, Englewood Cliffs, NJ, USA, 1980.

[2] M. Chilali and P. Gahinet, "H_∞ design with pole placement constraints: an LMI approach," *IEEE Transactions on Automatic Control*, vol. 41, no. 3, pp. 358–367, 1996.

[3] M. Chilali, P. Gahinet, and P. Apkarian, "Robust pole placement in LMI regions," *IEEE Transactions on Automatic Control*, vol. 44, no. 12, pp. 2257–2270, 1999.

[4] S. P. Boyd, *Linear Matrix Inequalities in System and Control Theory*, SIAM, 1994.

[5] D. Peaucelle, D. Arzelier, O. Bachelier, and J. Bernussou, "A new robust script D sign-stability condition for real convex polytopic uncertainty," *Systems and Control Letters*, vol. 40, no. 1, pp. 21–30, 2000.

[6] V. J. Leite and P. L. Peres, "An improved LMI condition for robust D-stability of uncertain polytopic systems," *IEEE Transactions on Automatic Control*, vol. 48, no. 3, pp. 500–504, 2003.

[7] W. J. Mao and J. Chu, "Quadratic stability and stabilization of dynamic interval systems," *IEEE Transactions on Automatic Control*, vol. 48, no. 6, pp. 1007–1012, 2003.

[8] D. Henrion, D. Peaucelle, D. Arzelier, and M. Sebek, "Ellipsoidal approximation of the stability domain of a polynomial," *IEEE Transactions on Automatic Control*, vol. 48, no. 12, pp. 2255–2259, 2003.

[9] D. Henrion, M. Sebek, and V. Kucera, "Positive polynomials and robust stabilization with fixed-order controllers," *IEEE Transactions on Automatic Control*, vol. 48, no. 7, pp. 1178–1186, 2003.

[10] F. Yang, M. Gani, and D. Henrion, "Fixed-order robust H_∞ controller design with regional pole assignment," *IEEE Transactions on Automatic Control*, vol. 52, no. 10, pp. 1959–1963, 2007.

[11] N. Maamri, O. Bachelier, and D. Mehdi, "Pole placement in a union of regions with prespecified subregion allocation," *Mathematics and Computers in Simulation*, vol. 72, no. 1, pp. 38–46, 2006.

[12] O. Rejichi, O. Bachelier, M. Chaabane, and D. Mehdi, "Robust root-clustering analysis in a union of subregions for descriptor systems," *IET Control Theory and Applications*, vol. 2, no. 7, pp. 615–624, 2008.

[13] B. Sari, O. Bachelier, J. Bosche, N. Maamri, and D. Mehdi, "Pole placement in non connected regions for descriptor models," *Mathematics and Computers in Simulation*, vol. 81, no. 12, pp. 2617–2631, 2011.

[14] S.-K. Yang, "Robust pole-clustering of structured uncertain systems in union of disjointed circular regions," *Applied Mathematics and Computation*, vol. 217, no. 18, pp. 7488–7495, 2011.

[15] S. Datta, B. Chaudhuri, and D. Chakraborty, "Partial pole placement with minimum norm controller," in *Proceedings of the 49th IEEE Conference on Decision and Control (CDC '10)*, pp. 5001–5006, Atlanta, Ga, USA, December 2010.

[16] S. Datta, D. Chakraborty, and B. Chaudhuri, "Partial pole placement with controller optimization," *IEEE Transactions on Automatic Control*, vol. 57, no. 4, pp. 1051–1056, 2012.