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Research Article

Novel Approach to Preview Control for a Class of Continuous-Time Systems

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This paper explicates a new method of designing a preview controller for a class of continuous-time systems. The augmented error system is constructed by the error system with the derivative of the tracking error signal, the state equation, and an identical equation of the derivative of the control input, which transforms a tracking problem into a regulation problem. Therefore, in the paper, the performance index contains the derivative of the control input. Based on the theory of optimal control, the regulator problem of the augmented error system is solved. Thus, the controller with preview compensation for the original system is deduced. The response speed of the closed-loop system is accelerated by the previewed demand output. A final numerical example is given to illustrate the validity of the proposed method.

1. Introduction

The ability to take the known future demand output or disturbance signal into consideration to design a controller with preview compensation is the greatest distinction of the preview control methods [1–3]. It has been proved that better output tracking can be obtained and the performance of the closed-loop system can be improved when the preview control method is applied; this has been demonstrated in many laboratory simulations such as electromechanical valve actuators in [4], networked control systems in [5], and vehicle active suspension systems in [6]. Recently, preview control has drawn greater attention [2, 7]. It has been integrated with other control theories to propose some new conceptions such as H_∞ preview control in [8, 9], FI (full-information) preview control in [9], and stochastic optimal preview control in [10, 11].

In the preview control method, the most important thing is that the known future demand output or disturbance signal affects the operation of the control system. As a typical method, the augmented error system plays a vital role in the design of the controller in discrete-time systems. Since the relationship between the system and the future signal was established by the difference operator, discrete-time systems have become very popular in many research fields [12, 13].

In continuous-time systems, differential operation is used instead of difference operation. Therefore, it is difficult to construct augmented error systems. In [14], reasonable assumptions about the known future signals are made. Derivatives are taken of both the tracking error signal and the state equation. Thus, the tracking error signal and the derivative of the state vector are put together as the state vector of the augmented error system. In [14], the optimal preview control problem of continuous-time constant systems is solved. In this paper, the augmented error system is constructed by the error system with the derivative of the tracking error signal, the state equation, and an identical equation of the derivative of the control input. Thus, the augmented state vector contains the tracking error signal, the state vector, and the control input. Based on optimal control theory, the optimal controller of the augmented error system is derived. A differential equation relevant to the control input of the original system is solved, and an optimal preview controller is deduced.

This paper is organized as follows. In Section 2, the continuous-time system, desired output, performance index,

and basic assumptions needed for the proof of the main results are given. In Section 3, the augmented error system is constructed, in which the control input is taken as a part of the augmented state vector. This is the greatest distinction from [14]. Then, an optimal controller for the augmented error system is deduced, based on the theory of optimal control. Furthermore, the optimal preview controller for the original system is obtained through integration. In Section 4, a numerical example is presented to illustrate the effectiveness of the proposed method. The paper ends with conclusions and cited references.

2. Problem Statement

We consider a class of linear continuous-time systems described by

$$\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t), \quad t \in [t_0, t_\alpha], \tag{1}$$

where $x(t) \in \mathbf{R}^n$, $u(t) \in \mathbf{R}^m$, and $y(t) \in \mathbf{R}^p$ are, respectively, the state vector, control input, and output vector. $A \in \mathbf{R}^{n\times n}$, $B \in \mathbf{R}^{n\times m}$, and $C \in \mathbf{R}^{p\times n}$ are all constant matrices. The positive real number t_0 denotes the initial time and t_α represents the terminal time. The vector $x(t_0) = x_0$ is the initial state of system (1). When $t \le t_0$, both the state vector and the controlled input are zero.

Let $r(t) \in \mathbf{R}^p$ be the desired output or the demand vector. The tracking error signal $e(t)$ is defined as the subtraction of the output vector $y(t)$ and the desired output $r(t)$; that is,

$$e(t) = y(t) - r(t). \tag{2}$$

Concerning the desired output $r(t)$, the following assumptions will be needed throughout the paper.

Assumption 1. The desired output $r(t)$ is piecewise differentiable. At some nondifferential points t, we take the left derivative $\dot{r}(t-0)$ or the right derivative $\dot{r}(t+0)$ instead of $\dot{r}(t)$.

Assumption 2. The desired output $r(t)$ is previewable in the sense that the future value of $r(\sigma)$ $(t \le \sigma \le t + l_r)$ is available at each instant of time t. The value of l_r is presented as the preview length of the desired output in [14].

Remark 3. In Assumption 1, $r(t)$ is in low requirement; hence our method can be used as widely as possible for more desired output. In fact, if $r(t)$ is continuously differentiable, the most commonly used step signal will be excluded.

The basic design problem considered in this paper is to find a controller such that the output vector $y(t)$ can track the desired output $r(t)$ without static error. Therefore, we wish to obtain the optimal controller $u(t)$ with preview compensation such that the performance index

$$\begin{aligned} J = {} & \frac{1}{2} e^{\mathrm{T}}(t_\alpha) e(t_\alpha) \\ & + \frac{1}{2}\int_{t_0}^{t_\alpha} \left[e^{\mathrm{T}}(t) Q_e e(t) + x^{\mathrm{T}}(t) Q_x x(t) \right. \\ & \left. + u^{\mathrm{T}}(t) Q_u u(t) + \dot{u}^{\mathrm{T}}(t) R \dot{u}(t) \right] \mathrm{d}t \end{aligned} \tag{3}$$

is minimized, where $Q_e \in \mathbf{R}^{p\times p}$ and $R \in \mathbf{R}^{m\times m}$ are both positive definite matrices and $Q_x \in \mathbf{R}^{n\times n}$ and $Q_u \in \mathbf{R}^{m\times m}$ are nonnegative definite matrices.

Remark 4. It is well known in [15] that introducing the derivative of the control input into the quadratic performance index J can allow the controller to contain integration terms, which may help the system reduce static errors. So $\dot{u}(t)$ is contained in J here. Additionally, it is realistic to bring $u^{\mathrm{T}}(t)Q_u u(t)$ into the quadratic performance index, which can limit $u(t)$.

We wish to design an optimal preview controller for system (1) under the performance index (3).

3. Design of the Optimal Preview Controller

In order to obtain the optimal preview controller for system (1), we construct an augmented error system that includes the future information on the desired output $r(t)$ as well as the tracking error signal $e(t)$, the state vector $x(t)$, and the controlled input $u(t)$.

Since the desired output $r(t)$ is contained in the tracking error signal $e(t)$ defined by (2), differentiate (2) and we can get

$$\dot{e}(t) = CAx(t) + CBu(t) - \dot{r}(t). \tag{4}$$

Combining (4) with the first formula of system (1) yields

$$\begin{bmatrix} \dot{e}(t) \\ \dot{x}(t) \end{bmatrix} = \begin{bmatrix} 0 & CA \\ 0 & A \end{bmatrix} x(t) + \begin{bmatrix} CB \\ B \end{bmatrix} u(t) + \begin{bmatrix} -I \\ 0 \end{bmatrix} \dot{r}(t). \tag{5}$$

Equation (5) is the error system of (1). Due to the derivative of the control input being contained in performance index (3), we consider taking $\dot{u}(t)$ as the control input of the augmented error system. Now define the augmented state vector

$$X(t) = \begin{bmatrix} e(t) \\ x(t) \\ u(t) \end{bmatrix} \in \mathbf{R}^{p+n+m}. \tag{6}$$

Putting (5) and $\dot{u}(t) = \dot{u}(t)$ together gives

$$\dot{X}(t) = \overline{A}X(t) + \overline{B}\dot{u}(t) + \overline{D}\dot{r}(t), \quad t \in [t_0, t_\alpha]. \tag{7}$$

The corresponding output equation is

$$e(t) = \overline{C}X(t), \tag{8}$$

where

$$\overline{A} = \begin{bmatrix} 0 & CA & CB \\ 0 & A & B \\ 0 & 0 & 0 \end{bmatrix} \in \mathbf{R}^{(p+n+m)\times(p+n+m)},$$

$$\overline{B} = \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix} \in \mathbf{R}^{(p+n+m)\times m}, \qquad \overline{D} = \begin{bmatrix} -I \\ 0 \\ 0 \end{bmatrix} \in \mathbf{R}^{(p+n+m)\times p},$$

$$\overline{C} = \begin{bmatrix} I & 0 & 0 \end{bmatrix}. \tag{9}$$

Noting the tracking error signal $e(t)$ of system (1) as being a part of the augmented state vector $X(t)$ in (7), performance index (3) can be expressed as

$$J = \frac{1}{2} X^{\mathrm{T}}(t_\alpha) E X(t_\alpha) + \frac{1}{2}\int_{t_0}^{t_\alpha} \left[X^{\mathrm{T}}(t)\overline{Q}X(t) + \dot{u}^{\mathrm{T}}(t) R \dot{u}(t) \right] \mathrm{d}t, \tag{10}$$

where

$$E = \begin{bmatrix} I & & \\ & 0 & \\ & & 0 \end{bmatrix}, \qquad \overline{Q} = \begin{bmatrix} Q_e & & \\ & Q_x & \\ & & Q_u \end{bmatrix}. \tag{11}$$

Therefore, the optimal controller can be derived by solving the control problem that minimizes the performance index J of (10),subject to the dynamic constraint of (7).

Equation (7) is the augmented error system that we need. It is obvious that the tracking error signal $e(t)$ is a part of the augmented state vector $X(t)$ in (7). So it is natural to conclude that the output vector $y(t)$ of (1) can track the desired output $r(t)$ with less static error if the controller of (7) makes the state vector $X(t)$ stable to zero.

Remark 5. Concerning the construction of the augmented error system, the innovative features of this paper mainly lie in the following two aspects, which are also the primary differences from [14]. First, we only take the derivative of the tracking error signal $e(t)$, whereas [14] took the derivatives of both the tracking error signal and the two sides of the state equation. Second, the control input is put into the augmented state vector. And then $\dot{u}(t)$ and $u(t)$ are both introduced into the performance index to restrict themselves within an allowable range. The method described in this paper can also ensure that the closed-loop system of (1) contains integration terms to reduce any static errors.

Next, the optimal preview controller $u(t)$ of system (1) will be deduced by solving the controlled input $\dot{u}(t)$ of (7) under performance index (10).

For system (7) and performance index (10), the Hamilton function is chosen as

$$H(X, \dot{u}, \lambda, t) = \frac{1}{2}\left[X^{\mathrm{T}}(t)\overline{Q}X(t) + \dot{u}^{\mathrm{T}}(t) R \dot{u}(t) \right] + \lambda^{\mathrm{T}}(t)\left[\overline{A}X(t) + \overline{B}\dot{u}(t) + \overline{D}\dot{r}(t) \right]. \tag{12}$$

According to the theory of optimal control [16], if we want to enforce optimal control on system (7), the Hamilton function $H(X, \dot{u}, \lambda, t)$ must satisfy the canonical equations:

$$\dot{\lambda}(t) = -\frac{\partial H}{\partial X}, \qquad \frac{\partial H}{\partial \dot{u}} = 0; \tag{13}$$

that is,

$$\begin{aligned} \dot{\lambda}(t) &= -\overline{Q}X(t) - \overline{A}^{\mathrm{T}}\lambda(t), \\ \dot{u}(t) &= -R^{-1}\overline{B}^{\mathrm{T}}\lambda(t), \end{aligned} \tag{14a}$$

and the boundary conditions

$$X(t_0) = X_0, \qquad \lambda(t_\alpha) = 0. \tag{14b}$$

As the constraint condition, Equation (7) needs to be met for $H(X, \dot{u}, \lambda, t)$. Combining (14a) and (14b) with (7) holds as follows:

$$\begin{aligned} \dot{X}(t) &= \overline{A}X(t) + \overline{B}\dot{u}(t) + \overline{D}\dot{r}(t), \\ \dot{\lambda}(t) &= -\overline{Q}X(t) - \overline{A}^{\mathrm{T}}\lambda(t), \\ \dot{u}(t) &= -R^{-1}\overline{B}^{\mathrm{T}}\lambda(t), \\ X(t_0) &= X_0, \qquad \lambda(t_\alpha) = 0. \end{aligned} \tag{15}$$

Eliminating $\dot{u}(t)$ from the first three equations of (15), we can get the following differential equations relevant to $X(t)$ and $\lambda(t)$:

$$\begin{bmatrix} \dot{X}(t) \\ \dot{\lambda}(t) \end{bmatrix} = \begin{bmatrix} \overline{A} & -\overline{B}R^{-1}\overline{B}^{\mathrm{T}} \\ -\overline{Q} & -\overline{A}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} X(t) \\ \lambda(t) \end{bmatrix} + \begin{bmatrix} \overline{D} \\ 0 \end{bmatrix} \dot{r}(t). \tag{16}$$

Then from the above differential equations and the conclusions in [14], we can apprehend that the relationship between $\lambda(t)$ and $X(t)$ can be expressed as

$$\lambda(t) = P(t)X(t) + g(t), \tag{17}$$

where the matrix $P(t)$ and the vector function $g(t)$, respectively, satisfy the following Riccati equation and linear differential equation:

$$\begin{aligned} &\dot{P}(t) + P(t)\overline{A} + \overline{A}^{\mathrm{T}}P(t) - P(t)\overline{B}R^{-1}\overline{B}^{\mathrm{T}}P(t) + \overline{Q} = 0, \\ &\dot{g}(t) + \left[\overline{A}^{\mathrm{T}} - P(t)\overline{B}R^{-1}\overline{B}^{\mathrm{T}}\right] g(t) + P(t)\overline{D}\dot{r}(t) = 0, \end{aligned} \tag{18}$$

with the boundary conditions $P(t_\alpha) = 0$, $g(t_\alpha) = 0$.

Substituting (17) into the third formula of (15) obtains Theorem 6.

Theorem 6. *Under performance index (10), the optimal control input of augmented error system (7) is*

$$\dot{u}(t) = -R^{-1}\overline{B}^{\mathrm{T}}P(t)X(t) - R^{-1}\overline{B}^{\mathrm{T}}g(t), \tag{19}$$

where $P(t)$ is the nonnegative definite solution of the Riccati equation:

$$\dot{P}(t) + P(t)\overline{A} + \overline{A}^{\mathrm{T}} P(t) - P(t)\overline{B}R^{-1}\overline{B}^{\mathrm{T}} P(t) + \overline{Q} = 0, \qquad P(t_\alpha) = 0, \tag{20}$$

and $g(t)$ is the solution of the following time-varying differential equation:

$$\dot{g}(t) + \left[\overline{A}^{\mathrm{T}} - P(t)\overline{B}R^{-1}\overline{B}^{\mathrm{T}}\right] g(t) + P(t)\overline{D}\dot{r}(t) = 0, \qquad g(t_\alpha) = 0. \tag{21}$$

Remark 7. Theoretically, we can obtain $g(t)$ in (21) after obtaining the nonnegative definite solution $P(t)$ of (20). Then substituting $g(t)$ into (19), we can deduce the expression of $\dot{u}(t)$. So, through integration, the optimal control input $u(t)$ with preview compensation of system (1) can be derived easily.

Remark 8. Practically, all variables mentioned in Remark 7 can be given in the form of numerical solutions. Consequently, the control law in Theorem 6 can be realized. But we still hope to get further results.

First, we solve $g(t)$ from (21). Let $A_c(t) = \overline{A} - \overline{B}R^{-1}\overline{B}^{\mathrm{T}}P(t)$. Equation (21) can be written as follows:

$$\dot{g}(t) = -A_c^{\mathrm{T}}(t)\, g(t) - P(t)\overline{D}\dot{r}(t), \qquad g(t_\alpha) = 0. \tag{22}$$

Suppose $\widetilde{\Phi}(t, t_0)$ is a fundamental matrix solution of the homogeneous equation $\dot{g}(t) = -A_c^{\mathrm{T}}(t)g(t)$; then based on the theory of differential equations [16], the solution of (22) is given by the formula

$$g(t) = \widetilde{\Phi}(t, t_0)\, g(t_0) - \int_{t_0}^{t} \widetilde{\Phi}(t, \tau) P(\tau)\overline{D}\dot{r}(\tau)\,\mathrm{d}\tau. \tag{23}$$

Thus the following formula holds:

$$g(t_\alpha) = \widetilde{\Phi}(t_\alpha, t)\, g(t) - \int_{t}^{t_\alpha} \widetilde{\Phi}(t_\alpha, \tau) P(\tau)\overline{D}\dot{r}(\tau)\,\mathrm{d}\tau. \tag{24}$$

Using $g(t_\alpha) = 0$, we get

$$g(t) = \widetilde{\Phi}^{-1}(t_\alpha, t) \int_{t}^{t_\alpha} \widetilde{\Phi}(t_\alpha, \tau) P(\tau)\overline{D}\dot{r}(\tau)\,\mathrm{d}\tau. \tag{25}$$

Considering the characteristics of the desired output $r(t)$ and $\widetilde{\Phi}(t, t_0)$, we realize that $g(t)$ can be given as

$$\begin{aligned} g(t) &= \widetilde{\Phi}(t, t_\alpha) \int_{t}^{t_\alpha \Lambda(t+l_r)} \widetilde{\Phi}(t_\alpha, \tau) P(\tau)\overline{D}\dot{r}(\tau)\,\mathrm{d}\tau \\ &= \int_{t}^{t_\alpha \Lambda(t+l_r)} \widetilde{\Phi}(t, \tau) P(\tau)\overline{D}\dot{r}(\tau)\,\mathrm{d}\tau. \end{aligned} \tag{26}$$

Further, we want to deduce the relationship among the derivative of the controlled input $\dot{u}(t)$, the tracking error signal $e(t)$, the state vector $x(t)$, and the desired output $r(t)$ of system (1). In view of $P(t)$ being $(p+n+m)\times(p+n+m)$ nonnegative definite matrices, partitioning $P(t)$ gives

$$\begin{aligned} P(t) &= \left[p_1(t) \quad p_2(t) \quad p_3(t)\right] \\ &= \begin{bmatrix} p_{11}(t) & p_{12}(t) & p_{13}(t) \\ p_{12}^{\mathrm{T}}(t) & p_{22}(t) & p_{23}(t) \\ p_{13}^{\mathrm{T}}(t) & p_{23}^{\mathrm{T}}(t) & p_{33}(t) \end{bmatrix}, \end{aligned} \tag{27}$$

where $p_{11}(t) \in \mathbf{R}^{p\times p}$, $p_{22}(t) \in \mathbf{R}^{n\times n}$, and $p_{33}(t) \in \mathbf{R}^{m\times m}$. Then, based on the structure of $X(t)$, $\overline{B}$ and $\overline{D}$, substituting (26) and (27) into (19) gives

$$\begin{aligned} \dot{u}(t) &= -R^{-1}\left[p_{13}^{\mathrm{T}}(t)\, e(t) + p_{23}^{\mathrm{T}}(t)\, x(t) + p_{33}(t)\, u(t)\right] \\ &\quad - \left[0 \quad 0 \quad R^{-1}\right] \int_{t}^{t_\alpha \Lambda(t+l_r)} \widetilde{\Phi}(t, \tau) P(\tau)\overline{D}\dot{r}(\tau)\,\mathrm{d}\tau; \end{aligned} \tag{28}$$

that is,

$$\begin{aligned} &\dot{u}(t) + R^{-1} p_{33}(t)\, u(t) \\ &\quad = -R^{-1}\left[p_{13}^{\mathrm{T}}(t)\, e(t) + p_{23}^{\mathrm{T}}(t)\, x(t)\right] \\ &\qquad - \left[0 \quad 0 \quad R^{-1}\right] \int_{t}^{t_\alpha \Lambda(t+l_r)} \widetilde{\Phi}(t, \tau) P(\tau)\overline{D}\dot{r}(\tau)\,\mathrm{d}\tau. \end{aligned} \tag{29}$$

Let $f(t) = \exp(R^{-1}\int_{t_0}^{t} p_{33}(\theta)\mathrm{d}\theta)$; we have $(\mathrm{d}/\mathrm{d}t)f(t) = f(t)R^{-1}p_{33}(t)$. So the above formula can be rewritten as

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t}\left[f(t)\, u(t)\right] \\ &\quad = -f(t)\, R^{-1} \\ &\qquad \cdot \Bigg[p_{13}^{\mathrm{T}}(t)\, e(t) + p_{23}^{\mathrm{T}}(t)\, x(t) \\ &\qquad\quad + \left[0 \quad 0 \quad I\right] \int_{t}^{t_\alpha \Lambda(t+l_r)} \widetilde{\Phi}(t, \tau) P(\tau)\overline{D}\dot{r}(\tau)\,\mathrm{d}\tau\Bigg]. \end{aligned} \tag{30}$$

Noticing $u(t) = 0$, $x(t) = 0$, and $r(t) = 0$ when $t \le t_0$, so we have

$$\begin{aligned} &u(t) \\ &= -f^{-1}(t) \\ &\quad \cdot \int_{t_0}^{t} \Bigg\{ f(\sigma)\, R^{-1} \Bigg[p_{13}^{\mathrm{T}}(\sigma)\, e(\sigma) \\ &\qquad\qquad + p_{23}^{\mathrm{T}}(\sigma)\, x(\sigma) + \left[0 \quad 0 \quad I\right] \\ &\qquad\qquad \cdot \int_{\sigma}^{t_\alpha \Lambda(\sigma+l_r)} \widetilde{\Phi}(\sigma, \tau) P(\tau)\overline{D}\dot{r}(\tau)\,\mathrm{d}\tau\Bigg]\Bigg\}\,\mathrm{d}\sigma. \end{aligned} \tag{31}$$

Therefore, the subsequent theorem to describe the controlled input $u(t)$ of continuous-time system (1) follows immediately.

Theorem 9. *Under performance index (3), the optimal preview controller of continuous-time system (1) is*

$$\begin{aligned} u(t) &= -f^{-1}(t) \\ &\cdot \int_{t_0}^{t} \Big\{ f(\sigma) R^{-1} \Big[p_{13}^{\mathrm{T}}(\sigma) e(\sigma) \\ &\quad + p_{23}^{\mathrm{T}}(\sigma) x(\sigma) + [0 \;\; 0 \;\; I] \\ &\quad \cdot \int_{\sigma}^{t_\alpha \Lambda(\sigma+l_r)} \widetilde{\Phi}(\sigma,\tau) P(\tau) \overline{D}\dot{r}(\tau)\, \mathrm{d}\tau \Big] \Big\} \mathrm{d}\sigma, \end{aligned} \tag{32}$$

where $f(t) = \exp(R^{-1}\int_{t_0}^{t} p_{33}(\theta)\,\mathrm{d}\theta)$, $p_{i3}(t)$ $(i = 1,2,3)$ *is shown as (27),* $P(t)$ *is the nonnegative definite solution of the Riccati equation (20) [14], and* $\widetilde{\Phi}(t,t_0)$ *is the fundamental matrix solution of the homogeneous equation of Equation (22).*

As in Theorem 9, it is easy to find that the controller of continuous-time system (1) is the combination of integral actions related to the tracking error signal, the state vector, and the desired output. Compared with [14], the controller described in this paper does not contain the state feedback but the integral of the state vector.

4. Numerical Example

Example 1. Consider a continuous-time system with single input and single output described by

$$\begin{gathered} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u, \\ y = [2 \;\; 4] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}. \end{gathered} \tag{33}$$

The coefficient matrices of system (1) are, respectively,

$$A = \begin{bmatrix} -3 & 2 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = [2 \;\; 4], \quad t_0 = 0. \tag{34}$$

The weight matrices in performance index (3) are

$$\begin{gathered} Q_e = 1, \qquad R = 16, \\ Q_x = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad Q_u = 1. \end{gathered} \tag{35}$$

The solution of Riccati equation

$$P\overline{A} + \overline{A}^{\mathrm{T}} P - P\overline{B}R^{-1}\overline{B}^{\mathrm{T}} P + \overline{Q} = 0 \tag{36}$$

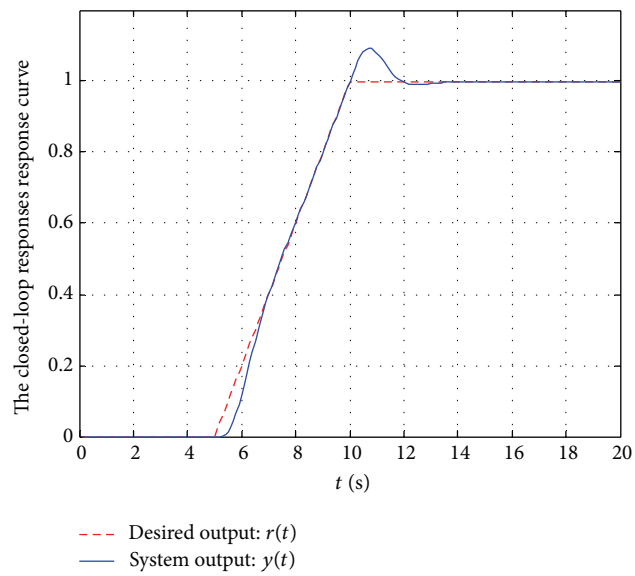

FIGURE 1: Closed-loop responses of system (33) without preview compensation.

is

$$P = \begin{bmatrix} 2.8410 & -2.1234 & 3.1238 & 4.0988 \\ -2.1234 & 9.0732 & 30.2384 & 15.6246 \\ 3.1238 & 30.2384 & 147.9250 & 88.5550 \\ 4.0988 & 15.6246 & 88.5550 & 59.3829 \end{bmatrix}. \tag{37}$$

And the matrix of A_c is

$$A_c = \begin{bmatrix} 0 & -2 & 8 & 4 \\ 0 & -3 & 2 & 0 \\ 0 & 1 & 1 & 1 \\ -0.2440 & -0.9300 & -5.2711 & -3.5347 \end{bmatrix}. \tag{38}$$

Let the desired output $r(t)$ be

$$r(t) = \begin{cases} 0, & 0 \le t \le 5; \\ 0.2(t-5), & 5 < t \le 10; \\ 1, & t > 10. \end{cases} \tag{39}$$

When there is no preview compensation, that is, $l_r = 0$ in the controller, the output response of system (33) is shown in Figure 1 through Matlab simulation. When the preview length is chosen as $l_r = 2$, the corresponding output responses of system (33) are shown, respectively, in Figure 2. Comparing Figures 1 and 2, we find that preview control for continuous-time systems can decrease the static error.

5. Conclusions

Based on the desired output being piecewise differentiable, we take only the derivative of the tracking error equation, the state equation, and an identical equation of the derivative of the control input as the new state equation of the augmented

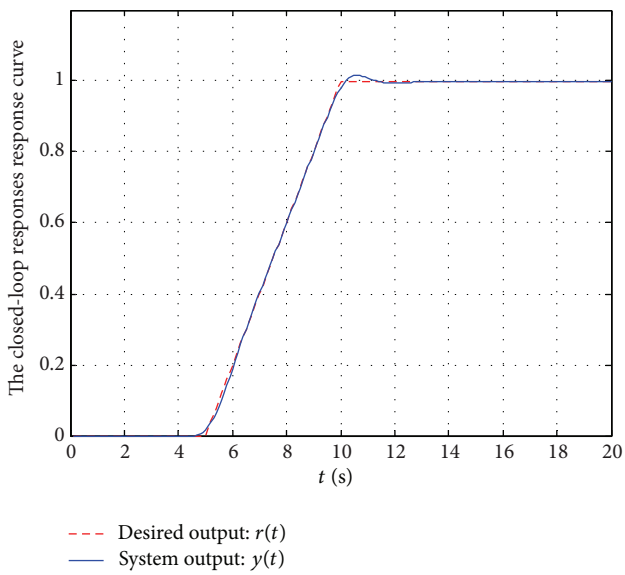

Figure 2: Closed-loop responses of system (33) with $l_r = 2$.

error system. Then, according to optimal control theory, a differential equation relevant to the control input is given. So the optimal preview controller for the original continuous-time system is deduced. The controller in this paper does not contain the state feedback, but rather, the desired output and the integrals of the tracking error signal and the state vector. The preview controller can also reduce static errors effectively.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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