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Research Article

Robust Adaptive Output Feedback Control Scheme for Chaos Synchronization with Input Nonlinearity

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This paper proposes a robust adaptive output feedback control strategy which can automatically regulate control gain for chaos synchronization. Chaotic systems with input nonlinearities, delayed nonlinear coupling, and external disturbance can achieve synchronization by applying this strategy. Utilizing Lyapunov method and LMI technique, the conditions ensuring chaos synchronization are obtained. Finally, simulations are given to show the effectiveness of our control strategy.

1. Introduction

Chaos synchronization has attracted a lot of interest due to its wide engineering application in various areas like secure communication [1–4], neural networks [5, 6], electronic engineering [7], and so on [8, 9]. Consider the fact that chaotic system is a class of nonlinear dynamical system which sensitively depends on initial conditions. It is necessary to solve the problem of chaos synchronization.

In the practical physical systems, physical limitations will lead to state nonlinearity and input nonlinearity, such as sector [10–12], saturation [13–15], and dead zones [16–19]. Considering state nonlinearity, [20] proposed an adaptive control strategy for multirate networked nonlinear systems. In [21], fuzzy control method has been applied to a class of nonlinear system. Filter design and H_∞ performance for nonlinear networked systems have been researched in [22]. A novel sliding mode observer approach has been proposed for a class of stochastic systems in [23]. Moreover, it should not be ignored that effect of nonlinear control inputs can cause serious degradation of synchronization performance even nonsynchronous. Thus, nonlinear control inputs should be considered in synchronization controller design of chaotic systems. Unfortunately, in [24, 25], input nonlinearity has not been considered.

As a source of nonsynchronism, time-varying delay has to be faced in many engineering synchronization systems, such as chemical processes [26, 27] and pneumatic systems [28]. Therefore, designing a controller for time-varying delay systems is necessary [29].

Recently, considering nonlinearly coupled chaotic systems, [30] proposed a state feedback controller to achieve synchronization. However, the input nonlinearity was not considered. To the authors' knowledge, synchronization for coupled chaotic systems with input nonlinearity and time delays has been rarely mentioned. Furthermore, in real application, only the output state is available. Therefore, it is necessary to design a synchronization controller in output feedback form.

Motivated by the previous discussions, we propose an adaptive output feedback controller to make chaotic systems synchronize. Input nonlinearities, nonlinear coupling, and time-varying delay have been taken into account. By utilizing Lyapunov method and LMI technique, the conditions ensuring synchronization are obtained.

In the rest of this paper, Section 2 provides systems and problem description. Then a robust adaptive output feedback control strategy is proposed for chaos synchronization in Section 3. In Section 4, simulations are given to demonstrate

effectiveness of this control strategy. Conclusion are collected in Section 5.

2. Problem Formulation

Consider chaotic systems as follows:

$$\begin{aligned}\dot{x}_m(t) &= Ax_m(t) + A_c x_m(t-d_1(t)) + Bf(t, x_m(t)) \\ &\quad + B_c g(t, x_m(t-d_1(t))) \\ &\quad - B_d k(t, x_s(t-d_2(t))), \\ y_m(t) &= Cx_m(t)\end{aligned} \tag{1}$$

$$\begin{aligned}\dot{x}_s(t) &= Ax_s(t) + A_c x_s(t-d_1(t)) + Bf(t, x_s(t)) \\ &\quad + B_c g(t, x_s(t-d_1(t))) \\ &\quad - B_d k(t, x_m(t-d_2(t))) + D\omega(t) \\ &\quad + E\Lambda(u(t)), \\ y_s(t) &= Cx_s(t),\end{aligned} \tag{2}$$

where $x_m \in \mathbb{R}^n$, $x_s \in \mathbb{R}^n$, $y_m \in \mathbb{R}^m$, and $y_s \in \mathbb{R}^m$ are the state vector and output vector for the drive and response system, respectively. $f(t,x), g(t,x), k(t,x) : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^m$ represent nonlinear vectors. $u(t) = [u_1(t) \cdots u_m(t)]^T \in \mathbb{R}^m$ is the control input vector; $\omega(t)$ denotes the external disturbance; time-varying delay $d_1(t)$ and $d_2(t)$ satisfy

$$d_{11} \le \dot{d}_1(t) \le d_{12}, \tag{3}$$

$$d_{21} \le \dot{d}_2(t) \le d_{22}. \tag{4}$$

$\Lambda(u(t)) = [\lambda_1(u_1(t)) \cdots \lambda_m(u_m(t))]^T$ is representing the nonlinear control input vector which satisfies the following inequality:

$$\nu_i(t)\lambda_i(\nu_i(t)) \ge \chi_i(\nu_i(t))^2. \tag{5}$$

χ_i is an unknown positive constant satisfying $\chi^* = \min \chi_i$. Constant matrices $A, A_c, B, B_c, B_d, C, D, E$ have appropriate dimensions.

Synchronization error can be defined as $e(t) = x_s(t) - x_m(t)$. Using (1) and (2), synchronization error can be obtained:

$$\begin{aligned}\dot{e}(t) &= Ae(t) + A_c e(t-d_1(t)) + B\Psi(t) + B_c\Upsilon(t, d_1) \\ &\quad + B_d\Xi(t, d_2) + D\omega(t) + E\Lambda(u(t)), \\ y_e(t) &= Ce(t),\end{aligned} \tag{6}$$

where

$$\begin{aligned}\Psi(t) &= f(t, x_s(t)) - f(t, x_m(t)), \\ \Upsilon(t, d_1) &= g(t, x_s(t-d_1(t))) - g(t, x_m(t-d_1(t))), \\ \Xi(t, d_2) &= k(t, x_s(t-d_2(t))) - k(t, x_m(t-d_2(t))).\end{aligned} \tag{7}$$

The objective is to make drive and response systems synchronize. Obviously, if $e(t) \to 0$, then $x_s(t) - x_m(t) \to 0$ and it means that system (1) and (2) is synchronized.

To obtain the synchronization conditions, the following lemma and assumptions will be used during the proof.

Lemma 1. *If matrix $H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}$, where H_{11} and H_{22} are square matrices, then the following inequalities are equivalent:*

(1) $H < 0$;

(2) $H_{11} < 0$, $H_{22} - H_{12}^T H_{11}^{-1} H_{12} < 0$;

(3) $H_{22} < 0$, $H_{11} - H_{12} H_{22}^{-1} H_{12}^T < 0$.

Assumption 2. The nonlinear function $\Psi(t)$, $\Upsilon(t, d_1)$, and $\Xi(t, d_2)$ satisfy the global Lipschitz condition:

$$\begin{aligned}\|\Psi(t)\| &\le L_1 \|x_s(t) - x_m(t)\|, \\ \|\Upsilon(t, d_1)\| &\le L_2 \|x_s(t-d_1(t)) - x_m(t-d_1(t))\|, \\ \|\Xi(t, d_2)\| &\le L_3 \|x_s(t-d_2(t)) - x_m(t-d_2(t))\|.\end{aligned} \tag{8}$$

Assumption 3. Matrix $P > 0$ and satisfies the following equation:

$$E^T P = C. \tag{9}$$

3. Robust Adaptive Controller Design Based on LMI

In order to make drive and response systems synchronize, the following adaptive controller is considered:

$$u(t) = -\frac{\nu}{2} Ce(t), \tag{10}$$

where ν is an adaptive control gain and is adjusted by the following adaptation law:

$$\dot{\nu} = \chi^* \rho \|Ce(t)\|^2, \quad \nu(0) > 0, \tag{11}$$

where ρ is a positive parameter.

By applying of the adaptive controller, synchronization errors will converge to zero asymptotically. In Theorems 4 and 5 the main results will be presented.

Theorem 4. *Consider the drive and response system (1) and (2) under $\omega(t) = 0$. By application of the adaptive control law (10) and (11), if existing symmetric and positive definite matrices P, W_1, W_2, and a scalar $\alpha > 0$, satisfying the following LMI:*

$$\begin{bmatrix} \Delta & PA_c & 0 & PB & PB_c & PB_d & L_1^T & 0 & 0 \\ * & -(1-d_{12})W_1 & 0 & 0 & 0 & 0 & 0 & L_2^T & 0 \\ * & * & -(1-d_{22})W_2 & 0 & 0 & 0 & 0 & 0 & L_3^T \\ * & * & * & -I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0, \tag{12}$$

where

$$\Delta = PA + A^T P - \alpha PEE^T P + W_1 + W_2, \tag{13}$$

the system (1) and (2) is synchronized.

Proof. We consider the following Lyapunov-Krasovskii functional:

$$V = e^T(t)Pe(t) + \int_{t-d_1(t)}^{t} e^T(v)W_1 e(v)\,dv + \int_{t-d_2(t)}^{t} e^T(v)W_2 e(v)\,dv + \frac{1}{2}\rho^{-1}\tilde{\nu}^2, \tag{14}$$

where $\tilde{\nu} = \nu^* - \nu$. ν^* and ρ are positive constants.

The derivative of V can be calculated as follows:

$$\begin{aligned} \dot{V} &= 2e^T(t)P\dot{e}(t) + e^T(t)W_1 e(t) - \left(1-\dot{d}_1(t)\right)e^T(t-d_1(t))W_1 e(t-d_1(t)) + e^T(t)W_2 e(t) - \left(1-\dot{d}_2(t)\right)e^T(t-d_2(t))W_2 e(t-d_2(t)) - \rho^{-1}\dot{\nu}\tilde{\nu} \\ &= 2e^T(t)P\left[Ae(t) + A_c e(t-d_1(t)) + B\Psi(t) + B_c\Upsilon(t,d_1) + B_d\Xi(t,d_2) + E\Lambda(u(t))\right] + e^T(t) \cdot W_1 e(t) - \left(1-\dot{d}_1(t)\right)e^T(t-d_1(t))W_1 e(t-d_1(t)) \\ &\quad + e^T(t)W_2 e(t) - \left(1-\dot{d}_2(t)\right)e^T(t-d_2(t))W_2 e(t-d_2(t)) - \rho^{-1}\dot{\nu}\tilde{\nu}. \end{aligned} \tag{15}$$

Incorporating (3) and (4), we can get

$$\begin{aligned} \dot{V} &\le e^T(t)\left(PA + A^T P - \alpha PEE^T P + W_1 + W_2\right)e(t) \\ &\quad + \alpha\left\|E^T Pe(t)\right\|^2 + 2e^T(t)PA_c e(t-d_1(t)) \\ &\quad + 2e^T(t)PB\Psi(t) + 2e^T(t)PB_c\Upsilon(t,d_1) \\ &\quad + 2e^T(t)PB_d\Xi(t,d_2) + 2e^T(t)PE\Lambda(u(t)) \\ &\quad - (1-d_{12})e^T(t-d_1(t))W_1 e(t-d_1(t)) \\ &\quad - (1-d_{22})e^T(t-d_2(t))W_2 e(t-d_2(t)) \\ &\quad - \rho^{-1}\dot{\nu}\tilde{\nu}. \end{aligned} \tag{16}$$

Using (8) leads to

$$\begin{aligned} \dot{V} &\le e^T(t)\left(PA + A^T P - \alpha PEE^T P + W_1 + W_2\right)e(t) \\ &\quad + \alpha\left\|E^T Pe(t)\right\|^2 + 2e^T(t)PA_c e(t-d_1(t)) \\ &\quad + 2e^T(t)PB\Psi(t) + 2e^T(t)PB_c\Upsilon(t,d_1) \\ &\quad + 2e^T(t)PB_d\Xi(t,d_2) + 2e^T(t)PE\Lambda(u(t)) \\ &\quad - (1-d_{12})e^T(t-d_1(t))W_1 e(t-d_1(t)) \\ &\quad - (1-d_{22})e^T(t-d_2(t))W_2 e(t-d_2(t)) \\ &\quad - \rho^{-1}\dot{\nu}\tilde{\nu} + e^T(t)L_1^T L_1 e(t) \\ &\quad + e^T(t-d_1(t))L_2^T L_2 e(t-d_1(t)) \\ &\quad + e^T(t-d_2(t))L_3^T L_3 e(t-d_2(t)) - \Psi^T(t)\Psi(t) \\ &\quad - \Upsilon^T(t,d_1)\Upsilon(t,d_1) - \Xi^T(t,d_2)\Xi(t,d_2). \end{aligned} \tag{17}$$

Assume $Ce(t) = Z(t)_{m\times 1}$; $Z_n(t)$ is the nth element of $Z(t)$. By using (11), it is easy to prove that $\nu > 0$. Based on (5), we need to consider the following two cases:

(1) When $Z_n(t) > 0$, we have $u_n(t) < 0$. Multiplying $Z_n(t) > 0$ and dividing $u_n(t) < 0$ by both sides of (5), we can obtain that $Z_n(t)\lambda_n(\nu_n(t)) \le \chi_n Z_n(t)\nu_n(t)$.

(2) When $Z_n(t) < 0$, we have $u_n(t) > 0$. Multiplying $Z_n(t) < 0$ and dividing $u_n(t) > 0$ by both sides of (5), we can obtain that $Z_n(t)\lambda_n(\nu_n(t)) \le \chi_n Z_n(t)\nu_n(t)$.

Form the previous discussion, we can prove that the following relation always holds:

$$Z_n(t)\lambda_n(\nu_n(t)) \le \chi_n Z_n(t)\nu_n(t). \tag{18}$$

Using Assumption 3 and $\chi^* = \min \chi_n$ we have

$$\begin{aligned} 2e^T(t)PE\Lambda(\nu(t)) &= 2\sum_{n=1}^{m} Z_n(t)\lambda_n(\nu_n(t)) \\ &\le 2\sum_{n=1}^{m}\chi_n Z_n(t)\nu_n(t) \\ &\le -\chi^*\nu\|Ce(t)\|^2. \end{aligned} \tag{19}$$

Let $\alpha = \chi^*\nu^*$; incorporating the previous result (19), we can obtain

$$\begin{aligned} \dot{V} \le{}& e^T(t) \\ &\cdot\left(PA + A^TP - \alpha PEE^TP + W_1 + W_2 + L_1^TL_1\right)e(t) \\ &+ \chi^*\nu^*\|Ce(t)\|^2 + 2e^T(t)PA_ce(t-d_1(t)) \\ &+ 2e^T(t)PB\Psi(t) + 2e^T(t)PB_c\Upsilon(t,d_1) + 2e^T(t) \\ &\cdot PB_d\Xi(t,d_2) - \chi^*\nu\|Ce(t)\|^2 - (1-d_{12}) \\ &\cdot e^T(t-d_1(t))W_1e(t-d_1(t)) - (1-d_{22}) \\ &\cdot e^T(t-d_2(t))W_2e(t-d_2(t)) - \chi^*(\nu^*-\nu) \\ &\cdot\|Ce(t)\|^2 + e^T(t-d_1(t))L_2^TL_2e(t-d_1(t)) \\ &+ e^T(t-d_2(t))L_3^TL_3e(t-d_2(t)) - \Psi^T(t)\Psi(t) \\ &- \Upsilon^T(t,d_1)\Upsilon(t,d_1) - \Xi^T(t,d_2)\Xi(t,d_2) = e^T(t) \\ &\cdot\left(PA + A^TP - \alpha PEE^TP + W_1 + W_2 + L_1^TL_1\right)e(t) \\ &+ e^T(t-d_1(t))\left(L_2^TL_2 - (1-d_{12})W_1\right) \\ &\cdot e(t-d_1(t)) + e^T(t-d_2(t)) \\ &\cdot\left(L_3^TL_3 - (1-d_{22})W_2\right)e(t-d_2(t)) + 2e^T(t) \\ &\cdot PA_ce(t-d_1(t)) + 2e^T(t)PB\Psi(t) + 2e^T(t) \\ &\cdot PB_c\Upsilon(t,d_1) + 2e^T(t)PB_d\Xi(t,d_2) - \Psi^T(t)\Psi(t) \\ &- \Upsilon^T(t,d_1)\Upsilon(t,d_1) - \Xi^T(t,d_2)\Xi(t,d_2) \end{aligned} \tag{20}$$

which further can be written as $\dot{V} \le \xi^T\Gamma_1\xi$, where

$$\xi^T = \left[e^T(t)\ \ e^T(t-d_1(t))\ \ e^T(t-d_2(t))\ \ \Psi^T(t)\ \ \Upsilon^T(t,d_1)\ \ \Xi^T(t,d_2)\right]^T, \tag{21}$$

$$\Gamma_1 = \begin{bmatrix} \Delta_1 & PA_c & 0 & PB & PB_c & PB_d \\ * & \Delta_2 & 0 & 0 & 0 & 0 \\ * & * & \Delta_3 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0, \tag{22}$$

where

$$\begin{aligned} \Delta_1 &= PA + A^TP - \alpha PEE^TP + L_1^TL_1 + W_1 + W_2, \\ \Delta_2 &= L_2^TL_2 - (1-d_{12})W_1, \\ \Delta_3 &= L_3^TL_3 - (1-d_{22})W_2. \end{aligned} \tag{23}$$

Using Lemma 1, (22) can be transformed to (12), which completed the proof of Theorem 4. □

In fact, disturbances always come from surrounding environment and communication channel. So, it is significant to solve the robustness problem in practical application. Theorem 5 will tackle this issue.

Theorem 5. *Consider the drive and response system (1) and (2) under $\omega(t) \ne 0$. By application of the adaptive control law (10) and (11), for given γ, if existing symmetric and positive definite matrices P, W_1, W_2, and a scalar $\alpha > 0$, satisfy the following LMI:*

$$\begin{bmatrix} \Pi & PA_c & 0 & PB & PB_c & PB_d & PD & L_1^T & 0 & 0 \\ * & -(1-d_{12})W_1 & 0 & 0 & 0 & 0 & 0 & 0 & L_2^T & 0 \\ * & * & -(1-d_{22})W_2 & 0 & 0 & 0 & 0 & 0 & 0 & L_3^T \\ * & * & * & -I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0, \tag{24}$$

where

$$\Pi = PA + A^T P - \alpha PEE^T P + W_1 + W_2 + C^T C, \tag{25}$$

the attention rate γ for H_∞ synchronization in the disturbance situation can be achieved.

Proof. With zero initial condition, let us introduce

$$J = \int_o^\infty \left[y_e^T(t)\, y_e(t) - \gamma^2 \omega^T(t)\, \omega(t) \right] dt \le 0. \tag{26}$$

For $\omega(t) \neq 0$, the following function can be obtained:

$$J \le \int_o^\infty \left[\dot{V}_d + y_e^T(t)\, y_e(t) - \gamma^2 \omega^T(t)\, \omega(t) \right] dt \le 0. \tag{27}$$

From the proof of Theorem 4, $\dot{V}_d$ can be obtained:

$$\begin{aligned} \dot{V}_d = {} & e^T(t) \\ & \cdot \left(PA + A^T P - \alpha PEE^T P + L_1^T L_1 + W_1 + W_2 \right) \\ & \cdot e(t) + e^T(t - d_1(t)) \left(L_2^T L_2 - (1-d_{12}) W_1 \right) \\ & \cdot e(t - d_1(t)) + e^T(t - d_2(t)) \\ & \cdot \left(L_3^T L_3 - (1-d_{22}) W_2 \right) e(t - d_2(t)) + 2e^T(t) \\ & \cdot PA_c e(t - d_1(t)) + 2e^T(t)\, PB\Psi(t) + 2e^T(t) \\ & \cdot PB_c \Upsilon(t, d_1) + 2e^T(t)\, PB_d \Xi(t, d_2) + 2e^T(t) \\ & \cdot PD\omega(t) - \Psi^T(t)\, \Psi(t) - \Upsilon^T(t, d_1)\, \Upsilon(t, d_1) \\ & - \Xi^T(t, d_2)\, \Xi(t, d_2). \end{aligned} \tag{28}$$

Let

$$\xi_d^T = \left[e^T(t) \;\; e^T(t - d_1(t)) \;\; e^T(t - d_2(t)) \;\; \omega^T(t) \;\; \Psi^T(t) \;\; \Upsilon^T(t, d_1) \;\; \Xi^T(t, d_2) \right]^T. \tag{29}$$

Equation (28) can be written as

$$J \le \xi_d^T \Gamma_2 \xi_d^T, \tag{30}$$

where

$$\Gamma_2 = \begin{bmatrix} \Pi_1 & PA_c & 0 & PB & PB_c & PB_d & PD \\ * & \Pi_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Pi_3 & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -I & 0 \\ * & * & * & * & * & 0 & -\gamma^2 I \end{bmatrix}, \tag{31}$$

$$\begin{aligned} \Pi_1 = {} & PA + A^T P - \alpha PEE^T P + L_1^T L_1 + W_1 + W_2 \\ & + C^T C, \end{aligned} \tag{32}$$

$$\begin{aligned} \Pi_2 &= L_2^T L_2 - (1-d_{12}) W_1, \\ \Pi_3 &= L_3^T L_3 - (1-d_{22}) W_2. \end{aligned} \tag{33}$$

Using Lemma 1, (31) can be transformed to (24), which completed the proof of Theorem 5. □

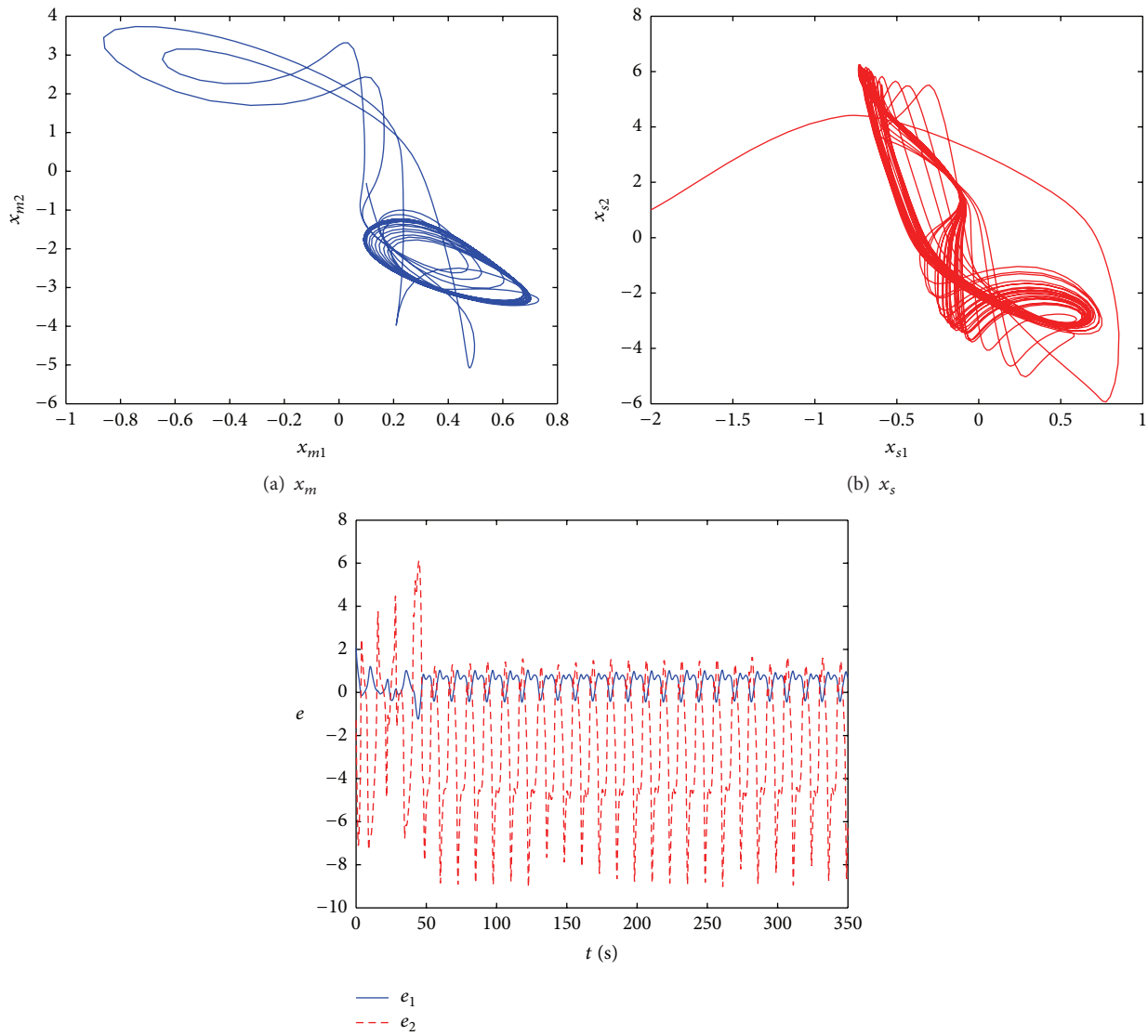


FIGURE 1: Behavior of the drive and response systems without any control: (a) phase trajectory of drive system, (b) phase trajectory of response system, and (c) synchronization errors.

4. Simulation Results

Consider the drive and response system (1) and (2) with parameters:

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix},$$

$$A_c = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 2 & -0.1 \\ -5 & 4.5 \end{bmatrix},$$

$$B_c = \begin{bmatrix} -1.5 & -0.1 \\ -0.2 & -4 \end{bmatrix},$$

$$B_d = \begin{bmatrix} 0.001 & 0 \\ 0 & 0.001 \end{bmatrix},$$

$$C = \begin{bmatrix} 1.5 & 0 \\ 0 & 0.5 \end{bmatrix},$$

$$E = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

$$f(t, x(t)) = \tanh(x(t)),$$

$$g(t, x(t - d_1(t))) = \tanh(x(t - d_1(t))),$$

$$h(t, x(t - d_2(t))) = x(t - d_2(t)) \sin(t),$$

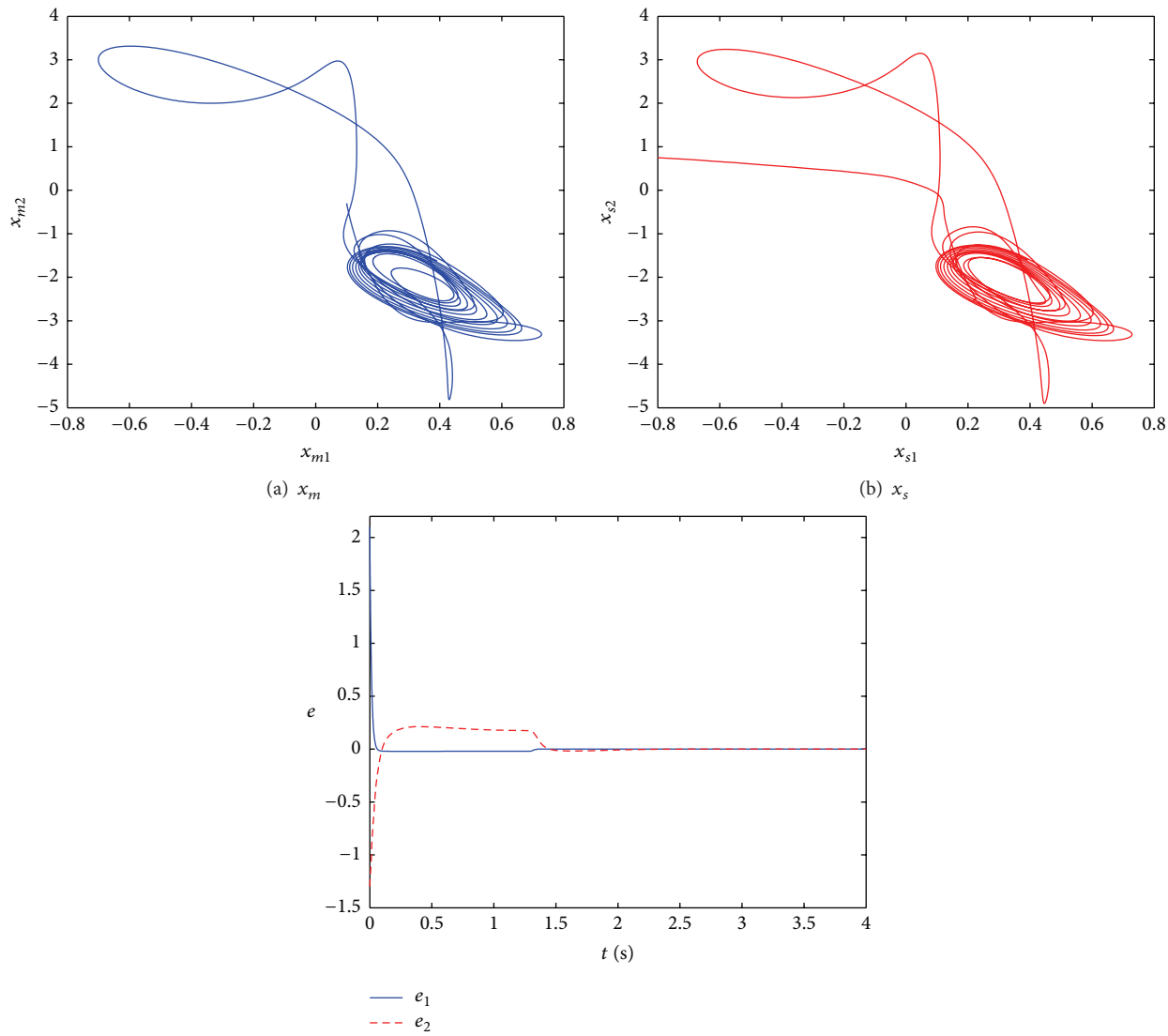


Figure 2: Behavior of the drive and response systems with controller: (a) phase trajectory of drive system, (b) phase trajectory of response system, and (c) synchronization errors.

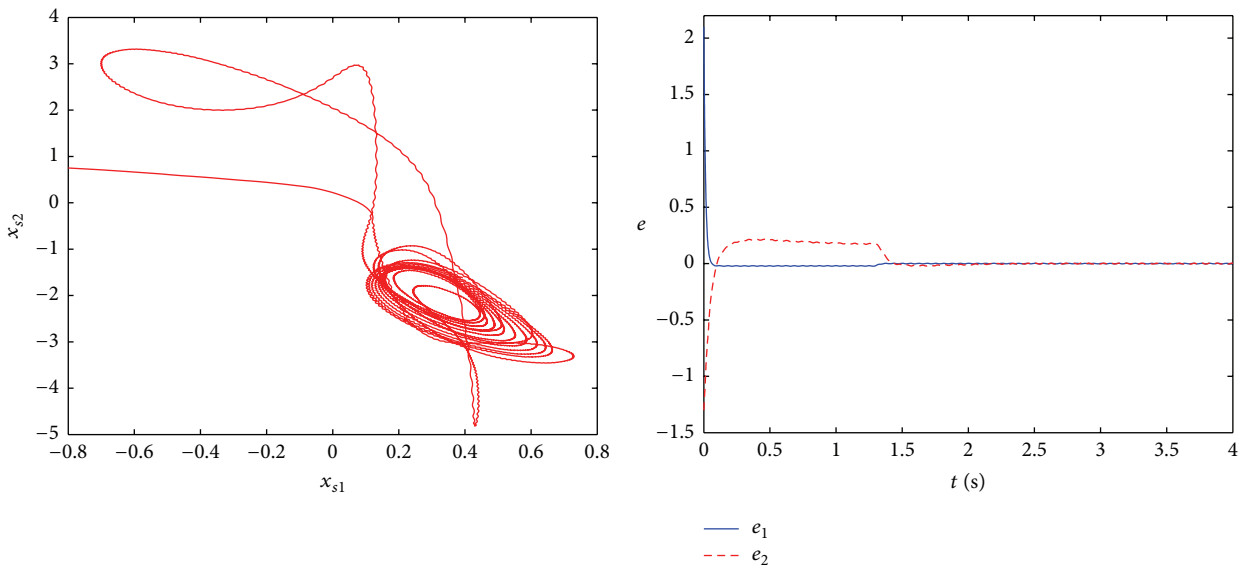
$$\begin{aligned} d_1(t) &= 1 + 0.3\sin(t), \\ d_2(t) &= 1 - 0.02\sin(10t), \\ \begin{bmatrix} \lambda(u_1(t)) \\ \lambda(u_2(t)) \end{bmatrix} &= \begin{bmatrix} 1 + 0.4\sin(u_1(t))\, u_1(t) \\ 1.2 + 0.2\cos(u_2(t))\, u_2(t) \end{bmatrix}, \\ \omega(t) &= [0.3\sin(100t) \quad 0.5\sin(110t)]^T. \end{aligned} \tag{34}$$

Initial conditions are chosen as

$$\begin{aligned} (x_{m1}(0), x_{m2}(0)) &= (0.1, -0.3), \\ (x_{s1}(0), x_{s2}(0)) &= (-2, 1), \\ \chi^* \rho &= 5, \\ \gamma &= 0.6. \end{aligned} \tag{35}$$

When $\omega(t) = 0$, using the LMI given in Theorem 4 (12), we can obtain

$$\begin{aligned} W_1 &= \begin{bmatrix} 29.1148 & 0.2902 \\ 0.2902 & 3.7117 \end{bmatrix}, \\ W_2 &= \begin{bmatrix} 27.6971 & 0.2399 \\ 0.2399 & 2.9992 \end{bmatrix}, \\ P &= \begin{bmatrix} 0.7500 & 0 \\ 0 & 0.2500 \end{bmatrix}, \\ \alpha &= 46.6242. \end{aligned} \tag{36}$$



(a) x_s (b) $e(t)$

Figure 3: Behavior of the drive and response systems with controller and disturbance: (a) phase trajectory of response system and (b) synchronization errors.

When $\omega(t) \neq 0$, using the LMI given in Theorem 5 (24), we can obtain

$$W_1 = \begin{bmatrix} 48.1311 & 0.2457 \\ 0.2457 & 6.9022 \end{bmatrix},$$
$$W_2 = \begin{bmatrix} 32.2655 & 0.2254 \\ 0.2254 & 6.8700 \end{bmatrix}, \quad (37)$$
$$P = \begin{bmatrix} 0.7500 & 0 \\ 0 & 0.2500 \end{bmatrix},$$
$$\alpha = 77.6427.$$

The phase trajectory of system (1) and (2) without any control is shown in Figures 1(a) and 1(b). The error between them is shown in Figure 1(c). It is obvious in Figure 1 that the systems are nonsynchronous. After application of the proposed controller, the phase trajectory of system (1) and (2) with nonlinear control inputs is shown in Figures 2(a) and 2(b). The error between them is shown in Figure 2(c). It is obvious in Figure 2 that the systems are synchronous. Figure 3(a) illustrates the phase trajectory of response system with nonlinear control inputs and disturbances after applying the proposed controller. Figure 3(b) illustrates that the error signal tends to zero in a short time, in spite of the disturbances.

5. Conclusion

We investigate the synchronization problem of chaotic systems with nonlinear control inputs. A robust adaptive controller has been established. By applying this controller, control gain can be regulated automatically and the synchronization of chaotic systems can be achieved. Then, considering external disturbances, we propose a new H_∞ synchronization method for chaotic systems. From the above simulation results, we can find that the error signal tends to zero in a short time. Therefore our control strategy is effective in synchronizing chaotic systems.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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