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*Research Article*

# Robust Control of Wind Turbines by Using Singular Perturbation Method and Linear Parameter Varying Model

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The maximum power point tracking problem of variable-speed wind turbine systems is studied in this paper. The wind conversion systems contain both mechanical part and electromagnetic part, which means the systems have time scale property. The wind turbine systems are modeled using singular perturbation methodology. A linear parameter varying (LPV) model is developed to approximate the nonlinear singularly perturbed model. Then stability and robust properties of the open-loop linear singularly perturbed system are analyzed using linear matrix inequalities (LMIs). An algorithm of designing a stabilizing state-feedback controller is proposed which can guarantee the robust property of the closed-loop system. Two numerical examples are provided to demonstrate the effectiveness of the control scheme proposed.

## 1. Introduction

Wind energy has become one of the fastest-growing energies during the last two decades [1–3]. Because the wind turbines are large, flexible structures operating in noisy environment can offer abundant energy to mankind, and control problems are necessary to be studied to improve the reliability and conversion efficiency of the wind energy conversion systems (WECS).

Generally, wind turbines can be classified into four categories by [4], namely, fixed-speed fixed-pitch turbines, fixed-speed variable-pitch turbines, variable-speed fixed-pitch turbines, and variable-speed variable-pitch turbines. Compared to fixed-speed wind turbines, variable-speed ones can capture more energy and achieve better dynamic loads alleviation and fewer grid connection power peaks [4, 5].

Even though a wind energy conversion system can operate over a wide range of wind speeds, basically it is only active in two regions including partial load and full load [2]. In the partial load region, the control objective is to track the desired rotor speed corresponding to varying wind speed $V(t)$ and maintain optimal tip-speed ratio [2, 6]. In the full load region, the generation goal is to limit the generated power to avoid overloading [4, 7]. Many researchers focus their interest on the control problems in the partial load region. In [8], an adaptive control scheme was developed for partial load region control of a variable-speed wind turbine, and the stability properties of the adaptive controller and the rotor speed were analyzed. Adaptive control approaches for maximizing power capture were also studied in [9, 10]. In [11], a two-mass model and a wind speed estimator were used to propose a nonlinear controller, which was aimed to optimize the wind power capture. A high-order sliding mode controller is designed based on a high gain observer to optimize the capture wind energy by tracking the optimal torque in [12, 13].

The LPV reformulation of gain scheduling has become very popular during the last decades [4]. Compared with switching linear time invariant (LTI) controllers, LPV method can achieve better stability property. The LPV model was firstly introduced by Shamma and Athans [14] and then has been widely applied to wind turbine systems [15–18].

As known to all, the wind conversion systems include mechanical part and electromagnetic part, and the time scales of mechanical dynamics (slow) and electrical dynamics (fast) are quite different from each other [2, 19]. So the wind conversion systems have two-time-scale property, which means these kinds of systems have high dimension and stiffness problems. The methodology of singular perturbation is always considered as a powerful tool to achieve dimension reduction, stiffness relief, and better control precision of multi-time scale systems in reality [20–23]. Therefore, the singular perturbation technique is introduced to reduce the stiffness and to reach better tracking precision in this paper.

Motivated by above, in this paper, the control problems of variable-speed variable-pitch turbines in the partial load region are considered. The rotor speed tracking problem of wind energy conversion system is studied to maintain the tip-speed ratio at optimal value $\lambda_{\text{opt}}$. Namely, our goal is to keep rotor speed $\omega_r(t)$ tracking the reference $\omega_{\text{ref}}(t) = \lambda_{\text{opt}} V(t)/R$ while the wind speed varies with time $t$, where $R$ is the radius of the wind rotor plane.

Firstly, the nonlinear model of the WECS is presented. By extracting a small singular perturbation parameter, the nonlinear singularly perturbed system is obtained from the original nonlinear system. Then, the nonlinear singularly perturbed system is approximated by a LPV model. The stability properties of the open-loop singularly perturbed LPV model are analyzed. Also, it is proved that $\|T_{\delta V \delta\omega_r}\| < \gamma$ holds for a given constant $\gamma > 0$, where $\delta V = V - \widehat{V}$, where $V$ is the wind speed, $\widehat{V}$ is the operating point, $\delta\omega_r = \omega_r(t) - \omega_{\text{ref}}(t)$ is the tracking error, and $\|T_{\delta V \delta\omega_r}\|$ is the transfer function from $\delta V$ to $\delta\omega_r$. Next, the controller design method is developed using LMI skills. Then, an algorithm is proposed to demonstrate the whole process of designing a feedback controller for the singularly perturbed nonlinear system. In the end, two numerical examples demonstrate the effectiveness of the algorithm obtained in this paper. The whole control scheme is depicted in Figure 1.

The main contributions of this paper are outlined as below:

(1) Singular perturbation methodology and LPV model are combined, for the first time, to solve the maximum power point tracking problem of wind energy conversion system. Compared to switching LTI controllers, LPV model can achieve better stability property. And, singular perturbation methodology is a wonderful tool to reduce the stiffness and improve the tracking precision.

(2) The stability and robust property of the singularly perturbed LPV systems are proved under certain conditions. Besides, an algorithm is presented to design a robust stabilizing control scheme for the singularly perturbed LPV system. Using LMI technology, the stability and robust property are proved for the closed-loop system.

The rest of this paper is organized as follows. In Section 2, the nonlinear model of the WECS is presented which involves the dynamic characteristics of the mechanical part and electrical part. Then, a singular perturbation parameter is extracted which leads to a singularly perturbed nonlinear model. The nonlinear singularly perturbed model is reformed into a singularly perturbed LPV model that contains a family of LTI models. Later, Lyapunov theory is utilized to prove

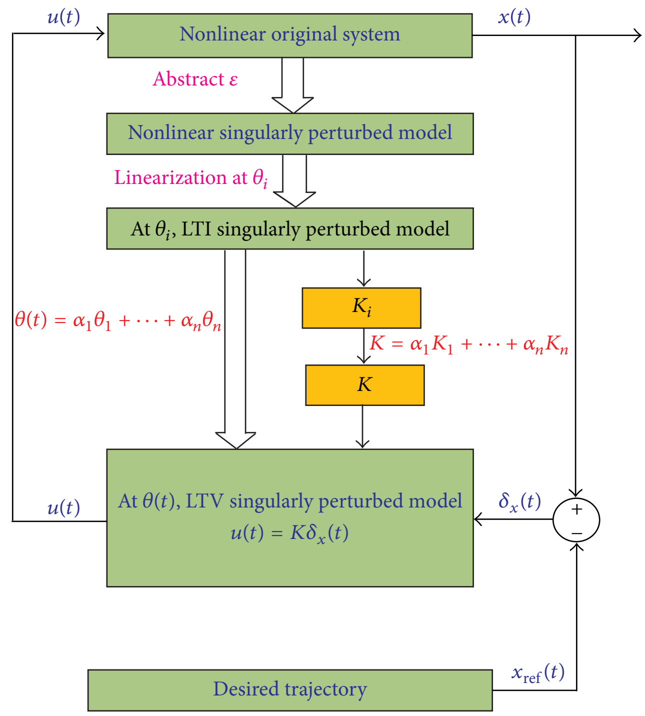


Figure 1: Control configuration of wind turbine system.

that the tracking error between the rotor speed $\omega_r(t)$ and the desired rotor speed $\omega_{\text{ref}}(t)$ decays to zero in Section 3. Furthermore, for a given constant $\gamma > 0$, $\|T_{\delta V \delta\omega_r}\| < \gamma$ is proved. In Section 4, the conditions in LMI forms are given to design a robust stabilizing parameter-dependent controller. Section 5 presents an algorithm to design a parameter-dependent controller. In Section 6, two numerical examples are given to illustrate the effectiveness of the results obtained. Finally, conclusions are drawn in Section 7.

## 2. System Description

In this section, the mathematical model of the wind turbine systems with permanent magnet synchronous generators is developed.

Because the electrical part of the system changes much faster than the mechanical part, namely, the states of the wind energy conversion systems have two different time scales. A singular perturbation parameter is extracted to obtain the singularly perturbed nonlinear model. The following LPV technique is used to linearize the singularly perturbed nonlinear system.

Our consideration is focused on partial load. In the partial load region, the wind rotor speed is adjusted to maintain the optimal tip-speed ratio as the wind speed changes. For this control purpose, only aerodynamics, drive train dynamics, and generator dynamics are taken into account.

Commonly, the aerodynamic torque $T_r$ is given as follows [24, 25]:

$$T_r = \frac{1}{2}\rho\pi R^3 C_Q(\lambda) V^2, \quad (1)$$

where $\rho$ is the air density, $V$ is the wind speed, $R$ is the radius of the wind rotor plane, and power coefficient $C_Q(\lambda)$ is approximated by a second-order polynomial of tip-speed ratio $\lambda$ [4] as below:

$$C_Q(\lambda) = C_{Q\max} - k_Q(\lambda - \lambda_{Q\max})^2, \tag{2}$$

where $C_{Q\max}$ is the maximum power coefficient, $\lambda_{Q\max}$ is the optimal tip-speed ratio corresponding to $C_{Q\max}$, and $\lambda$ is defined as

$$\lambda = \frac{\omega_r R}{V}, \tag{3}$$

where $\omega_r$ is the wind rotor speed.

The drive train block has the model as below [2, 26]:

$$\begin{aligned}
\dot{\omega}_r &= -\frac{i}{\eta J_r} T_H + \frac{1}{J_r} T_r, \\
\dot{\omega}_g &= \frac{1}{J_g} T_H - \frac{1}{J_g} T_g, \\
\dot{T}_H &= iK_g \omega_r - K_g \omega_g - B_g\left(\frac{1}{J_g} + \frac{i^2}{\eta J_r}\right) T_H + \frac{iB_g}{J_r} T_r \\
&\quad + \frac{B_g}{J_g} T_g,
\end{aligned} \tag{4}$$

where $\omega_g$ is the generator speed, $T_H$ is the internal torque, $J_r$ is the wind rotor inertia, $J_g$ is the generator inertia, $K_g$ is the stiffness coefficient of the high-speed shaft (the generator shaft), $B_g$ is the damping coefficient of the high-speed shaft (the generator shaft), $i$ is the gearbox ratio, and $\eta$ is the gearbox efficiency.

Then, the generator dynamics are modeled as [2, 25]

$$\begin{aligned}
\dot{i}_d &= -\frac{R_s}{L_d} i_d + \frac{pL_q}{L_d} i_q \omega_g - \frac{1}{L_d} u_d, \\
\dot{i}_q &= -\frac{R_s}{L_q} i_q - \frac{p}{L_q}(L_d i_d - \phi_m)\omega_g - \frac{1}{L_q} u_q, \\
T_g &= p\phi_m i_q,
\end{aligned} \tag{5}$$

where $T_g$ is the generator electromagnetic torque, $i_d$, $L_d$, $u_d$ and $i_q$, $L_q$, $u_q$ are the $d$ and $q$ components of the stator current, inductance, and voltage, respectively, $R_s$ is the stator resistance, $P$ is the number of pole pairs, and $\phi_m$ is the flux.

By combining (4) and (5), the complete nonlinear model of the wind energy conversion system is obtained. Next, we will introduce the singular perturbation method.

Considering the order of magnitude of $L_d$ and $L_q$, we define the so-called singular perturbation parameter $\varepsilon = 1 \times 10^{-2}$ and obtain

$$\begin{aligned}
\varepsilon \dot{i}_d &= -\frac{R_s}{\overline{L}_d} i_d + \frac{pL_q}{\overline{L}_d} i_q \omega_g - \frac{1}{\overline{L}_d} u_d, \\
\varepsilon \dot{i}_q &= -\frac{R_s}{\overline{L}_q} i_q - \frac{p}{\overline{L}_q}(L_d i_d - \phi_m)\omega_g - \frac{1}{\overline{L}_q} u_q,
\end{aligned} \tag{6}$$

where $\overline{L}_d = L_d \times 10^2$, $\overline{L}_q = L_q \times 10^2$.

Now the singularly perturbed nonlinear system is obtained by uniting (4) and (6).

And now we are ready to derive the linear model using LPV method. Choose an operating point $\theta_1 = [\widehat{\omega}_r\ \widehat{V}\ \widehat{\omega}_g\ \widehat{i}_d\ \widehat{i}_q]^T$ and linearize the nonlinear parts in the singularly perturbed nonlinear system at point $\theta_1$:

$$\begin{aligned}
T_r(\lambda, V) - T_r(\widehat{\lambda}, \widehat{V}) &= -B_r(\theta_1)\delta\omega_r + K_{rv}(\theta_1)\delta V, \\
\omega_g i_q - \widehat{\omega}_g \widehat{i}_q &= B_{gq}(\theta_1)\delta i_q + B_{qg}(\theta_1)\delta\omega_g, \\
\omega_g i_d - \widehat{\omega}_g \widehat{i}_d &= B_{gd}(\theta_1)\delta i_d + B_{dg}(\theta_1)\delta\omega_g,
\end{aligned} \tag{7}$$

where

$$\begin{aligned}
\delta\omega_r &= \omega_r - \widehat{\omega}_r, \\
\delta V &= V - \widehat{V},
\end{aligned} \tag{8}$$

$$\begin{aligned}
\delta i_q &= i_q - \widehat{i}_q, \\
\delta i_d &= i_d - \widehat{i}_d,
\end{aligned} \tag{9}$$

$$\begin{aligned}
\delta\omega_g &= \omega_g - \widehat{\omega}_g, \\
\widehat{\lambda} &= \frac{R\widehat{\omega}_r}{\widehat{V}},
\end{aligned} \tag{10}$$

$$\begin{aligned}
B_r(\theta_1) &= -\left.\frac{\partial T_r}{\partial \omega_r}\right|_{(\widehat{\omega}_r, \widehat{V})} = -\frac{T_r(\widehat{\lambda}, \widehat{V})}{\widehat{\omega}_r} \left.\frac{\partial C_Q/\partial\lambda}{C_Q/\lambda}\right|_{(\widehat{\lambda}, \widehat{V})} \\
&= \rho\pi R^4 k_Q (R\widehat{\omega}_r - \lambda_{Q\max}\widehat{V}),
\end{aligned} \tag{11}$$

$$\begin{aligned}
K_{rv}(\theta_1) &= \left.\frac{\partial T_r}{\partial V}\right|_{(\widehat{\omega}_r, \widehat{V})} \\
&= \frac{T_r(\widehat{\lambda}, \widehat{V})}{\widehat{V}} \left(2 - \left.\frac{\partial C_Q/\partial\lambda}{C_Q/\lambda}\right|_{(\widehat{\lambda}, \widehat{V})}\right) \\
&= \rho\pi R^4 k_Q \left(R\widehat{\omega}_r - \left(1 - \frac{C_{Q\max}}{k_Q \lambda_{Q\max}^2}\right)\lambda_{Q\max}\widehat{V}\right),
\end{aligned} \tag{12}$$

$$\begin{aligned}
B_{gq}(\theta_1) &= \widehat{\omega}_g, \\
B_{qg}(\theta_1) &= \widehat{i}_q,
\end{aligned} \tag{13}$$

$$\begin{aligned}
B_{gd}(\theta_1) &= \widehat{\omega}_g, \\
B_{dg}(\theta_1) &= \widehat{i}_d.
\end{aligned} \tag{14}$$

By substituting (7) into (4) and (6), we have the following singularly perturbed linear system at the operating point $\theta_1$:

$$\begin{bmatrix} \delta\dot{\omega}_r \\ \delta\dot{\omega}_g \\ \delta\dot{T}_H \\ \varepsilon\delta\dot{i}_d \\ \varepsilon\delta\dot{i}_q \end{bmatrix} = A(\theta_1) \begin{bmatrix} \delta\omega_r \\ \delta\omega_g \\ \delta T_H \\ \delta i_d \\ \delta i_q \end{bmatrix} + B(\theta_1) \begin{bmatrix} \delta V \\ \delta u_d \\ \delta u_q \end{bmatrix}, \tag{15}$$

where

$$A(\theta_1) = \begin{bmatrix} -\frac{B_r(\theta_1)}{J_r} & 0 & -\frac{i}{\eta J_r} & 0 & 0 \\ 0 & 0 & \frac{1}{J_g} & 0 & -\frac{1}{J_g} P\phi_m \\ iK_g - \frac{iB_g B_r(\theta_1)}{J_r} & -K_g & -B_g\left(\frac{1}{J_g} + \frac{i^2}{\eta J_r}\right) & 0 & \frac{B_g}{J_g} P\phi_m \\ 0 & \frac{PL_q}{\overline{L}_d} B_{qg}(\theta_1) & 0 & -\frac{R_s}{\overline{L}_d} & \frac{PL_q}{\overline{L}_d} B_{gq}(\theta_1) \\ 0 & \frac{P}{\overline{L}_q}\left(\phi_m - L_d B_{dg}(\theta_1)\right) & 0 & -\frac{P}{\overline{L}_q} L_d B_{gd}(\theta_1) & -\frac{R_s}{\overline{L}_q} \end{bmatrix},$$
$$B(\theta_1) = \begin{bmatrix} \frac{1}{J_r} K_{rv}(\theta_1) & 0 & 0 \\ 0 & 0 & 0 \\ \frac{iB_g K_{rv}(\theta_1)}{J_r} & 0 & 0 \\ 0 & -\frac{1}{\overline{L}_d} & 0 \\ 0 & 0 & -\frac{1}{\overline{L}_q} \end{bmatrix}. \quad (16)$$

Furthermore, note $x = [\delta\omega_r \;\; \delta\omega_g \;\; \delta T_H]^T$, $z = [\delta i_d \;\; \delta i_q]^T$, $u = [\delta u_d \;\; \delta u_q]^T$, and rewrite (15) as

$$E_\varepsilon \begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = A(\theta_1) \begin{bmatrix} x \\ z \end{bmatrix} + B(\theta_1) \begin{bmatrix} \delta V(t) \\ \delta u_d \\ \delta u_q \end{bmatrix}$$
$$= A(\theta_1) \begin{bmatrix} x \\ z \end{bmatrix} + B_1(\theta_1)\,\delta V + B_2 u, \quad (17)$$

where

$$E_\varepsilon = \begin{bmatrix} I_{3\times 3} & 0 \\ 0 & \varepsilon I_{2\times 2} \end{bmatrix};$$
$$B(\theta_1) = [B_1(\theta_1) \;\; B_2]. \quad (18)$$

Then, by appropriately choosing operating points $\theta_i$,

$$\Theta = \mathrm{Co}\{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5\} \quad (19)$$

is a convex polytope with $\theta_i$ being vertices, $i = 1, \ldots, 5$. Note that the LPV model (17), with $B_r(\theta)$ and $K_{rv}(\theta)$ approximated by (11) and (12), is affine in the parameters. Namely, there exist scalars $a$, $b$, $c$, and $d$ such that $B_r(\theta) = a\theta + b$ and $K_{rv}(\theta) = c\theta + d$. Hence, it is easy to see that, for any $\theta \in \Theta$, there exists a set of positive numbers $\alpha_i > 0$, $i = 1, \ldots, 5$, such that

$$A(\theta) = \sum_{i=1}^{5} \alpha_i A(\theta_i);$$
$$B_1(\theta) = \sum_{i=1}^{5} \alpha_i B_1(\theta_i), \quad (20)$$

where $\sum_{i=1}^{5} \alpha_i = 1$.

Therefore, for any $\theta \in \Theta$, we have derived the singularly perturbed LPV model as below:

$$E_\varepsilon \begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = A(\theta) \begin{bmatrix} x \\ z \end{bmatrix} + B_1(\theta)\,\delta V + B_2 u. \quad (21)$$

*Remark 1.* For the details of skills to choose operating points appropriately, please refer to [4].

*Remark 2.* The wind conversion system considered here contains both mechanical and electrical parts which are of different time scales. Singularly perturbed model is developed of this system and LPV method is used to reform the model. The combination of singular perturbation theory and LPV method is novel for the maximum power point tracking problem.

## 3. Stability Analysis of Open-Loop System

In this paper, the objective is to maintain the optimal tip-speed ratio by adjusting wind rotor speed as the wind speed changes. So the operating points are chosen such that the tip-speed ratio is optimal. If the states $x$ and $z$ in (21) decay to zero, it means that the errors between the actual states and the desired states tend to zero. In this case, the wind turbine runs to extract all the available power.

And, this section will analyze the stability of the singularly perturbed system (21) when the control input $u = 0$ based on LMI technique.

When $u(t) = 0$, from (21) we can have

$$E_\varepsilon \begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = A(\theta) \begin{bmatrix} x \\ z \end{bmatrix} + B_1(\theta)\,\delta V(t). \quad (22)$$

Before the main result, Schur Complement Lemma is given below.

**Lemma 3** (Schur Complement Lemma). *The partitioned matrix*

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{12}^T & A_{22} \end{bmatrix} < 0 \tag{23}$$

*if and only if*

$$\begin{aligned} A_{11} &< 0, \\ A_{22} - A_{12}^T A_{11}^{-1} A_{12} &< 0, \\ \text{or } A_{22} &< 0, \\ A_{11} - A_{12} A_{22}^{-1} A_{12}^T &< 0. \end{aligned} \tag{24}$$

**Theorem 4.** *For the singularly perturbed system (22), given a positive scalar $\gamma > 0$, if there exist five positive matrices $P_i > 0$ satisfying the following conditions:*

$$\begin{bmatrix} A^T(\theta_i) P_i E_\varepsilon + E_\varepsilon^T P_i A(\theta_i) & * & * \\ B_1^T(\theta_i) P_i E_\varepsilon & -\gamma^2 I & * \\ C & 0 & -I \end{bmatrix} < 0, \tag{25}$$

*where $\alpha_i > 0$, $i = 1, \ldots, 5$, and*

$$\begin{aligned} \theta &= \sum_{i=1}^{5} \alpha_i \theta_i; \\ 1 &= \sum_{i=1}^{5} \alpha_i; \\ A(\theta) &= \sum_{i=1}^{5} \alpha_i A(\theta_i); \\ B_1(\theta) &= \sum_{i=1}^{5} \alpha_i B_1(\theta_i). \end{aligned} \tag{26}$$

*Then, the equilibrium point of system (22) at $\theta$ point is asymptotically stable, and $\|C(sE_\varepsilon - A)^{-1} B_1\| < \gamma$ is satisfied.*

*Proof.* Construct a Lyapunov function

$$W(X) = X^T E_\varepsilon^T P E_\varepsilon X, \tag{27}$$

where $X^T = \begin{bmatrix} x^T & z^T \end{bmatrix}^T$, $P = \sum_{i=1}^{5} \alpha_i P_i$. Since $P_i > 0$ and $\alpha_i > 0$, we have $P > 0$, and obviously $W(X) > 0$ holds.

Firstly, the asymptotic stability of the system is proved under conditions in Theorem 4 when the disturbance $\delta V(t)$ is zero. Derive $W(X)$ with the respect to $t$ along the trajectory of (22), with $\delta V(t) = 0$, and we obtain

$$\dot{W}(X) = X^T \left\{ A^T(\theta) P E_\varepsilon + E_\varepsilon^T P A(\theta) \right\} X. \tag{28}$$

From the LMI (25), it is not difficult to have the following inequality:

$$A^T(\theta_i) P_i E_\varepsilon + E_\varepsilon^T P_i A(\theta_i) < 0. \tag{29}$$

And, by adding the weight values (i.e., $\alpha_i$) of $\theta_i$ to (29), we can get

$$\begin{aligned} &\left\{ \sum_{i=1}^{5} \alpha_i A^T(\theta_i) \right\} \left\{ \sum_{i=1}^{5} \alpha_i P_i \right\} E_\varepsilon \\ &\quad + E_\varepsilon^T \left\{ \sum_{i=1}^{5} \alpha_i P_i \right\} \left\{ \sum_{i=1}^{5} \alpha_i A(\theta_i) \right\} = A^T(\theta) P E_\varepsilon \\ &\quad + E_\varepsilon^T P A(\theta) < 0. \end{aligned} \tag{30}$$

Therefore,

$$\dot{W}(X) < 0. \tag{31}$$

Hence, the system of (22) at point $\theta$ is asymptotically stable when $\delta V(t) = 0$.

Next, the robust property of system (22) will be proved when the disturbance $\delta V(t) \neq 0$.

Because $\alpha_i > 0$, using the LMI (25) it can be obtained that

$$\sum_{i=1}^{5} \alpha_i \sum_{i=1}^{5} \alpha_i \begin{bmatrix} A^T(\theta_i) P E_\varepsilon + E_\varepsilon^T P A(\theta_i) & * & * \\ B_1^T(\theta_i) P E_\varepsilon & -\gamma^2 I & * \\ C & 0 & -I \end{bmatrix} < 0 \tag{32}$$

which leads to

$$\begin{aligned} &\begin{bmatrix} \left( \sum_{i=1}^{5} \alpha_i A^T(\theta_i) \right) \left( \sum_{i=1}^{5} \alpha_i P_i \right) E_\varepsilon + E_\varepsilon^T \left( \sum_{i=1}^{5} \alpha_i P_i \right) \left( \sum_{i=1}^{5} \alpha_i A(\theta_i) \right) & * & * \\ \left( \sum_{i=1}^{5} \alpha_i B_1^T(\theta_i) \right) \left( \sum_{i=1}^{5} \alpha_i P_i \right) E_\varepsilon & -\gamma^2 I & * \\ C & 0 & -I \end{bmatrix} = \begin{bmatrix} A^T(\theta) P E_\varepsilon + E_\varepsilon^T P A(\theta) & * & * \\ B_1^T(\theta) P E_\varepsilon & -\gamma^2 I & * \\ C & 0 & -I \end{bmatrix} \\ &\quad < 0. \end{aligned} \tag{33}$$

Then by using *Schur Complement Lemma* twice, LMI (33) can be transformed into inequality below:

$$A^T(\theta) PE_\varepsilon + E_\varepsilon^T P^T A(\theta) + C^T C + \frac{1}{\gamma^2} E_\varepsilon^T P^T B_1(\theta) B_1^T(\theta) PE_\varepsilon < 0. \tag{34}$$

Derive $W(X)$ along the trajectory of (22) and use the inequality (34); $\dot{W}(X)$ becomes

$$\begin{aligned} &\dot{W}(X) \\ &= X^T \left\{A^T(\theta) PE_\varepsilon + E_\varepsilon^T PA(\theta)\right\} X \\ &\quad + \delta V^T B_1^T(\theta) PE_\varepsilon X + X^T E_\varepsilon^T PB_1(\theta)\delta V \\ &< X^T \left\{-C^T C - \frac{1}{\gamma^2} E_\varepsilon^T P^T B_1(\theta) B_1^T(\theta) PE_\varepsilon\right\} X \\ &\quad + \delta V^T B_1^T(\theta) PE_\varepsilon X + X^T E_\varepsilon^T PB_1(\theta)\delta V \\ &= -y^T y + \gamma^2 \delta V^T \delta V \\ &\quad - \gamma^2 \left(\delta V - \frac{1}{\gamma^2} B_1^T PE_\varepsilon X\right)^T \left(\delta V - \frac{1}{\gamma^2} B_1^T PE_\varepsilon X\right). \end{aligned} \tag{35}$$

Based on the asymptotic stability proved at the first part of this proof, we have $X(\infty) = 0$. Assume $X(0) = 0$, and integrate both sides of (35) from $t = 0$ to $t = \infty$; we can have

$$\begin{aligned} 0 &< -\int_0^\infty y^T(t)\, y(t)\, dt + \int_0^\infty \gamma^2 \delta V^T(t)\, \delta V(t)\, dt \\ &\quad - \int_0^\infty \gamma^2 \left(\delta V - \gamma^{-2} B_1^T(\theta) PE_\varepsilon X\right)^T \\ &\quad \cdot \left(\delta V - \gamma^{-2} B_1^T(\theta) PE_\varepsilon X\right) dt. \end{aligned} \tag{36}$$

Then, (36) can be rearranged to have

$$\begin{aligned} &\int_0^\infty y^T(t)\, y(t)\, dt - \int_0^\infty \gamma^2 \delta V^T(t)\, \delta V(t)\, dt \\ &\quad < -\int_0^\infty \gamma^2 \left(\delta V - \gamma^{-2} B_1^T(\theta) PE_\varepsilon X\right)^T \\ &\quad \cdot \left(\delta V - \gamma^{-2} B_1^T(\theta) PE_\varepsilon X\right) dt. \end{aligned} \tag{37}$$

Therefore, as can be seen from the inequality above,

$$\int_0^\infty y^T(t)\, y(t)\, dt - \int_0^\infty \gamma^2 V^T(t)\, V(t)\, dt < 0. \tag{38}$$

Now, it is easy to see that $\|C(sE_\varepsilon - A(\theta))^{-1} B_1(\theta)\| < \gamma$ is satisfied. This completes the proof. □

*Remark 5.* Even though Theorem 4 can guarantee the stability and robust property of system (22), the LMI (25) is dependent on small parameter $\varepsilon$, so it might be singular and difficult to solve.

The following result improves the ill-condition inequality.

**Theorem 6.** *For the singularly perturbed system (22), given a positive scalar $\gamma > 0$, if there exist five matrices $P_i$ satisfying the following conditions,*

$$E_\varepsilon^T P_i = P_i^T E_\varepsilon > 0, \tag{39}$$

$$\begin{bmatrix} A^T(\theta_i) P_i + P_i A(\theta_i) & * & * \\ B_1^T(\theta_i) P_i & -\gamma^2 I & * \\ C & 0 & -I \end{bmatrix} < 0, \tag{40}$$

*where $\alpha_i > 0$, $i = 1, \ldots, 5$, and*

$$\begin{aligned} \theta &= \sum_{i=1}^5 \alpha_i \theta_i; \\ 1 &= \sum_{i=1}^5 \alpha_i; \\ A(\theta) &= \sum_{i=1}^5 \alpha_i A(\theta_i); \\ B_1(\theta) &= \sum_{i=1}^5 \alpha_i B_1(\theta_i), \end{aligned} \tag{41}$$

*then, the equilibrium point of system (22) at point $\theta$ is asymptotically stable, and $\|C(sE_\varepsilon - A(\theta))^{-1} B_1(\theta)\| < \gamma$ holds.*

*Proof.* Define a Lyapunov function as below:

$$W(X) = X^T E_\varepsilon^T PX, \tag{42}$$

where $X = \begin{bmatrix} x^T & z^T \end{bmatrix}^T$ and $P = \sum_{i=1}^5 \alpha_i P_i$.

According to condition (39), we have

$$E_\varepsilon^T P = P^T E_\varepsilon > 0. \tag{43}$$

As a consequence, $W(X) > 0$ is satisfied.

Then, the time-derivative of $W(X)$ along the solution of (21) is given by

$$\begin{aligned} \dot{W}(X) &= X^T \left\{A^T(\theta) P + P^T A(\theta)\right\} X \\ &\quad + \delta V^T B_1^T(\theta) PX + X^T P^T B_1(\theta)\delta V. \end{aligned} \tag{44}$$

The following part is similar to the proof of Theorem 4 and is consequently omitted here. □

## 4. Controller Design

In this section, a robust state-feedback controller is designed for system (21), and the stability property of the closed-loop system is analyzed.

Since the coefficients of system (21) depend on $\theta$, it is reasonable to design a controller whose feedback gain matrix also depends on $\theta$.

As can be seen from (20) and (22), at any point $\theta \in \Theta$, the dynamic equation of the nonlinear singularly perturbed system can be expressed as a weight-sum of the dynamic equations at the vertices $\theta_i$ of $\Theta$, $i = 1, \ldots, 5$. Therefore, we will design controllers for the LTI systems operating at the vertices $\theta_i$ of $\Theta$, $i = 1, \ldots, 5$, and use a weight-sum of the controllers at vertices as the control input to the system at point $\theta \in \Theta$.

At the vertex point $\theta_i$, $i \in \{1, 2, 3, 4, 5\}$, design a robust state-feedback controller as below:

$$u_i = K(\theta_i) X. \tag{45}$$

The closed-loop system of (22) at the vertex point $\theta_i$ with (45) as control input is as follows:

$$E_\varepsilon \dot{X} = A_c(\theta_i) X + B_1(\theta_i) \delta V, \tag{46}$$

where $A_c(\theta_i) = A(\theta_i) + B_2 K(\theta_i)$.

Then, for the nonlinear singularly perturbed system (21) at $\theta$, the state-feedback controller is as follows:

$$u(\theta) = K(\theta) X = \sum_{i=1}^{5} \alpha_i K(\theta_i) X, \tag{47}$$

where $\alpha_i > 0$ and $\sum_{i=1}^{5} \alpha_i = 1$.

Applying (47) to system (21), the closed-loop system is obtained as below:

$$E_\varepsilon \begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = (A(\theta) + B_2 K(\theta)) \begin{bmatrix} x \\ z \end{bmatrix} + B_1(\theta) V. \tag{48}$$

The properties of closed-loop system of (48) are analyzed in Theorem 8. To prove Theorem 8, the following lemma from [27] is needed.

**Lemma 7** (see [27]). *Let $X, Y \in \mathbb{R}^{m\times n}$, $H > 0$, and $\delta > 0$ be a scalar; then*

$$X^T H Y + Y^T H X \le \delta X^T H X + \delta^{-1} Y^T H Y. \tag{49}$$

**Theorem 8.** *The closed-loop system of (21) with (47) as input is asymptotically stable; namely, (48) system is asymptotically stable, and $\|C(sE_\varepsilon - A_c(\theta))^{-1} B_1(\theta)\| < \gamma$ is satisfied for a given $\gamma > 0$, if there exist matrices $P_i$ and $K(\theta_i)$ of appropriate dimension, such that the following LMIs hold:*

$$E_\varepsilon^T P_i = P_i^T E_\varepsilon > 0, \tag{50}$$

$$\begin{bmatrix} A^T(\theta_i) P_i + P_i^T A(\theta_i) & * & * & * & * \\ B_2 K(\theta_i) & -I & * & * & * \\ P_i & 0 & -I & * & * \\ B_1^T(\theta_i) P_i & 0 & 0 & -\gamma^2 I & * \\ C & 0 & 0 & 0 & -I \end{bmatrix} < 0, \tag{51}$$

*where $i = 1, \ldots, 5$.*

*Proof.* Similar to the proof of Theorem 4, $\alpha_i > 0$ and

$$\begin{aligned} A(\theta) &= \sum_{i=1}^{n} \alpha_i A(\theta_i); \\ B_1(\theta) &= \sum_{i=1}^{n} \alpha_i B_1(\theta_i); \\ K(\theta) &= \sum_{i=1}^{n} \alpha_i K_1(\theta_i); \\ \sum_{i=1}^{n} \alpha_i &= 1. \end{aligned} \tag{52}$$

According to (51), we can have

$$\begin{bmatrix} A^T(\theta) P_i + P_i^T A(\theta) & * & * & * & * \\ B_2 K(\theta) & -I & * & * & * \\ P_i & 0 & -I & * & * \\ B_1^T(\theta) P_i & 0 & 0 & -\gamma^2 I & * \\ C & 0 & 0 & 0 & -I \end{bmatrix} < 0. \tag{53}$$

Let $P = \sum_{i=1}^{n} \alpha_i P_i$. From (50) and (53) we can obtain $E_\varepsilon^T P = P^T E_\varepsilon > 0$, and

$$\begin{bmatrix} A^T(\theta) P + P^T A(\theta) & * & * & * & * \\ B_2 K(\theta) & -I & * & * & * \\ P & 0 & -I & * & * \\ B_1^T(\theta) P & 0 & 0 & -\gamma^2 I & * \\ C & 0 & 0 & 0 & -I \end{bmatrix} < 0. \tag{54}$$

By applying Schur Complement Lemma four times to (54), it yields

$$\begin{aligned} &A^T(\theta) P + P^T A(\theta) + (B_2 K(\theta))^T (B_2 K(\theta)) + P^T P \\ &\quad + C^T C + \frac{1}{\gamma^2} P^T B_1(\theta) B_1^T(\theta) P < 0. \end{aligned} \tag{55}$$

According to Lemma 7, it is easy to obtain

$$\begin{aligned} &(B_2 K(\theta))^T P + P^T B_2 K(\theta) \\ &\quad \le (B_2 K(\theta))^T B_2 K(\theta) + P^T P. \end{aligned} \tag{56}$$

Hence, from (55) and (56), we can get

$$\begin{aligned} &A^T(\theta) P + P^T A(\theta) + (B_2 K(\theta))^T P + P^T (B_2 K(\theta)) \\ &\quad + C^T C + \frac{1}{\gamma^2} P^T B_1(\theta) B_1^T(\theta) P < 0 \end{aligned} \tag{57}$$

which is equal to

$$\begin{aligned} &(A(\theta) + B_2 K(\theta))^T P + P^T (A(\theta) + B_2 K(\theta)) + C^T C \\ &\quad + \frac{1}{\gamma^2} P^T B_1(\theta) B_1^T(\theta) P < 0. \end{aligned} \tag{58}$$

Define a Lyapunov function as below:

$$W(X) = X^T E_\varepsilon^T P X, \tag{59}$$

where $X = \begin{bmatrix} x^T & z^T \end{bmatrix}^T$. Then, the time-derivative of $W(X)$ along the solution of (21) is given by

$$\begin{aligned}\dot{W}(X) = X^T &\left\{ (A(\theta) + B_2 K(\theta))^T P \right. \\ &+ \left. P^T (A(\theta) + B_2 K(\theta)) \right\} X + V^T B_1^T(\theta) P X \\ &+ X^T P^T B_1(\theta) V.\end{aligned} \tag{60}$$

The following part is similar to the proof of Theorem 4 and is consequently omitted here. □

## 5. Algorithm of Synthesis

In order to clarify the whole process of designing a parameter-dependent controller for the original nonlinear system (4) and (5), the following algorithm is presented.

*Step 1.* Choose five operating points $\theta_1, \ldots, \theta_5$.

*Step 2.* Abstract singular perturbation parameter $\varepsilon$, and get the nonlinear singularly perturbed systems (4) and (6).

*Step 3.* Linearize the nonlinear singularly perturbed systems (4) and (6) at $\theta_i$, and obtain the linear parameter-dependent coefficients $A(\theta_i)$ and $B(\theta_i)$, where $i = 1, \ldots, 5$.

*Step 4.* For a given $\gamma$, at each operating point $\theta_i$, LMIs (50) and (51) are solved to obtain control gain matrices $K_i$, $i = 1, \ldots, 5$.

*Step 5.* At time $t_k$, the variable $\theta(t_k) = \theta_k$ is measured, and the weighting coefficients $\alpha_i$ satisfying

$$\theta_k = \sum_{i=1}^{5} \alpha_i(t_k)\theta_i \tag{61}$$

are computed with $0 \le \alpha_i \le 1$ and $\sum_{i=1}^{5} \alpha_i = 1$.

*Step 6.* The control gain matrix at time $t_k$ is obtained as below:

$$K(\theta(t_k)) = \sum_{i=1}^{5} \alpha_i(t_k) K_i. \tag{62}$$

*Step 7.* Apply the controller (62) to the original nonlinear system (4) and (5).

*Step 8.* At time $t_{k+1}$, repeat Step 5 to Step 8.

*Remark 9.* This algorithm can only be applied to the point $\theta \in \Theta$, where $\Theta$ is defined by (19). If $\theta \notin \Theta$, it can not be guaranteed that we can compute $\alpha_i > 0$, $i = 1, \ldots, 5$, with $\sum_{i=1}^{5} \alpha_i = 1$ satisfied.

*Remark 10.* Since the operating point $\theta = \begin{bmatrix} \widehat{\omega}_r & \widehat{V} & \widehat{\omega}_g & \widehat{i}_d & \widehat{i}_q \end{bmatrix}^T$ involves wind speed $V$, $\Theta$ implies the range of the wind speed within which our algorithm is effective.

Table 1: Parameters of wind turbine.

| Descriptions | Notions | Values |
|---|---|---|
| Cut-in wind speed | $V$ | 4 m/s |
| Mean wind speed | $V_R$ | 15 m/s |
| Optimal tip-speed ratio | $\lambda_{\text{opt}}$ | 6.2 |
| Rotor radius | $R$ | 2.5 m |
| Optimal power coefficient | $C_{P\max}$ | 0.4633 |
| Air density | $\rho$ | 0.98 Kg/m$^3$ |
| Gearbox efficiency | $\eta$ | 1 |
| Wind rotor inertia | $J_r$ | 3.88 Kg·m$^2$ |
| Generator inertia | $J_g$ | 0.22 Kg·m$^2$ |
| Number of pole pairs | $P$ | 3 |
| Gearbox ratio | $i$ | 6 |
| Flux linkage | $\phi_m$ | 0.4382 wb |
| Shaft damping coefficient | $B_g$ | 0.3 Kg·m$^2$/s |
| Shaft stiffness coefficient | $K_g$ | 75 Nm/rad |
| Stator $d$-axis inductance | $L_d$ | 41.56 mH |
| Stator $q$-axis inductance | $L_q$ | 41.56 mH |
| Stator resistance | $R_s$ | 3.3 Ω |

*Remark 11.* In [12, 13], a high-order sliding mode control strategy is proposed based on a high gain observer to optimize the maximum power point tracking problem of wind energy conversion system. Their method presents chattering-free, behavior, finite reaching time and robustness. However, high-order controller and observer need more online computation. In this paper, the wind energy conversion system is modeled using singular perturbation theory which considers the generator dynamics as fast subsystem and the drive train block as slow subsystem. This method combines the singular perturbation methodology and LPV model for the first time to solve the MPPT problem.

## 6. Numerical Examples

In this paper, the goal is to track the desired wind rotor speed and maintain the optimal tip-speed ratio when the wind speed changes. The simulation study is performed to verify the effectiveness of the proposed control algorithm. The two examples consider the CART 3-blades wind turbine taken from [20] as an objective. The parameters of the wind turbine are given in Table 1. The experiment is carried out on the MATLAB FAST® software which is developed by the American National Renewable Energy Laboratory.

*Example 12.* Consider a wind turbine system with the parameters depicted in Table 1. The wind speed is assumed as a constant 9 m/s in this example. The tracking result controlled by the algorithm developed in Section 5 is compared with that of the optimal torque (OT) method, and the compared rotor tracking results are shown in Figure 2.

It can be seen that the tracking error between actual rotor speed controlled by the algorithm presented here and the desired rotor speed decays to zero after a small overshoot. But

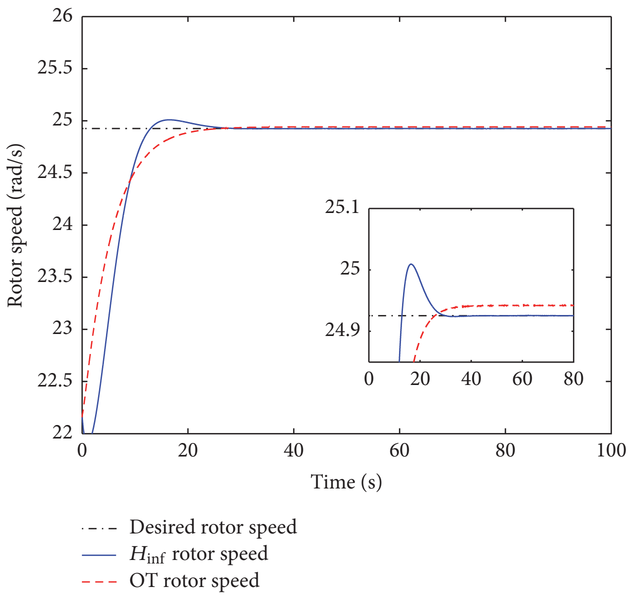


Figure 2: Rotor speed constant tracking.

the tracking error between the desired rotor speed and the rotor speed controlled by the optimal torque method can not decay to zero with time, which means a static error exists. So this example illustrates that the algorithm developed in this paper is more effective than the optimal torque method.

*Example 13.* In this example, a turbulence of 600 seconds produced by using TurbSim software is applied. This turbulence satisfies the IEC-61400-1 standard and the turbulence is shown in Figure 3.

The rotor speeds controlled by the algorithm presented in this paper and the optimal torque method are shown in Figure 4. It is obvious that the rotor speed controlled by the $H_\infty$ algorithm can track the desired rotor speed much better than that of the optimal torque method.

Furthermore, the wind power capture efficiency and energy conversion efficiency controlled by this novel method and optimal torque method are compared in Table 2. It can be seen from Table 2 that the $H_\infty$ method developed can obtain better wind power capture efficiency and energy conversion efficiency.

Table 2: Compared power conversion efficiency.

| Methods | Wind power Capture efficiency | Energy conversion Efficiency |
|---|---|---|
| $H_\infty$ method | 0.4485 | 96.97% |
| Optimal torque | 0.4455 | 96.26% |

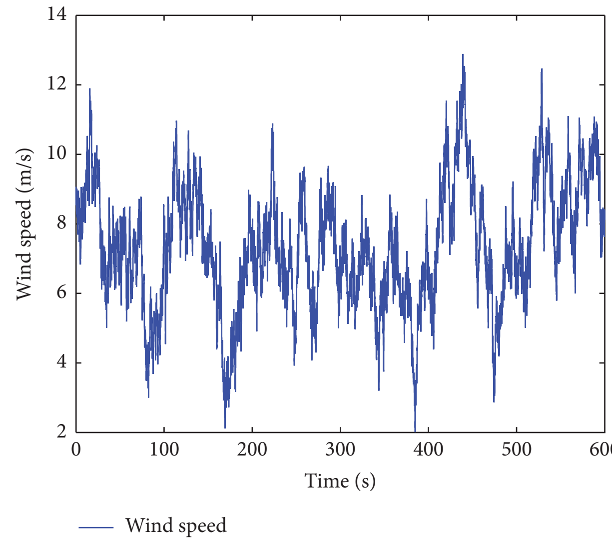


Figure 3: Wind turbulence.

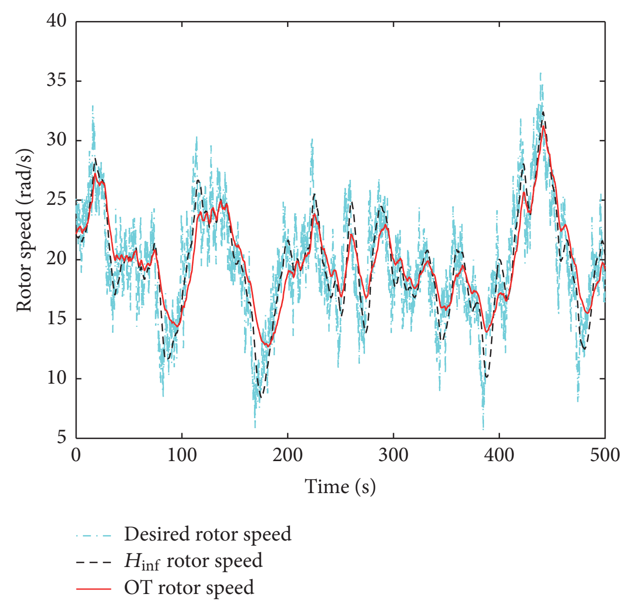


Figure 4: Controlled results by $H_\infty$ method and OT method.

## 7. Conclusions

This paper extended the continuous-time infinite horizon nonlinear quadratic optimal control problem of NSPSs to discrete-time version with the weight matrices dependent on the states in the cost function. For a class of discrete-time NSPSs in this paper, we used the theory of singular perturbations and time scales to decouple the original high-order NSPS into order-reduced slow and fast (boundary layer) subsystems. Then, via the state-dependent Riccati equation, suboptimal controllers for the two subsystems are designed with the weight matrices varying with states in the cost functions. A composite controller consisting of two suboptimal controllers is developed for the original system. It is proved that the equilibrium point of the original closed-loop system with a composite controller is locally asymptotically stable. In the end, an example is given to show the effectiveness of the results obtained.

## Competing Interests

The authors declare that they have no competing interests.

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