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Research Article

Two Novel Grey System Models and Their Applications on Landslide Forecasting

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For the small sample poor information, grey model is one of the good forecasting models. However, the simulation curve of original data is not consistent with that of the data by translations. In this paper, we present two novel grey system models, that is, generalized grey model and generalized discrete grey model. Compared with grey model, we prove that the simulation curve of original data is consistent with that of the new data by translations for the novel grey model, which was also demonstrated by the results of practical numerical examples.

1. Introduction

There are many uncertain problems in our lives. In order to analyze deeply these problems, grey model ($GM(1,1)$) was proposed by Deng [1]. Many researchers focused on the study of grey model since it was presented. $GM(1,1)$ has a wide field of application. Based on the characteristic of different data sequence, lots of researchers improve the fitting accuracy of $GM(1,1)$ from various angles. Besides $GM(1,1)$, Connotation grey model was also proposed by Deng [2]. The relation between Connotation grey model and $GM(1,1)$ was further studied. When development coefficient is small, the above two models are interchangeable. Moreover, the uniform upper bounds of relative error were given by Liu et al. [3].

Later on, $GM(1,1)$ was extended to discrete grey system ($DGM(1,1)$) by Xie and Liu [4]. Because $DGM(1,1)$ does not need the discrete approximation, it usually gets small simulation error. After exploring the effect of original data tiny disturbance in depth, Wu et al. [5] proposed the fractional order accumulation grey model which could effectively reduce disturbance of the grey model. Fractional order grey model was proposed by Mao et al. [6] and the corresponding image equation was extended to the fractional order differential equation. $GM(1,1)$ was extended to partial grey model by Liu [7, 8] and the corresponding property was studied. By extreme learning machines, grey model was extended to GrELM by Liu and Fu [9] and solved the volatility forecasting problems of interbank offered rate. $GM(1,1)$ was also extended to other models in [10, 11].

However, for the aforementioned models, it is usually supposed that original data are nonnegative. If the original data contain negative and each number plus a constant, the effect of the simulation forecasting of $GM(1,1)$ was researched by Li [12]. Feng [13] proved that the different initial data yield different simulation data. Li [14] showed that the original data consisting of negative are suitable for $GM(1,1)$ through multiply transform.

Observation data, for example, landslide observation data, are usually recorded relative to a reference point. Besides, studying landslide does not need to start with the first data every time. Sometimes the situation after a period of landslide also deserves to be investigated. In this case, we should choose a reference point. However, different reference point has different simulation data of $GM(1,1)$. This will reduce the reliability and consistency.

In this paper, in order to ensure that the simulation curve of original data is consistent with that of the data by translations, we propose generalized grey system (GGM(1, 1)) and generalized discrete grey system (GDGM(1, 1)). Compared with grey model, we prove that the simulation curve of original data is consistent with that of the data by translations for the novel grey model. At last, example on landslide has further demonstrated that the simulation effects of two novel grey models have no relation with reference point.

The remainder of this paper is organized as follows. In Section 2, the definitions of GM(1, 1) and DGM(1, 1) are introduced. GGM(1, 1) and GDGM(1, 1) are presented and their translation consistence is proved in Sections 3 and 4, respectively. In Section 5, through three examples, we compare the simulation effect of new GGM(1, 1) and GDGM(1, 1) with that of GM(1, 1) and DGM(1, 1), respectively. An empirical analysis of landslide forecasting in BaZiMen area is discussed in Section 6. In the last section we give the conclusions of this paper.

2. Preliminaries

For quick later reference we collect some notations and basic facts about grey system. Good general reference for the theory of grey system is the book of Deng [2].

Let the original data sequence be

$$X^{(0)} = \left\{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\right\}, \tag{1}$$

where $x^{(0)}(k) \geq 0$, $k = 1, 2, \dots, n$. Then the first order accumulated sequence of the original data sequence is

$$X^{(1)} = \left\{x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)\right\}, \tag{2}$$

where $x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i)$, $k = 1, 2, \dots, n$.

As a consequence, the mean generated sequence of the first order accumulated sequence could be expressed as

$$Z^{(1)} = \left\{z^{(1)}(2), z^{(1)}(3), \dots, z^{(1)}(n)\right\}, \tag{3}$$

where $z^{(1)}(k) = (1/2)(x^{(1)}(k) + x^{(1)}(k-1))$, $k = 2, 3, \dots, n$.

Based on the above notations, we introduce GM(1, 1) and its character as follows.

Definition 1 (see [15]). $x^{(0)}(k) + az^{(1)}(k) = b$ is called mean form of GM(1, 1).

Definition 2 (see [15]). $dx^{(1)}/dt + ax^{(1)} = b$ is called Whitenization equation of GM(1, 1).

By the definition of the grey model, we could easily derive the following results.

Lemma 3 (see [15]). *The following equation*

$$\widehat{x}^{(1)}(k) = \left(x^{(0)}(1) - \frac{b}{a}\right)e^{-a(k-1)} + \frac{b}{a}, \quad k = 1, 2, \dots, n \tag{4}$$

is called the time response equation of GGM(1, 1).

Lemma 4 (see [15]). *Let* $\widehat{\beta} = (a, b)^T$ *be a sequence of parameters. Denote*

$$Y = \begin{pmatrix} x^{(1)}(2) \\ x^{(1)}(3) \\ \vdots \\ x^{(1)}(n) \end{pmatrix}, \quad B = \begin{pmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{pmatrix}. \tag{5}$$

Then the least squares estimate parameter of GM(1, 1) equation

$$x^{(0)}(k) + az^{(1)}(k) = b \tag{6}$$

satisfies $\widehat{\beta} = (B^T B)^{-1} B^T Y$.

In what follows, we introduce the definition of DGM(1, 1) and its characters.

Definition 5 (see [4]). $x^{(1)}(k+1) = \beta_1 x^{(1)}(k) + \beta_2$ is called DGM(1, 1).

Lemma 6 (see [4]). *The following equation*

$$\widehat{x}^{(1)}(k) = \left(x^{(0)}(1) - \frac{\beta_2}{1-\beta_1}\right)\beta_1^k + \frac{\beta_2}{1-\beta_1}, \quad k = 1, 2, \dots, n \tag{7}$$

is called the time response equation of DGM(1, 1).

Lemma 7 (see [4]). *If* $\widehat{\beta} = (\beta_1, \beta_2)^T$ *is a sequence of parameters and*

$$Y = \begin{pmatrix} x^{(1)}(2) \\ x^{(1)}(3) \\ \vdots \\ x^{(1)}(n) \end{pmatrix}, \quad B = \begin{pmatrix} x^{(1)}(1) & 1 \\ x^{(1)}(2) & 1 \\ \vdots & \vdots \\ x^{(1)}(n-1) & 1 \end{pmatrix}, \tag{8}$$

then the least squares estimate parameter of DGM(1, 1) equation

$$x^{(1)}(k+1) = \beta_1 x^{(1)}(k) + \beta_2 \tag{9}$$

satisfies $\widehat{\beta} = (B^T B)^{-1} B^T Y$.

3. The Proposed Generalized Grey Model (GGM(1, 1))

In this section, we are going to investigate the proposed forecasting model GGM(1, 1). We first give the definition of GGM(1, 1) and then derive its time response equation.

Assume $h \in R$. Let the original data sequence be

$$X^{*(0)} = \left\{x^{*(0)}(1), x^{*(0)}(2), \ldots, x^{*(0)}(n)\right\}, \tag{10}$$

where $x^{*(0)}(k) = x^{(0)}(k) + h$, $k = 1, 2, \ldots, n$. Then we have the first order accumulated sequence of the original data as follows:

$$X^{*(1)} = \left\{x^{*(1)}(1), x^{*(1)}(2), \ldots, x^{*(1)}(n)\right\}, \tag{11}$$

where $x^{*(1)}(k) = \sum_{i=1}^{k} x^{*(0)}(i)$, $k = 1, 2, \ldots, n$. As a result, the mean generated sequence of the first order accumulated sequence writes as

$$Z^{*(1)} = \left\{z^{*(1)}(2), z^{*(1)}(3), \ldots, z^{*(1)}(n)\right\}, \tag{12}$$

where $z^{*(1)}(k) = (1/2)(x^{*(1)}(k) + x^{*(1)}(k-1))$, $k = 2, \ldots, n$.

Given the above notations, we now present the definition of GGM(1, 1).

Definition 8. The following equation

$$x^{*(0)}(k) + a^* z^{*(1)}(k) = b^* + h + a^* h(k - 0.5) \tag{13}$$

is called mean form of GGM(1, 1).

Definition 9. Whitenization differential equation of GGM(1, 1) is

$$\frac{dx^{*(1)}}{dt} + ax^{*(1)} = b^* + h + a^* ht. \tag{14}$$

Based on the above definitions, we could derive the following.

Theorem 10. *The following equation,*

$$\hat{x}^{*(1)}(k) = \frac{b^*}{a^*} + \left(x^{*(1)}(1) - h - \frac{b^*}{a^*}\right)e^{-a^*(k-1)} + hk, \quad k = 1, 2, \ldots, n, \tag{15}$$

is called the time response equation of GGM(1, 1).

Proof. The solution of differential equation $dx^{*(1)}/dt + ax^{*(1)} = b^* + h + a^* ht$ is $\hat{x}^{*(1)}(t) = b^*/a^* + ce^{-a^* t} + ht$. If initial datum satisfies $\hat{x}^{*(1)}(1) = x^{*(1)}(1)$, then $\hat{x}^{*(1)}(t) = b^*/a^* + (x^{*(1)}(1) - h - b^*/a^*)e^{-a^*(t-1)} + ht$.

It follows that

$$\hat{x}^{*(1)}(k) = \frac{b^*}{a^*} + \left(x^{*(1)}(1) - h - \frac{b^*}{a^*}\right)e^{-a^*(k-1)} + hk, \quad k = 1, 2, \ldots, n. \tag{16}$$

This finishes the proof of Theorem 10. □

Theorem 11. *If $x^{(0)}(k) \geq 0$, $k = 1, 2, \ldots, n$, and $h \geq 0$, then GGM(1, 1) is reduced to GM(1, 1).*

In other words, If $x^{*(0)}(k) = x^{(0)}(k) + h$, $k = 1, 2, \ldots, n$, and $x^{(0)}(k) \geq 0$, $k = 1, 2, \ldots, n$, $h \geq 0$, then $a^* = a$, $b^* = b$, and $\widehat{X}^{*(0)} = \widehat{X}^{(0)} + h$.

Proof. If $x^{*(0)}(k) = x^{(0)}(k) + h$, $k = 1, 2, \ldots, n$, then $x^{*(1)}(k) = x^{(1)}(k) + kh$, $k = 1, 2, \ldots, n$.

In view of (12), we have

$$\begin{aligned} z^{*(1)}(k) &= 0.5\left(x^{*(1)}(k) + x^{*(1)}(k-1)\right) \\ &= 0.5\left(x^{(1)}(k) + x^{(1)}(k-1)\right) + (k - 0.5)h \\ &= z^{(1)}(k) + (k - 0.5)h, \quad k = 2, 3, \ldots, n. \end{aligned} \tag{17}$$

Plugging (17) into (13), we arrive at

$$\begin{aligned} &x^{(0)}(k) + h + a^* z^{(1)}(k) + a^* h(k - 0.5) \\ &\quad = b^* + h + a^* h(k - 0.5). \end{aligned} \tag{18}$$

Therefore,

$$x^{(0)}(k) + a^* z^{(1)}(k) = b^*. \tag{19}$$

This equation is the same as the mean form of GM(1, 1). That is, $a^* = a$ and $b^* = b$. By Theorem 10, one gets

$$\begin{aligned} \hat{x}^{*(1)}(k) &= \frac{b^*}{a^*} + \left(x^{*(1)}(1) - h - \frac{b^*}{a^*}\right)e^{-a^*(k-1)} \\ &\quad + hk, \\ a^* &= a, \\ b^* &= b, \\ x^{*(1)}(1) &= x^{(1)}(1) + h, \\ \text{so } \hat{x}^{*(1)}(k) &= \hat{x}^{(1)}(k) + hk. \end{aligned} \tag{20}$$

Then

$$\begin{aligned} \hat{x}^{*(0)}(k) &= \hat{x}^{*(1)}(k) - \hat{x}^{*(1)}(k-1) \\ &= \hat{x}^{(1)}(k) + hk - \hat{x}^{(1)}(k-1) - h(k-1) \\ &= \hat{x}^{(0)}(k) + h. \end{aligned} \tag{21}$$

Therefore, $\widehat{X}^{*(0)} = \widehat{X}^{(0)} + h$ and this finishes the proof of Theorem 11. □

Remark 12. By Theorem 11, it is clear that the simulation curve of GGM(1, 1) is consistent with that of GM(1, 1) for the nonnegative original data.

4. The Proposed Generalized Discrete Grey Model (GDGM(1, 1))

In this section, we will investigate the proposed forecasting model GDGM(1, 1). We first give the definition of GDGM(1, 1) and then derive its time response equation.

Definition 13. The following equation,

$$x^{*(1)}(k+1)=\beta_1^* x^{*(1)}(k)+\beta_2^*-\beta_1^* kh+(k+1)h, \quad (22)$$

is called mean form of GDGM$(1,1)$.

The following theorem is provided to discuss the time response equation of GDGM$(1,1)$.

Theorem 14. *The following equation,*

$$\begin{aligned}\widehat{x}^{*(1)}(k+1)&=\beta_1^{*k}x^{*(1)}(1)+\frac{1-\beta_1^{*k}}{1-\beta_1^*}(h+\beta_2^*)\\&\quad-\beta_1^{*k}h+(k+1)h,\\&\qquad k=1,2,\ldots,n-1,\end{aligned} \quad (23)$$

is called the time response equation of GDGM(1, 1).

Proof.

$$\begin{aligned}\widehat{x}^{*(1)}(k+1)&=\beta_1^*\widehat{x}^{*(1)}(k)+\beta_2^*-\beta_1^*kh+(k+1)h\\&=\beta_1^*\left[\widehat{x}^{*(1)}(k-1)+\beta_2^*-\beta_1^*(k-1)h+kh\right]+\beta_2^*\\&\quad-\beta_1^*kh+(k+1)h=\cdots\\&=\beta_1^{*k}x^{*(1)}(1)+\frac{1-\beta_1^{*k}}{1-\beta_1^*}\beta_2^*-\beta_1^*h+(k+1)h,\\&\qquad k=1,2,\ldots,n-1.\end{aligned} \quad (24)$$

This proves Theorem 14. □

To obtain the relation between GDGM$(1,1)$ and DGM$(1,1)$, we have the following.

Theorem 15. *If $x^{(0)}(k)\geq 0$, $k=1,2,\ldots,n$, and $h\geq 0$, then GDGM(1, 1) is reduced to DGM(1, 1).*

In other words, provided $x^{*(0)}(k)=x^{(0)}(k)+h$, $k=1,2,\ldots,n$, and $x^{(0)}(k)\geq 0$, $k=1,2,\ldots,n$, $h\geq 0$, then

$$\begin{aligned}\beta_1^*&=\beta_1,\\\beta_2^*&=\beta_2,\\\widehat{x}^{*(0)}(k)&=\widehat{x}^{(0)}(k)+h.\end{aligned} \quad (25)$$

Proof. The equation of GDGM$(1,1)$ is $x^{*(1)}(k+1)=\beta_1^*x^{*(1)}(k)+\beta_2^*-\beta_1^*kh+(k+1)h$, and $x^{*(1)}(k)=x^{(1)}(k)+kh$; then,

$$\begin{aligned}&x^{(1)}(k+1)+(k+1)h\\&\quad=\beta_1^*\left(x^{(1)}(k)+kh\right)+\beta_2^*-\beta_1^*kh+(k+1)h.\end{aligned} \quad (26)$$

Thus $x^{(1)}(k+1)=\beta_1^*x^{(1)}(k)+\beta_2^*$, which implies $\beta_1^*=\beta_1$, $\beta_2^*=\beta_2$.

By Theorem 14, that is, $\widehat{x}^{*(1)}(k+1)=\beta_1^{*k}x^{*(1)}(1)+((1-\beta_1^{*k})/(1-\beta_1^*))(h+\beta_2^*)-\beta_1^{*k}h+(k+1)h$, and $x^{*(1)}(1)=x^{(1)}(1)+h$, we have

$$\begin{aligned}\widehat{x}^{*(1)}(k+1)&=\beta_1^k x^{(1)}(1)+\frac{1-\beta_1^k}{1-\beta_1}(h+\beta_2)\\&\quad+(k+1)h=\widehat{x}^{(1)}(k+1)+(k+1)h,\end{aligned} \quad (27)$$

and therefore $\widehat{x}^{*(0)}(k)=\widehat{x}^{(0)}(k)+h$. We complete the proof of Theorem 15. □

Remark 16. From Theorem 15, it is clear that the simulation curve of GDGM$(1,1)$ is consistent with DGM$(1,1)$ for the nonnegative original data.

5. Verification of GGM(1, 1) and GDGM(1, 1)

The simulation effects of GGM$(1,1)$ and GDGM$(1,1)$ are evaluated via the following three real cases in this section.

Case 1 (the example for numeric data). Li [16] used the numeric data to discuss the conflict between desire for a good smoothing effect and desire to give additional weight to the recent change, but the author does not give a method for solving such a conflict. Wu et al. [17] presented GDES$(1,1)$ to solve the above problem. In this paper, we do not plan to discuss the conflict in [16]. The main purpose of this paper is to investigate that the simulation curve of original data is consistent with that of the new data by translations. From Table 1, if each original datum is translated by 100, we see that the new data simulation value of GGM$(1,1)$ does consist with its corresponding original data simulation curve, but for GM$(1,1)$, there is not such consistency.

Case 2 (the example for the incidence of Hepatitis B). The following actual value data come from [17] which compared the simulation values of GM$(1,1)$, improved model [18], Holt-Winters model [18], and GDES and DES models [17], and obtained the idea that the value of MAPE for DES model is minimum. Here, by the aid of the data in [17], we mainly consider the curve consistency. From Table 2, if each original datum is translated by 100000, as in Case 1, we also see that the new data simulation curve of GGM$(1,1)$ does consist with its corresponding original data simulation curve, but for GM$(1,1)$, there is not such consistency.

Case 3 (the example for comparing DGM$(1,1)$ and GDGM$(1,1)$). These data come from [4]. The authors [4] compared the simulation values of GM$(1,1)$, DGM$(1,1)$, and OSDGM$(1,1)$ and got that the value of MAPE for OSDGM$(1,1)$ is minimum. Here, by using the data in [4], we mainly consider the curve consistency. From Table 3, if each original datum is translated by 50, we see that the new data simulation curve of GDGM$(1,1)$ does consist with its corresponding original data simulation curve, but for DGM$(1,1)$, there is not such consistency.

Table 1: Simulation by GM(1, 1) and GGM(1, 1).

Time	Actual value	GM(1, 1)	Actual value + 100	GM(1, 1)	GGM(1, 1)
1	6	6	106	106	106
2	4	4.50	104	104.15	104.50
3	7	5.03	107	104.93	105.03
4	5	5.62	105	105.72	105.62
5	6	6.28	106	106.51	106.28
6	4	7.02	104	107.31	107.02
7	10	7.85	110	108.12	107.85
8	9	8.77	109	108.93	108.77
9	11	9.80	111	109.75	109.80
10	10	10.96	110	110.57	110.96
MAPE (%)		1.36		0.098	0.10

Table 2: Simulation by GM(1, 1) and GGM(1, 1).

Month	Actual value	GM(1, 1)	Actual value − 100000	GM(1, 1)	GGM(1, 1)
Jan	162818	162818	62818	62818	62818
Feb	214523	200801	114523	102462	100801
Mar	201184	193817	101184	94260	93817
Apr	155942	187077	55942	86714	87077
May	183216	180571	83216	79773	80571
Jun	165935	174291	65935	73386	74291
Jul	175836	168229	75836	67512	68229
Aug	170885	162379	70885	62107	62379
MAPE (%)		0.67		1.39	1.43

Table 3: Simulation by DGM(1, 1) and GDGM(1, 1).

Actual value	DGM(1, 1)	Actual value − 50	DGM(1, 1)	GDGM(1, 1)
21.1	21.1	−28.9	−28.9	−28.9
26.6	22.90	−23.4	18.81	−27.10
36.1	34.79	−13.9	22.95	−15.21
52.3	52.85	2.3	28.01	2.85
80.1	80.30	30.1	34.17	30.30
126.8	122.00	76.8	41.70	72.00
MAPE (%)	0.48		2.74	0.54

6. Landslide Forecasting in BaZiMen Area

Landslide is one of universal geological hazards. The people of all countries would like to prevent and control it. The formation of landslide is complicated and related to many factors. Through long-term monitoring on landslide, one finds that forecasting landslide is possible by using statistics method, which offers a useful reference for decision-maker.

Based on the observation data of the landslide involving BaZiMen in three-gorge reservoir area, this paper studies the laws of landslide for a specific period. Let us point out that the aforementioned observation data in terms of the landslide are usually relative to a reference point. Obviously, different reference point has different observation data.

Table 4 is the result of analyzing landslide data between April and September in 2004. The new data simulation curve of GGM(1, 1) coincides with its corresponding original data simulation curve, but for GM(1, 1), there is not such kind of consistency. That is to say, the result of proposed GGM(1, 1) has no relation with reference point but the result of GM(1, 1) is connected with reference point. Thus, GGM(1, 1) improves reliability of simulation forecasting result, which contributes grey system method to landslide forecasting.

7. Conclusion

This paper proposes two novel grey models, GGM(1, 1) and GDGM(1, 1). They are the generalization of GM(1, 1) and DGM(1, 1), respectively. Furthermore, we obtain the simulation forecasting curves of two novel grey models which have no relation with translation. The results of practical numerical examples have further demonstrated that the simulation effects of two novel grey models have no relation with reference point.

Table 4: The result of landslide forecasting in BaZiMen area.

YYYYMM	Actual value	GM(1, 1)	Actual value + 100	GM(1, 1)	GGM(1, 1)
2004.04	140.7	140.7	240.7	240.7	240.7
2004.05	144.4	152.72	244.4	249.79	252.72
2004.06	177.8	180.43	277.8	280.41	280.43
2004.07	218.5	213.17	318.5	314.78	313.17
2004.08	270.4	251.85	370.4	353.36	351.85
2004.09	285.2	297.54	385.2	396.68	397.54
MAPE	—	0.69	—	0.40	0.46
2004.10	300.0	351.53	400.0	445.30	451.53

Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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