Research Article

Impulsive Control Strategy for a Nonautonomous Food-Chain System with Multiple Delays

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A nonautonomous food-chain system with Holling II functional response is studied, in which multiple delays of digestion are also considered. By applying techniques in differential inequalities, comparison theorem in ordinary differential equations, impulsive differential equations, and functional differential equations, some effective control strategies are obtained for the permanence of the system. Furthermore, effects of some important coefficients and delays on the permanence of the system are intuitively and clearly shown by series of numerical examples.

1. Introduction

From the beginning of the early 21st century, people began to study the complexity of biology by using a variety of mathematical models and methods; then the development of mathematical biology entered a new period, especially in the field of population dynamics (see [1]) and epidemic dynamics. Moreover, some emerging theoretical tools such as the complex network were also introduced to study mathematical models in biology (see [2–4]) and so forth.

Meanwhile, it is known that many evolution processes are characterized by the fact at certain moments when they experience a change of state abruptly. These processes are subject to short-term perturbations whose duration is negligible in comparison with the duration of the process. Thus, models involving impulsive effects seem to be a hot research field (see [5–8]), which could describe the real relationship among the species more accurately. On the other hand, when prey-predator system is referred, sometimes there is a digest and absorption time (which is always called the digest delay) during the predation instead of transforming the food into growth rate immediately. Hence, in order to model the relationship between the predator and the prey more accurately, it is more reasonable to introduce time delay into the model.

Enlightened by above ecological backgrounds and based on model (1.1) in [5], we consider a nonautonomous three-species food-chain system with multiple delays and impulsive perturbation in this paper, and the model is described as the following impulsive differential equations:

$$\frac{dx}{dt} = r_1(t)x(t) \left(1 - \frac{x(t-\tau_1)}{k_1(t)} \right) - \frac{a_1(t)x(t)y(t)}{b_1(t)+x(t)} - \frac{a_2(t)x(t)z(t)}{b_2(t)+x(t)},$$

$$\frac{dy}{dt} = r_2(t)y(t) \left(1 - \frac{y(t-\tau_2)}{k_2(t)} \right) + \frac{e_1(t)a_1(t)x(t-\tau_4)y(t)}{b_1(t)+x(t-\tau_4)} - \frac{a_3(t)y(t)z(t)}{b_3(t)+y(t)},$$

$$\frac{dz}{dt} = z(t) \left(\frac{e_2(t)a_2(t)x(t-\tau_5)}{b_2(t)+x(t-\tau_5)} + \frac{e_3(t)a_3(t)y(t-\tau_6)}{b_3(t)+y(t-\tau_6)} - m(t)z(t-\tau_3) \right),$$

$$t \neq t_k,$$

$$x(t^+) = (1 + q_{1k})x(t),$$

$$y(t^+) = (1 + q_{2k}) y(t),$$

$$z(t^+) = (1 + q_{3k}) z(t),$$

$$t = t_k, \quad k \in N^*, \quad (1)$$

where $x(t)$, $y(t)$, and $z(t)$ denote the population of the prey and the lower and higher predator at time t , respectively. $r_1(t)$ and $r_2(t)$ are the intrinsic growth rate of the prey $x(t)$ and the lower predator $y(t)$; $k_1(t)$ and $k_2(t)$ denote the environmental carrying capacity of the prey and the lower predator. $a_i(t)$ and $b_i(t)$ are the coefficient of the functional response, $m(t)$ is the density dependent coefficient of the higher predator $z(t)$, and $e_i(t)$ ($i = 1, 2, 3$) represents the transform coefficients during the predation. More details of the background of this model can be found in [5].

Let $I = \{t_k\} \in R : t_k < t_{k+1}, k \in N^*, \lim_{k \rightarrow \pm\infty} t_k = \pm\infty$, and the set of all sequences are bounded and strictly increasing. Let $\Omega \subset R, \Omega \neq \emptyset, \tau = \max_{1 \leq i \leq 6} \{\tau_i\}, \xi_0 \in R$. $PC(\xi_0)$ is the space of all function $\phi : [\xi_0 - \tau, \xi_0] \rightarrow \Omega$ having points of discontinuity at $\mu_1, \mu_2, \dots \in [\xi_0 - \tau, \xi_0]$ of the first kind and left continuous at these points.

For $J \subset R$, $PC(J, R)$ is the space of all piecewise continuous functions from J to R with points of discontinuity of the first kind t_k at which it is left continuous.

Let $\phi_i \in PC(0)$ ($i = 1, 2, 3$), denote $x(t) = x(t; 0, \phi_1)$, $y(t) = y(t; 0, \phi_2)$, $z(t) = z(t; 0, \phi_3)$, and $(x(t), y(t), z(t))^T$ is the solution of system (1) satisfying the following initial conditions:

$$\begin{aligned} 0 &\leq x(t; 0, \phi_1) = \phi_1(s) \leq \infty, \\ s &\in [-\tau, 0], \quad x(0+0; 0, \phi_1) = \phi_1(0) > 0; \\ 0 &\leq y(t; 0, \phi_2) = \phi_2(s) \leq \infty, \\ s &\in [-\tau, 0], \quad y(0+0; 0, \phi_2) = \phi_2(0) > 0; \\ 0 &\leq z(t; 0, \phi_3) = \phi_3(s) \leq \infty, \\ s &\in [-\tau, 0], \quad z(0+0; 0, \phi_3) = \phi_3(0) > 0. \end{aligned} \quad (2)$$

It is easy to verify that solutions of system (1) with above initial conditions are positive for all $t \geq 0$.

Furthermore, throughout the present paper, we assume

- (H1) $0 < t_1 < t_2 < \dots < t_k < \dots$ are fixed impulsive points with $\lim_{k \rightarrow \infty} t_k = \infty$;
- (H2) q_{ik} are real sequences with $q_{ik} > -1$;
- (H3) there exist some positive constants M_i and m_i such that

$$m_i \leq \prod_{0 < t_k < t} (1 + q_{ik}) \leq M_i, \quad i = 1, 2, 3, \quad (3)$$

where $k \in N^*$; N^* denotes set of the positive integer.

For a real positive and continuous function $f(t)$, we denote

$$\begin{aligned} f^M &= \max_{t \in [0, \infty)} f(t), \\ f^L &= \min_{t \in [0, \infty)} f(t). \end{aligned} \quad (4)$$

2. Preliminaries

In this section, we will consider the following impulsive system (5) with time delay and give two important lemmas:

$$\begin{aligned} u'(t) &= u(t) (A - Bu(t - \tau)), \quad t \neq t_k, \\ u(t^+) &= (1 + q_{ik}) u(t), \quad t = t_k, \quad k \in N^*. \end{aligned} \quad (5)$$

Under the hypotheses (H1)–(H3), the corresponding nonimpulsive system of system reads

$$v'(t) = v(t) \left(A - B \prod_{0 < t_k < t} (1 + q_{ik}) v(t - \tau) \right). \quad (6)$$

Lemma 1 (see [9]). Assume that (H1)–(H3) hold:

- (1) If $v(t)$ is a solution of system (6) on $[-\tau, \infty)$, then

$$u(t) = \prod_{0 < t_k < t} (1 + q_{ik}) v(t) \quad (7)$$

is a solution of system (5) on $[-\tau, \infty)$.

- (2) If $u(t)$ is a solution of system (5) on $[-\tau, \infty)$, then

$$v(t) = \prod_{0 < t_k < t} (1 + q_{ik})^{-1} u(t) \quad (8)$$

is a solution of system (6) on $[-\tau, \infty)$.

The proof of this lemma can be found in [10], and we omit it here.

In the following, we will give the other important lemma, which was introduced by Gopalsamy in [11] (page 57: Theorem 1.4.3).

For a delay logistic equation of the form

$$N'(t) = N(t) (b - a_1 N(t - \tau_1) - a_2 N(t - \tau_2)), \quad (9)$$

with initial conditions,

$$\begin{aligned} N(s) &= \varphi(s) \geq 0, \\ s &\in [-\tau, 0], \quad \tau = \max\{\tau_1, \tau_2\}, \quad \varphi \in C([-\tau, 0], R_+), \quad \varphi(0) > 0, \end{aligned} \quad (10)$$

where $b, a_1, a_2, \tau_1, \tau_2 \in (0, +\infty)$.

Lemma 2. For the delay logistic equation (9) with initial conditions (10), if condition

$$b(\tau_1 + \tau_2) \exp[b(\tau_1 + \tau_2)] < 1 \quad (11)$$

holds, then

$$\lim_{t \rightarrow \infty} N(t) = \frac{b}{a_1 + a_2}. \quad (12)$$

By the above lemma, we have the following corollary.

Corollary 3. For any solution $u(t)$ of the functional differential equation

$$u'(t) = u(t)(A - Bu(t - \tau)), \quad (13)$$

with initial condition

$$u(s) = \varphi(s) \in C([- \tau, 0], R^+), \quad \varphi(0) > 0, \quad (14)$$

if condition

(H4)

$$A\tau e^{A\tau} < 1 \quad (15)$$

holds, then

$$\lim_{t \rightarrow \infty} u(t) = \frac{A}{B}. \quad (16)$$

3. Main Results

Theorem 4. Assume that (H1)–(H4) hold; then for any solution of system (1) there exist $T_0 > 0$ and $\eta_i > 0$ such that $x(t) \leq \eta_1$, $y(t) \leq \eta_2$, $z(t) \leq \eta_3$ for $t > T_0$.

Proof. From the first equation of system (1), we have

$$x'(t) \leq r_1^M x(t) \left(1 - \frac{x(t - \tau_1)}{k_1^M} \right), \quad t \neq t_k, \quad (17)$$

$$x(t^+) = (1 + q_{1k}) x(t), \quad t = t_k, \quad k \in N^*,$$

whose corresponding comparison impulsive system is

$$u_1'(t) = r_1^M u_1(t) \left(1 - \frac{u_1(t - \tau_1)}{k_1^M} \right), \quad t \neq t_k, \quad (18)$$

$$u_1(t^+) = (1 + q_{1k}) u_1(t), \quad t = t_k, \quad k \in N^*.$$

According to Lemma 1, the corresponding nonimpulsive equation of (18) reads

$$\begin{aligned} v_1'(t) &= r_1^M v_1(t) \left(1 - \frac{1}{k_1^M} \prod_{0 < t_k < t} (1 + q_{1k}) v_1(t - \tau_1) \right). \end{aligned} \quad (19)$$

By hypothesis (H3), it follows from (19) that

$$v_1'(t) \leq r_1^M v_1(t) \left(1 - \frac{m_1}{k_1^M} v_1(t - \tau_1) \right), \quad (20)$$

whose corresponding comparison functional differential system is

$$w_1'(t) = r_1^M w_1(t) \left(1 - \frac{m_1}{k_1^M} w_1(t - \tau_1) \right). \quad (21)$$

By the corollary, it follows from (21) that

$$\lim_{t \rightarrow \infty} w_1(t) = \frac{k_1^M}{m_1} = \Delta_1; \quad (22)$$

then, for any small positive $\varepsilon_1 > 0$, there exists $T_1 > 0$, such that $w_1(t) < \Delta_1 + \varepsilon_1$ for $t > T_1$.

On the other hand, by hypothesis (H3), the comparison theorem on impulsive differential equation (see [12]), and the comparison theorem on functional differential equation (see [13]), we have

$$\begin{aligned} x(t) &\leq u_1(t) = \prod_{0 < t_k < t} (1 + q_{1k}) v_1(t) \leq M_1 v_1(t) \\ &\leq M_1 w_1(t) \leq M_1 (\Delta_1 + \varepsilon_1) = \eta_1. \end{aligned} \quad (23)$$

Also, from the second and the third equations of system (1), when $t \geq T_1$ we have

$$\begin{aligned} y'(t) &\leq y(t) \left(r_2^M + \frac{e_1^M a_1^M \eta_1}{b_1^L + \eta_1} - \frac{r_2^M y(t - \tau_2)}{k_2^M} \right), \\ &t \neq t_k, \end{aligned} \quad (24)$$

$$y(t^+) = (1 + q_{2k}) y(t), \quad t = t_k, \quad k \in N^*,$$

whose comparison system is

$$\begin{aligned} u_2'(t) &= u_2(t) \left(r_2^M + \frac{e_1^M a_1^M \eta_1}{b_1^L + \eta_1} - \frac{r_2^M u_2(t - \tau_2)}{k_2^M} \right), \\ &t \neq t_k, \end{aligned} \quad (25)$$

$$u_2(t^+) = (1 + q_{2k}) u_2(t), \quad t = t_k, \quad k \in N^*.$$

And the corresponding nonimpulsive system of (25) is

$$\begin{aligned} v_2'(t) &= v_2(t) \left(r_2^M + \frac{e_1^M a_1^M \eta_1}{b_1^L + \eta_1} \right. \\ &\quad \left. - \frac{r_2^M}{k_2^M} \prod_{0 < t_k < t} (1 + q_{2k}) v_2(t - \tau_2) \right) \leq v_2(t) \left(r_2^M \right. \\ &\quad \left. + \frac{e_1^M a_1^M \eta_1}{b_1^L + \eta_1} - \frac{m_2 r_2^M}{k_2^M} v_2(t - \tau_2) \right), \end{aligned} \quad (26)$$

whose comparison system is

$$\begin{aligned} w_2'(t) &= w_2(t) \left(r_2^M + \frac{e_1^M a_1^M \eta_1}{b_1^L + \eta_1} - \frac{m_2 r_2^M}{k_2^M} w_2(t - \tau_2) \right). \end{aligned} \quad (27)$$

By the corollary again, it follows from (27) that

$$\lim_{t \rightarrow \infty} w_2(t) = \frac{r_2^M + e_1^M a_1^M \eta_1 / (b_1^L + \eta_1)}{m_2 r_2^M / k_2^M} = \Delta_2; \quad (28)$$

FIGURE 1: Permanence of system (1) with the impulsive control strategy with $\phi_1(s) = x(0) = 0.5$, $\phi_2(s) = y(0) = 0.8$, $\phi_3(s) = z(0) = 0.6$, $s \in [-\tau, 0]$.

then, for any small positive $\varepsilon_2 > 0$, there exists $T_2 > 0$, such that $w_2(t) < \Delta_2 + \varepsilon_2$ for $t > T_2$.

Thus, by hypothesis (H3) and the comparison theorem we have

$$\begin{aligned} y(t) &\leq u_2(t) = \prod_{0 < t_k < t} (1 + q_{2k}) v_2(t) \leq M_2 v_2(t) \\ &\leq M_2 w_2(t) \leq M_2 (\Delta_2 + \varepsilon_2) = \eta_2. \end{aligned} \quad (29)$$

Finally, when $t \geq T_2$ it follows from the third equation of system (1) that we have

$$z'(t) \leq z(t) \left(\frac{e_2^M a_2^M \eta_1}{b_2^L + \eta_1} + \frac{e_3^M a_3^M \eta_2}{b_3^L + \eta_2} - m^L z(t - \tau_3) \right), \quad t \neq t_k, \quad (30)$$

$$z(t^+) = (1 + q_{3k}) z(t), \quad t = t_k, \quad k \in N^*.$$

Repeating the above process, we can derive that, for any small positive $\varepsilon_2 > 0$, there exists $T_0 > 0$, such that

$$z(t) \leq M_3 (\Delta_3 + \varepsilon_3) = \eta_3 \quad \text{for } t > T_0, \quad (31)$$

where

$$\Delta_3 = \frac{1}{m^L m_3} \left(\frac{e_2^M a_2^M \eta_1}{b_2^L + \eta_1} + \frac{e_3^M a_3^M \eta_2}{b_3^L + \eta_2} \right). \quad (32)$$

□

Theorem 5. If (H1)–(H4) hold, further assume that

(H5)

$$r_1^L > \frac{a_1^M \eta_1 \eta_2}{b_2^L + \eta_1} + \frac{a_2^M \eta_1 \eta_3}{b_2^L + \eta_1}, \quad (33)$$

(H6)

$$r_2^L + \frac{a_2^L e_2^L \xi_1}{b_1^M + \xi_1} > \frac{a_3^M \eta_3}{b_3^L} \quad (34)$$

FIGURE 2: Extinction of the higher predator of system (1) with $\phi_1(s) = x(0) = 0.5$, $\phi_2(s) = y(0) = 0.8$, $\phi_3(s) = z(0) = 0.6$, $s \in [-\tau, 0]$.

hold; then for any solution of system (1) there exist $T > T_0$ and $\xi_i > 0$ ($i = 1, 2, 3$) such that

$$\begin{aligned} x(t) &\geq \xi_1, \\ y(t) &\geq \xi_2, \\ z(t) &\geq \xi_3, \end{aligned} \quad (35)$$

for $t > T$.

Proof. From the first equation of system (1), when $t > T_0$,

$$\begin{aligned} x'(t) &\geq x(t) \left(r_1^L - \frac{a_1^M \eta_1 \eta_2}{b_1^L + \eta_1} - \frac{a_2^M \eta_1 \eta_3}{b_2^L + \eta_1} - \frac{r_1^M}{k_1^L} x(t - \tau_1) \right), \\ x(t^+) &= (1 + q_{1k}) x(t), \quad t = t_k, \quad k \in N^*, \end{aligned} \quad (36)$$

$t \neq t_k$,

whose corresponding comparison impulsive system is

$$\begin{aligned} u_3'(t) &= u_3(t) \cdot \left(r_1^L - \frac{a_1^M \eta_1 \eta_2}{b_1^L + \eta_1} - \frac{a_2^M \eta_1 \eta_3}{b_2^L + \eta_1} - \frac{r_1^M}{k_1^L} u_3(t - \tau_1) \right), \\ u_3(t^+) &= (1 + q_{1k}) u_3(t), \quad t = t_k, \quad k \in N^*. \end{aligned} \quad (37)$$

$t \neq t_k$,

By Lemma 1, the corresponding nonimpulsive equation of system (37) reads

$$\begin{aligned} v_3'(t) &= v_3(t) \left(r_1^L - \frac{a_1^M \eta_1 \eta_2}{b_1^L + \eta_1} - \frac{a_2^M \eta_1 \eta_3}{b_2^L + \eta_1} \right. \\ &\quad \left. - \frac{r_1^M}{k_1^L} \prod_{0 < t_k < t} (1 + q_{1k}) v_3(t - \tau_1) \right) \geq v_3(t) \left(r_1^L \right. \\ &\quad \left. - \frac{a_1^M \eta_1 \eta_2}{b_1^L + \eta_1} - \frac{a_2^M \eta_1 \eta_3}{b_2^L + \eta_1} - \frac{r_1^M M_1}{k_1^L} v_3(t - \tau_1) \right), \end{aligned} \quad (38)$$

FIGURE 3: Extinction of the prey of system (1) with $\phi_1(s) = x(0) = 0.5$, $\phi_2(s) = y(0) = 0.8$, $\phi_3(s) = z(0) = 0.6$, $s \in [-\tau, 0]$.

and the comparison equation of (38) is

$$w_3'(t) = w_3(t) \cdot \left(r_1^L - \frac{a_1^M \eta_1 \eta_2}{b_1^L + \eta_1} - \frac{a_2^M \eta_1 \eta_3}{b_2^L + \eta_1} - \frac{r_1^M M_1}{k_1^L} w_3(t - \tau_1) \right). \quad (39)$$

For the functional differential equation (39), it follows from the corollary that $\lim_{t \rightarrow \infty} w_3(t) = \delta_1$, where

$$\delta_1 = \frac{k_1^L}{r_1^M M_1} \left(r_1^L - \frac{a_1^M \eta_1 \eta_2}{b_1^L + \eta_1} - \frac{a_2^M \eta_1 \eta_3}{b_2^L + \eta_1} \right). \quad (40)$$

Then, for any small positive $\varepsilon_4 > 0$, there exists $T_3 > T_0$, such that $w_3(t) > \delta_1 - \varepsilon_4$ for $t > T_0$.

On the other hand, by hypothesis (H3) and the comparison theorem again we have

$$\begin{aligned} x(t) &\geq u_3(t) = \prod_{0 < t_k < t} (1 + q_{1k}) v_3(t) \geq m_1 v_3(t) \\ &\leq m_1 w_3(t) \geq m_1 (\delta_1 - \varepsilon_4) = \xi_1. \end{aligned} \quad (41)$$

Consider the second equation of system (1); when $t > T_3$ we have

$$\begin{aligned} y'(t) &\geq y(t) \left(r_2^L + \frac{a_1^L e_1^L \xi_1}{b_1^M + \xi_1} - \frac{a_3^M \eta_3}{b_3^L + \eta_3} - \frac{r_2^M}{k_2^L} y(t - \tau_2) \right), \\ &t \neq t_k, \end{aligned} \quad (42)$$

$$y(t^+) = (1 + q_{2k}) y(t), \quad t = t_k, \quad k \in N^*.$$

Repeating the above process, we can derive that, for any small positive $\varepsilon_5 > 0$, there exists $T_4 > T_3 > T_0$, such that

$$y(t) \geq m_2 (\delta_2 - \varepsilon_5) = \xi_2, \quad (43)$$

for $t > T_4$, where

$$\delta_2 = \frac{k_2^L}{r_2^M M_2} \left(r_2^L + \frac{a_1^L e_1^L \xi_1}{b_1^M + \xi_1} - \frac{a_3^M \eta_3}{b_3^L + \eta_3} \right). \quad (44)$$

FIGURE 4: Extinction of the lower predator of system (1) with $\phi_1(s) = x(0) = 0.5$, $\phi_2(s) = y(0) = 0.8$, $\phi_3(s) = z(0) = 0.6$, $s \in [-\tau, 0]$.

Finally, considering the third equation of system (1), when $t > T_4$ we have

$$z'(t) \geq z(t) \left(\frac{a_2^L e_2^L \xi_1}{b_2^M + \xi_1} + \frac{a_3^L e_3^L \xi_2}{b_3^M + \xi_2} - m^M z(t - \tau_3) \right), \quad t \neq t_k, \quad (45)$$

$$z(t^+) = (1 + q_{3k}) z(t), \quad t = t_k, \quad k \in N^*.$$

In the same way, we have that, for any small positive $\varepsilon_6 > 0$, there exists $T > T_4$, such that

$$z(t) \geq m_3 (\delta_3 - \varepsilon_6) = \xi_3, \quad \text{for } t > T, \quad (46)$$

where

$$\delta_3 = \frac{1}{m^M M_3} \left(r_2^L + \frac{a_2^L e_2^L \xi_1}{b_2^M + \xi_1} + \frac{a_3^L e_3^L \xi_2}{b_3^M + \xi_2} \right). \quad (47)$$

□

Theorem 6. *If conditions (H1)–(H6) hold, then system (1) is permanent.*

Proof. If we denote $\xi = \min\{\xi_1, \xi_2, \xi_3\}$, $\eta = \max\{\eta_1, \eta_2, \eta_3\}$. Further, if conditions (H1)–(H6) hold, then any solution

$(x(t), y(t), z(t))^T$ of system (1) will eventually enter and remain in the compact set $K = \{\xi \leq x, y, z \leq \eta\}$.

That is to say, system (1) is permanent. □

Remark 7. From the above theorems, we know that if we choose suitable control conditions (H1)–(H6), then the system will be controlled to be permanent. These results may provide some reasonable control strategies for relevant ecological departments.

4. Numerical Simulations and Discussions

In the above section, we focused our attention on the permanence of the food-chain system with multiple delays in theory; some control conditions have been obtained to guarantee the permanence of system (1). In this section we will give some numerical examples and simulations and then make some discussions. We denote

$$\begin{aligned} f &:= r_1^L - \left(\frac{a_1^M \eta_1 \eta_2}{b_2^L + \eta_1} + \frac{a_2^M \eta_1 \eta_3}{b_2^L + \eta_1} \right); \\ g &:= r_2^L + \frac{a_2^L e_2^L \xi_1}{b_1^M + \xi_1} - \frac{a_3^M \eta_3}{b_3^L}. \end{aligned} \quad (48)$$

FIGURE 5: Effect of delays on the dynamical behavior of system (1) with $\phi_1(s) = x(0) = 0.5$, $\phi_2(s) = y(0) = 0.8$, $\phi_3(s) = z(0) = 0.6$, $s \in [-\tau, 0]$.

According to Theorem 5, we can see that the signs of f and g are very important factors for controlling the permanence of system (1). In order to verify this point, we only change one or two parametric values based on Case 1 as follows.

Case 1. We consider the following choice of parametric values:

$$r_1(t) = 4 + \sin(t),$$

$$r_2(t) = b + \cos(t),$$

$$k_1(t) = 2 + 0.1 \sin(t),$$

$$k_2(t) = 3,$$

$$b_1(t) = 3 + \sin(t),$$

$$b_2(t) = 4 - 0.1 \cos(t),$$

$$b_3(t) = 4,$$

$$m(t) = 0.2 + 0.1 \cos(2t),$$

$$a_1(t) = 1,$$

$$a_2(t) = 0.2,$$

$$a_3(t) = 2,$$

$$e_1(t) = 18,$$

$$e_2(t) = 3,$$

$$e_3(t) = 6,$$

$$q_{1k} = 0.1,$$

$$q_{2k} = 0.2,$$

$$q_{3k} = 0.2,$$

$$k = 1, 2, \dots$$

(49)

with initial conditions

$$\phi_1(s) = x(0) = 0.5,$$

$$\phi_2(s) = y(0) = 0.8,$$

$$\phi_3(s) = z(0) = 0.6,$$

$$s \in [-\tau, 0].$$

(50)

The delays are given as

$$\tau_1 = \tau_2 = \tau_3 = 0.01,$$

$$\tau_4 = \tau_5 = \tau_6 = 0.02.$$

(51)

On the one hand, one can verify that conditions (H1)–(H6) are satisfied. According to Theorem 5, system (1) is permanent. On the other hand, system (1) is numerically solved

for the above choice of parameters and initial conditions. It is obvious that the system is permanent and has quasiperiodic solutions; it is clear to see the time-series and phase portrait intuitively in Figure 1.

Case 2. If we increase the value of a_1^M , for example, if we let $a_1(t) = 3.5$ while other parametric values are the same as Case 1, then system (1) is numerically solved in Figure 2. One can find that the highest predator z will be extinct finally, and the density of the prey will decrease to zero periodically, while the middle predator can be permanent.

Case 3. When we go on increasing the value of a_1^M such that $a_1(t) = 3.9$, one can see that both the highest predator z and the prey x will be extinct finally, while the middle predator y can be permanent at the moment (see Figure 3).

Case 4. If we increase the value of a_3^M , for example, we let $a_3(t) = 2.3$, while other parametric values are the same as Case 1. It is very strange and interesting that both the highest predator z and the prey x can be permanent, while the middle predator y will be extinct gradually (see Figure 4).

Case 5. In the following, we will study effects of the delays on the dynamical behavior of the system. To this end, we enlarge the value of all of the delays and choose

$$\tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5 = \tau_6 = 0.2, \quad (52)$$

while the other parameters and initial conditions are the same as Case 1. It is easy to verify that condition (H4) does not hold any more. By the numerical simulations, we can see that the prey x and the higher predator z will be permanent while the density of the lower predator $y(t)$ oscillates, and it shows a strange dynamic characteristic of intermittent extinction (see Figure 5).

By the above theoretical analysis and the numerical experiments, it is shown that we can seek some reasonable control strategies so that the system can be controlled to be permanent, such as changing values of some important parameters of the system. On the other hand, by the numerical simulations in Case 5, when the delays are too long, the density of the species will oscillate and leads to strange phenomena of intermittent extinction, which can explain the complexity of biological systems. In addition, we can also seek some efficient measures to guarantee that some of the species (beneficial insects) in the system can be permanent while the other species (pest insects) will be extinct finally. And this method can be extended to the study of the other systems with variable coefficients, such as epidemic systems and neural network systems.

Competing Interests

The authors declare that they have no competing interests.

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