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Research Article

Nonfragile H_∞ Filtering for Nonlinear Markovian Jumping Systems with Mode-Dependent Time Delays and Quantization

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This paper is about the nonfragile H_∞ filtering problem for a class of Markovian jumping systems (MJSs) subject to mode-dependent time delays and quantization. Mode switching is considered not only in the system parameters but also in the time delays. Signal quantization is taken into account to reflect the phenomenon of incomplete measurement. Norm bound uncertainty is utilized to indicate the filter gain variation due to the inaccuracy of actual filtering realization. The purpose of this paper is to design a nonfragile filter so that the filtering error dynamics is stochastically stable with a prescribed disturbance attenuation level. By the Lyapunov stability theory, stochastic analysis theory, and linear matrix inequalities (LMIs) technique, some new sufficient conditions are derived for the existence of the desired nonfragile filter which ensures the stochastic stability and H_∞ performance of the filtering error dynamics. The expression of the filter can be obtained via solving the feasible solution to the LMIs. A simulation example is presented to illustrate the performance of the proposed filtering scheme.

1. Introduction

Recently, as a special class of stochastic systems, Markovian jumping systems (MJSs) have been employed to model many kinds of dynamic systems such as manufacturing system, networked control systems (NCSs) and financial market system. MJSs can reflect the abrupt phenomena in the aforementioned systems including variation of system structures and parameters, component, or interconnection failures and sudden environment disturbances. Thus, a large number of results have been reported in the literature about MJSs on stability analysis [1–3], control [4–6], and filtering [7, 8] problems. To mention a few, the sliding mode controller has been designed in [4] for a kind of uncertain MIMO linear MJSs; authors of [2] studied the stability and stabilization problem of continuous-time stochastic MJSs with random switching signals, while the event-triggered H_∞ filtering problem for networked MaJSs is investigated in [7]. On the other hand, time delays exist extensively in practical applications, which may cause oscillation or even instability of MJSs and increase the complexity of system analysis and design; thus some efforts have been made on the MJSs subject to time delays [1, 8]. For example, the globally exponential stability and stabilization problems have been dealt with in [1] for interconnected MJSs with mode-dependent time delays. H_∞ deconvolution filter has been designed in [8] for singular MJSs with time-varying delays and parameter uncertainties.

In the field of filtering for MJSs, the existing results generally assume that the filter parameters can be implemented precisely. Nevertheless, the assumption is not always reasonable as there exists inevitable parameters variation in actual systems due to a number of reasons such as finite precision and rounding errors. To avoid the undesirable consequences that resulted from the inaccurate execution of filters/estimators/controllers parameters, some research motivation has been stimulated on the nonfragile characteristics which can tolerate a degree of gain variations, and some relevant papers have appeared on this topic [6, 9–13]. On the other hand, the H_∞ filter can make the H_∞ norm of the transfer function from the noise signal to the filtering error less than a given

index, and different from the common Kalman filtering approach, the H_∞ one can be applied to the situation with unknown statistic characteristics of noise signals. However, it should be noted that nonfragile H_∞ filtering for MJSs with mode-dependent time delays is still an open issue. Therefore, one of the motivations of this paper is to focus on deriving the sufficient conditions which guarantee both the stochastic stability and the prescribed disturbance attenuation performance of the filtering error dynamics against the gain uncertainties.

Nowadays, more and more research attention has been paid on the network-induced phenomena caused by the limited bandwidth of the communication channel, for the reason that they may degrade the performance of the NCSs. So far, to better depict and analyze the real condition of NCSs, network-induced phenomena such as time delays [14, 15], channel fading [16], data missing [17–19], and quantization effect [20, 21] have been incorporated into system models. Among them, it becomes essential to investigate the occurring of incomplete measurements, such as measurement quantization, which originates from the imperfect communications in numerous engineering practices particularly in networked environments, and several literatures have been accessible. For example, in [20], the phenomena of missing measurements, random occurring sensor saturation, and quantization have been described within a unified expression for complex networks. The H_∞ filter has been designed in [22] for time-varying systems with measurement dropouts and quantization. A necessary and sufficient condition is first proposed which illustrates that the expected expression of the time-varying filter can be determined by solving the coupled recursive Riccati difference equations (RDEs).

In fact, almost all practical systems contain nonlinearities [14, 23, 24], which would arise in a probabilistic way and change randomly on the aspects of type and intensity, and this phenomenon is called randomly occurring nonlinearities (RONs). RONs can be utilized to reflect the network-induced congestions caused by the limitations of bandwidth when there are a great many packets transmitted in the communication channel at one instant, and the probability of occurring would be obtained by experiments [16]. Different functions or constraint conditions, such as sigmoid functions, nonmonotonic functions, sector-bounded condition, and Lipschitz condition, have been selected to describe nonlinearities. In [25], nonmonotonic nonlinear functions have been used which quantify their lower and upper bounds precisely/tightly, and this is good for employing the method based on linear matrix equalities (LMIs) to decrease the potential conservatism. Nevertheless, to the best of the authors' knowledge, the nonfragile H_∞ filtering issue for MJSs with RONs, time delays, and quantization has not been studied yet, which constitutes the main motivation of this paper.

In this paper, the nonfragile H_∞ filtering problem is investigated for a class of discrete-time MJSs with mode-dependent time delays, quantization, and RONs. Taking these factors into account, the nonlinear MJSs model becomes complicated. Therefore, the mathematical analysis will be more complex and challenging. After deriving the augmented system of the original state and the filtering error, Lyapunov stability theorem and matrix analysis technology are utilized to obtain the sufficient conditions for designing the filter. The gain of the filter can be known in terms of the feasible solution to the LMIs. A simulation example is presented to verify the usefulness of the addressed method. The main novelties of this paper are summarized as follows. (1) The nonfragile filtering issue is studied for a class of discrete-time MJSs with mode-dependent time delays, randomly occurring gain variations, randomly occurring nonlinearities, and measurement quantization. (2) Sufficient conditions are derived via Lyapunov theory and stochastic analysis technique, which ensure the existence of nonfragile filter and guarantee the stochastic stability of the filtering error dynamics with a prescribed disturbance attenuation level.

Notation. Throughout this paper, M^T means the transpose of M. $\mathbb{R}^n$ means the n dimensional Euclidean space and $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices. The set of all nonpositive integers is denoted by $\mathbb{Z}^-$. I and 0 denote the identity matrix and zero matrix, respectively. The notation $P > 0$ means that P is a real symmetric and positive definite matrix. $\mathbb{E}\{x\}$ and $\mathbb{E}\{x \mid y\}$ represent, respectively, the expectation of x and the expectation of x conditional on y. $\|x\|$ stands for the Euclidean norm of a vector x. In symmetric block matrices, the shorthand $\mathrm{diag}\{A_1, A_2, \ldots, A_n\}$ represents a block diagonal matrix with diagonal blocks being the matrices $A_1, \ldots, A_n$, and the symbol $*$ denotes an ellipsis for terms induced by symmetry. Matrices without explicitly stated dimensions are supposed to be compatible for matrix operations.

2. Problem Formulation and Preliminaries

$r(k)$ $(k \geq 0)$, which satisfies $r(k) \in S = \{1, 2, \ldots, s\}$, describes the Markov chain whose transition probability matrix $\Xi = [\delta_{ij}]_{s\times s}$ is

$$\mathrm{Prob}\{r(k+1) = j \mid r(k) = i\} = \delta_{ij}, \quad \forall i, j \in S, \tag{1}$$

where $\delta_{ij} \geq 0$ $(i, j \in S)$ denotes the transition probability from mode i to mode j with $\sum_{j=1}^{s} \delta_{ij} = 1, \forall i \in S$.

Consider the following discrete-time MJSs with mode-dependent time delays and RONs:

$$\begin{aligned} x(k+1) &= A(r(k))\,x(k) + B(r(k))\,x(k-d(r(k))) \\ &\quad + \alpha(k)\,h(x(k)) + C(r(k))\,w(k), \\ y(k) &= D(r(k))\,x(k) + E(r(k))\,w(k), \\ z(k) &= H(r(k))\,x(k), \end{aligned} \tag{2}$$

where $x(k) = \left[x_1^T(k)\ x_2^T(k)\ \cdots\ x_n^T(k)\right]^T \in \mathbb{R}^n$ is the state vector; $d(r(k))$ denotes the discrete-time delays; $h(x(k))$ describes the nonlinear disturbance with $h(0) = 0$; $w(k)$ shows the external disturbance and belongs to $l_2([0,\infty)$; $y(k) \in \mathbb{R}^r$ is measurement output; $z(k)$ is the output of the system, which is the linear combination of $x_1(k)$, $x_2(k)$, ... and $x_n(k)$.

The Bernoulli-distributed white sequence $\alpha(k) \in \mathbb{R}$, which represents the phenomenon of RONs, conforms to the following probability distribution:

$$\begin{aligned} &\text{Prob}\{\alpha(k) = 1\} = \overline{\alpha}, \\ &\text{Prob}\{\alpha(k) = 0\} = 1 - \overline{\alpha}, \end{aligned} \tag{3}$$

where $\overline{\alpha} \in [0, 1]$ is a known constant.

In the whole paper, the following assumption is made.

Assumption 1. For any $\tau_1, \tau_2 \in \mathbb{R}$, $\tau_1 \neq \tau_2$, the nonlinear function $h(\cdot)$ satisfies

$$\varrho_i^- \leq \frac{h_i(\alpha_1) - h_i(\alpha_2)}{\alpha_1 - \alpha_2} \leq \varrho_i^+, \quad (i = 1, 2, \ldots, n), \tag{4}$$

where ϱ_i^- and ϱ_i^+ are known constant scalars.

The set S shows s modes of equation (2), for the ith mode ($r(k) = i$), system matrices, and time delays, are represented by A_i, B_i, C_i, D_i, E_i, H_i, and d_i.

The signal quantization may greatly influence the performance of the filters. The quantizer $q(\cdot)$ is denoted by

$$q(\vartheta) = [q_1(\vartheta_1) \;\; q_2(\vartheta_2) \;\; \cdots \;\; q_r(\vartheta_r)]^T, \quad \forall \vartheta \in \mathbb{R}^r. \tag{5}$$

According to [20], the logarithmic typed $q(\cdot)$ is selected here. For each $q_j(\cdot)$ $(1 \leq j \leq r)$, quantization levels are shown by

$$\begin{aligned} \mathcal{U}_j = \left\{\pm u_i^{(j)}, u_i^{(j)} = \rho_j^i u_0^{(j)}, \; i = 0, \pm 1, \pm 2, \ldots\right\} \cup \{0\}, \\ 0 \leq \rho_j \leq 1, \; u_0^{(j)} > 0. \end{aligned} \tag{6}$$

The logarithmic quantizer $q_j(\cdot)$ is defined as

$$q_j(\vartheta_j) = \begin{cases} u_i^{(j)}, & \dfrac{1}{1+\kappa_j} u_i^{(j)} \leq \vartheta_j \leq \dfrac{1}{1-\kappa_j} u_i^{(j)}, \\ 0, & \vartheta_j = 0, \\ -q_j(-\vartheta_j), & \vartheta_j \leq 0 \end{cases} \tag{7}$$

with $\kappa_j = (1 - \rho_j)/(1 + \rho_j)$. It follows from [20] that

$$q_j(\vartheta_j) = \left(1 + \Lambda_k^j\right)\vartheta_j \quad \text{such that } \left|\Lambda_k^j\right| \leq \kappa_j. \tag{8}$$

Letting $\Lambda_k = \text{diag}\{\Lambda_k^1, \Lambda_k^2, \ldots, \Lambda_k^r\}$, the measurement with quantization is described as

$$\widetilde{y}(k) = (I + \Lambda_k)\, y(k). \tag{9}$$

Denoting $\overline{\Lambda} = \text{diag}\{\kappa_1, \ldots, \kappa_r\}$ and setting $F_1(k) = \Lambda_k \overline{\Lambda}^{-1}$, then the quantization effect is shown by

$$\begin{aligned} \widetilde{y}(k) &= \left(I + F_1(k)\overline{\Lambda}\right) y(k), \\ F_1(k) F_1^T(k) &= F_1^T(k) F_1(k) \leq I. \end{aligned} \tag{10}$$

Considering the filter gain variations in actual implementation, the following structure of filter is constructed:

$$\begin{aligned} \widehat{x}(k+1) &= A_i \widehat{x}(k) + B_i \widehat{x}(k - d_i) + \overline{\alpha} h(\widehat{x}(k)) \\ &\quad + \left(K_i + \sigma(k)\Delta K_i\right)\left[\widetilde{y}(k) - D_i \widehat{x}(k)\right], \\ z(k) &= H_i \widehat{x}(k), \end{aligned} \tag{11}$$

where $\widehat{x}(k) \in \mathbb{R}^n$ is filter state and K_i is the matrix to be computed. The random variable $\sigma(k)$ is employed to regulate the occurrence of such filter gain variations, whose expectation and variance are $\overline{\sigma}$ and $\widetilde{\sigma}^2$, respectively, and $\sigma(k)$ is uncorrelated with $\alpha(k)$. ΔK_i determines the gain uncertainties of the norm-bounded multiplicative form as follows:

$$\Delta K_i = K_i M F_2(k) N, \tag{12}$$

where M and N are known matrices of proper dimensions and $F_2(k)$ is the unknown matrix with $F_2^T(k) F_2(k) \leq I$.

Remark 2. In practical applications, the filter gain variations occur inevitably for the reason of rounding errors and the demands for readjusting filter gains. Furthermore, random and sudden shifts caused by random changes of loads in NCSs can influence the actual values of the filter under implementation. In (11), the stochastic variable $\sigma(k)$ is employed to characterize the random occurring of this kind of filter gain uncertainties. Next, we are to obtain a filter with satisfactory filtering performance in spite of stochastic gain drifts on the expected values and stochastic analysis technique will be adopted to deal with it.

Denoting $e(k) = x(k) - \widehat{x}(k)$, $\widetilde{\sigma}(k) = \sigma(k) - \overline{\sigma}$, and $\widetilde{z}(k) = z(k) - \widehat{z}(k)$, from (2), (9), and (11), the following error dynamics is obtained:

$$\begin{aligned} e(k+1) &= A_i e(k) + B_i e(k - d_i) \\ &\quad + \overline{\alpha}\left(h(x(k)) - h(\widehat{x}(k))\right) \\ &\quad + \widetilde{\alpha}(k) h(x(k)) + C_i w(k) \\ &\quad - \left(K_i + \overline{\sigma}\Delta K_i\right) E_i w(k) \\ &\quad - \left(K_i + \overline{\sigma}\Delta K_i\right) D_i e(k) \\ &\quad - \left(K_i + \overline{\sigma}\Delta K_i\right) \Lambda_k D_i x(k) \\ &\quad - \widetilde{\sigma}(k) \Delta K_i \Lambda_k D_i x(k) \\ &\quad - \left(K_i + \overline{\sigma}\Delta K_i\right) \Lambda_k E_i w(k) \\ &\quad - \widetilde{\sigma}(k) \Delta K_i D_i e(k) - \widetilde{\sigma}(k) \Delta K_i E_i w(k) \\ &\quad - \widetilde{\sigma}(k) \Delta K_i \Lambda_k E_i w(k), \\ \widetilde{z}(k) &= H_i e(k). \end{aligned} \tag{13}$$

Noting $\eta(k) = \left[x^T(k) \;\; e^T(k)\right]^T$, $\widetilde{h}(e(k)) = h(x(k)) - h(\widehat{x}(k))$, and $h(\eta(k)) = \left[h^T(x(k)) \;\; \widetilde{h}^T(e(k))\right]^T$ and combining

the filter error (13) with system (2), we have the following augmented system:

$$\begin{aligned}\eta(k+1) &= \left(\overline{A}_i + \overline{K}_i\widetilde{D}_i + \overline{K}_i\overline{D}_i\right)\eta(k)\\ &\quad + \widetilde{\sigma}(k)\widetilde{K}_i\left(\widetilde{D}_i + \overline{D}_i\right)\eta(k)\\ &\quad + \overline{B}_i\eta(k-d_i) + \overline{\alpha}S_1h(\eta(k))\\ &\quad + \widetilde{\alpha}(k)S_2h(\eta(k))\\ &\quad + \left(\overline{C}_i + \overline{K}_i\overline{E}_i + \overline{K}_i\widetilde{E}_i\right)w(k)\\ &\quad + \widetilde{\sigma}(k)\left(\widetilde{K}_i\overline{E}_i + \widetilde{K}_i\widetilde{E}_i\right)w(k),\\ \widetilde{z}(k) &= \overline{H}_i\eta(k),\end{aligned} \tag{14}$$

where

$$\begin{aligned}\overline{A}_i &= \begin{bmatrix} A_i & 0\\ 0 & A_i\end{bmatrix},\\ \overline{B}_i &= \begin{bmatrix} B_i & 0\\ 0 & B_i\end{bmatrix},\\ S_1 &= \begin{bmatrix} I & 0\\ 0 & I\end{bmatrix},\\ S_2 &= \begin{bmatrix} I & 0\\ I & 0\end{bmatrix},\\ \overline{C}_i &= \begin{bmatrix} C_i\\ C_i\end{bmatrix},\\ \overline{K}_i &= \begin{bmatrix} K_i + \overline{\sigma}\Delta K_i & 0\\ 0 & K_i + \overline{\sigma}\Delta K_i\end{bmatrix},\\ \overline{D}_i &= \begin{bmatrix} 0 & 0\\ 0 & -D_i\end{bmatrix},\\ \widetilde{D}_i &= \begin{bmatrix} 0 & 0\\ -\Lambda_k D_i & 0\end{bmatrix},\\ \overline{E}_i &= \begin{bmatrix} 0\\ -E_i\end{bmatrix},\\ \widetilde{K}_i &= \begin{bmatrix} \Delta K_i & 0\\ 0 & \Delta K_i\end{bmatrix},\\ \widetilde{E}_i &= \begin{bmatrix} 0\\ -\Lambda_k E_i\end{bmatrix},\\ \overline{H}_i &= \begin{bmatrix} 0 & H_i\end{bmatrix}^T.\end{aligned} \tag{15}$$

Our objective is to establish a nonfragile H_∞ filtering approach for MJSs (2) with measurement quantization (9). To be specific, we are interested in designing a filter in the form of (11) with admissible gain variations of the form (12) so that the following requirements are satisfied:

(a) The augmented system (14) is stochastically stable.

(b) The filtering error from (14) satisfies the H_∞ performance constraint as follows:

$$\sum_{k=0}^{\infty}\mathbb{E}\left\{\|\widetilde{z}(k)\|^2\right\} \le \gamma^2\sum_{k=0}^{\infty}\mathbb{E}\left\{\|w(k)\|^2\right\}. \tag{16}$$

By defining Lyapunov-Krasovskii function, sufficient conditions for the existence of the desired filter (11) for MJSs (2) will be given and the gain matrix K_i ($i \in S$) can also be calculated explicitly.

3. Main Results

The following lemmas are employed for the derivation of the main results.

Lemma 3 (Schur complement [26]). *Given constant matrices S_1, S_2, and S_3 where $S_1 = S_1^T$ and $0 < S_2 = S_2^T$, then $S_1 + S_3^TS_2^{-1}S_3 < 0$ if and only if*

$$\begin{bmatrix} S_1 & S_3^T\\ S_3 & -S_2\end{bmatrix} < 0, \qquad \begin{bmatrix} -S_2 & S_3\\ S_3^T & S_1\end{bmatrix} < 0. \tag{17}$$

Lemma 4 (S-procedure [26]). *Let $L = L^T$, M, and N be real matrices of appropriate dimensions with F satisfying $F^TF \le I$; then*

$$L + MFN + N^TF^TM^T < 0 \tag{18}$$

if and only if there exists a positive scalar μ such that

$$L + \mu^{-1}MM^T + \mu N^TN < 0 \tag{19}$$

or, equivalently,

$$\begin{bmatrix} L & M & \mu N^T\\ M^T & -\mu I & 0\\ \mu N & 0 & -\mu I\end{bmatrix} < 0. \tag{20}$$

Lemma 5 (see [25]). *Suppose that $\mathscr{B} = \mathrm{diag}\{\beta_1, \beta_2, \ldots, \beta_n\}$ is a positive-semidefinite diagonal matrix. Let $x = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n$ and $\mathscr{H}(x) = [h_1(x_1), h_2(x_2), \ldots, h_n(x_n)]^T$ be a continuous nonlinear function satisfying*

$$l_i^- \le \frac{h_i(m)}{m} \le l_i^+, \quad m \ne 0,\ m \in \mathbb{R},\ i = 1, 2, \ldots, n \tag{21}$$

with l_i^- and l_i^+ being constant scalars. Then

$$\begin{bmatrix} x\\ \mathscr{H}(x)\end{bmatrix}^T \begin{bmatrix} \mathscr{B}L_1 & -\mathscr{B}L_2\\ -\mathscr{B}L_2 & \mathscr{B}\end{bmatrix}\begin{bmatrix} x\\ \mathscr{H}(x)\end{bmatrix} \le 0 \tag{22}$$

or

$$x^T \mathscr{B} L_1 x - 2x^T \mathscr{B} L_2 \mathscr{H}(x) + \mathscr{H}^T(x) \mathscr{B} \mathscr{H}(x) \le 0, \quad (23)$$

where $L_1 = \operatorname{diag}\{l_1^+ l_1^-, l_2^+ l_2^-, \ldots, l_n^+ l_n^-\}$ *and* $L_2 = \operatorname{diag}\{(l_1^+ + l_1^-)/2, (l_2^+ + l_2^-)/2, \ldots, (l_n^+ + l_n^-)/2\}$.

Denoting $d_M = \max\{d_i \mid i \in S\}$, $d_m = \min\{d_i \mid i \in S\}$ and $\underline{\delta} = \min\{\delta_{ii} \mid i \in S\}$.

In the following theorem, the sufficient conditions which guarantee the stochastic stability and the H_∞ performance of (14) will be derived.

Theorem 6. *Let the parameters* $\overline{K}_i$ $(i \in S)$ *be known. The augmented system (14) is stochastically stable if there exist a set of matrices* $P_i > 0$ $(i \in S)$, *matrix* $Q > 0$, *and diagonal matrices* X *and* Y *satisfying*

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} \\ * & \Pi_{22} & \Pi_{23} & \Pi_{24} \\ * & * & \Pi_{33} & \Pi_{34} \\ * & * & * & \Pi_{44} \end{bmatrix} < 0, \quad (24)$$

where

$$\begin{aligned}
&\overline{P}_i = \sum_{j=1}^{s} \delta_{ij} P_j = \operatorname{diag}\left\{P_{1_i}, P_{1_i}\right\}, \\
&\xi(k,i) = \left[\eta^T(k) \;\; \eta^T(k-d_i) \;\; h^T(\eta(k))\right]^T, \\
&\Pi_{11} \\
&\quad = \left(\overline{A}_i + \overline{K}_i \widetilde{D}_i + \overline{K}_i \overline{D}_i\right)^T \overline{P}_i \left(\overline{A}_i + \overline{K}_i \widetilde{D}_i + \overline{K}_i \overline{D}_i\right) \\
&\qquad + \tilde{\sigma}^2 \left(\widetilde{K}_i \left(\widetilde{D}_i + \overline{D}_i\right)\right)^T \overline{P}_i \left(\widetilde{K}_i \left(\widetilde{D}_i + \overline{D}_i\right)\right) - \Lambda_{11} \\
&\qquad + \left[(1-\underline{\delta})(d_M - d_m) + 1\right] Q, \\
&\Pi_{12} = \left(\overline{A}_i + \overline{K}_i \widetilde{D}_i + \overline{K}_i \overline{D}_i\right)^T \overline{P}_i \overline{B}_i, \\
&\Pi_{13} = \left(\overline{A}_i + \overline{K}_i \widetilde{D}_i + \overline{K}_i \overline{D}_i\right)^T \overline{P}_i \overline{\alpha} S_1 - \Lambda_{21}, \\
&\Pi_{14} \\
&\quad = \left(\overline{A}_i + \overline{K}_i \widetilde{D}_i + \overline{K}_i \overline{D}_i\right)^T \overline{P}_i \left(\overline{C}_i + \overline{K}_i \overline{E}_i + \overline{K}_i \widetilde{E}_i\right) \\
&\qquad + \tilde{\sigma}^2 \left(\widetilde{K}_i \left(\widetilde{D}_i + \overline{D}_i\right)\right)^T \overline{P}_i \left(\widetilde{K}_i \overline{E}_i + \widetilde{K}_i \widetilde{E}_i\right), \\
&\Pi_{22} = \overline{B}_i^T \overline{P}_i \overline{B}_i - Q, \\
&\Pi_{23} = \overline{B}_i^T \overline{P}_i \overline{\alpha} S_1, \\
&\Pi_{24} = \overline{B}_i^T \overline{P}_i \left(\overline{C}_i + \overline{K}_i \overline{E}_i + \overline{K}_i \widetilde{E}_i\right), \\
&\Pi_{33} = \overline{\alpha}^2 S_1^T \overline{P}_i S_1 + \tilde{\alpha}^2 S_2^T \overline{P}_i S_2 - \overline{X}, \\
&\Pi_{34} = \overline{\alpha} S_1^T \overline{P}_i \left(\overline{C}_i + \overline{K}_i \overline{E}_i + \overline{K}_i \widetilde{E}_i\right), \\
&\tilde{\alpha}^2 = \overline{\alpha}(1 - \overline{\alpha}), \\
&\Pi_{44} \\
&\quad = \left(\overline{C}_i + \overline{K}_i \overline{E}_i + \overline{K}_i \widetilde{E}_i\right)^T \overline{P}_i \left(\overline{C}_i + \overline{K}_i \overline{E}_i + \overline{K}_i \widetilde{E}_i\right) \\
&\qquad + \tilde{\sigma}^2 \left(\widetilde{K}_i \overline{E}_i + \widetilde{K}_i \widetilde{E}_i\right)^T \overline{P}_i \left(\widetilde{K}_i \overline{E}_i + \widetilde{K}_i \widetilde{E}_i\right), \\
&L_1 = \operatorname{diag}\left\{\varrho_1^+ \varrho_1^-, \varrho_2^+ \varrho_2^-, \ldots, \varrho_n^+ \varrho_n^-\right\}, \\
&L_2 = \operatorname{diag}\left\{\frac{\varrho_1^+ + \varrho_1^-}{2}, \frac{\varrho_2^+ + \varrho_2^-}{2}, \ldots, \frac{\varrho_n^+ + \varrho_n^-}{2}\right\}, \\
&\Lambda_{11} = \begin{bmatrix} XL_1 & 0 \\ 0 & YL_1 \end{bmatrix}, \\
&\Lambda_{21} = \begin{bmatrix} -XL_2 & 0 \\ 0 & -YL_1 \end{bmatrix}, \\
&\overline{X} = \begin{bmatrix} X & 0 \\ 0 & Y \end{bmatrix}.
\end{aligned} \quad (25)$$

Proof. Construct the following Lyapunov function:

$$\begin{aligned} V(\eta(k), k, r(k)) &= V_1(\eta(k), k, r(k)) \\ &\quad + V_2(\eta(k), k, r(k)), \end{aligned} \quad (26)$$

where

$$\begin{aligned}
&V_1(\eta(k), k, r(k)) = \eta^T(k) P(r(k)) \eta(k), \\
&V_2(\eta(k), k, r(k)) = \sum_{l=k-d(r(k))}^{k-1} \eta^T(l) Q \eta(l) + (1-\underline{\delta}) \\
&\quad \cdot \sum_{m=d_m}^{d_M-1} \sum_{l=k-m}^{k-1} \eta^T(l) Q \eta(l).
\end{aligned} \quad (27)$$

For $i \in S$, we have

$$\begin{aligned}
&\mathbb{E}\left\{V_1(\eta(k+1), k+1, r(k+1)) \mid \eta(k), r(k) = i\right\} \\
&\quad - V_1(\eta(k), k, i) = \eta^T(k) \\
&\quad \cdot \left[\left(A_i + \overline{K}_i \widetilde{D}_i + \overline{K}_i \overline{D}_i\right)^T \overline{P}_i \left(A_i + \overline{K}_i \widetilde{D}_i + \overline{K}_i \overline{D}_i\right)\right. \\
&\quad \left. + \tilde{\sigma}^2 \left(\widetilde{K}_i \left(\widetilde{D}_i + \overline{D}_i\right)\right)^T \overline{P}_i \left(\widetilde{K}_i \left(\widetilde{D}_i + \overline{D}_i\right)\right)\right] \eta(k) \\
&\quad - \eta^T(k) P_i \eta(k) + 2\eta^T(k) \left(A_i + \overline{K}_i \widetilde{D}_i + \overline{K}_i \overline{D}_i\right)^T \\
&\quad \cdot \overline{P}_i \overline{B}_i \eta(k - d_i) + 2\eta^T(k) \left(A_i + \overline{K}_i \widetilde{D}_i + \overline{K}_i \overline{D}_i\right)^T
\end{aligned}$$

$$\cdot \overline{P}_i\overline{\alpha}S_1h(\eta(k)) + 2\eta^T(k)$$
$$\cdot \left[\left(A_i + \overline{K}_i\widetilde{D}_i + \overline{K}_i\overline{D}_i\right)^T \overline{P}_i\left(\overline{C}_i + \overline{K}_i\overline{E}_i + \overline{K}_i\widetilde{E}_i\right)\right.$$
$$\left. + \widetilde{\sigma}^2\left(\widetilde{K}_i\left(\widetilde{D}_i + \overline{D}_i\right)\right)^T \overline{P}_i\left(\widetilde{K}_i\left(\widetilde{E}_i + \overline{E}_i\right)\right)\right] w(k)$$
$$+ \eta^T(k-d_i)\overline{B}_i^T\overline{P}_i\overline{B}_i\eta(k-d_i) + 2\eta^T(k-d_i)$$
$$\cdot \overline{B}_i^T\overline{P}_i\overline{\alpha}S_1h(\eta(k)) + 2\eta^T(k-d_i)\overline{B}_i^T\overline{P}_i\left(\overline{C}_i + \overline{K}_i\overline{E}_i\right.$$
$$\left. + \overline{K}_i\widetilde{E}_i\right) w(k) + h^T(\eta(k))\left[\overline{\alpha}^2S_1^T\overline{P}_iS_1 + \widetilde{\alpha}^2S_2^T\overline{P}_iS_2\right]$$
$$\cdot h(\eta(k)) + 2h^T(\eta(k))\overline{\alpha}S_1^T\overline{P}_i\left(\overline{C}_i + \overline{K}_i\overline{E}_i + \overline{K}_i\widetilde{E}_i\right)$$
$$\cdot w(k) + w^T(k)$$
$$\cdot \left[\left(\overline{C}_i + \overline{K}_i\overline{E}_i + \overline{K}_i\widetilde{E}_i\right)^T \overline{P}_i\left(\overline{C}_i + \overline{K}_i\overline{E}_i + \overline{K}_i\widetilde{E}_i\right)\right.$$
$$\left. + \widetilde{\sigma}^2\left(\widetilde{K}_i\overline{E}_i + \widetilde{K}_i\widetilde{E}_i\right)^T \overline{P}_i\left(\widetilde{K}_i\overline{E}_i + \widetilde{K}_i\widetilde{E}_i\right)\right] w(k), \tag{28}$$

$$\mathbb{E}\{V_2(\eta(k+1), k+1, r(k+1)) \mid \eta(k), r(k) = i\}$$
$$- V_2(\eta(k), k, i) \le \left[(1-\underline{\delta})(d_M - d_m) + 1\right]\eta^T(k) \tag{29}$$
$$\cdot Q\eta(k) - \eta^T(k-d_i)Q\eta(k-d_i).$$

From Assumption 1 and Lemma 5, it can be obtained that

$$\begin{bmatrix} x(k) \\ h(x(k)) \end{bmatrix}^T \begin{bmatrix} XL_1 & -XL_2 \\ -XL_2 & X \end{bmatrix} \begin{bmatrix} x(k) \\ h(x(k)) \end{bmatrix} \le 0, \tag{30}$$

$$\begin{bmatrix} e(k) \\ \widetilde{h}(e(k)) \end{bmatrix}^T \begin{bmatrix} YL_1 & -YL_2 \\ -YL_2 & Y \end{bmatrix} \begin{bmatrix} e(k) \\ \widetilde{h}(e(k)) \end{bmatrix} \le 0. \tag{31}$$

Combining (30) and (31), we get

$$\begin{bmatrix} \eta(k) \\ h(\eta(k)) \end{bmatrix}^T \begin{bmatrix} \Lambda_{11} & \Lambda_{21} \\ \Lambda_{21} & \overline{X} \end{bmatrix} \begin{bmatrix} \eta(k) \\ h(\eta(k)) \end{bmatrix} \le 0. \tag{32}$$

According to (28)–(32), we have

$$\mathbb{E}\{V(\eta(k+1), k+1, r(k+1)) \mid \eta(k), r(k)$$
$$= i,\ w(k) = 0\} - V(\eta(k), k, i \mid w(k) = 0)$$
$$\le \eta^T(k)$$
$$\cdot \left[\left(A_i + \overline{K}_i\widetilde{D}_i + \overline{K}_i\overline{D}_i\right)^T \overline{P}_i\left(A_i + \overline{K}_i\widetilde{D}_i + \overline{K}_i\overline{D}_i\right)\right.$$
$$\left. + \widetilde{\sigma}^2\left(\widetilde{K}_i\left(\widetilde{D}_i + \overline{D}_i\right)\right)^T \overline{P}_i\left(\widetilde{K}_i\left(\widetilde{D}_i + \overline{D}_i\right)\right)\right]\eta(k)$$
$$- \eta^T(k)P_i\eta(k) + 2\eta^T(k)\left(A_i + \overline{K}_i\widetilde{D}_i + \overline{K}_i\overline{D}_i\right)^T$$
$$\cdot \overline{P}_i\overline{B}_i\eta(k-d_i) + 2\eta^T(k)\left(A_i + \overline{K}_i\widetilde{D}_i + \overline{K}_i\overline{D}_i\right)^T$$
$$\cdot \overline{P}_i\overline{\alpha}S_1h(\eta(k)) + \eta^T(k-d_i)\overline{B}_i^T\overline{P}_i\overline{B}_i\eta(k-d_i)$$
$$+ 2\eta^T(k-d_i)\overline{B}_i^T\overline{P}_i\overline{\alpha}S_1h(\eta(k)) + h^T(\eta(k))$$
$$\cdot \left[\overline{\alpha}^2S_1^T\overline{P}_iS_1 + \widetilde{\alpha}^2S_2^T\overline{P}_iS_2\right]h(\eta(k)) - \begin{bmatrix} \eta(k) \\ h(\eta(k)) \end{bmatrix}^T$$
$$\cdot \begin{bmatrix} \Lambda_{11} & \Lambda_{21} \\ \Lambda_{21} & \overline{X} \end{bmatrix} \begin{bmatrix} \eta(k) \\ h(\eta(k)) \end{bmatrix} = \xi^T(k,i)\widetilde{\Pi}\xi(k,i), \tag{33}$$

where

$$\widetilde{\Pi} = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} \\ * & \Pi_{22} & \Pi_{23} \\ * & * & \Pi_{33} \end{bmatrix}. \tag{34}$$

By applying Lemma 3 to (24), it can be deduced that $\widetilde{\Pi} < 0$ $(i \in S)$; then we have the conclusion that system (14) with $w(k) = 0$ is stochastically stable.

Then we proceed to analyze the H_∞ performance for system (14). Construct a cost function as follows:

$$J(n) = \sum_{k=0}^{n}\mathbb{E}\left[\|\widetilde{z}(k)\|^2 - \gamma^2\|w(k)\|^2\right]. \tag{35}$$

Combining (28), (29), and (33), we can acquire that

$$J(n) = \sum_{k=0}^{n}\mathbb{E}\left[\|\widetilde{z}(k)\|^2 - \gamma^2\|w(k)\|^2 + \Delta V(k)\right]$$
$$- V(n+1) = \zeta^T(k)\Pi\zeta(k), \tag{36}$$

where $\zeta(k) = \left[\xi^T(k)\ \ \omega^T(k)\right]^T$. According to (24), we know that $J(n) < 0$. Letting $n \to \infty$, we have

$$\sum_{k=0}^{\infty}\mathbb{E}\|\widetilde{z}(k)\|^2 \le \gamma^2\sum_{k=0}^{\infty}\|\omega(k)\|^2, \tag{37}$$

which completes the proof. □

Now we will deal with the filter design problem. From Theorem 6, the following results are derived.

Theorem 7. *There exists a nonfragile H_∞ filter (11) such that the augmented system (14) is stochastically stable if there exist two sets of matrices $P_{1_i} > 0$, X_i, positive constant scalars $\varepsilon_{1i}, \varepsilon_{2i}, \varepsilon_{3i}$ $(i \in S)$, matrix $Q > 0$, and diagonal matrices X and Y satisfying*

$$\widetilde{\Phi} = \begin{bmatrix} \Phi & * \\ \widetilde{\Phi}_{21} & \widetilde{\Phi}_{22} \end{bmatrix} < 0, \tag{38}$$

where

$$
\begin{aligned}
\Phi &= \begin{bmatrix} \Gamma & * \\ \Phi_{21} & \Phi_{22} \end{bmatrix}, \\
\Gamma &= \begin{bmatrix} -P_i - \Lambda_{11} + \overline{H}_i^T \overline{H}_i^T + \left(\left(1-\underline{\delta}\right)\left(d_M - d_m\right)+1\right)Q & 0 & -\Lambda_{21} & 0 \\ 0 & -Q & 0 & 0 \\ -\Lambda_{21} & 0 & -\overline{X} & 0 \\ 0 & 0 & 0 & -\gamma^2 I \end{bmatrix}, \\
\Phi_{21} &= \begin{bmatrix} \overline{P}_i\overline{A}_i + \mathcal{X}_i\overline{D}_i & \overline{P}_i\overline{B}_i & \overline{P}_i\overline{\alpha}S_1 & \overline{P}_i\overline{C}_i + \mathcal{X}_i\overline{E}_i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \widetilde{\alpha}\overline{P}_i S_2 & 0 \end{bmatrix}, \\
\Phi_{22} &= \operatorname{diag}\left\{-\overline{P}_i, -\overline{P}_i, -\overline{P}_i\right\}, \\
\widetilde{\Phi}_{21} &= \begin{bmatrix} 0 & 0 & 0 & \varepsilon_{1i}V_1^T & \varepsilon_{2i}V_2^T & \varepsilon_{3i}V_3^T \\ W_1^T & W_2^T & W_3^T & 0 & 0 & 0 \end{bmatrix}^T, \\
W_1 &= \begin{bmatrix} \mathcal{X}_i^T & 0 & 0 \end{bmatrix}, \\
\mathcal{X}_i &= \operatorname{diag}\left\{X_i, X_i\right\}, \\
\widetilde{\Phi}_{22} &= \operatorname{diag}\left\{-\varepsilon_1 I, -\varepsilon_2 I, -\varepsilon_3 I, -\varepsilon_1 I, -\varepsilon_2 I, -\varepsilon_3 I\right\}, \\
W_2 &= W_3 = \begin{bmatrix} \overline{\sigma}\widehat{M}^T\mathcal{X}_i^T & \widetilde{\sigma}\widehat{M}^T\mathcal{X}_i^T & 0 \end{bmatrix}, \\
V_1 &= V_2 = \begin{bmatrix} \widehat{D}_i & 0 & 0 & \widehat{E}_i \end{bmatrix}, \\
V_3 &= \begin{bmatrix} \widehat{N}\overline{D}_i & 0 & 0 & \widehat{N}\overline{E}_i \end{bmatrix}.
\end{aligned} \tag{39}
$$

Furthermore, the gain of the filter is given by $K_i = P_{1_i}^{-1}X_i$ $(i \in S)$.

Proof. From Theorem 6, it is derived that

$$\Pi = \Gamma + \Gamma_1^T\overline{P}_i\Gamma_1 + \Gamma_2^T\overline{P}_i\Gamma_2 + \Gamma_3^T\overline{P}_i\Gamma_3 < 0, \tag{40}$$

where

$$
\begin{aligned}
\Gamma_1 &= \begin{bmatrix} \overline{A}_i + \overline{K}_i\overline{D}_i & \overline{B}_i + \overline{K}_i\widetilde{D}_i & \overline{\alpha}S_1 & \overline{C}_i + \overline{K}_i\overline{E}_i + \overline{K}_i\widetilde{E}_i \end{bmatrix}, \\
\Gamma_2 &= \begin{bmatrix} \widetilde{\sigma}\widetilde{K}_i\left(\overline{D}_i + \widetilde{D}_i\right) & 0 & 0 & \widetilde{\sigma}\left(\widetilde{K}_i\overline{E}_i + \widetilde{K}_i\widetilde{E}_i\right) \end{bmatrix}, \\
\Gamma_3 &= \begin{bmatrix} 0 & 0 & \widetilde{\alpha}S_2 & 0 \end{bmatrix}.
\end{aligned} \tag{41}
$$

Then, using the Schur complement Lemma to (40) and splitting the obtained inequality, we have

$$\overline{\Phi} + \Delta\Phi < 0 \tag{42}$$

with

$$
\begin{aligned}
\overline{\Phi} &= \begin{bmatrix} \Gamma & * \\ \overline{\Phi}_{21} & \Phi_{22} \end{bmatrix}, \\
\overline{\Phi}_{21} &= \begin{bmatrix} \overline{P}_i\overline{A}_i + \overline{P}_i\overline{K}_i\overline{D}_i & \overline{P}_i\overline{B}_i & \overline{P}_i\overline{\alpha}S_1 & \overline{P}_i\overline{C}_i + \overline{P}_i\widehat{K}_i\overline{E}_i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \widetilde{\alpha}\overline{P}_i S_2 & 0 \end{bmatrix}, \\
\Delta\Phi &= \begin{bmatrix} 0 & * \\ \Delta\Phi_{21} & 0 \end{bmatrix}, \\
\Delta\Phi_{21} &= \begin{bmatrix} \widehat{K}_i\widetilde{D}_i + \overline{\sigma}\widetilde{K}_i\widetilde{D}_i + \overline{\sigma}\widetilde{K}_i\overline{D}_i & 0 & 0 & \overline{\sigma}\widetilde{K}_i\overline{E}_i + \widehat{K}_i\widetilde{E}_i + \overline{\sigma}\widetilde{K}_i\widetilde{E}_i \\ \widetilde{\sigma}\widetilde{K}_i\widetilde{D}_i + \widetilde{\sigma}\widetilde{K}_i\overline{D}_i & 0 & 0 & \widetilde{\sigma}\widetilde{K}_i\overline{E}_i + \widetilde{\sigma}\widetilde{K}_i\widetilde{E}_i \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\
\widetilde{D}_i &= \widetilde{F}_1(k)\widehat{D}_i, \\
\widetilde{E}_i &= \widetilde{F}_1(k)\widehat{E}_i, \\
\widetilde{F}_1(k) &= \operatorname{diag}\left\{F_1(k), F_1(k)\right\},
\end{aligned}
$$

$$\widehat{K}_i = \operatorname{diag}\{K_i, K_i\},$$
$$\widehat{D}_i = \begin{bmatrix} 0 & 0 \\ -\overline{\Lambda} D_i & 0 \end{bmatrix},$$
$$\widehat{E}_i = \begin{bmatrix} 0 \\ -\overline{\Lambda} E_i \end{bmatrix}. \tag{43}$$

After some matrix analysis with (42), the following inequality can be derived:

$$\begin{aligned} &\overline{\Phi} + \begin{bmatrix} 0 \\ \widehat{W}_1 \end{bmatrix} \widetilde{F}_1(k) \begin{bmatrix} V_1 & 0 \end{bmatrix} \\ &\quad + \left(\begin{bmatrix} 0 \\ \widehat{W}_1 \end{bmatrix} \widetilde{F}_1(k) \begin{bmatrix} V_1 & 0 \end{bmatrix} \right)^T \\ &\quad + \begin{bmatrix} 0 \\ \widehat{W}_2 \end{bmatrix} \widetilde{F}_2(k) \widehat{N} \widetilde{F}_1(k) \begin{bmatrix} V_2 & 0 \end{bmatrix} \\ &\quad + \left(\begin{bmatrix} 0 \\ \widehat{W}_2 \end{bmatrix} \widetilde{F}_2(k) \widehat{N} \widetilde{F}_1(k) \begin{bmatrix} V_2 & 0 \end{bmatrix} \right)^T \\ &\quad + \begin{bmatrix} 0 \\ \widehat{W}_3 \end{bmatrix} \widetilde{F}_2(k) \begin{bmatrix} V_3 & 0 \end{bmatrix} \\ &\quad + \left(\begin{bmatrix} 0 \\ \widehat{W}_3 \end{bmatrix} \widetilde{F}_2(k) \begin{bmatrix} V_3 & 0 \end{bmatrix} \right)^T < 0, \end{aligned} \tag{44}$$

where

$$\begin{aligned} \widehat{M} &= \operatorname{diag}\{M, M\}, \\ \widehat{N} &= \operatorname{diag}\{N, N\}, \\ \widetilde{F}_2(k) &= \operatorname{diag}\{F_2(k), F_2(k)\}, \\ \widehat{W}_1 &= \begin{bmatrix} \widehat{K}_i^T \overline{P}_i & 0 & 0 \end{bmatrix}, \\ \widehat{W}_2 = \widehat{W}_3 &= \begin{bmatrix} \left(\overline{\sigma} \widehat{K}_i \widehat{M}\right)^T \overline{P}_i & \left(\widetilde{\sigma} \widehat{K}_i \widehat{M}\right)^T \overline{P}_i & 0 \end{bmatrix}. \end{aligned} \tag{45}$$

According to Lemmas 4 and 3, (44) holds if and only if the following inequality holds:

$$\begin{bmatrix} \overline{\Phi} & * \\ \widehat{\Phi}_{21} & \widetilde{\Phi}_{22} \end{bmatrix} < 0, \tag{46}$$

where

$$\widehat{\Phi}_{21} = \begin{bmatrix} 0 & 0 & 0 & \varepsilon_{1i} V_1^T & \varepsilon_{2i} V_2^T & \varepsilon_{3i} V_3^T \\ \widehat{W}_1^T & \widehat{W}_2^T & \widehat{W}_3^T & 0 & 0 & 0 \end{bmatrix}^T. \tag{47}$$

Letting $\mathcal{X}_i = \overline{P}_i \widehat{K}_i$ $(i \in S)$, it is convenient to see that (46) is equivalent to (38) and the proof is now complete. □

Remark 8. In the context of MJSs, different modes need to be dealt with which are decided by the probability transition matrix. Consequently, the mathematical analysis becomes more complicated when dealing with the nonlinear MJSs. For example, meanwhile in the constructing of Lyapunov function, positive definite matrix $P(r(k))$ under different modes is considered.

Remark 9. The filter parameters can be obtained by Theorem 7 which is derived on the basis of Theorem 6. Through solving the feasible solution to certain LMIs with Matlab, the desired nonfragile filter gain can be calculated explicitly. In Theorem 7, sufficient conditions are demonstrated which guarantee that the augmented system (14) is stochastically stable and satisfies the H_∞ performance constraint (16). It should be mentioned that the parameters of the system, the delay info, the mode switching probabilities, and matrices of the gain variations and quantization are contained in Theorem 7. It can be seen that the existence of the nonfragile H_∞ filter is based on the feasible solution to the LMI (38). A simulation example is used to check the developed method in the next section.

4. Numerical Example

In this section, an example is presented to illustrate the addressed filtering method. Consider the MJSs (2) with parameters as follows:

$$A(1) = \begin{bmatrix} 0.24 & -0.18 \\ 0.36 & 0.3 \end{bmatrix},$$
$$A(2) = \begin{bmatrix} 0.2 & -0.1 \\ 0.2 & 0.3 \end{bmatrix},$$
$$B(1) = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix},$$
$$B(2) = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix},$$
$$C(1) = \begin{bmatrix} 0.5 \\ 0.4 \end{bmatrix},$$
$$C(2) = \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix},$$
$$D(1) = \begin{bmatrix} 0.3 & 0.3 \end{bmatrix},$$
$$D(2) = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix},$$
$$E(1) = 0.2,$$
$$E(2) = 0.3,$$
$$H(1) = \begin{bmatrix} 0.3 & 0.1 \end{bmatrix},$$
$$H(2) = \begin{bmatrix} 0.2 & 0.2 \end{bmatrix},$$

$$\Xi = \begin{bmatrix} 0.6 & 0.4 \\ 0.5 & 0.5 \end{bmatrix},$$
$$M = 1,$$
$$N = 0.4,$$
$$d_m = 1,$$
$$d_M = 5,$$
$$\gamma = 0.8,$$
$$\overline{\Lambda} = 0.25,$$
$$\overline{\alpha} = 0.8,$$
$$\overline{\sigma} = 0.65,$$
$$\tilde{\sigma}^2 = 0.15. \tag{48}$$

The parameters of the quantizer are selected as $\kappa_1 = 0.25$ and $u_0^1 = 3$. Meanwhile, the nonlinear function $h(x(k))$ is chosen as

$$h_1(x(k)) = -\tanh(0.4x_1(k)),$$
$$h_2(x(k)) = 0.2\tanh(x_2(k)). \tag{49}$$

We can obtain that $L_1 = \mathrm{diag}\{0, 0\}$ and $L_2 = \mathrm{diag}\{0.2, 0.1\}$. By the LMI Toolbox in Matlab, the LMIs (38) are solved with the above parameters, and the following feasible solution is acquired:

$$P_{1_1} = \begin{bmatrix} 13.5379 & 0.0931 \\ 0.0931 & 0.0886 \end{bmatrix},$$
$$P_{1_2} = \begin{bmatrix} 0.1103 & 0.0894 \\ 0.0894 & 13.1531 \end{bmatrix},$$
$$Z_1 = \begin{bmatrix} 0.0299 \\ 0.0461 \end{bmatrix},$$
$$Z_2 = \begin{bmatrix} 0.0730 \\ 0.0799 \end{bmatrix},$$
$$Q = \begin{bmatrix} 0.0118 & 0.0108 & -0.0028 & 0.0028 \\ 0.0108 & 0.0228 & 0.0024 & 0.0009 \\ -0.0028 & 0.0024 & 0.0119 & 0.0084 \\ 0.0028 & 0.0009 & 0.0084 & 0.0210 \end{bmatrix},$$
$$X_1 = 16.8313,$$
$$X_2 = 17.7341,$$

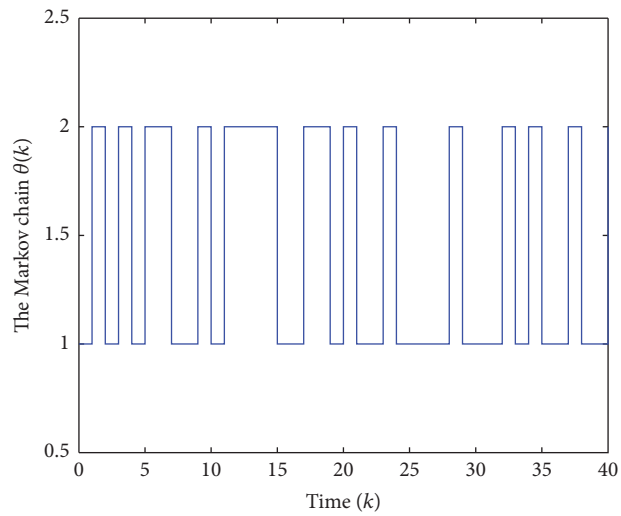

Figure 1: Modes evolution.

$$\varepsilon_{11} = 0.7779,$$
$$\varepsilon_{21} = 0.7112,$$
$$\varepsilon_{31} = 0.5948,$$
$$\varepsilon_{12} = 2.3668,$$
$$\varepsilon_{22} = 1.7724,$$
$$\varepsilon_{32} = 1.1124. \tag{50}$$

According to Theorem 7, (11) is the desired filter of the MJSs (2) with the aforementioned parameters, and the gain matrix of the filter is figured out as

$$K_1 = \begin{bmatrix} -0.0014 \\ 0.5223 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} 0.6604 \\ 0.0016 \end{bmatrix}. \tag{51}$$

Then, we will present a simulation to further testify the stability of system (14) as well as the H_∞ performance of the filter (11). Select the noise signal as $\omega(k) = 5\cos(0.5k)\exp(-0.2k)$, and set $x(0) = [0.2 \;\; 0.3]^T$, $\hat{x}(0) = [0.5 \;\; 0.1]^T$. Figure 1 plots the evolution of the system modes. Figure 2 shows the real measurement signal with quantization and ideal measurement outputs. Figures 3 and 4 depict the states of the MJSs $(x_1(k), x_2(k))$ and the filter $(\hat{x}_1(k), \hat{x}_2(k))$. Figure 5 represents the estimation error $\tilde{z}(k)$, and we can confirm that the nonfragile H_∞ filter works effectively with a relatively fine performance.

Different from the filtering method presented in [27, 28], the established method here can be applied to the complex networks with time delays, quantization, and RONs and can tolerate a degree of stochastic filter gain variation. The stochastic stability and H_∞ performance of the filtering error dynamics can be satisfied simultaneously.

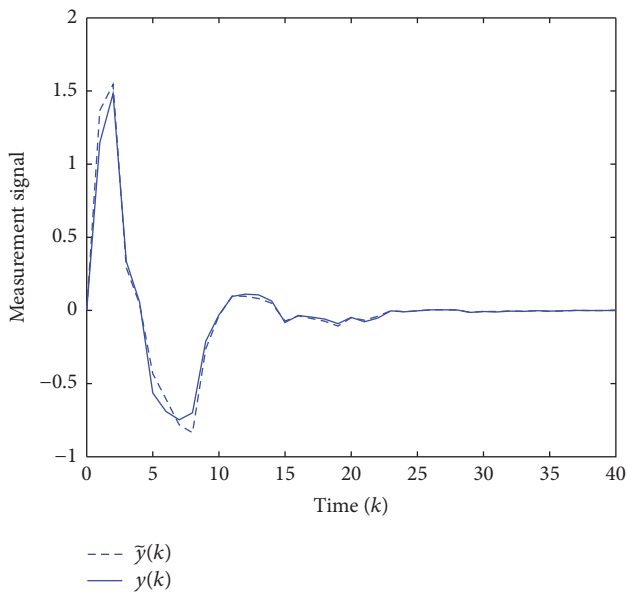

Figure 2: Real and ideal measurement outputs.

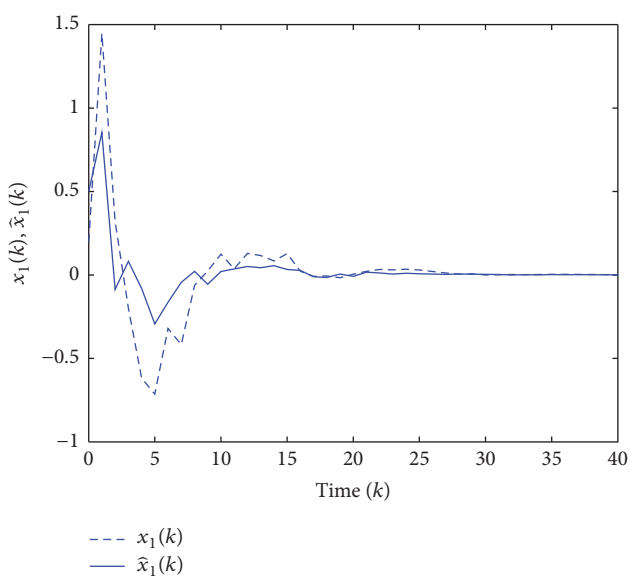

Figure 3: $x_1(k)$ and its estimate $\hat{x}_1(k)$.

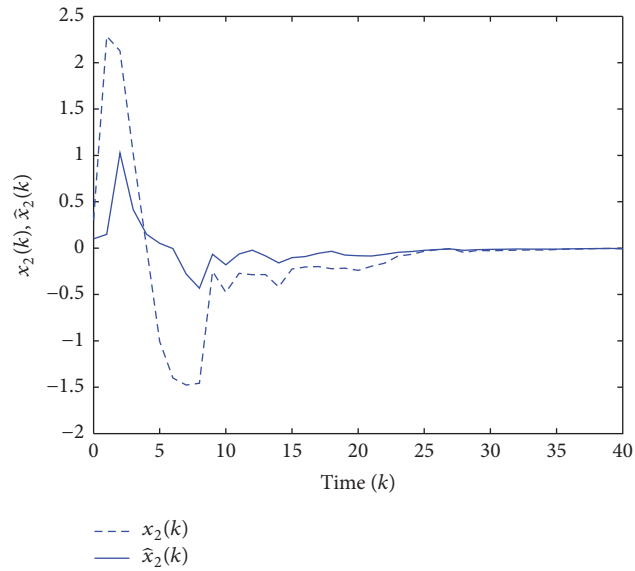

Figure 4: $x_2(k)$ and its estimate $\hat{x}_2(k)$.

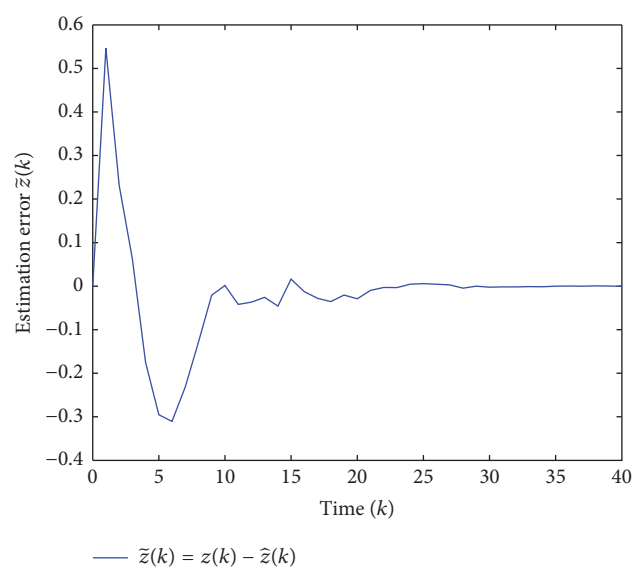

Figure 5: Estimation error $\tilde{z}(k)$.

5. Conclusions

In this paper, the nonfragile H_∞ filtering method has been proposed for MJSs with mode-dependent time delays, measurement quantization, RONs, and gain variations. Lyapunov stability theory and stochastic analysis theory associated with LMIs technique have been adopted in the analysis process. Sufficient conditions have been derived for obtaining the nonfragile filter which can guarantee the stochastic stability and H_∞ performance of the filtering error dynamics. Via solving the feasible solution to certain LMIs, the gain of the desired nonfragile filter can be calculated. The numerical simulation has verified the performance of the nonfragile filter. The present results could be extended to complex networks [11], fuzzy systems [12], and sensor networks [29], and so forth.

Competing Interests

The authors declare that they have no competing interests.

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