Research Article

Generalized Minimum Variance Control for MDOF Structures under Earthquake Excitation

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Control of a multi-degree-of-freedom structural system under earthquake excitation is investigated in this paper. The control approach based on the Generalized Minimum Variance (GMV) algorithm is developed and presented. Our approach is a generalization to multivariable systems of the GMV strategy designed initially for single-input-single-output (SISO) systems. Kanai-Tajimi and Clough-Penzien models are used to generate the seismic excitations. Those models are calculated using the specific soil parameters. Simulation tests using a 3DOF structure are performed and show the effectiveness of the control method.

1. Introduction

During the last decades, there have been many attempts in the field of structural control to develop new algorithms according to the increasing size and flexibility of building structures. This is done especially for multi-degree-of-freedom (MDOF) structures subjected to strong vibrations. A MDOF structure can be considered as a large-scale system with interconnected subsystems, which are story units. Therefore, control techniques initially designed for SDOF structures cannot be directly applied to MDOF structures. The problem of interconnection between story units and the structural properties of the building are considered as a whole system and must be addressed. The interconnection effects can also be introduced in control law formulation or considered as external disturbances to be compensated by control actions. Another problem encountered in MDOF structural control is the number and location of actuators. In fact it seems to be more efficient to place an actuator in each story unit, but the cost of implementation may be very expensive.

Several structural models have been investigated in the literature for an active control perspective. Indeed, many researchers have used an active control strategy based on state-space model to develop and design a controller for structural vibration [1–6]. Nonlinear model has been used and presented for Stochastic Optimal Control of Structural

Systems [1, 7]. Saaed Alqado et al. [8] have developed five methods for structural identification approach, specifically ARX, ARMAX, BJ, OE, and state-space models, and have implemented them for the identification process. Furthermore, the paper shows that the ARMAX and the output error models have indicated an excellent performance to predict the mathematical models of vibration's propagation in the building. Also, it has been shown by Guenfaf and Azira [9] that the ARMAX model can give an interesting representation of the system for a digital control perspective. In this case the system is characterized as stochastic process described by a linear model subjected to external disturbances.

Vigorous researches on structural control have given more interesting solutions [10–12]; some of them have already been adopted in actual building structures. Guenfaf et al. [13] have presented GMV algorithm based on ARMAX model of SDOF structure with introducing Kanai-Tajimi and Clough-Penzien seismic excitation within the structural model. The results show that the relative displacement of the structure and the control effort are significantly reduced. Lei et al. [14] have proposed an algorithm for the decentralized structural control of tall shear-type buildings under unknown earthquake excitation. A decentralized control algorithm based on the instantaneous optimal control scheme has been developed with limited measurements of structural absolute acceleration responses. The interconnection effect

between adjacent substructures has been treated as “additional unknown disturbances” at substructural interfaces to each substructure. However, the control algorithm is based on the minimization of some cost functions such as linear quadratic regulator which requires a solution of the Riccati equation that is subjected to a boundary condition at the terminal time. That leads to a suboptimal solution. So, in such perturbed systems, the controller performances and robustness may be weak [15].

The purpose of this paper is to develop a control strategy applied to an earthquake excited 3DOF structure. This algorithm attempts to minimize a generalized cost function including variance of both output and control effort. For this purpose a specific model of the system to be controlled and its environment will be developed.

The GMV algorithm has been widely studied in the literature. It was introduced by Clarke [16, 17] as an extended version of the Minimum Variance (MV) algorithm initially developed by Åström [18].

The paper is organized as follows. Section 2 deals with the development of the dynamic model of the structure, while Section 3 deals with presentation of the MIMO ARMAX model. The GMV algorithm is introduced in Section 4. The seismic excitation models are presented in Section 5. In Section 6, simulation results show the effectiveness of the developed algorithms. Finally, some conclusions are given.

2. Dynamical Model of a Multi-Degree-of-Freedom Structure

The purpose of this section is to formulate dynamical equations of motion of a multi-degree-of-freedom structure under seismic excitation. Figure 1(a) is a schematic representation of the structure. It is a multistory building with active tendon controllers in each story unit. In order to establish the dynamical model, the following assumptions are considered [19]:

- (1) Each story is supposed to be a lumped mass in the girder.
- (2) The two vertical axes between two adjacent floors are weightless and inextensible in the vertical direction.

Figure 1(b) is a representation of the dynamic force equilibration at i th story unit, which can be written as

$$\begin{aligned} f_{I_i}(t) + f_{A_i}(t) + f_{E_i}(t) - f_{A_{i+1}}(t) - f_{E_{i+1}}(t) \\ = u_i(t) - u_{i+1}(t), \end{aligned} \quad (1)$$

where

$$\begin{aligned} f_{I_i}(t) \text{ is the inertial force of the mass } m_i \\ f_{I_i}(t) = m_i \ddot{x}_i^t(t), \end{aligned} \quad (2)$$

$$\begin{aligned} f_{A_i}(t) \text{ is the damping force of } i\text{th story} \\ f_{A_i}(t) = c_i (\dot{x}_i(t) - \dot{x}_{i-1}(t)), \end{aligned} \quad (3)$$

$f_{E_i}(t)$ is the elastic force of i th story

$$f_{E_i}(t) = k_i (x_i(t) - x_{i-1}(t)), \quad (4)$$

where relative displacement is $x_i(t)$.

$u_i(t)$ is the control force from the controller installed between $(i-1)$ th story and i th story.

Absolute displacement $x_i^t(t)$ is defined as

$$x_i^t(t) = x_i(t) + x_g(t), \quad (5)$$

where $x_g(t)$ is the ground motion.

i th story unit is characterized by its characteristic parameters. In this system, it is assumed that the structural mass, m_i , and the elastic stiffness, k_i , have been concentrated in floors and columns, respectively. Internal viscous damping, c_i , is also a parameter that describes the structural behaviour.

Substituting (2), (3), (4), and (5) into (1), we obtain

$$\begin{aligned} m_i (\ddot{x}_i + \ddot{x}_g) + c_i (\dot{x}_i - \dot{x}_{i-1}) + k_i (x_i - x_{i-1}) \\ - c_{i+1} (\dot{x}_{i+1} - \dot{x}_i) - k_{i+1} (x_{i+1} - x_i) = u_i - u_{i+1} \end{aligned} \quad (6)$$

$i = \overline{1, n},$

where $c_{n+1} = 0, k_{n+1} = 0, x_0 = 0, x_{n+1} = 0, u_{n+1} = 0$, and $\ddot{x}_g(t)$ represents the ground acceleration.

Equation (6) can be written in matrix form as

$$M \ddot{X}(t) + C \dot{X}(t) + KX(t) = LU(t) - MI_v \ddot{x}_g(t), \quad (7)$$

where $X^T(t) = [x_1(t) \ x_2(t) \ \cdots \ x_n(t)]$ is the structural displacement vector, $U^T(t) = [u_1(t) \ u_2(t) \ \cdots \ u_n(t)]$ control vector, M is the $(n \times n)$ mass matrix of the structure, C is the $(n \times n)$ damping matrix of the structure, K is the $(n \times n)$ stiffness matrix of the structure, L is the $(n \times 1)$ matrix indicating the location of actuators, and $I_v = [1 \ 1 \ \cdots \ 1]^T$ unity vector of dimension $(n \times 1)$.

3. ARMAX Model of the Structure

We are interested in this section to derive the discrete ARMAX model of the MDOF structure. Consider the state-space equation of the MDOF structure which can be obtained from (7) by choosing $Z^T(t) = [Z_1^T(t) \ Z_2^T(t)] = [X^T(t) \ \dot{X}^T(t)]$ as a state vector:

$$\begin{aligned} \dot{Z}(t) &= AZ(t) + BU(t) + E\ddot{x}_g(t), \\ Y(t) &= DZ(t), \end{aligned} \quad (8)$$

where $A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}$ of dimension $(2n \times 2n)$, $B = \begin{bmatrix} 0 \\ M^{-1}L \end{bmatrix}$ of dimension $(2n \times n)$, $E = \begin{bmatrix} 0 \\ I_v \end{bmatrix}$ of dimension $(2n \times 1)$, $D = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$ of dimension $(2n \times 2n)$, I identity matrix of

FIGURE 1: Schematic representation of a multi-degree-of-freedom structure under seismic excitation. (a) Motion of the structure; (b) forces equilibrium of i th story unit.

dimension $(n \times n)$, $\underline{0}$ null matrix of dimension $(n \times n)$, and $\underline{0}$ null vector of dimension $(n \times 1)$.

The digital control of the system needs the knowledge of the discrete representation model. Thus, the continuous model can be written in the following form [20–22]:

$$\begin{aligned} & \begin{bmatrix} A_{11}(q^{-1}) & \cdots & A_{1n}(q^{-1}) \\ \vdots & \ddots & \vdots \\ A_{n1}(q^{-1}) & \cdots & A_{nn}(q^{-1}) \end{bmatrix} X(t) \\ &= \begin{bmatrix} B_{11}(q^{-1}) & \cdots & B_{1n}(q^{-1}) \\ \vdots & \ddots & \vdots \\ B_{n1}(q^{-1}) & \cdots & B_{nn}(q^{-1}) \end{bmatrix} U(t) \\ &+ \begin{bmatrix} C_{11}(q^{-1}) & \cdots & C_{1n}(q^{-1}) \\ \vdots & \ddots & \vdots \\ C_{n1}(q^{-1}) & \cdots & C_{nn}(q^{-1}) \end{bmatrix} \underline{e}(t), \end{aligned} \quad (9)$$

where $X(t) = [x_1(t) \cdots x_n(t)]^T$ is the $n \times 1$ output vector, $U(t) = [u_1(t) \cdots u_n(t)]^T$ is the $n \times 1$ input vector, $\underline{e}(t) = [e_1(t) \cdots e_n(t)]^T$ is the $n \times 1$ zero-mean white noise vector with covariance matrix $E[e(t)e^T(t)] = r * I$, $A_{ii}(q^{-1}) = 1 + \sum_{l=1}^{n_{ii}} a_{ii,l} q^{-l}$, $A_{ij}(q^{-1}) = \sum_{l=1}^{n_{ij}} a_{ij,l} q^{-l}$, $B_{ij}(q^{-1}) = q^{-d_{ij}} \sum_{l=1}^{m_{ij}} b_{ij,l} q^{-l}$, $C_{ii}(q^{-1}) = 1 + \sum_{l=1}^{n_{ci}} c_{ii,l} q^{-l}$, $C_{ij}(q^{-1}) = \sum_{l=1}^{n_{ci}} c_{ij,l} q^{-l}$, for $i = 1, n$,

$j = \overline{1, n}$, $d_{ij} \geq 0$, q^{-1} shift operator defined as $q^{-1} x_i(t+1) = x_i(t)$.

3.1. Multivariable Model. We intend in this section to derive the complete multivariable ARMAX model of the MDOF structure without neglecting the coupling between stories. We can perform this calculation using the equation of motion of the structural system. Consider (6) which can be rewritten in the following form:

$$\begin{aligned} m_i \ddot{x}_i + (c_i + c_{i+1}) \dot{x}_i + (k_i + k_{i+1}) x_i - c_i \dot{x}_{i-1} - k_i x_{i-1} \\ - c_{i+1} \dot{x}_{i+1} - k_{i+1} x_{i+1} = u_i - u_{i+1} - m_i \ddot{x}_g. \end{aligned} \quad (10)$$

Now applying the Laplace transform to this equation,

$$\begin{aligned} & \left(s^2 + \frac{c_i + c_{i+1}}{m_i} s + \frac{k_i + k_{i+1}}{m_i} \right) X_i(s) \\ & - \left(\frac{c_i}{m_i} s + \frac{k_i}{m_i} \right) X_{i-1}(s) \\ & - \left(\frac{c_{i+1}}{m_i} s + \frac{k_{i+1}}{m_i} \right) X_{i+1}(s) \\ & = \frac{1}{m_i} U_i(s) - \frac{1}{m_i} U_{i+1}(s) - \ddot{X}_g(s). \end{aligned} \quad (11)$$

Thus, we can deduce the continuous ARMAX model given in the following:

$$\overline{A}(s) X(s) = \overline{B}(s) U(s) + \overline{C}(s) I_v \ddot{X}_g(s), \quad (12)$$

where

$$\begin{aligned}
 \bar{A}(s) &= \begin{bmatrix} \bar{A}_{11}(s) & \bar{A}_{12}(s) & 0 \\ \bar{A}_{21}(s) & \bar{A}_{22}(s) & \bar{A}_{23}(s) \\ & \ddots & \\ 0 & \bar{A}_{n(n-1)}(s) & \bar{A}_{nn}(s) \end{bmatrix}, \\
 \bar{B}(s) &= \begin{bmatrix} \bar{B}_{11}(s) & \bar{B}_{12}(s) & 0 \\ & \bar{B}_{22}(s) & \bar{B}_{23}(s) \\ & \ddots & \\ 0 & \bar{B}_{n(n-1)}(s) & \bar{B}_{nn}(s) \end{bmatrix}, \\
 \bar{C}(s) &= \begin{bmatrix} \bar{C}_{11}(s) & 0 \\ & \ddots \\ 0 & \bar{C}_{nn}(s) \end{bmatrix}, \\
 X(s) &= [X_1(s) \cdots X_n(s)]^T, \\
 U(s) &= [U_1(s) \cdots U_n(s)]^T
 \end{aligned} \tag{13}$$

with $\bar{A}_{ii}(s) = s^2 + ((c_i + c_{i+1})/m_i)s + (k_i + k_{i+1})/m_i$, $\bar{A}_{i(i-1)}(s) = -((c_i/m_i)s + k_i/m_i)$, $\bar{A}_{i(i+1)}(s) = -((c_{i+1}/m_i)s + k_{i+1}/m_i)$, and $\bar{B}_{ii}(s) = 1/m_i$, $\bar{B}_{i(i+1)}(s) = -1/m_i$, $\bar{C}_{ii}(s) = -1$.

To obtain the discrete ARMAX model, we use (11) written as

$$\begin{aligned}
 X_i(s) &+ \frac{\bar{A}_{i(i-1)}(s)}{\bar{A}_{ii}(s)} X_{i-1}(s) + \frac{\bar{A}_{i(i+1)}(s)}{\bar{A}_{ii}(s)} X_{i+1}(s) \\
 &= \frac{\bar{B}_{ii}(s)}{\bar{A}_{ii}(s)} U_i(s) + \frac{\bar{B}_{i(i+1)}(s)}{\bar{A}_{ii}(s)} U_{i+1}(s) \\
 &+ \frac{\bar{C}_{ii}(s)}{\bar{A}_{ii}(s)} \ddot{X}_g(s).
 \end{aligned} \tag{14}$$

Using discretization methods, we deduce the discrete form of (14):

$$\begin{aligned}
 x_i(t) &+ \frac{\bar{A}_{i(i-1)}(q^{-1})}{\bar{A}_{ii}(q^{-1})} x_{i-1}(t) + \frac{\bar{A}_{i(i+1)}(q^{-1})}{\bar{A}_{ii}(q^{-1})} x_{i+1}(t) \\
 &= \frac{\bar{B}_{ii}(q^{-1})}{\bar{A}_{ii}(q^{-1})} u_i(t) + \frac{\bar{B}_{i(i+1)}(q^{-1})}{\bar{A}_{ii}(q^{-1})} u_{i+1}(t) \\
 &+ \frac{\bar{C}_{ii}(q^{-1})}{\bar{A}_{ii}(q^{-1})} \ddot{x}_g(t).
 \end{aligned} \tag{15}$$

The discrete multivariable ARMAX model is then obtained. It is given by

$$\bar{A}(q^{-1}) X(t) = \bar{B}(q^{-1}) U(t) + \bar{C}(q^{-1}) I_v \ddot{x}_g(t), \tag{16}$$

where

$$\begin{aligned}
 \bar{A}(q^{-1}) &= \begin{bmatrix} \bar{A}_{11}(q^{-1}) & \bar{A}_{12}(q^{-1}) & 0 \\ \bar{A}_{21}(q^{-1}) & \bar{A}_{22}(q^{-1}) & \bar{A}_{23}(q^{-1}) \\ & \ddots & \\ 0 & \bar{A}_{n(n-1)}(q^{-1}) & \bar{A}_{nn}(q^{-1}) \end{bmatrix}, \\
 \bar{B}(q^{-1}) &= \begin{bmatrix} \bar{B}_{11}(q^{-1}) & \bar{B}_{12}(q^{-1}) & 0 \\ & \bar{B}_{22}(q^{-1}) & \bar{B}_{23}(q^{-1}) \\ & \ddots & \\ 0 & \bar{B}_{n(n-1)}(q^{-1}) & \bar{B}_{nn}(q^{-1}) \end{bmatrix}, \\
 \bar{C}(q^{-1}) &= \begin{bmatrix} \bar{C}_{11}(q^{-1}) & 0 \\ & \ddots \\ 0 & \bar{C}_{nn}(q^{-1}) \end{bmatrix}.
 \end{aligned} \tag{17}$$

Remark 1. We can introduce here the seismic excitation model in the derivation of the multivariable ARMAX model. By considering the seismic excitation as a filtered white noise, we follow the same calculations as done in Section 3.1, to obtain the discrete multivariable ARMAX model of the structure under a specified excitation model [23].

4. Generalized Minimum Variance Control Algorithm

The Generalized Minimum Variance (GMV) algorithm was introduced by Clarke [16, 17] to control nonminimum phase systems. It is an extension of the Minimum Variance algorithm [18] which, by choosing a certain performance criterion and a certain model of the controlled system and its perturbations, attempts to minimize the variance of the output. A generalization of the GMV algorithm for multi-input-multi-output (MIMO) systems has been proposed in the context of self-tuning control [24–26]. The controlled system in this case is assumed to be described by a linear vector difference equation including a moving average of white noise.

A Generalized Minimum Variance strategy for multivariable systems will be now presented. Consider the system described by the linear vector difference equation (multivariable ARMAX model):

$$A(q^{-1}) y(t) = q^{-d} B(q^{-1}) u(t) + C(q^{-1}) e(t), \tag{18}$$

where $y(t)$ $n \times 1$ output vector, $u(t)$ $n \times 1$ input vector, $e(t)$ $n \times 1$ zero-mean white noise vector with covariance $E[e(t)e^T(t)] = r$, $d \geq 0$ is the smallest time delay of the system [20], $A(q^{-1}) = I + A_1 q^{-1} + \cdots + A_{n_a} q^{-n_a}$, $B(q^{-1}) = B_1 q^{-1} + \cdots + B_{n_b} q^{-n_b}$,

B_1 is nonsingular, and $C(q^{-1}) = I + C_1 q^{-1} + \dots + C_{n_c} q^{-n_c}$, where

$$\begin{aligned} A_i &= \begin{bmatrix} A_{11}(q^{-1}) & \dots & A_{1n}(q^{-1}) \\ \vdots & \ddots & \vdots \\ A_{n1}(q^{-1}) & \dots & A_{nn}(q^{-1}) \end{bmatrix}, \\ B_i &= \begin{bmatrix} B_{11}(q^{-1}) & \dots & B_{1n}(q^{-1}) \\ \vdots & \ddots & \vdots \\ B_{n1}(q^{-1}) & \dots & B_{nn}(q^{-1}) \end{bmatrix}, \\ C_i &= \begin{bmatrix} C_{11}(q^{-1}) & \dots & C_{1n}(q^{-1}) \\ \vdots & \ddots & \vdots \\ C_{n1}(q^{-1}) & \dots & C_{nn}(q^{-1}) \end{bmatrix}. \end{aligned} \quad (19)$$

We can summarize the basic assumptions on the system in the following:

The number of outputs is equal to the number of inputs.

B_1 is nonsingular.

The criterion to be minimized is

$$\begin{aligned} J &= E \left[\left\| P\underline{y}(t+d+1) - R_w \underline{w}(t+d+1) \right\|^2 \right. \\ &\quad \left. + \left\| Q'_m \underline{u}(t) \right\|^2 \right], \end{aligned} \quad (20)$$

where E is expected value, $\underline{w}(t+d+1)$ is the $n \times 1$ vector defining the reference signal, P , R_w , and Q'_m are weighing polynomial matrices of dimension $n \times n$ each, and $\|\cdot\|$ vectorial norm is defined as $\|V\|^2 = V^T V$.

As in the monovariate case we have first to derive the optimal predictor $\varphi^*(t+d+1)$ of $\varphi(t+d+1) = P\underline{y}(t+d+1)$, since it is a future information. It is given by (see Appendix A)

$$\begin{aligned} \varphi^*(t+d+1) &= \tilde{C}^{-1}(q^{-1}) \left[\tilde{F}(q^{-1}) B^*(q^{-1}) \underline{u}(t) + \tilde{G}(q^{-1}) \underline{y}(t) \right], \end{aligned} \quad (21)$$

where $\tilde{C}$, $\tilde{F}$, and $\tilde{G}$ are polynomial matrices defined in Appendix A and $B^*(q^{-1}) = qB(q^{-1})$.

Using (18) and (21) it is shown that the control strategy is given by (see Appendix B)

$$\begin{aligned} H(q^{-1}) \underline{u}(t) &= \tilde{C}(q^{-1}) R_w(q^{-1}) \underline{w}(t+d+1) \\ &\quad - \tilde{G}(q^{-1}) \underline{y}(t), \end{aligned} \quad (22)$$

where $H(q^{-1}) = \tilde{F}(q^{-1}) B^*(q^{-1}) + \tilde{C}(q^{-1}) Q'_m(q^{-1})$.

5. Mathematical Model of Earthquake Ground Motion

The earthquake ground acceleration is modeled as a uniformly modulated nonstationary random process [19, 27]:

$$\ddot{x}_g(t) = \psi(t) \ddot{x}_s(t), \quad (23)$$

where $\psi(t)$ is a deterministic nonnegative envelope function and $\ddot{x}_s(t)$ is a stationary random process with zero mean and a Kanai-Tajimi power spectral density:

$$\phi_g(\omega) = \left[\frac{1 + 4\xi_g^2 (\omega/\omega_g)^2}{(1 - (\omega/\omega_g)^2)^2 + 4\xi_g^2 (\omega/\omega_g)^2} \right] S_0^2, \quad (24)$$

where ξ_g , ω_g are filter parameters and S_0 is the constant spectral density of the white noise. However, it can be shown that the velocity and displacement spectra, which are derived from the acceleration spectra described by (24), have strong singularities at zero frequency. These singularities can be removed by using high-pass filter, as suggested by Clough-Penzien [19].

Using such a second high-pass filter, the Kanai-Tajimi spectrum is modified as follows to obtain the Clough-Penzien spectrum:

$$\begin{aligned} \phi_c(\omega) &= \left[\frac{1 + 4\xi_g^2 (\omega/\omega_g)^2}{(1 - (\omega/\omega_g)^2)^2 + 4\xi_g^2 (\omega/\omega_g)^2} \right] \\ &\quad \cdot \left[\frac{(\omega/\omega_c)^4}{(1 - (\omega/\omega_c)^2)^2 + 4\xi_c^2 (\omega/\omega_c)^2} \right] S_0^2. \end{aligned} \quad (25)$$

A particular envelope function $\psi(t)$ given in the following will be used:

$$\psi(t) = \begin{cases} 0 & \text{for } t < 0 \\ \left(\frac{t}{t_1} \right)^2 & \text{for } 0 \leq t \leq t_1 \\ 1 & \text{for } t_1 \leq t \leq t_2 \\ \exp[-a(t-t_2)] & \text{for } t \geq t_2, \end{cases} \quad (26)$$

where t_1 , t_2 , and a are parameters that should be selected appropriately to reflect the shape and duration of the earthquake ground acceleration. Numerical values of parameters are $t_1 = 3$ s, $t_2 = 13$ s, $a = 0.26$, $\xi_g = 0.65$, $\omega_g = 19$ rad/s, $\xi_c = 0.6$, $\omega_c = 2$ rad/s, and $S_0 = 0.8 \cdot 10^{-2}$ m/s.

The Kanai-Tajimi and Clough-Penzien ground accelerations have been simulated and are presented in Figure 2.

6. Simulation Results

Simulation tests are performed using a 3DOF structure with structural parameters [28]

$$m_1 = 2100 \text{ Kg}, \quad k_1 = 1262450 \text{ N/m}, \quad \text{and } c_1 = 3675 \text{ Ns/m},$$

$$m_2 = 2100 \text{ Kg}, \quad k_2 = 2607500 \text{ N/m}, \quad \text{and } c_2 = 10500 \text{ Ns/m},$$

FIGURE 2: Simulated ground accelerations.

$m_3 = 2100 \text{ Kg}$, $k_3 = 2607500 \text{ N/m}$, and $c_3 = 10500 \text{ Ns/m}$.

An active tendon controller is installed in every story unit and the angle of incline of the tendons with respect to the floor is 25° . Thus, the control force vector from the controllers is $u/\cos 25^\circ$. Thus, we can suppose that the force is applied at the top of each story and assumed to be activated externally by an independent power supply.

To implement the multivariable GMV algorithm, a sampling period $T_s = 0.02 \text{ s}$ is used. Parameters of the multivariable ARMAX model of the structure are given in the following:

$$\begin{aligned}
 \bar{A}_1 &= \begin{bmatrix} -1,225 & -0,3059 & 0 \\ -0,2906 & -0,9905 & -0,2906 \\ 0 & -0,318 & -1,451 \end{bmatrix}, \\
 \bar{A}_2 &= \begin{bmatrix} 0,8737 & -0,1309 & 0 \\ -0,1236 & 0,8187 & -0,1236 \\ 0 & -0,1354 & 0,9048 \end{bmatrix}, \\
 \bar{B}_1 &= \begin{bmatrix} 8,568 & -8,568 & 0 \\ 0 & 8,215 & -8,215 \\ 0 & 0 & 8,842 \end{bmatrix} \cdot 10^{-8}, \\
 \bar{B}_2 &= \begin{bmatrix} 8,182 & -8,182 & 0 \\ 0 & 7,667 & -7,667 \\ 0 & 0 & 8,547 \end{bmatrix} \cdot 10^{-8}, \\
 \bar{C}_1 &= \begin{bmatrix} 0,955 & 0 & 0 \\ 0 & 0,933 & 0 \\ 0 & 0 & 0,966 \end{bmatrix}.
 \end{aligned} \tag{27}$$

Ponderation polynomials are

$$\begin{aligned}
 Q_m(q^{-1}) &= \begin{bmatrix} 2 \cdot 10^{-7} & 0 & 0 \\ 0 & 2 \cdot 10^{-7} & 0 \\ 0 & 0 & 2 \cdot 10^{-7} \end{bmatrix}, \\
 P(q^{-1}) &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} -0,5 & 0 & 0 \\ 0 & -0,5 & 0 \\ 0 & 0 & -0,5 \end{bmatrix} q^{-1} \\
 &\cdot R_w = I.
 \end{aligned} \tag{28}$$

$P(q^{-1})$ is the output signal filter transfer function given with $P(1)$ set to unity to ensure offset-free control. For process control, if the process output has a large overshoot to set-point and load changes, the transfer function $P(q^{-1})$ can be used to penalize this overshoot. The output signal filter influences both the reference tracking and the disturbance rejection properties. When it is applied, the control can be slower, the overshoots can be damped, and the changes in the manipulated variable can be suppressed.

R_w is the reference weighting filter. Higher weighting leads to faster control (with more overshoot) and to higher control signal increments. On the other hand, Smaller weighting leads to slower control (with less overshoot) and to smaller changes in the manipulated signals. The reference signal filter does not influence the disturbance rejection or a plant/model mismatch. When it is applied, the control can be slower, overshoots can be damped, and the changes in the manipulated variable can be suppressed.

Q_m is the weighting of the control increments. Higher weighting leads to slower control (with less overshoot) and to smaller changes in the manipulated variable. However, smaller weighting leads to faster control (with more overshoot) and to bigger changes in the manipulated variable.

FIGURE 3: Structural response to Kanai-Tajimi ground acceleration. (a) 1st story, (b) 2nd story, and (c) 3rd story.

Table 1 gives the output variance for each story.

Responses of the structure to Kanai-Tajimi and Clough-Penzien excitation models are shown in Figures 3 and 4, respectively.

Generalized Minimum of Variance (GMV) control strategy for MDOF structure using Kanai-Tajimi and Clough-Penzien seismic models is presented in Figures 3 and 4. GMV algorithm is based on the minimization of a quadratic cost function, and the solution of the Diophantine equation is required. We compare the displacement responses without control and the ones controlled by the GMV algorithm. The control parameters are chosen to give the optimal performances according to the effect of each parameter discussed ($P(q^{-1})$, R_w , and Q_m). The results show a considerable reduction of the structural vibration under the applied seismic load.

TABLE 1: Output variance.

	Kanai-Tajimi excitation model	
	No control	GMV control
1st story	$8,4792 \cdot 10^{-7}$	$1,4868 \cdot 10^{-8}$
2nd story	$1,5669 \cdot 10^{-6}$	$1,4473 \cdot 10^{-8}$
3rd story	$2,0394 \cdot 10^{-6}$	$7,9022 \cdot 10^{-9}$
	Clough-Penzien excitation model	
	No control	GMV control
1st story	$8,5882 \cdot 10^{-7}$	$1,4698 \cdot 10^{-8}$
2nd story	$1,5872 \cdot 10^{-6}$	$1,4287 \cdot 10^{-8}$
3rd story	$2,066 \cdot 10^{-6}$	$7,7709 \cdot 10^{-9}$

The proposed control strategy has proved the considerable efficiency for the mitigation of the structural response under

FIGURE 4: Structural response to Clough-Penzien ground acceleration. (a) 1st story, (b) 2nd story, and (c) 3rd story.

seismic ground motion. The ARMAX model presents an interesting form to formulate a MDOF structural model for a control perspective in the earthquake engineering problem.

In the last decade, several works have shown the effectiveness of the active control strategies in the earthquake engineering problem. Guenfaf and Allaoua [29] have investigated a flux oriented control of Active Mass Damper (AMD) induction motor for buildings under seismic ground motion. The LQR control algorithm has been used for displacement reduction of the structure. The simulation results show that the torque supplied by the induction motor pursues its reference provided by the LQR control algorithm. Significant reductions of 65% and 78%, respectively, in the roof acceleration and roof displacement have been reached. Xu et al. [30] have proposed a real-time Active Mass Damper (AMD) subsystem testing method, which has been applied

to verify the performance of an innovative AMD control system driven by a linear motor. Yamamoto and Sone [31] have presented the feasibility, stability, and reliability of the integrated AMD subsystem, including hardware and software. The main purpose of an AMD that was installed in a high-rise building in Tokyo is to maintain human comfort in two horizontal directions under the strong earthquakes. The control performance of the AMD was confirmed. The AMD controlled the building acceleration such that it was 60% of its maximum derivation during the Tohoku earthquake 2011. Ikeda [32] has presented the practical applications and verification of the active and the semiactive control strategy for the structural control problem. Practical applications of active and semiactive vibration control of buildings in Japan are reported, and the installation tendency is described in relation to research phases of structural control.

Control verification methods are introduced to link structural control with monitoring and system identification. The state of the art shows that the use of structural control and its related technologies have been expanding in the last two decades since the late Professor Kobori suggested the importance of active structural control in the civil engineering field in the mid-1980s.

7. Conclusions

This paper investigated GMV control technique applied to earthquake excited MDOF structures. Our control approach was developed and presented based on the ARMAX model calculation. The multivariable GMV have demonstrated good performances and the structural responses are significantly reduced. The control strategy used in this paper is an extension to the SDOF case of the classical GMV algorithm. It is designed for the structure as a multivariable system.

The multivariable GMV algorithm is an efficient control technique with a well-established theoretical development. However, it has the disadvantage of the great amount of calculations and matrix manipulation especially in the case of tall buildings with a large number of degrees of freedom. Nevertheless, it takes into account the coupling terms between different storey buildings, and this gives more realistic results due to the nonlinear nature of this coupling terms.

Appendix

A. Derivation of the Optimal Predictor for MIMO Systems

We first introduce the polynomial matrix equation:

$$\bar{C}(q^{-1}) = A(q^{-1})F(q^{-1}) + q^{-(d+1)}G(q^{-1}), \quad (\text{A.1})$$

where

$$\begin{aligned} F(q^{-1}) &= F_0 + F_1 q^{-1} + \dots + F_d q^{-d}, \\ G(q^{-1}) &= G_0 + G_1 q^{-1} + \dots + G_{ng} q^{-ng}, \end{aligned} \quad (\text{A.2})$$

where $ng = \max[l + np - d - 1, n - 1]$ and $np = \deg[P(q^{-1})]$,

$$\bar{C}(q^{-1}) = \bar{P}(q^{-1})C(q^{-1}), \quad (\text{A.3})$$

$$\bar{A}(q^{-1})P(q^{-1}) = \bar{P}(q^{-1})A(q^{-1}), \quad (\text{A.4})$$

where $\det \bar{A}(q^{-1}) = \det A(q^{-1})$ and $\bar{A}(0) = I$.

There exist nonunique matrices $\bar{F}$ and $\bar{G}$ verifying

$$\bar{F}(q^{-1})G(q^{-1}) = \bar{G}(q^{-1})F(q^{-1}), \quad (\text{A.5})$$

where $\det \bar{F}(q^{-1}) = \det F(q^{-1})$.

We introduce the polynomial matrix $\tilde{C}$:

$$\tilde{C}(q^{-1}) = \bar{F}(q^{-1})A(q^{-1}) + q^{-(d+1)}\tilde{G}(q^{-1}). \quad (\text{A.6})$$

From (12), (A.4), (A.3), (A.5), and (A.6) we demonstrate that

$$\begin{aligned} \tilde{C}(q^{-1})P(q^{-1}) &= \tilde{F}(q^{-1})A(q^{-1}) \\ &\quad + q^{-(d+1)}\tilde{G}(q^{-1}), \end{aligned} \quad (\text{A.7})$$

$$\tilde{C}(q^{-1})F(q^{-1}) = \tilde{F}(q^{-1})C(q^{-1}),$$

where $\tilde{F}(q^{-1}) = \bar{F}(q^{-1})\bar{P}(q^{-1})$ and $\tilde{G}(q^{-1}) = \bar{G}(q^{-1})P(q^{-1})$.

Equation (18) can be written as

$$\begin{aligned} A(q^{-1})\underline{y}(t+d+1) &= B^*(q^{-1})\underline{u}(t) \\ &\quad + C(q^{-1})\underline{e}(t+d+1). \end{aligned} \quad (\text{A.8})$$

Multiplying (A.8) by $\tilde{F}$ and after some calculations using (A.7) we obtain

$$\begin{aligned} \varphi(t+d+1) &= P(q^{-1})\underline{y}(t+d+1) = \tilde{C}^{-1}(q^{-1}) \\ &\quad \cdot [\tilde{G}(q^{-1})\underline{y}(t) + \tilde{F}(q^{-1})B^*(q^{-1})\underline{u}(t)] \\ &\quad + F(q^{-1})\underline{e}(t+d+1). \end{aligned} \quad (\text{A.9})$$

We remark that $F(q^{-1})\underline{e}(t+d+1) = F_0 e(t+d+1) + F_1 e(t+d) + \dots + F_d e(t+1)$ is the future information. Thus, the optimal predictor of $\varphi(t+d+1)$ is given by

$$\begin{aligned} \varphi^*(t+d+1) &= \tilde{C}^{-1}(q^{-1}) \\ &\quad \cdot [\tilde{G}(q^{-1})\underline{y}(t) + \tilde{F}(q^{-1})B^*(q^{-1})\underline{u}(t)]. \end{aligned} \quad (\text{A.10})$$

B. Derivation of the GMV Control for MIMO Systems

Using (A.10), the criterion to be minimized (20) becomes

$$\begin{aligned} J &= E \left[\left\| \varphi^*(t+d+1) - R_w(q^{-1})\underline{w}(t+d+1) \right. \right. \\ &\quad \left. \left. + F(q^{-1})\underline{e}(t+d+1) \right\|^2 + \left\| Q'_m(q^{-1})\underline{u}(t) \right\|^2 \right]. \end{aligned} \quad (\text{B.1})$$

The term $F(q^{-1})\underline{e}(t+d+1)$ is uncorrelated with $\underline{u}(t-i)$, $\underline{w}(t-i)$, and $\underline{y}(t-i)$ for $0 \leq i \leq t$. Thus, J can be written as

$$\begin{aligned} J &= E \left[\left\| \varphi^*(t+d+1) - R_w(q^{-1})\underline{w}(t+d+1) \right\|^2 \right. \\ &\quad \left. + \left\| Q'_m(q^{-1})\underline{u}(t) \right\|^2 \right] \\ &\quad + E \left[\left\| F(q^{-1})\underline{e}(t+d+1) \right\|^2 \right]. \end{aligned} \quad (\text{B.2})$$

The minimum of J is found by forming the gradient of J with respect to $\underline{u}(t)$:

$$\begin{aligned} \frac{\partial J}{\partial \underline{u}(t)} &= 2E \left[\left(\frac{\partial \varphi^*(t+d+1)}{\partial \underline{u}(t)} \right)^T \right. \\ &\quad \cdot \left(\varphi^*(t+d+1) - R_w(q^{-1})\underline{w}(t+d+1) \right) \\ &\quad \left. + Q'_m(0)^T Q'_m(q^{-1})\underline{u}(t) \right] = 0 \end{aligned} \quad (\text{B.3})$$

or

$$\begin{aligned} \varphi^*(t+d+1) - R_w(q^{-1})\underline{w}(t+d+1) \\ + Q_m(q^{-1})\underline{u}(t) = 0, \end{aligned} \quad (\text{B.4})$$

where $Q_m(q^{-1}) = [(P_0 B_1)^T]^{-1} Q'_m(0)^T Q'_m(q^{-1})$.

Substituting (A.10) into (B.4), we obtain the control strategy:

$$\begin{aligned} H(q^{-1})\underline{u}(t) = \tilde{C}(q^{-1})R_w(q^{-1})\underline{w}(t+d+1) \\ - \tilde{G}(q^{-1})\underline{y}(t), \end{aligned} \quad (\text{B.5})$$

where $H(q^{-1}) = \tilde{F}(q^{-1})B^*(q^{-1}) + \tilde{C}(q^{-1})Q_m(q^{-1})$.

Competing Interests

The authors declare that there are no competing interests regarding the publication of this paper.

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