

Research Article

A Nonlinear Robust Controller Design for Ship Dynamic Positioning Based on L_2 -Gain Disturbance Rejection

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In ship motion control process, it is difficult to design ship controller due to the effects of environmental disturbances such as wind, waves, current, and unmodelled dynamics. In order to solve these problems, a nonlinear robust controller based on L_2 -gain disturbance rejection is proposed in this paper. To steer ships to the desired position, an error feedback control law on account of Lyapunov functions is designed. Then, to satisfy the L_2 -gain disturbance rejection, proper parameters are chosen based on the system dissipative property. In order to verify the performance of the proposed controller, the MATLAB simulation results in two situations which are without and with the effects of environmental disturbances are demonstrated.

1. Introduction

With the exploration and exploitation of ocean resource and improvement of modern technology, ship Dynamic Positioning (DP) control system is increasingly utilized on floating vessel or platform control systems. DP is defined as follows: “Only depend on the propeller can automatically keep the position of floating structure (maintaining in fixed position or presetting tracked for equipment or ship)” [1]. DP control process has experienced several stages, from Proportion Integration Differentiation (PID) control, advanced output feedback control, to nonlinear adaptive control and hybrid control, especially for hybrid DP control which automatically switched among controller set, improving the performance and maneuverability of DP ships [2].

But in ship motion control process, it is difficult to design ship controller due to effects of environmental disturbances such as wind, waves, current, and unmodelled dynamics. And among control methods of DP system, fuzzy control, backstepping control, neural network control, and hybrid control (see, e.g., [3–10]) are commonly used. Not depending on ship's accurate model, robust fuzzy control by using optimal H^∞ control technique on DP ships was designed and had good performance in disturbance rejection and fast response and had good robustness [4]. In order to eliminate the effects

of uncertainties, backstepping control method was applied to construct appropriate virtual control law. And neural network control of DP ships was utilized to solve the problem of approximate of uncertainties and unmodelled items [5]. Bateman et al. [6] developed a backstepping method improving ability of control response and disturbance rejecting performance under the low and high gain. And the article [11] designed a recursive backstepping controller to maintain ship in fixed position and the controller had good performance of response and maneuvering motion. Chen et al. [12–14] proposed the finite time tracking control method to handle the problem of external disturbances and had good performance. The hybrid control was designed to switch the control situation automatically and had good performance to reject changes of external environment [3]. All of these papers proposed kind of methods to solve the problem of environmental disturbances.

In addition, the key to these designed control methods is to ensure system stabilization. L_2 -gain disturbance rejection is especially used to reduce effects of disturbances by choosing proper control schemes and has good performance and robustness. An optimal L_2 -gain disturbance rejection control for linear system was designed based on Lyapunov-Krasovskii theories [15]. López-Martínez et al. [16] applied the problem of L_2 disturbance rejection to a twin-rotor

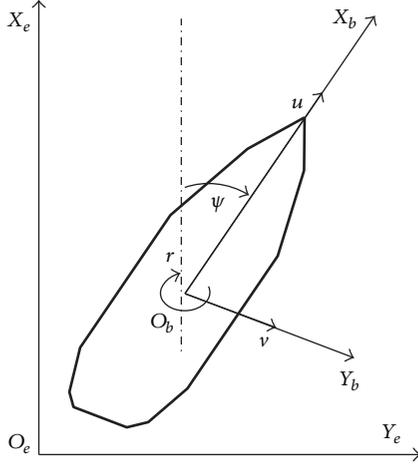


FIGURE 1: Relationship between Earth-fixed frame and Body-fixed frame.

system and designed the L_2 control law with PID control to make system stability in variable speed rotors. Paper [17] designed an uncertain switched nonlinear system to solve the problem of L_2 -gain. In order to handle the external disturbance and input constraint, Xiao et al. [18] proposed an adaptive L_2 -gain controller in the condition of attitude tracking in flexible spacecraft. Ahmed [19] proposed an L_2 -gain disturbance rejection neural controller for nonlinear system. Paper [20] designed a disturbance rejection control method in the base of the theories of L_2 -gain and port-controlled Hamiltonian system. In this paper, based on L_2 -gain disturbance rejection, the nonlinear robust controller is designed to steer the DP ships to the desired position and solve problems of uncertainties and unmodelled dynamics.

The structure of this paper is organized as follows. Section 2 states the control problem of DP ships subject to uncertain terms and disturbances. Section 3 presents the design of the nonlinear robust controller based on L_2 -gain disturbance rejection. MATLAB simulation results without and with the effects of environmental disturbances are demonstrated to verify the effectiveness of the proposed controller.

2. Problem Formulation

2.1. DP Ships' Kinematics and Dynamics. In order to analyze DP ships' kinematics, two reference coordinate systems which include Earth-fixed frame O_e - X_e - Y_e - Z_e and Body-fixed frame O_b - X_b - Y_b - Z_b , are introduced in this paper. 3-Degree-Of-Freedom (3 DOF) horizontal motion (surge, sway, and yaw) is usually researched in DP control system. And the relationship between Earth-fixed frame and Body-fixed frame is shown as Figure 1.

For the DP control system, the kinematics and dynamics in low speed condition [21] are described as follows:

$$\begin{aligned} \dot{\eta} &= R(\psi) \nu, \\ M\dot{\nu} + D\nu &= \tau_c + \Delta, \end{aligned} \quad (1)$$

where, $\eta = [x, y, \psi]^T \in \mathfrak{R}^{3 \times 1}$ is the vehicle's position vector in Earth-fixed frame. $\nu = [u, v, r]^T \in \mathfrak{R}^{3 \times 1}$ represents the velocities in Body-fixed frame. $M \in \mathfrak{R}^{3 \times 3}$ is the system inertial matrix, $D \in \mathfrak{R}^{3 \times 3}$ represents the strictly positive linear damping matrix, and $\tau_c \in \mathfrak{R}^{3 \times 1}$ is the forces and moments vector produced by thrusters. $\Delta \in \mathfrak{R}^{3 \times 1}$ is a vector of forces and moments produced by slowly varying environmental disturbances and unmodelled items. $R(\psi) \in \mathfrak{R}^{3 \times 3}$ is the transformation matrix between Earth-fixed frame and Body-fixed frame expressed by

$$R(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (2)$$

2.2. Control Objective. The DP control system is to steer the ship to maintain a fixed position or preset tracked position. The desired position vector is $\eta_d = [x_d, y_d, \psi_d]^T$. The control objective is to design a nonlinear robust control law τ and choose proper parameters which satisfy the L_2 -gain disturbance rejection constrains. Thus the position η is accurately approached to the desired position η_d .

3. Nonlinear Robust Controller Design Based on L_2 -Gain Disturbance Rejection

In this section, in order to design a backstepping nonlinear robust controller based on L_2 disturbance rejection and achieve the control object, the nonlinear robust controller is designed to steer the DP ship to the desired position and solve problems of uncertainties and unmodelled dynamics. The following dynamic equation of DP ship is derived from (1):

$$MR^{-1}(\psi) \ddot{\eta} + MR_{\dot{\text{dot}}} \dot{\eta} + DR^{-1}(\psi) \dot{\eta} = \tau_c + \Delta. \quad (3)$$

With defining the error $e \in \mathfrak{R}^{3 \times 1}$ as $e = \eta - \eta_d$, the following error dynamic equation based on (3) is written as

$$\begin{aligned} MR(\psi)^{-1} \ddot{e} &+ \left(-MR^{-1}(\psi) \dot{R}(\psi) R^{-1}(\psi) + DR^{-1}(\psi) \right) \dot{e} \\ &= MR(\psi)^{-1} \ddot{\eta}_d + \tau_c + \Delta \\ &+ \left(-MR^{-1}(\psi) \dot{R}(\psi) R^{-1}(\psi) + DR^{-1}(\psi) \right) \dot{\eta}_d. \end{aligned} \quad (4)$$

Simply, let

$$R_{\dot{\text{dot}}} = -R^{-1}(\psi) \dot{R}(\psi) R^{-1}(\psi). \quad (5)$$

Thus, the error dynamic equation (4) is rewritten as

$$\begin{aligned} MR(\psi)^{-1} \ddot{e} + \left(MR_{\dot{\text{dot}}} + DR^{-1}(\psi) \right) \dot{e} \\ = MR(\psi)^{-1} \ddot{\eta}_d + \left(MR_{\dot{\text{dot}}} + DR^{-1}(\psi) \right) \dot{\eta}_d + \tau_c + \Delta. \end{aligned} \quad (6)$$

3.1. L_2 -Gain Disturbance Rejection. L_2 -gain disturbance rejection is used to reduce the effect of disturbances by choosing proper parameters. In order to analyze the L_2 -gain disturbance rejection of a control system, the definitions of L_2 -gain and system dissipation are introduced in following part. And L_2 -gain can be determined by considering the following nonlinear system [15]:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u, \\ y &= h(x), \end{aligned} \quad (7)$$

where, $x \in \mathfrak{R}^n$ is the state vector of system; $u \in \mathfrak{R}^p$ represents the input signal; $y \in \mathfrak{R}^p$ is output vector; $f(x)$, $h(x)$, and $g(x)$ represent the given function vector or matrix.

Assumption 1. The system is a free system and has an equilibrium point, that is $f(0) = 0$ when $x = 0$. At the same time, assume $h(0) = 0$; thus for any $u \in L_2$, $y \in L_2$, there exists $\gamma \in \mathfrak{R}$ when L_2 -norm $\|H_{yu}\|$ satisfied the condition of

$$\|H_{yu}\| = \sup_{\|u\|_2 \neq 0} \frac{\|y\|_2}{\|u\|_2} \leq \gamma. \quad (8)$$

Therefore, L_2 -norm $\|H_{yu}\|$ is called L_2 -gain which is less than or equal to γ . And γ is called disturbance rejection factor.

Assumption 2. If a positive semidefinite function $V(x)$ ($V: x \rightarrow R$), $V(0) = 0$, the following inequality is established when

$$V(x(T)) - V(x(0)) \leq \int_0^T s(u(\tau), y(\tau)) d\tau. \quad (9)$$

For any $T > 0$ and any input signal $u \in R^p$, the system is called passive system when inequality (9) is established. And inequality (9) is called dissipative inequality, $s(u, y)$ is energy storage function, and $V(x)$ is the energy storage function.

Consider the following system:

$$\begin{aligned} \dot{x} &= f_1(x, y), \\ \dot{y} &= f_2(x, y) + g_1(x, y)u + g_2(x, y)w, \\ z &= h(x, y), \end{aligned} \quad (10)$$

where $x \in \mathfrak{R}^n$ is the state vector of system; $u \in \mathfrak{R}^p$ represents input signal; $y \in \mathfrak{R}^p$ is output vector; $w \in \mathfrak{R}^p$ represents disturbance input signal; $z \in \mathfrak{R}^p$ represents evaluation function with characterization of disturbance rejection performance; $f_1(*)$, $f_2(*)$, $g_1(*)$, $g_2(*)$, and $h(*)$ are smooth functions with appropriate dimension or matrix. And the dissipative inequality of system (10) is

$$\dot{V}(x, y) \leq \frac{1}{2}(\gamma^2 \|w\|^2 - \|z\|^2), \quad (11)$$

where $V(x, y)$ is energy storage function with the property of positive semidefinite. Thus the L_2 disturbance rejection problem is solved by inequality of (11).

3.2. DP Nonlinear Backstepping Controller Design. DP's nonlinear control law is designed based on backstepping theory. The control law τ is designed by error dynamic equation (6) as

$$\begin{aligned} \tau &= -MR(\psi)^{-1} \ddot{\eta}_e - (MR_{\dot{\text{dot}}} + DR^{-1}(\psi)) \dot{\eta}_e \\ &\quad - MR^{-1}(\psi) K\dot{e} - (MR_{\dot{\text{dot}}} + DR^{-1}(\psi)) Ke + u. \end{aligned} \quad (12)$$

Thus (6) can be rewritten as

$$\begin{aligned} (\ddot{e} + K\dot{e}) &= -M^{-1}R(\psi) (MR_{\dot{\text{dot}}} + DR^{-1}(\psi)) \\ &\quad * (\dot{e} + Ke) + M^{-1}R(\psi)u + M^{-1}R(\psi)\Delta. \end{aligned} \quad (13)$$

In order to design the disturbance rejection control law u , define the error varieties $x_1, x_2 \in \mathfrak{R}^{3 \times 1}$ as

$$\begin{aligned} x_1 &= e, \\ x_2 &= \dot{e} + Kx_1, \end{aligned} \quad (14)$$

where $K \in \mathfrak{R}^{3 \times 3}$ is to be designed as parameter matrix; at the same time, let

$$\begin{aligned} F &= -M^{-1}R(\psi) (MR_{\dot{\text{dot}}} + DR^{-1}(\psi)), \\ G &= M^{-1}R(\psi). \end{aligned} \quad (15)$$

Thus, combining (13), (14), and (15), the derivative of x_1 and x_2 is determined as

$$\begin{aligned} \dot{x}_1 &= -Kx_1 + x_2, \\ \dot{x}_2 &= Fx_2 + Gu + G\Delta. \end{aligned} \quad (16)$$

In order to prove the stability of subsystem x_1 in (14), Lyapunov function V_1 is defined as

$$V_1 = \frac{1}{2}x_1^T x_1. \quad (17)$$

Thus, the deviation of V_1 is determined as

$$\dot{V}_1 = x_1^T \dot{x}_1 = -x_1^T Kx_1 + x_1^T x_2. \quad (18)$$

$\dot{V}_1 = -x_1^T Kx_1 \leq 0$, when $x_2 = 0$, so the subsystem with x_1 is stabilization. And to prove the stability of x_2 , Lyapunov function V_2 is defined as

$$V_2 = \frac{1}{2}x_2^T x_2 + V_1 = \frac{1}{2}x_2^T x_2 + \frac{1}{2}x_1^T x_1. \quad (19)$$

Thus deviation of V_1 is determined as

$$\dot{V}_2 = x_1^T \dot{x}_1 + x_2^T \dot{x}_2 = -x_1^T Kx_1 + x_2^T (\dot{x}_2 + x_1). \quad (20)$$

Combine (16) and (20), so

$$\dot{V}_2 = -x_1^T Kx_1 + x_2^T (Fx_2 + Gu + G\Delta + x_1). \quad (21)$$

Design the feedback control law as

$$u = G^{-1}(-Fx_2 - x_1 - Kx_2), \quad (22)$$

where $k > 0$ is to be designed as parameter. And plug (22) into (21), so (21) is rewritten as

$$\dot{V}_2 = -x_1^T K x_1 - k x_2^T x_2 + x_2^T G \Delta. \quad (23)$$

Thus, if $\Delta = 0$

$$\dot{V}_2 = -x_1^T K x_1 - k x_2^T x_2 \leq 0. \quad (24)$$

If $\Delta \neq 0$, consider the following inequality:

$$x_2^T G \Delta \leq a \|x_2^T G\|^2 + \frac{\|\Delta\|^2}{4a} \leq a \|G\|^2 \|x_2\|^2 + \frac{\|\Delta\|^2}{4a}. \quad (25)$$

Let $k = k' + a\|G\|$, then

$$\begin{aligned} \dot{V}_2 &\leq -x_1^T K x_1 - (k' + a\|G\|) x_2^T x_2 + a\|G\|^2 x_2^2 \\ &\quad + \frac{\|\Delta\|^2}{4a} \\ &= -x_1^T K x_1 - k' x_2^2 - a\|G\|^2 x_2^2 + a\|G\|^2 x_2^2 + \frac{\|\Delta\|^2}{4a} \\ &\leq -x_1^T K x_1 - k' \|x_2\|^2 + \frac{\|\Delta\|^2}{4a}. \end{aligned} \quad (26)$$

$\dot{V}_2 \leq 0$ when the following error inequality is workable:

$$\|x_2\| > \frac{\rho}{2\sqrt{ak'}}, \quad (27)$$

where ρ is the upper bound of Δ . Thus the subsystem with x_2 is stabilization. Thus the Lyapunov stability can be proved based on the above equations from (13) to (26). And the control law is to be designed to make the system stable under L_2 disturbance rejection.

3.3. Efficiency Proof of L_2 -Gain Disturbance Rejection. The control law is derived above without considering the capacity of disturbance rejection. And L_2 -gain disturbance rejection of the robust nonlinear controller in this section is proved.

First, evaluation function z mentioned in Section 3.2 is shown as

$$z = \begin{bmatrix} h_1 & 0 \\ 0 & h_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad (28)$$

where h_1, h_2 are the weighted factors. In order to prove the efficiency of L_2 -gain disturbance rejection, the following dissipative inequality needs to be proved:

$$\dot{V}_2 \leq \frac{1}{2} (\gamma^2 \|\Delta\|^2 - \|z\|^2). \quad (29)$$

Define auxiliary function T as

$$T = 2\dot{V}_2 - \gamma^2 \|\Delta\|^2 + \|z\|^2. \quad (30)$$

Combining (29) with (30), the auxiliary function T is rewritten as

$$\begin{aligned} T &= -2x_1^T K x_1 - 2k x_2^T x_2 + 2x_2^T G \Delta - \gamma^2 \|\Delta\|^2 + \|z\|^2 \\ &\leq -x_1^T (K - h_1^T h_1) x_1 - x_2^T \left(2kI - h_2^T h_2 - \frac{1}{\gamma^2} I \right) x_2. \end{aligned} \quad (31)$$

If the parameters are designed to satisfy the following inequality:

$$\begin{aligned} K - h_1^T h_1 &\geq 0, \\ 2kI - h_2^T h_2 - \frac{1}{\gamma^2} I &\geq 0. \end{aligned} \quad (32)$$

Thus $T \leq 0$; that is,

$$2\dot{V}_2 - \gamma^2 \|\Delta\|^2 + \|z\|^2 \leq 0. \quad (33)$$

Therefore,

$$\dot{V}_2 \leq \frac{1}{2} (\gamma^2 \|\Delta\|^2 - \|z\|^2). \quad (34)$$

Parameters should satisfy inequality (32); thus the efficiency of L_2 disturbance rejection can be proved. Therefore the control law is designed as

$$\begin{aligned} \tau &= -MR^{-1}(\psi) \\ &\quad \cdot (\ddot{\eta}_e + (K + kI)(\dot{\eta} - \dot{\eta}_e) + (kK + I)(\eta - \eta_e)) \\ &\quad - (MR_{\dot{\text{dot}}} + DR^{-1}(\psi)) \\ &\quad \cdot (-\dot{\eta} - 2\dot{\eta}_e) - 2K(\eta - \eta_e). \end{aligned} \quad (35)$$

4. Simulation Results

To verify the performance of proposed robust nonlinear controller based on L_2 -gain disturbance rejection, it is assessed in the MATLAB simulation environment with a 3-DOF nonlinear model of DP ship. The desired position vector can be given as follows:

$$\eta_{d0} = \left[2, 3, 5 * \frac{\pi}{180^\circ} \right]^T. \quad (36)$$

The initial position and attitude vector is

$$\eta = [0, 0, 0]^T. \quad (37)$$

The inertia matrix and damping matrix are given as

$$\begin{aligned} M &= \begin{bmatrix} 2.769e+07 & 0 & 0 \\ 0 & 5.160e+07 & 4.836e+08 \\ 0 & 4.836e+08 & 4.510e+10 \end{bmatrix}, \\ D &= \begin{bmatrix} 8.743e+04 & 0 & 0 \\ 0 & 1.373e+05 & -3.118e+06 \\ 0 & 2.904e+06 & 7.264e+08 \end{bmatrix}. \end{aligned} \quad (38)$$

The simulating time is 100 s and the step time is 0.2 s. The desired position is a step signal and its derivate is not to be determined. In order to generate the smoothly desired path, the 2-order filter is applied as

$$\begin{aligned} \dot{\eta}_d &= \eta_d, \\ \ddot{\eta}_d &= \omega_n^2 \eta_{d0} - \omega_n^2 \eta_d - \delta |\eta_d| \eta_d - 2\zeta \omega_n \dot{\eta}_d, \end{aligned} \quad (39)$$

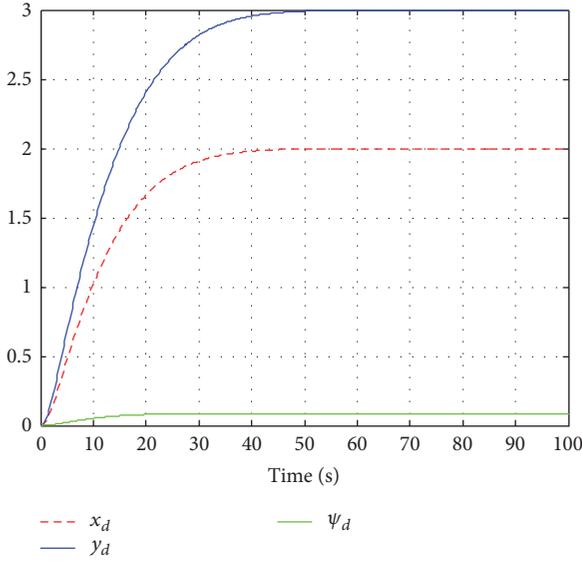


FIGURE 2: The desired position curves.

where ω_n is the natural frequency, ζ is the relative damping ratio, and δ is the designed parameter. Thus the desired position η_d is generated smoothly and the figure of the desired path is shown as Figure 2.

And to validate the effectiveness of controller, two groups of simulating results are demonstrated without and with the effects of environmental disturbances. And let the slowly varying environmental disturbances Δ be

$$\Delta = (\sin(0.02t) + \cos(0.02t)) * \text{diag}(1e6 \ 1e6 \ 1e6). \quad (40)$$

In one situation, without the effects of environmental disturbances, the outputs of ship motion curves (such as the position, position error, and control law τ) are shown as Figures 3, 4, and 5. And the parameters in the designed control law are chosen as

$$K = \text{diag}(0.1 \ 0.1 \ 0.1), \quad (41)$$

$$k = 1.8.$$

From Figures 3, 4, and 5, the desired path is well approached by the real simulating motion path. And the designed control law τ effectively steers the ship motion to the desired position though the position error changing in some upper and lower bounds.

The other situation is with the effects of environmental disturbances; the outputs of ship motion curves are demonstrated from Figures 6, 7, and 8. In this condition, the desired path is well approached by the real simulating motion path, and the designed control law τ effectively controls the ship motion to the desired position although the control law is in shock at the first 10 s. And the position error also can be changed in some acceptable ranges.

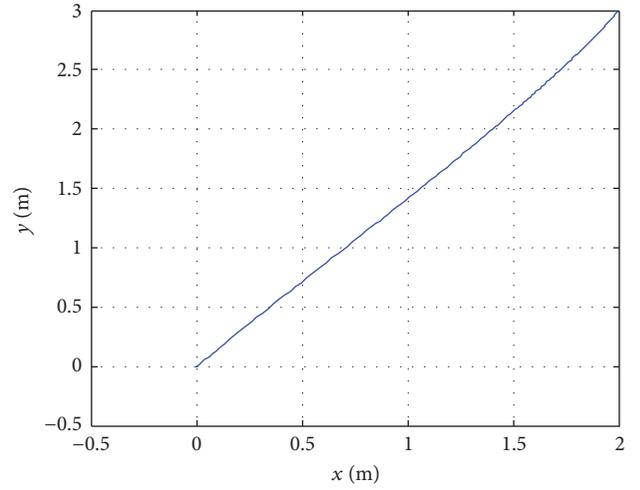


FIGURE 3: Ship horizontal motion without environmental disturbances.

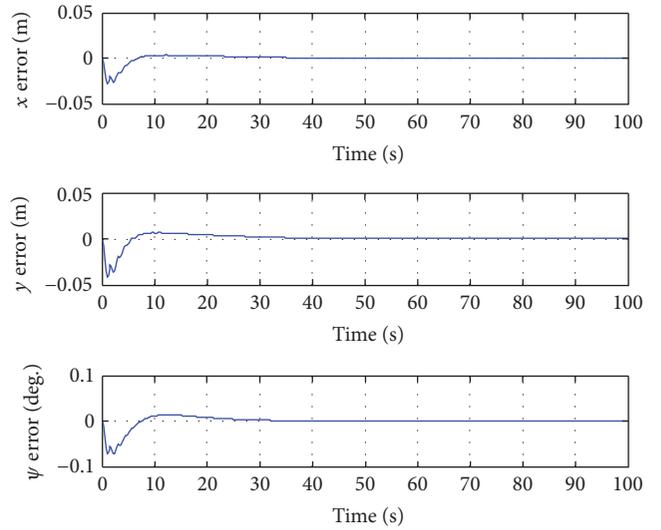


FIGURE 4: The position error of ship without environmental disturbances.

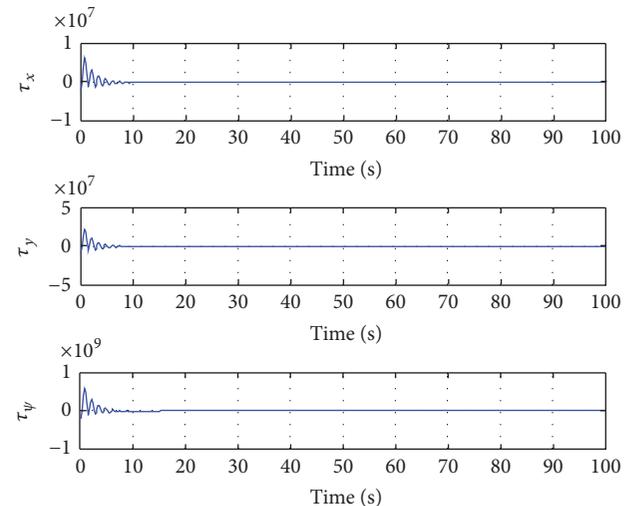


FIGURE 5: The curves of τ without environmental disturbances.

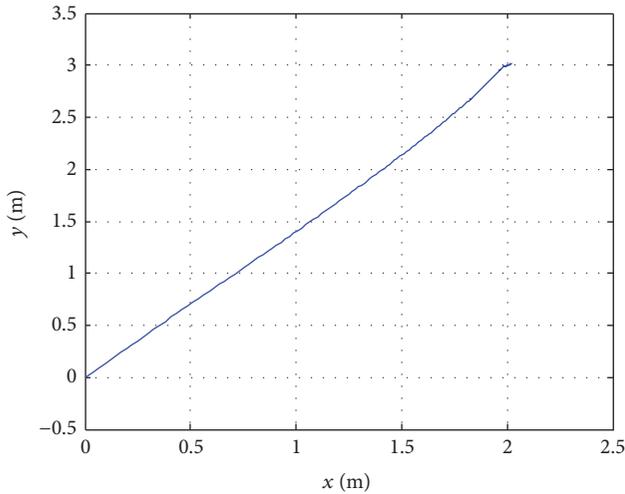


FIGURE 6: Ship horizontal motion with environmental disturbances.

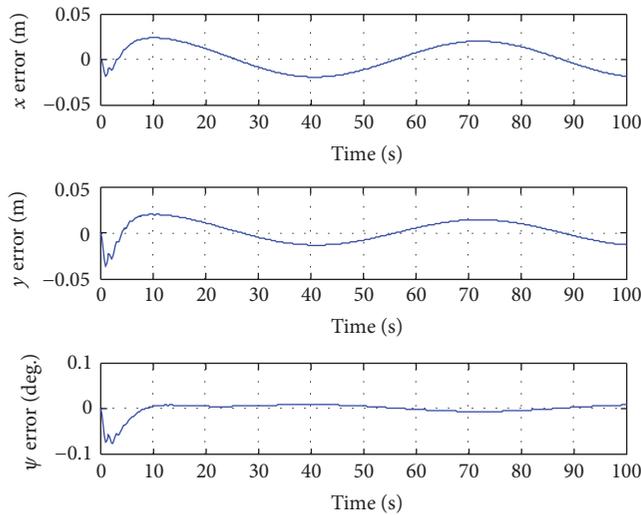


FIGURE 7: The position error of ship with environmental disturbances.

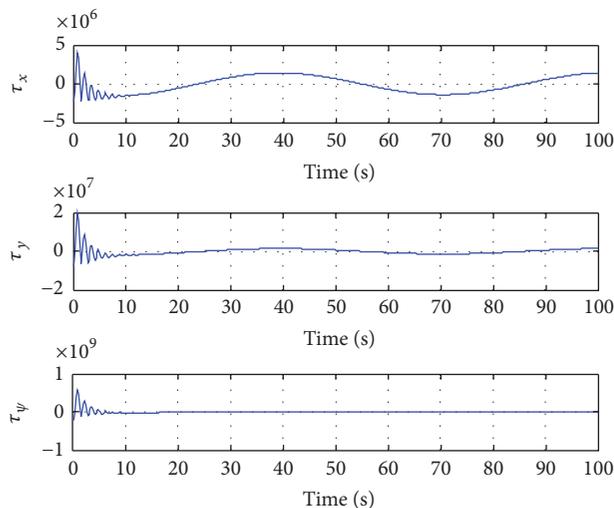


FIGURE 8: The curves of τ with environmental disturbances.

5. Conclusions

In this paper, the problem of Dynamic Positioning for offshore structures has been researched under the nonlinear robust controller based on L_2 -gain disturbance rejection. At first, the nonlinear controller is designed based on robust nonlinear control theories and the efficiency of L_2 disturbance rejection is proved by designing the parameters. Then, two groups of simulating results are demonstrated without and with the effects of environmental disturbances. And the simulating results validate the performance of the controller.

Competing Interests

The authors declare that there are no competing interests regarding the publication of this paper.

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