

Research Article

An Accelerating Iterative Learning Control Based on an Adjustable Learning Interval

Dongqi Ma and Hui Lin

School of Automation, Northwestern Polytechnical University, Xi'an, Shaanxi 710129, China

Correspondence should be addressed to Dongqi Ma; madongqi0499@163.com

Received 8 December 2016; Accepted 13 February 2017; Published 2 March 2017

Academic Editor: William MacKunis

Copyright © 2017 Dongqi Ma and Hui Lin. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

An iterative learning control algorithm with an adjustable interval is proposed for nonlinear systems to accelerate the convergence rate of iterative learning control. For λ -norm, the monotonic convergence of ILC was analyzed, and the corresponding convergence conditions were obtained. The results showed that the convergence rate was mainly determined by the controlled object, the control law gain, the correction factor, and the iteration interval size and that the control law gain was corrected in real time in the modified interval and the modified interval shortened as the number of iterations increased, further accelerating the convergence. The numerical simulation shows the effectiveness of the proposed method.

1. Introduction

Iterative learning control (ILC) [1], as a model-free control, has a simple structure and does not require specific model parameters. It can force the operation to meet the designed requirements within a finite interval after sufficient rounds of iterations. Because of the above attributes, ILC has become popular among many scholars in recent years.

Existing studies on ILC theory mainly focus on proving the convergence of the learning algorithm, learning speed, and learning law structure and the robustness, analysis method, initial value, and application of the learning process [2–8]. The convergence rate is an important indicator for evaluating the performance of an algorithm and also applies to iterative learning algorithms. Since the beginning of their research on ILC theory, Kawamura et al. [9] have proposed using the information obtained in previous learning to construct control inputs for the subsequent learning to improve the learning speed. Later, researchers found that initial controls that were often overlooked had a great impact on the convergence rate. Arif et al. [10] used databases to store previous information to construct new initial control inputs that greatly reduce the number of iterations required and improve the learning speed. Based on the knowledge in

databases, Yang et al. [11] used a linear weighting algorithm and curve fitting method to construct new initial control inputs to reduce the number of iterations and improve the convergence rate of the learning law. Wang et al. [12] proposed a Radial Basis Function Network- (RBFN-) based iterative learning algorithm to construct initial control inputs based on experience to minimize the distance between the initial output and the expected trajectory. The disadvantages of the above-mentioned algorithms in the literature are increased complexity of the control structure and a lack of rigorous proof as to whether they can improve the convergence rate or not. Bien and Huk [13] used a high-order iterative control algorithm to construct the initial control input. However, the initial inputs constructed using this method contain redundant information. Therefore, this approach actually decreases the system's learning speed to some extent.

Lin et al. [14] proposed the concept and analysis method of learning speed and performed a detailed analysis of the main factors that affect learning speed for the case of an open-closed P-type learning law. They found that convergence rate is related not only to the initial deviation, initial control variables, and learning gain of each run but also to the system's dynamic equations, operation interval size, and dimensions. Piao et al. [15] studied the

convergence rate of linear time invariant systems using D-type and P-type learning laws as examples. In their study, the sufficient conditions for the convergence of the system were obtained using time-weighted norms, and the quantitative relation between the number of iterations and the convergence condition was given; meanwhile, the Frobenius norm properties were used together with the gradient method to solve for the optimum gain matrix for iterative learning laws. Xu and Tan [16] adopted the Q-factor as the performance indicator of convergence rate and theoretically expanded the convergence rate comparison method compared with the method of using the convergence condition in [15, 16].

The focus of previous work has been to analyze the effects of the initial control and convergence condition on convergence rate. However, there have been few findings on control law design that accelerates the convergence rate. In the literature [14], using exponential gain to accelerate learning algorithms for P-type and D-type closed-loop learning has been proposed. A large number of simulation experiments have shown that the convergence rate of this design is sufficient for use in applications. In contrast to traditional iterative learning control, in which the learning is repeated in a fixed time interval, the control algorithm proposed in this paper has the following characteristics: a modified interval that decreases as the number of iterations increases is designed at the time axis, and the learning gain is also modified in real time; beyond the modified interval, the error is within the tolerance range, and no correction is needed. That is, the interval that requires learning is continuously shortened until the learning is completed. The overall computational load of this algorithm is reduced, which accelerates the learning. In this paper, a rigorous mathematical proof will be given, and a simulation experiment will be conducted using numerical examples to validate the effectiveness and correctness of the theory.

2. Mathematical Knowledge

The mathematical knowledge required to analyze the problem is given in this section.

Define a continuous vector function $f: [0, T_L] \rightarrow R^n$; $f(t) = [f^1(t), f^2(t), \dots, f^n(t)]^T$, where λ is a positive real number. Then, the λ -norm of the vector-valued function can be expressed as

$$\|f(\cdot)\|_\lambda = \sup_{0 \leq t \leq T} e^{-\lambda t} \left(\max_{1 \leq i \leq n} |f^i(t)| \right), \quad (1)$$

where $f[0, T_L]$ is a function defined on the interval $[0, T_L]$.

Lemma 1 (see [17]). *If $f_1(t)$, $g_1(t)$, and $h_1(t)$ are nonnegative continuous functions over the interval $[0, T_L]$ and there exists a nonnegative constant η such that*

$$f_1(t) \leq h_1(t) + \int_0^t \eta f_1(\tau) d\tau + \int_0^t g_1(\tau) d\tau, \quad (2)$$

then

$$f_1(t) \leq h_1(t) + \int_0^t \exp(\eta \cdot (t - \tau)) [\eta h_1(\tau) + g_1(\tau)] d\tau. \quad (3)$$

3. Problem Description and Analysis

Consider a class of nonlinear systems

$$\begin{aligned} \dot{x}_k(t) &= f(x_k(t), t) + Bu_k(t), \\ y_k(t) &= Cx_k(t) + Du_k(t), \end{aligned} \quad (4)$$

where $t \in [0, T_L]$; k is the number of iterations; $x_k \in R^n$, $u_k(t) \in R^r$, and $y_k(t) \in R^m$ are, respectively, the state, control, and output of the system; and B , C , and D are matrices with the corresponding dimension.

For the convenience of the proof, reasonable assumptions regarding the algorithm are given as follows.

Assumption 2. The unknown nonlinear function $f(\cdot, \cdot)$ satisfies the Lipschitz condition; namely, there exists a constant $L > 0$ such that

$$\|f(x_2(t), t) - f(x_1(t), t)\| \leq L \|x_2(t) - x_1(t)\|. \quad (5)$$

Assumption 3. There exists a unique ideal control $u_d(t)$ such that the state and output of the system are equal to designed $x_d(t)$ and $y_d(t)$.

In the proposed algorithm, the interval $[0, T_L]$ is divided equally into N subintervals with length h such that

$$\begin{aligned} t_0 &= 0, \\ t_{N+1} &= T_L, \\ t_{i+1} &= t_i + h, \quad i = 0, 1, 2, \dots, N. \end{aligned} \quad (6)$$

Assume that system (4) uses a closed-loop P-type learning law:

$$\begin{aligned} u_{k+1}(t) &= u_k(t) + \Delta u_{k+1}(t), \\ \Delta u_{k+1}(t) &= \begin{cases} 0, & 0 \leq t \leq t_{i_k}, \\ K_P e_{k+1}(t) e^{-K_E((t_{i_k}-t)/T_L)}, & t_{i_k} < t \leq T_L, \end{cases} \end{aligned} \quad (7)$$

where $K_P > 0$ is the proportional learning gain, $K_E > 0$ is the exponential correction factor, and $t_{i_k} \in [0, T_L]$ is the current correction starting time (note that in the $(k+1)$ th iteration, the error in interval $[0, t_{i_k}]$ is within the tolerance range, and the error in the remaining interval $[t_{i_k}, T_L]$ is not within the tolerance range).

Let $\delta_{[t_{i_k}, T_L]}$ be the demonstrative function; that is,

$$\delta_{[t_{i_k}, T_L]}(t) = \begin{cases} 0, & t \notin [t_{i_k}, T_L], \\ 1, & t \in [t_{i_k}, T_L]. \end{cases} \quad (9)$$

Then, based on (9), (7) can be written as follows:

$$u_{k+1}(t) = u_k(t) + K_P e^{-K_E((t_k - t)/T_L)} \delta_{[t_k, T_L]} e_{k+1}(t), \quad (10)$$

$$e_{k+1}(t) = \begin{cases} 0, & 0 \leq t \leq t_k, \\ y_d(t) - y_{k+1}(t), & t_k \leq t \leq T_L, \end{cases} \quad (11)$$

$$L_{i_k} = \{l \mid \|e_k[t_{i_k+l}, t_{i_k+l+1}]\|_\lambda \neq 0, 0 \leq l \leq N - i_k\}, \quad (12)$$

$$l_{i_k} = \begin{cases} \min L_{i_k}, & L_{i_k} \neq \phi, \\ 0, & L_{i_k} = \phi, \end{cases} \quad (13)$$

$$t_{i_{k+1}} = t_{i_k + l_{i_k}}. \quad (14)$$

The accelerated closed-loop P-type iterative learning control law (10) with gain correction that we constructed above can be described as follows: assume that the input of the first run is $u_0[0, T_L]$ and its value is arbitrarily assigned in the entire interval of $[0, T_L]$. Thus, the error of the first run is $e_0[0, T_L]$. The purpose of introducing l_{i_k} into (13) is to obtain the learning boundary point $t_{i_k + l_{i_k}}$ for the next iteration; in the $(k+1)$ th iteration, no learning is needed before $t = t_{i_k + l_{i_k}}$, and the corrections are only needed for interval $[t_{i_k + l_{i_k}}, T_L]$. The error in interval $[t_{i_k}, T_L]$ can be calculated using the designed trajectory $y_d[t_{i_k}, T_L]$ and the output $y_k[t_{i_k}, T_L]$. The construction of control law (7) shows that learning is only needed for intervals that do not match the designed trajectory and not for the interval that has met it. Thus, the size of the interval that requires learning gradually approaches zero as the number of iterations increases.

Theorem 4. *The iterative learning system described by (4) and (7) indicates that if the conditions,*

$$(I) \rho = \|(I + DK_P e^{K_E})^{-1}\| < 1,$$

$$(II) x_k(0) = x_d(0), (k = 0, 1, 2, \dots),$$

are satisfied, then

$$\|\delta_{[t_{i_k}, T_L]} e_{k+1}(t)\|_\lambda \leq \rho \|\delta_{[t_{i_k}, T_L]} e_k(t)\|_\lambda. \quad (15)$$

As $k \rightarrow \infty$, the output $y_k(t)$ of the system converges to the designed trajectory in $[0, T_L]$; that is,

$$\lim_{k \rightarrow \infty} y_k(t) = y_d(t) \quad (t_i \in [0, T_L]). \quad (16)$$

Proof. Assuming that, in the $(k+1)$ th iteration, the control time point stops at $t = t_{i_k}$, it is clear from (11) that $y_{k+1}(t) = y_k(t)$ ($t \in [0, t_{i_k}]$). Therefore, when $t \in [t_{i_k}, T_L]$, the error can be written as

$$\delta_{[t_{i_k}, T_L]} e_{k+1}(t) = y_d(t) - y_{k+1}(t). \quad (17)$$

Then, when $t \in [t_{i_k}, T_L]$, the following can be derived from (4) and condition (II):

$$\begin{aligned} e_{k+1}(t) &= y_d(t) - y_{k+1}(t) \\ &= (y_d(t) - y_k(t)) - (y_{k+1}(t) - y_k(t)) \\ &= e_k(t) - C(x_{k+1}(t) - x_k(t)) \\ &\quad - D(u_{k+1}(t) - u_k(t)) \\ &= e_k(t) \\ &\quad - C \int_0^t (f(x_{k+1}(\tau), \tau) - f(x_k(\tau), \tau)) d\tau \\ &\quad - CB \int_0^t (u_{k+1}(\tau) - u_k(\tau)) d\tau \\ &\quad - D(u_{k+1}(t) - u_k(t)) \\ &= e_k(t) \\ &\quad - C \int_0^t (f(x_{k+1}(\tau), \tau) - f(x_k(\tau), \tau)) d\tau \\ &\quad - CB \int_0^t K_P e^{-K_E((t_k - \tau)/T_L)} \delta_{[t_k, T_L]} e_{k+1}(\tau) d\tau \\ &\quad - DK_P e^{-K_E((t_k - t)/T_L)} \delta_{[t_k, T_L]} e_{k+1}(t). \end{aligned} \quad (18)$$

Equation (18) can be reorganized as

$$\begin{aligned} \delta_{[t_{i_k}, T_L]} e_{k+1}(t) &= (I + DK_P e^{-K_E((t_k - t)/T_L)})^{-1} \\ &\quad \cdot \delta_{[t_{i_k}, T_L]} e_k(t) - (I + DK_P e^{-K_E((t_k - t)/T_L)})^{-1} \\ &\quad \cdot \int_{t_{i_k}}^t C(f(x_{k+1}(\tau), \tau) - f(x_k(\tau), \tau)) d\tau \\ &\quad - (I + DK_P e^{-K_E((t_k - t)/T_L)})^{-1} \\ &\quad \cdot CBK_P \int_{t_{i_k}}^t e^{-K_E((t_k - \tau)/T_L)} \delta_{[t_{i_k}, T_L]} e_{k+1}(\tau) d\tau. \end{aligned} \quad (19)$$

Take the λ -norm for both sides of (19) and multiply by $e^{-\lambda t}$ ($\lambda > 0$), and then take the maximum value in $t \in [0, T_L]$ and combine with Lipschitz conditions to derive the following:

$$\begin{aligned} &\|\delta_{[t_{i_k}, T_L]} e_{k+1}(t)\|_\lambda \\ &\leq \rho \|\delta_{[t_{i_k}, T_L]} e_k(t)\|_\lambda \\ &\quad + \rho cL \frac{1 - e^{-\lambda T_L}}{\lambda} \|x_{k+1}(t) - x_k(t)\|_\lambda \\ &\quad + \rho b \frac{1 - e^{-\lambda T_L}}{\lambda} \|\delta_{[t_{i_k}, T_L]} e_{k+1}(t)\|_\lambda, \end{aligned} \quad (20)$$

where

$$\begin{aligned}\rho &= \left\| \left(I + DK_P e^{K_E} \right)^{-1} \right\|, \\ c &= \|C\|, \\ b &= \sup_{t \in [0, T]} \|CBK_P\| e^{-K_E((t_k - t)/T_L)}.\end{aligned}\quad (21)$$

From (4),

$$\begin{aligned}x_{k+1}(t) - x_k(t) &= \int_0^t (f(x_{k+1}(\tau), \tau) - f(x_k(\tau), \tau)) d\tau \\ &\quad + B \int_0^t (u_{k+1}(\tau) - u_k(\tau)) d\tau \\ &= \int_0^t (f(x_{k+1}(\tau), \tau) - f(x_k(\tau), \tau)) d\tau \\ &\quad + B \int_0^t K_P e^{-K_E((t_k - \tau)/T_L)} \delta_{[t_k, T_L]} e_{k+1}(\tau) d\tau.\end{aligned}\quad (22)$$

λ -norm is taken on both sides of (22) and combined with Lipschitz conditions to derive

$$\begin{aligned}\|x_{k+1}(t) - x_k(t)\| &\leq \int_0^t L \|x_{k+1}(\tau) - x_k(\tau)\| d\tau \\ &\quad + \|BK_P e^{K_E}\| \int_0^t \|\delta_{[t_k, T_L]} e_{k+1}(\tau)\| d\tau.\end{aligned}\quad (23)$$

The following is derived from Lemma 1:

$$\begin{aligned}\|x_{k+1}(t) - x_k(t)\| &\leq \|BK_P e^{K_E}\| \int_{t_k}^t e^{L(t-\tau)} \|\delta_{[t_k, T_L]} e_{k+1}(\tau)\| d\tau.\end{aligned}\quad (24)$$

Both sides of the above equation are multiplied by $e^{-\lambda t}$ ($\lambda > 0$), and the maximum value in $t \in [t_k, T_L]$ is taken:

$$\begin{aligned}\|x_{k+1}(t) - x_k(t)\|_\lambda &\leq \|BK_P e^{K_E}\| \int_{t_k}^t e^{(L-\lambda)(t-\tau)} d\tau \|\delta_{[t_k, T_L]} e_{k+1}(t)\|_\lambda \\ &\leq a \frac{1 - e^{(L-\lambda)T_L}}{\lambda - L} \|\delta_{[t_k, T_L]} e_{k+1}(t)\|_\lambda,\end{aligned}\quad (25)$$

where $a = \|BK_P e^{K_E}\|$. \square

In interval $[t_k, T_L]$, the following can be derived by substituting (25) into (20):

$$\begin{aligned}\|\delta_{[t_k, T_L]} e_{k+1}(t)\|_\lambda &\leq \rho \|\delta_{[t_k, T_L]} e_k(t)\|_\lambda \\ &\quad + \left(a \frac{1 - e^{(L-\lambda)T_L}}{\lambda - L} + \rho b \frac{1 - e^{T_L}}{\lambda} \right) \|\delta_{[t_k, T_L]} e_{k+1}(t)\|_\lambda.\end{aligned}\quad (26)$$

Select a large enough λ that

$$\|\delta_{[t_k, T_L]} e_{k+1}(t)\|_\lambda \leq \rho \|\delta_{[t_k, T_L]} e_k(t)\|_\lambda. \quad (27)$$

The following can be obtained after multiple iterations:

$$\|\delta_{[t_k, T_L]} e_{k+1}(t)\|_\lambda \leq \rho^{k-1} \|\delta_{[t_k, T_L]} e_1(t)\|_\lambda. \quad (28)$$

Therefore,

$$\lim_{k \rightarrow \infty} \sup_{t \in [t_k, T_L]} \|\delta_{[t_k, T_L]} e_k(t)\|_\lambda = 0. \quad (29)$$

It is shown that adopting a scheme to shorten the learning interval of control law (7) can ensure its monotonic convergence in λ -norm, and when λ is large enough, the size of the interval that requires learning decreases and becomes infinitely close to zero by iterations. Eventually, the system output follows the designed trajectory over the entire interval.

Remark 5. From (18), (19), (20), (22), (23), (24), and (25), we can see that as time t increases, the convergence becomes faster.

Remark 6. The learning process indicates that as the number of iterations increases, the length of the learning interval is shortened, which improves the learning efficiency.

4. Simulation Results and Analysis

Consider the following SISO nonlinear continuous time-varying system:

$$\begin{aligned}\dot{x}_1(t) &= (\sin x_1 + 2 \sin x_2) \frac{1}{1+t} + 3tu, \\ \dot{x}_2(t) &= 0.3 \sin x_1 + \frac{\sin x_2}{1+t} u \\ y(t) &= \sin x_1 + 0.5 \sin x_2 + 0.5u.\end{aligned}\quad (30)$$

The initial conditions are $x_1(0) = 0.5$ and $x_2(0) = 0.5$, and it is required to follow the designed trajectory within the time interval $[0, 6]$.

$$y_d(t) = 0.4t^2 - 0.2t^3. \quad (31)$$

The values in $t \in [0, 6]$ for the first iteration of the controller $u_k(t)$ were generated randomly using the rand function. The learning gain in the control law (7) was set to $K_P = 5$, and the exponential correction parameter was set to $K_E = 0.5$. The convergence condition of control law (7) was calculated to be $\rho \leq 0.1952 < 1$. The convergence condition of the traditional closed-loop P-type control law was $\rho_0 = 0.2857 < 1$. Obviously, $\rho < \rho_0$, indicating that control law (7) had a faster convergence rate than the conventional closed-loop P-type control law. Figures 1(a), 1(b), 1(c), and 1(d) represent the course of the system output and the designed trajectory in the 1st, 3rd, 5th, and 7th iterations, respectively, in which the red solid line is the

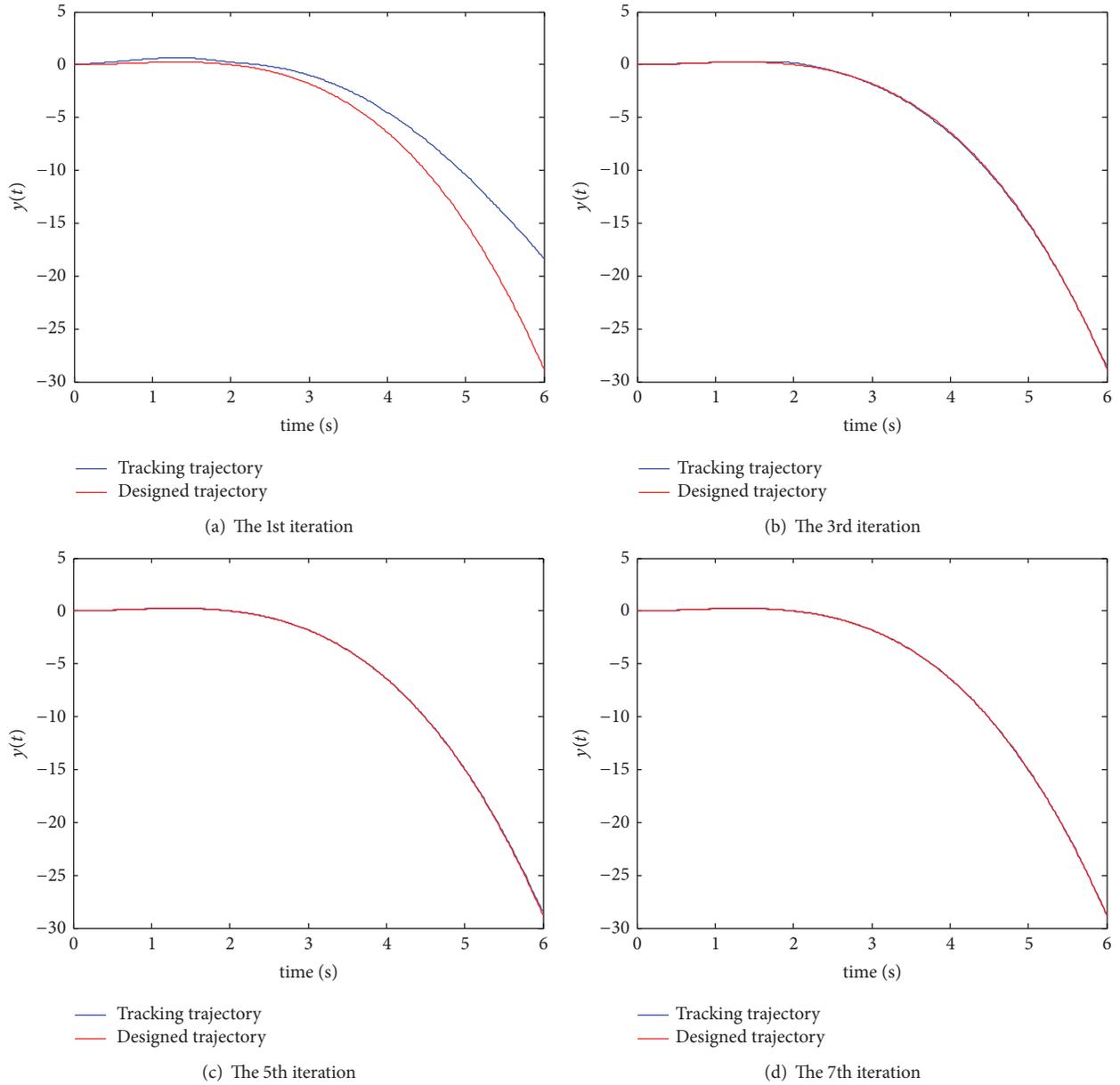


FIGURE 1: Tracking of the system.

designed trajectory and the blue solid line is the tracking trajectory. The curves in Figure 1 show that the length of the interval requiring system correction gradually decreased with increasing number of iterations, and eventually the system output completely followed the designed trajectory over the entire interval.

For the convenience of comparative analysis, numerical simulation was conducted using the proposed algorithm and the conventional closed-loop P-type iterative learning algorithm. The tracking errors over the number of iterations are shown in Figure 2, in which the iterative tracking error takes the form $\int |e_k(t)|^2 dt$.

Figure 2 shows that the modified P-type closed-loop control law converged to be within the error band (0.01) after

6 iterations, while the traditional control law took 11 iterations to converge to the above-mentioned error band. It is clear that the control algorithm proposed in this paper significantly improved the learning speed compared with the traditional learning law. It gave a much smaller error after the same number of iterations than the traditional learning law.

5. Conclusions

In this paper, a novel iterative learning algorithm was designed for nonlinear systems. The learning interval of this algorithm is adjustable, and the learning gain is modified in real time. As the number of iterations increases, the interval that actually needs correction shortens, accelerating

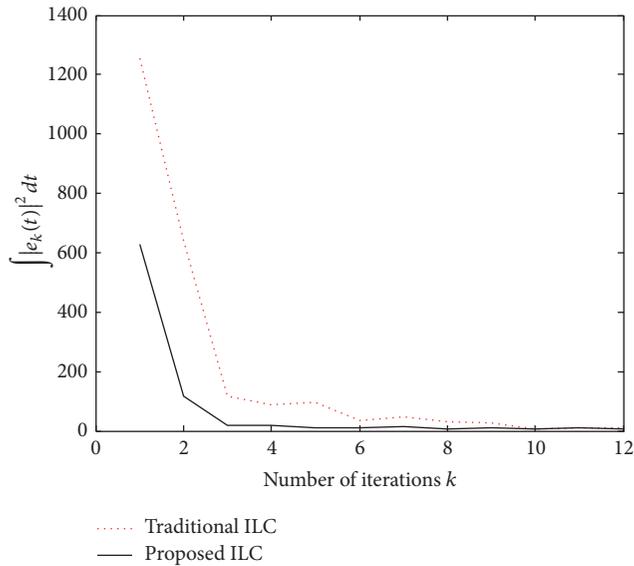


FIGURE 2: Comparison of tracking error.

the convergence rate. The convergence rate was discussed for the case of a closed-loop P-type iterative learning law, and its convergence conditions were obtained in λ -norm. The results indicated that the convergence condition of this algorithm was determined not only by the proportional learning gain but also by the learning interval and exponential factor.

Competing Interests

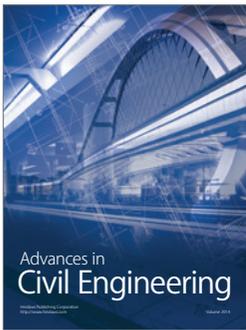
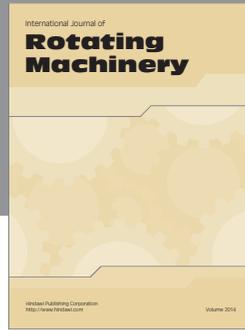
The authors declare that there is no conflict of interests regarding the publication of this article.

Acknowledgments

This research was supported by the National Natural Science Fund of China (Grant 51407143), the Specialized Research Fund for the Doctoral Program of Higher Education of China (Grant 20136102120049), and the Natural Science Foundation of Shaanxi Province, China (2015JM5227).

References

- [1] S. Arimoto, S. Kawamura, and F. Miyazaki, "Bettering operation of Robots by learning," *Journal of Robotic Systems*, vol. 1, no. 2, pp. 123–140, 1984.
- [2] N. Zsiga, S. Van Dooren, P. Elbert, and C. H. Onder, "A new method for analysis and design of iterative learning control algorithms in the time-domain," *Control Engineering Practice*, vol. 57, pp. 39–49, 2016.
- [3] D. Zhao and Y. Yang, "An iterative learning control design method for nonlinear discrete-time systems with unknown iteration-varying parameters and control direction," *Mathematical Problems in Engineering*, Article ID 8971407, 7 pages, 2016.
- [4] L. Zhang, W. Chen, J. Liu, and C. Wen, "A robust adaptive iterative learning control for trajectory tracking of permanent-magnet spherical actuator," *IEEE Transactions on Industrial Electronics*, vol. 63, no. 1, pp. 291–301, 2016.
- [5] Q. Zhu, J.-X. Xu, D. Huang, and G.-D. Hu, "Iterative learning control design for linear discrete-time systems with multiple high-order internal models," *Automatica. A Journal of IFAC, the International Federation of Automatic Control*, vol. 62, pp. 65–76, 2015.
- [6] Q. X. Yu, Z. S. Hou, and R. H. Chi, "Adaptive iterative learning control for nonlinear uncertain systems with both state and input constraints," *Journal of the Franklin Institute*, vol. 353, no. 15, pp. 3920–3943, 2016.
- [7] X. Li, J.-X. Xu, and D. Huang, "Iterative learning control for nonlinear dynamic systems with randomly varying trial lengths," *International Journal of Adaptive Control and Signal Processing*, vol. 29, no. 11, pp. 1341–1353, 2015.
- [8] L. Yan and J. Wei, "Fractional order nonlinear systems with delay in iterative learning control," *Applied Mathematics and Computation*, vol. 257, pp. 546–552, 2015.
- [9] S. Kawamura, F. Miyazaki, and S. Arimoto, "Intelligent control of robot motion based on learning method," *Memoirs of the Research Institute of Science and Engineering Ritumeikan University* 46, 1987.
- [10] M. Arif, T. Ishihara, and H. Inooka, "Iterative learning control using information database (ILCID)," *Journal of Intelligent and Robotic Systems: Theory and Applications*, vol. 25, no. 1, pp. 27–41, 1999.
- [11] S.-Y. Yang, X.-P. Fan, and A. Luo, "Experience based acquisition of the initial value for the iterative learning control inputs," *Kongzhi yu Juce/Control and Decision*, vol. 19, no. 1, pp. 27–35, 2004.
- [12] X.-S. Wang, G.-Z. Peng, and Y.-H. Cheng, "Iterative learning controller for manipulators based on the RBF network," *Transaction of Beijing Institute of Technology*, vol. 24, no. 6, pp. 512–515, 2004.
- [13] Z. Bien and K. M. Huk, "Higher-order iterative control algorithm," *IEEE Proceedings Part D: Control Theory and Applications*, vol. 136, no. 3, pp. 105–112, 1989.
- [14] H. Lin and L. Wang, *Iterative Learning Control Theory*, Northwestern Polytechnical University Press, Xi'an, China, 1998.
- [15] F. X. Piao, Q. L. Zhang, and Z. F. Wang, "Analysis of convergence rate for iterative learning control," *Journal of Northeastern University*, vol. 27, no. 8, pp. 835–838, 2006.
- [16] J.-X. Xu and Y. Tan, "Robust optimal design and convergence properties analysis of iterative learning control approaches," *Automatica*, vol. 38, no. 11, pp. 1867–1880, 2002.
- [17] X.-E. Ruan and J.-Y. Zhao, "Pulse compensated iterative learning control to nonlinear systems with initial state uncertainty," *Kongzhi Lilun Yu Yingyong/Control Theory and Applications*, vol. 29, no. 8, pp. 993–1000, 2012.



Hindawi

Submit your manuscripts at
<https://www.hindawi.com>

