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Research Article

Input-Output Finite-Time Control of Positive Switched Systems with Time-Varying and Distributed Delays

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The problem of input-output finite-time control of positive switched systems with time-varying and distributed delays is considered in this paper. Firstly, the definition of input-output finite-time stability is extended to positive switched systems with time-varying and distributed delays, and the proof of the positivity of such systems is also given. Then, by constructing multiple linear copositive Lyapunov functions and using the mode-dependent average dwell time (MDADT) approach, a state feedback controller is designed, and sufficient conditions are derived to guarantee that the corresponding closed-loop system is input-output finite-time stable (IO-FTS). Such conditions can be easily solved by linear programming. Finally, a numerical example is given to demonstrate the effectiveness of the proposed method.

1. Introduction

Positive switched systems are a class of dynamics whose state and output are nonnegative whenever the initial conditions and inputs are nonnegative. In the last decades, the research of positive switched systems is a hot topic due to their many practical applications in communication networks [1], the viral mutation dynamics under drug treatment [2], formation flying [3], systems theory [4, 5], and so forth. The stability analysis and controller synthesis of positive switched systems have been highlighted by many researchers [6–10]. However, most of the results mentioned above discussed the asymptotic stability (Lyapunov stability).

Contrary to asymptotic stability, some researchers focus on finite-time stability (FTS) of positive switched systems: that is, given a bound on the initial condition, the system state does not exceed a certain threshold during a specified time interval. Some works dealing with analysis and controller design of FTS control problems have been published [11–16]. Recently, the definition of IO-FTS is proposed by [17]; this definition is that a system is said to be IO-FTS if, given a class of norm bounded input signals over a specified time interval $[0, T]$, the outputs of the system are also norm bounded over $[0, T]$. IO-FTS involves signals defined over a finite-time interval and does not necessarily require the inputs and outputs to belong to the same class, and IO-FTS constraints permit specifying quantitative bounds on the controlled variables to be fulfilled during the transient response. This definition of IO-FTS is fully consistent with the definition of FTS. Some related results have been obtained in [18–25]. These results about IO-FTS are mainly involved in impulsive systems [18], linear systems [19–22], stochastic systems [23], Markovian jump systems [24], and impulsive switched linear systems [25]. It is worth noting that the results mentioned above are mainly concerned with nonpositive systems. Huang et al. [26] extended IO-FTS to a class of discrete-time positive switched systems with state delays via average dwell time (ADT), but the time-varying delay and distributed delay were not considered. As we know, multiple time delays, such as time-varying delays and distributed delays, usually occur in many practical systems and may result in system performance deterioration, even instability. Therefore, they must be taken into account in analyzing and implementing any controller scheme. Moreover, MDADT approach allows that every subsystem has its own ADT to make the individual properties of each subsystem unneglected, which is more applicable and less conservative compared with ADT. Then, it is necessary to investigate IO-FTS analysis of positive switched systems with time-varying and distributed delays via MDADT approach. However, this problem is still open.

Motivated by the aforementioned factors, the problem of IO-FTS of positive switched systems with time-varying and distributed delays is considered. The main contributions of this paper are as follows: (1) the positivity of positive switched systems with time-varying and distributed delays is proved; (2) the definition of IO-FTS is for the first time extended to positive switched systems with time-varying and distributed delays; (3) by using multiple copositive type Lyapunov function and MDADT approach, a state feedback controller is designed and sufficient conditions for IO-FTS of the corresponding closed-loop system are given. Such conditions can be easily solved by linear programming. The rest of this paper is organized as follows. In Section 2, problem statements and necessary lemmas are given. Main results are given in Section 3. A numerical example is provided in Section 4. Section 5 concludes this paper.

Notations. The representation $A \succ 0$ ($\succeq 0, \prec 0, \preceq 0$) means that $a_{ij} > 0$ ($\geq 0, < 0, \leq 0$), which is applicable to a vector. $A \succ B$ ($A \succeq B$) means that $A - B \succ 0$ ($A - B \succeq 0$). R_+^n is the n-dimensional nonnegative (positive) vector space. $R^{n\times n}$ denotes the space of $n \times n$ matrices with real entries. A^T denotes the transpose of matrix A. Let $x \in R^n$; 1-norm $\|x\|$ is defined by $\|x\| = \sum_{k=1}^{n} |x_k|$. $L_{1,[0,T]}$ denotes the space of absolute integrable vector-valued functions on the interval $[0, T]$; that is, $s(t) \in L_{1,[0,T]}$ if $\int_0^T \|s(t)\| dt < \infty$ holds. $L_{\infty,[0,T]}$ denotes the space of the uniformly bounded vector-valued functions on the interval $[0, T]$; that is, $s(t) \in L_{\infty,[0,T]}$ if $\max_{t\in[0,T]} \|w(t)\| < \infty$ holds. Matrices are assumed to have compatible dimensions for calculating if their dimensions are not explicitly stated.

2. Preliminaries and Problem Statements

Consider the following positive switched systems with time-varying and distributed delays:

$$\begin{aligned}
\dot{x}(t) &= A_{\sigma(t)} x(t) + A_{d\sigma(t)} x(t - d(t)) \\
&\quad + A_{b\sigma(t)} \int_{t-\tau_2}^{t} x(s)\, ds + G_{\sigma(t)} u(t) \\
&\quad + B_{\sigma(t)} w(t) \\
y(t) &= C_{\sigma(t)} x(t) \\
x(\theta) &= \varphi(\theta), \quad \theta \in [-\tau_1, 0],
\end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the system state and $u(t) \in R^m$ and $y(t) \in R^s$ represent the control input and output. $\sigma(t) : [0, \infty) \to \underline{M} = \{1, 2, \ldots, M\}$ is the system switching signal, where M is the number of subsystems; $\forall p \in \underline{M}$, A_p, A_{dp}, A_{bp}, B_p, C_p, and G_p are constant matrices with appropriate dimensions, $d(t) \geq 0$ denotes time-varying delay, which satisfies $\dot{d}(t) \leq h < 1$, and τ_2 is the distributed delay, where τ_2 and h are known positive constants. $\varphi(\theta)$ is the initial condition on $[-\tau_1, 0]$, $\tau_1 = \max\{d(t), \tau_2\}$. $w(t) \in R^l$ is the exogenous disturbance and defined as

$$W_1 = \left\{ w(\cdot) \in L_{1,[0,T]} : \int_0^T \|w(t)\|\, dt \leq d \right\} \tag{2}$$

with a known scalar $d > 0$.

Next, we will present some definitions and lemmas for the positive switched system (1) to our further study.

Definition 1 (see [14]). System (1) is said to be positive if for any switching signals $\sigma(t)$, any initial conditions $\varphi(\theta) \succeq 0$, $\theta \in [-\tau_1, 0]$, and any disturbance input $w(t) \succeq 0$, the corresponding trajectory satisfies $x(t) \succeq 0$ and $y(t) \succeq 0$ for all $t \geq 0$.

Definition 2 (see [14]). A is called a Metzler matrix if the off-diagonal entries of matrix A are nonnegative.

Definition 3 (see [16]). For any switching signal $\sigma(t)$ and any $t_2 \geq t_1 \geq 0$, let $N_{\sigma p}(t_1, t_2)$ denote the switching numbers where the pth subsystem is activated over the interval $[t_1, t_2)$ and $T_p(t_1, t_2)$ denote the total running time of the pth subsystem over the interval $[t_1, t_2)$. If there exist $N_{0p} \geq 0$ and $T_{\alpha p} > 0$ such that

$$N_{\sigma p}(t_1, t_2) \leq N_{0p} + \frac{T_p(t_1, t_2)}{T_{\alpha p}}, \quad \forall t_2 \geq t_1 \geq 0,\ \forall p \in \underline{M} \tag{3}$$

then $T_{\alpha p}$ and N_{0p} are called MDADT and mode-dependent chattering bounds, respectively. Generally, we choose $N_{0p} = 0$.

Lemma 4 (see [16]). *A matrix $A \in R^{n\times n}$ is a Metzler matrix if and only if there exists a positive constant η such that $A + \eta I_n \succeq 0$.*

Lemma 5 (see [9]). *Let $A \in R^{n\times n}$. Then $e^{At} \succeq 0$, $\forall t \geq 0$, if and only if A is a Metzler matrix.*

Lemma 6. *System (1) is positive if and only if A_p, $\forall p \in \underline{M}$, are Metzler matrices and $\forall p \in \underline{M}$, $A_{dp} \succeq 0$, $A_{bp} \succeq 0$, $B_p \succeq 0$, $C_p \succeq 0$, and $G_p \succeq 0$.*

Proof.

Sufficiency. For any $T > 0$, $t_0 = 0$, let $t_1, t_2, \ldots, t_{N_\sigma(t_0,T)}$ denote the switching instants on the interval $[t_0, T)$. From system (1), $\forall t \in [t_0, t_1)$, we have

$$\begin{aligned}
\exp\left\{-A_{\sigma(t_0)} t\right\} \left(\dot{x}(t) - A_{\sigma(t_0)} x(t)\right) &= \exp\left\{-A_{\sigma(t_0)} t\right\} \\
&\quad \cdot \left(A_{d\sigma(t_0)} x(t - d(t)) + A_{b\sigma(t_0)} \int_{t_0-\tau_2}^{t_0} x(s)\, ds \right. \\
&\quad \left. + G_{\sigma(t_0)} u(t) + B_{\sigma(t_0)} w(t) \right) \\
y(t) &= C_{\sigma(t_0)} x(t).
\end{aligned} \tag{4}$$

Integrating both sides of the first equation of the above equations from t_0 to t, it yields

$$\begin{aligned}\exp\left\{-A_{\sigma(t_0)}t\right\}x(t) &= \int_{t_0}^{t}\Bigg(\exp\left\{-A_{\sigma(t_0)}s\right\} \\ &\cdot\Bigg(A_{d\sigma(t_0)}x(s-d(s)) + A_{b\sigma(t_0)}\int_{t_0-\tau_2}^{t_0}x(s)\,ds \\ &+ G_{\sigma(t_0)}u(s) + B_{\sigma(t_0)}w(s)\Bigg)\Bigg)ds + x(t_0).\end{aligned} \tag{5}$$

It leads to

$$\begin{aligned}x(t) &= \int_{t_0}^{t}\Bigg(\exp\left\{A_{\sigma(t_0)}(t-s)\right\}\Bigg(A_{d\sigma(t_0)}x(s-d(s)) \\ &+ A_{b\sigma(t_0)}\int_{t_0-\tau_2}^{t_0}x(s)\,ds + G_{\sigma(t_0)}u(s) \\ &+ B_{\sigma(t_0)}w(s)\Bigg)\Bigg)ds + \exp\left\{A_{\sigma(t_0)}t\right\}\varphi(0).\end{aligned} \tag{6}$$

By Lemma 5, if $A_{\sigma(t_0)}$ is a Metzler matrix, $A_{d\sigma(t_0)} \succeq 0$, $A_{b\sigma(t_0)} \succeq 0$, $B_{\sigma(t_0)} \succeq 0$, $C_{\sigma(t_0)} \succeq 0$, and $G_{\sigma(t_0)} \succeq 0$, then it is easy to obtain that $x(t) \succeq 0$, $y(t) \succeq 0$, $\forall t \in [t_0, t_1)$, and $x(t_1) = x(t_1^-) \succeq 0$. Similar to the above process, $\forall t \in [t_1, t_2)$, it follows that

$$\begin{aligned}x(t) &= \int_{t_1}^{t}\Bigg(\exp\left\{A_{\sigma(t_1)}(t-s)\right\}\Bigg(A_{d\sigma(t_1)}x(s-\mathrm{d}(s)) \\ &+ A_{b\sigma(t_1)}\int_{t_1-\tau_2}^{t_1}x(s)\,ds + G_{\sigma(t_1)}u(s) \\ &+ B_{\sigma(t_1)}w(s)\Bigg)\Bigg)ds + \exp\left\{A_{\sigma(t_1)}t\right\}\varphi(t_1) \succeq 0 \\ y(t) &= C_{\sigma(t_1)}x(t) \succeq 0\end{aligned} \tag{7}$$

and $x(t_2) = x(t_2^-) \succeq 0$. Recursively, if A_i, $\forall i \in \underline{M}$, are Metzler matrices, $A_{di} \succeq 0$, $A_{bi} \succeq 0$, $B_i \succeq 0$, $C_i \succeq 0$, and $G_i \succeq 0$, then one has that $x(T) \succeq 0$, $y(T) \succeq 0$, $\forall t > 0$.

Necessary. Conversely, suppose that there exists an element $a_{iqg} < 0$, $i \in \underline{M}$, $q \neq g$, where a_{iqg} is in the qth row and gth column of A_i. From system (1), we can obtain

$$\begin{aligned}\dot{x}_q(t) &= \sum_{k=1,k\neq q,k\neq g}^{n} a_{iqk}x_k(t) + a_{iqq}x_q(t) + a_{iqg}x_g(t) \\ &+ \sum_{k=1}^{n} a_{diqk}x_k(t-d(t)) \\ &+ \sum_{k=1}^{n} a_{biqk}\int_{t-\tau_2}^{t}x_k(s)\,ds + \sum_{k=1}^{n} b_{iqk}w_k(t) \\ &+ \sum_{k=1}^{n} g_{iqk}u_k(t),\end{aligned} \tag{8}$$

where $x_k(t)$, $w_k(t)$, and $u_k(t)$ represent the kth elements of $x(t)$, $w(t)$, and $u(t)$, respectively. Then, if $x_g(t) \neq 0$, it is obvious that $\dot{x}_q(t) < 0$ is possible whenever $x_q(t) = 0$, which means that $x_q(t^+) < 0$. It follows that system (1) is not positive.

Then, suppose that A_{di} has an element $a_{diqg} < 0$, $i \in \underline{M}$; we obtain

$$\begin{aligned}\dot{x}_q(t) &= \sum_{k=1,k\neq q}^{n} a_{iqk}x_k(t) + a_{iqq}x_q(t) \\ &+ \sum_{k=1,k\neq g}^{n} a_{diqk}x_k(t-d(t)) \\ &+ a_{diqg}x_g(t-d(t)) + \sum_{k=1}^{n} a_{biqk}\int_{t-\tau_2}^{t}x_k(s)\,ds \\ &+ \sum_{k=1}^{n} b_{iqk}w_k(t) + \sum_{k=1}^{n} g_{iqk}u_k(t).\end{aligned} \tag{9}$$

It can be obtained that $x_q(t^+) < 0$ is possible whenever $x_q(t) = 0$. It shows that system (1) is not positive.

In the same way, if A_{bi} has an element $a_{biqg} < 0$, B_i has an element $b_{iqg} < 0$, or G_i has an element $g_{iqg} < 0$, $i \in \underline{M}$, then system (1) is exactly not positive.

Finally, assume that there exists an element $C_{iqg} < 0$, $i \in \underline{M}$, $q \neq g$, where C_{iqg} is in the qth row and gth column of C_i. According to system (1),

$$y_q(t) = \sum_{k=1,k\neq q,k\neq g}^{n} c_{iqk}x_k(t) + c_{iqq}x_q(t) + c_{iqg}x_g(t). \tag{10}$$

It is easy to obtain that $y_q(t) < 0$ is possible whenever $x_g(t) > 0$; it means that system (1) is not positive.

From the above, system (1) is positive under any switching signals if and only if A_i are Metzler matrices, $A_{di} \succeq 0$, $A_{bi} \succeq 0$, $B_i \succeq 0$, $C_i \succeq 0$, and $G_i \succeq 0$, $\forall i \in \underline{M}$.

The proof is completed. □

Next, we will give the definitions of input-output finite-time stability for the positive switched system (1).

Definition 7 (IO-FTS). Consider zero initial condition ($x(0) = 0$), for a given time constant T_f, disturbances signals W_1 defined by (2), and a vector $\delta \succ 0$; system (1) is said to be IO-FTS with respect to $(\delta, T_f, d, \sigma(t))$, if

$$\begin{gathered}w(t) \in W_1 \Longrightarrow \\ y^T\delta \le 1, \\ \forall t \in \left[0, T_f\right].\end{gathered} \tag{11}$$

If the disturbance $w(t)$ satisfies $w(t) \in W_2$, W_2 is defined as

$$W_2 = \left\{w(\cdot) \in L_{\infty,[0,T]} : \max_{t\in[0,T]}\|w(t)\|\,dt \le d\right\}; \tag{12}$$

then we give Definition 8.

Definition 8 (IO-FTS). Consider zero initial condition ($x(0) = 0$), for a given time constant T_f, disturbances signals W_2, and a vector $\delta \succ 0$; system (1) is said to be IO-FTS with respect to $(\delta, T_f, d, \sigma(t))$, if

$$\begin{aligned} &w(t) \in W_2 \Longrightarrow \\ &y^T\delta \le 1, \\ &\qquad \forall t \in [0, T_f]. \end{aligned} \tag{13}$$

Remark 9. [26] gives two definitions of discrete positive switched systems for two classes of exogenous disturbances, respectively. Similarly, for continuous positive switched systems, two classes of exogenous disturbances for IO-FTS problem can be also proposed, that is, norm bounded integrable signals $w(t)$ ($w(t) \in L_{1,[0,T]}$, $\int_0^T \|w(t)\|dt \le d$) and the uniformly bounded signals $w(t)$ ($w(t) \in L_{\infty,[0,T]}$, $\max_{t\in[0,T]}\|w(t)\| \le d$). Because of the similar process, in this paper, we only focus on the former.

The aim of this paper is to design a state feedback controller $u(t)$ and find a class of switching signals $\sigma(t)$ for the positive switched system (1) such that the corresponding closed-loop system is IO-FTS.

3. Main Results

3.1. IO-FTS Analysis. In this subsection, we will focus on the problem of IO-FTS of positive switched system (1) with $u(t) \equiv 0$. The following theorem gives sufficient conditions of IO-FTS of system (1) via the MDADT approach.

Theorem 10. *Consider system (1) with $u(t) \equiv 0$. Given positive constants T_f, λ_p, $p \in \underline{M}$, and γ and a vector $\delta \succ 0$, if there exist positive vectors ν_p, ρ_p, ϱ_p, $p \in \underline{M}$, the following inequalities hold:*

$$\Psi = \operatorname{diag}\left\{\psi_{p1}, \psi_{p2}, \ldots, \psi_{pn}, \dot{\psi}_{p1}, \dot{\psi}_{p2}, \ldots, \dot{\psi}_{pn}, \ddot{\psi}_{p1}, \ddot{\psi}_{p2}, \ldots, \ddot{\psi}_{pn}\right\} \le 0 \tag{14}$$

$$b_{pr}^T \nu_p < \gamma \tag{15}$$

$$\gamma d < e^{-\lambda_p T_f} \tag{16}$$

$$c_{pr}^T \delta \prec \nu_p \tag{17}$$

$$\begin{aligned} &\nu_p \preceq \mu_p \nu_q, \\ &\rho_p \preceq \mu_p \rho_q, \\ &\varrho_p \preceq \mu_p \varrho_q, \end{aligned} \tag{18}$$

where $\psi_{pr} = a_{dpr}^T \nu_{pr} - \lambda_p \nu_{pr} + \rho_{pr} + \tau_1 \varrho_{pr}$, $\dot{\psi}_{pr} = a_{dpr}^T \nu_{pr} - (1-h)\rho_{pr}$, $\ddot{\psi}_{pr} = a_{bpr}^T \nu_{pr} - \varrho_{pr}$, $a_{pr}(a_{dpr}, a_{bpr}, b_{pr}, c_{pr})$ represents the rth column vector of the matrix $A_p(A_{dp}, A_{bp}, B_p, C_p)$, $\nu_p = [\nu_{p1}, \nu_{p2}, \ldots, \nu_{pn}]$, $\rho_p = [\rho_{p1}, \rho_{p2}, \ldots, \rho_{pn}]$, $\varrho_p = [\varrho_{p1}, \varrho_{p2}, \ldots, \varrho_{pn}]$, and ν_{pr}, ρ_{pr}, and ϱ_{pr} represent the rth elements of the vectors ν_p, ρ_p, and ϱ_p, respectively. μ_p ($\mu_p \ge 1$) satisfies (18); then system (1) is IO-FTS for any switching signal $\sigma(t)$ with the MDADT

$$T_\alpha > T_\alpha^* = \frac{T_f \ln \mu_p}{\ln\left(e^{-\lambda_p T_f}\right) - \ln(\gamma d)}. \tag{19}$$

Proof. Construct the multiple copositive type Lyapunov-Krasovskii functional for system (1) as follows:

$$\begin{aligned} V_{\sigma(t)}(t) &= V_{\sigma(t)}(t, x(t)) \\ &= x^T(t)\nu_p + \int_{t-d(t)}^t e^{\lambda_p(t-s)} x^T(s)\rho_p ds \\ &\quad + \int_{-\tau_1}^0 \int_{t+\theta}^t e^{\lambda_p(t-s)} x^T(s)\varrho_p ds\, d\theta, \end{aligned} \tag{20}$$

where ν_p, ρ_p, and $\varrho_p \in R_+^n$, $\forall p \in \underline{M}$.

Along the trajectory of system (1) with $u(t) \equiv 0$, we have

$$\begin{aligned} \dot{V}_{\sigma(t)}(t) &= x^T(t) A_p^T \nu_p + x^T(t-d(t)) A_{dp}^T \nu_p \\ &\quad + \int_{t-\tau_2}^t x^T(s) A_{bp}^T \nu_p ds + w^T(t) B_p^T \nu_p \\ &\quad + \lambda_p \int_{t-d(t)}^t e^{\lambda_p(t-s)} x^T(s)\rho_p ds + x^T(s)\rho_p \\ &\quad - \left(1 - \dot{d}(t)\right) e^{\lambda_p d(t)} x^T(t-d(t))\rho_p \\ &\quad + \lambda_p \int_{-\tau_1}^0 \int_{t+\theta}^t e^{\lambda_p(t-s)} x^T(s)\varrho_p ds\, d\theta \\ &\quad + \tau_1 x^T(t)\varrho_p - \int_{-\tau_1}^0 e^{-\lambda_p\theta} x^T(t+\theta)\varrho_p d\theta \\ &\le x^T(t) A_p^T \nu_p + x^T(t-d(t)) A_{dp}^T \nu_p \\ &\quad + \int_{t-\tau_2}^t x^T(s) A_{bp}^T \nu_p ds + w^T(t) B_p^T \nu_p \\ &\quad + \lambda_p \int_{t-d(t)}^t e^{\lambda_p(t-s)} x^T(s)\rho_p ds + x^T(s)\rho_p \\ &\quad - (1-h) e^{\lambda_p d(t)} x^T(t-d(t))\rho_p \\ &\quad + \lambda_p \int_{-\tau_1}^0 \int_{t+\theta}^t e^{\lambda_p(t-s)} x^T(s)\varrho_p ds\, d\theta \\ &\quad + \tau_1 x^T(t)\varrho_p - \int_{t-\tau_1}^t x^T(s)\varrho_p ds. \end{aligned} \tag{21}$$

Combining (20) and (21) yields

$$\begin{aligned}\dot{V}_{\sigma(t)}(t)-\lambda_p V_{\sigma(t)}(t) &\le x^T(t)\left(A_p^T\nu_p-\lambda_p\nu_p+\rho_p+\tau_1\varrho_p\right)\\ &\quad+x^T(t-d(t))\left(A_{dp}^T-(1-h)\rho_p\right)\\ &\quad+\int_{t-\tau_2}^{t}x^T(s)A_{bp}^T\nu_p ds-\int_{t-\tau_1}^{t}x^T(s)\varrho_p ds\\ &\quad+w^T(t)B_p^T\nu_p.\end{aligned}\tag{22}$$

Noting that $\tau_1=\max\{d(t),\tau_2\}$, we have

$$\begin{aligned}&\int_{t-\tau_2}^{t}x^T(s)A_{bp}^T\nu_p ds-\int_{t-\tau_1}^{t}x^T(s)\varrho_p ds\\ &\quad\le\int_{t-\tau_1}^{t}x^T(s)A_{bp}^T\nu_p ds-\int_{t-\tau_1}^{t}x^T(s)\varrho_p ds\\ &\quad=\int_{t-\tau_1}^{t}x^T(s)\left(A_{bp}^T\nu_p-\varrho_p\right)ds.\end{aligned}\tag{23}$$

From (22) and (23), we obtain

$$\begin{aligned}\dot{V}_{\sigma(t)}(t)-\lambda_p V_{\sigma(t)}(t) &\le x^T(t)\left(A_p^T\nu_p-\lambda_p\nu_p+\rho_p+\tau_1\varrho_p\right)\\ &\quad+w^T(t)B_p^T\nu_p\\ &\quad+x^T(t-d(t))\left(A_{dp}^T-(1-h)\rho_p\right)\\ &\quad+\int_{t-\tau_1}^{t}x^T(s)\left(A_{bp}^T\nu_p-\varrho_p\right)ds.\end{aligned}\tag{24}$$

From (14), (15), and (24), we have

$$\dot{V}_{\sigma(t)}(t)-\lambda_p V_{\sigma(t)}(t)\le w^T(t)B_p^T\nu_p\le\gamma w^T(t).\tag{25}$$

Integrating both sides of (25) during the period $[t_k,t)$ for $t\in[t_k,t_{k+1})$ leads to

$$\begin{aligned}V_{\sigma(t)}(t)&\le e^{\lambda_{\sigma(t_k)}(t-t_k)}V_{\sigma(t_k)}(t_k)\\ &\quad+\gamma\int_{t_k}^{t}e^{\lambda_{\sigma(t_k)}(t-s)}w^T(s)ds.\end{aligned}\tag{26}$$

On the other hand, from (18) and (20), one can easily obtain

$$V_{\sigma(t_k)}(t_k)\le\mu_{\sigma(t_k)}V_{\sigma(t_k^-)}(t_k^-).\tag{27}$$

Let N be the switching number of $\sigma(t)$ over $[0,T_f)$, and denote $t_1,\dots,t_k,\dots,t_N$ as the switching instants over the interval $[0,T_f)$. Combining with (2), (26), and (27), we have

$$\begin{aligned}V_{\sigma(t)}(t)&\le e^{\lambda_{\sigma(t_k)}(t-t_k)}V_{\sigma(t_k)}(t_k)\\ &\quad+\gamma\int_{t_k}^{t}e^{\lambda_{\sigma(t_k)}(t-s)}w^T(s)ds\\ &\le\mu_{\sigma(t_k)}e^{\lambda_{\sigma(t_k)}(t-t_k)}V_{\sigma(t_k^-)}(t_k^-)\\ &\quad+\gamma\int_{t_k}^{t}e^{\lambda_{\sigma(t_k)}(t-s)}w^T(s)ds\\ &\le\mu_{\sigma(t_k)}e^{\lambda_{\sigma(t_k)}(t-t_k)}\Big[e^{\lambda_{\sigma(t_{k-1})}(t_k-t_{k-1})}V_{\sigma(t_{k-1})}(t_{k-1})\\ &\quad+\gamma\int_{t_{k-1}}^{t_k}e^{\lambda_{\sigma(t_{k-1})}(t_k-s)}w^T(s)ds\Big]\\ &\quad+\gamma\int_{t_k}^{t}e^{\lambda_{\sigma(t_k)}(t-s)}w^T(s)ds\le\cdots\le\left(\prod_{i=1}^{k}\mu_{\sigma(t_i)}\right)\\ &\quad\cdot e^{\lambda_{\sigma(t_k)}(t-t_k)}e^{\sum_{i=1}^{k-1}\lambda_{\sigma(t_i)}(t_i-t_{i-1})}V_{\sigma(0)}(0)+\left(\prod_{i=1}^{k}\mu_{\sigma(t_i)}\right)\\ &\quad\cdot\gamma\int_{0}^{t_1}e^{\lambda_{\sigma(0)}(t_1-s)}w^T(s)ds+\left(\prod_{i=2}^{k}\mu_{\sigma(t_i)}\right)\\ &\quad\cdot\gamma\int_{t_1}^{t_2}e^{\lambda_{\sigma(t_1)}(t_2-s)}w^T(s)ds+\cdots\\ &\quad+\gamma\int_{t_k}^{t}e^{\lambda_{\sigma(t_k)}(t-s)}w^T(s)ds\le\left(\prod_{p=1}^{M}\mu_p^{N_{\sigma p}(0,t)}\right)\\ &\quad\cdot e^{\sum_{p=1}^{M}\lambda_p T_p(0,t)}V_{\sigma(0)}(0)\\ &\quad+\gamma\int_{0}^{t}\left(\prod_{p=1}^{M}\mu_p^{N_{\sigma p}}(0,t)\right)e^{\sum_{p=1}^{M}\lambda_p T_p(s,t)}w^T(s)ds.\end{aligned}\tag{28}$$

According to (3) and choosing $\lambda=\max_{p\in\underline{M}}(\lambda_p+\ln\mu_p/T_{\alpha p})$, for $t\in[0,T_f)$, one can obtain from (28)

$$\begin{aligned}V_{\sigma(t)}(t)&\le\left(\prod_{p=1}^{M}\mu_p^{T_p(0,t)/T_{\alpha p}}\right)e^{\sum_{p=1}^{M}\lambda_p T_p(0,t)}\left(V_{\sigma(0)}(0)+\gamma d\right)\\ &=e^{\sum_{p=1}^{M}(\lambda_p+\ln\mu_p/T_{\alpha p})T_p(0,t)}\left(V_{\sigma(0)}(0)+\gamma d\right)\\ &\le e^{\lambda T_f}\left(V_{\sigma(0)}(0)+\gamma d\right).\end{aligned}\tag{29}$$

Under the zero initial condition $x(0)=0$, noting the definition of $V_{\sigma(t)}(t)$ and (29), we have

$$\begin{aligned}V_{\sigma(t)}(t)&\ge x^T(t)\nu_p,\\ V_{\sigma(t)}(t)&\le e^{\lambda T_f}\gamma d.\end{aligned}\tag{30}$$

From (3) and (30), one has

$$x^T(t)\nu_p \le e^{\lambda T_f}\gamma d. \quad (31)$$

From (17) and (31), we have

$$y^T(t)\delta = x^T(t)C^T_{\sigma(t)}\delta < x^T(t)\nu_p \le e^{\lambda T_f}\gamma d. \quad (32)$$

From (19) and (32), we obtain

$$y^T(t)\delta < 1. \quad (33)$$

Thus, the proof is completed. □

3.2. IO-FTS Controller Design. In this subsection, the state feedback controller $u(t) = K_{\sigma(t)}x(t)$ will be designed. For system (1), under the controller $u(t)$, the corresponding closed-loop system is given by

$$\begin{aligned} \dot{x}(t) &= \left(A_{\sigma(t)} + G_{\sigma(t)}K_{\sigma(t)}\right)x(t) + A_{d\sigma(t)}x(t-d(t)) \\ &\quad + A_{b\sigma(t)}\int_{t-\tau_2}^{t} x(s)\,ds + B_{\sigma(t)}w(t) \\ y(t) &= C_{\sigma(t)}x(t) \\ x(\theta) &= \varphi(\theta), \quad \theta \in [-\tau_1, 0]. \end{aligned} \quad (34)$$

According to Lemma 4, to guarantee the positivity of system (34), $A_p + G_pK_p$ should be Metzler matrices, $\forall p \in \underline{M}$. Theorem 11 gives some sufficient conditions to guarantee that the closed-loop system (34) is IO-FTS.

Theorem 11. *Consider the positive switched system (34). Given positive constants T_f, λ_p, and γ and a vector $\delta \succ 0$, if there exists a set of positive vectors ν_p, ρ_p, ϱ_p, g_p, $p \in \underline{M}$, (15)–(17) and the following conditions hold:*

$$A_p + G_pK_p \text{ are Metzler matrices.} \quad (35)$$

$$\Phi = \operatorname{diag}\{\phi_{p1}, \phi_{p2}, \ldots, \phi_{pn}, \dot{\phi}_{p1}, \dot{\phi}_{p2}, \ldots, \dot{\phi}_{pn}, \ddot{\phi}_{p1}, \ddot{\phi}_{p2}, \ldots, \ddot{\phi}_{pn}\} \le 0, \quad (36)$$

where $\phi_{pr} = a^T_{dpr}\nu_{pr} - \lambda_p\nu_{pr} + \rho_{pr} + \tau_1\varrho_{pr} + g_{pr}$, $\dot{\phi}_{pr} = a^T_{dpr}\nu_{pr} - (1-h)\rho_{pr}$, $\ddot{\phi}_{pr} = a^T_{bpr}\nu_{pr} - \varrho_{pr}$, $g_p = K^T_pG^T_p\nu_p$, $a_{pr}(a_{dpr}, a_{bpr}, b_{pr}, c_{pr})$ represents the rth column vector of the matrix $A_p(A_{dp}, A_{bp}, B_p, C_p)$, g_{pr} represents the rth elements of vector g_p, $\nu_p = [\nu_{p1}, \nu_{p2}, \ldots, \nu_{pn}]$, $\rho_p = [\rho_{p1}, \rho_{p2}, \ldots, \rho_{pn}]$, $\varrho_p = [\varrho_{p1}, \varrho_{p2}, \ldots, \varrho_{pn}]$, and ν_{pr}, ρ_{pr}, and ϱ_{pr} represent the rth elements of the vectors ν_p, ρ_p, and ϱ_p, respectively. μ_p ($\mu_p \ge 1$) satisfies (18); then the closed-loop system (34) is IO-FTS for any switching signal $\sigma(t)$ with MDADT (19).

Proof. According to Lemma 4, we get that $A_p + G_pK_p$ is a Metzler matrix for each $p \in \underline{M}$. Replacing A_p in (14) with $A_p + G_pK_p$, then letting $g_p = K^T_pG^T_p\nu_p$, similar to Theorem 10, we can get (36) and the resulting closed-loop system (34) is IO-FTS with the MDADT (19).

The proof is completed. □

Next, an algorithm is presented to obtain the feedback gain matrices K_p, $p \in \underline{M}$.

Step 1. By adjusting the parameters λ_p and solving (15)–(18) and (36) via linear programming, positive vectors ν_p, ρ_p, ϱ_p, and g_p can be obtained.

Step 2. Substituting ν_p and g_p into $g_p = K^T_pG^T_p\nu_p$, K_p can be obtained.

Step 3. The gain K_p is substituted into $A_p + G_pK_p$. If $A_p + G_pK_p$ are Metzler matrices, then K_p are admissible. Otherwise, return to Step 1.

4. Numerical Example

Consider the positive switched system (1) with the parameters as follows:

$$A_1 = \begin{bmatrix} -4 & 1 & 2 \\ 1 & -3 & 2 \\ 1 & 2 & -3.5 \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} 0.1 & 0.2 & 0.2 \\ 0.2 & 0.1 & 0.2 \\ 0.1 & 0.1 & 0.2 \end{bmatrix},$$

$$A_{b1} = \begin{bmatrix} 0.1 & 0.2 & 0.3 \\ 0.3 & 0.1 & 0.1 \\ 0.2 & 0.1 & 0.1 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.3 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -5 & 2 & 2 \\ 3 & -5 & 2 \\ 1 & 2 & -3 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} 0.2 & 0.2 & 0.1 \\ 0.2 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.1 \end{bmatrix},$$

$$A_{b2} = \begin{bmatrix} 0.2 & 0.1 & 0.3 \\ 0.1 & 0.2 & 0.1 \\ 0.1 & 0.1 & 0.2 \end{bmatrix},$$

$$G_2 = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.3 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.2 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0.2 \\ 0.1 \\ 0.2 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.2 & 0.3 & 0.2 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.3 & 0.2 & 0.2 \end{bmatrix}. \quad (37)$$

Let $d(t) = 0.1 + 0.1\sin(t)$, then we get $\tau_2 = 0.1$, $\tau_1 = \max\{d(t), \tau_2\} = 0.2$, $h = 0.1$. Choose $\gamma = 5$, $\lambda_1 = 0.18$, $\lambda_2 = 0.22$, $\mu_1 = 1.1$, $\mu_2 = 1.21$, $w(t) = 0.05e^{-0.5t}$, $d = 0.01$, $\delta = [1.73 \ \ 2.28]^T$. Solving the inequalities in Theorem 11 by linear programming, we have

$$\nu_1 = \begin{bmatrix} 6.7833 \\ 9.4931 \\ 10.2540 \end{bmatrix},$$

$$\rho_1 = \begin{bmatrix} 4.4016 \\ 4.6341 \\ 6.0117 \end{bmatrix},$$

$$\varrho_1 = \begin{bmatrix} 9.9060 \\ 5.5548 \\ 7.1065 \end{bmatrix},$$

$$g_1 = \begin{bmatrix} -1.2804 \\ -2.9308 \\ -2.3817 \end{bmatrix},$$

$$\nu_2 = \begin{bmatrix} 8.1623 \\ 8.6606 \\ 11.9049 \end{bmatrix},$$

$$\rho_2 = \begin{bmatrix} 4.3576 \\ 5.5309 \\ 5.7083 \end{bmatrix},$$

$$\varrho_2 = \begin{bmatrix} 9.9693 \\ 6.1282 \\ 7.5314 \end{bmatrix},$$

$$g_2 = \begin{bmatrix} -1.8343 \\ -2.4662 \\ -2.7991 \end{bmatrix}. \quad (38)$$

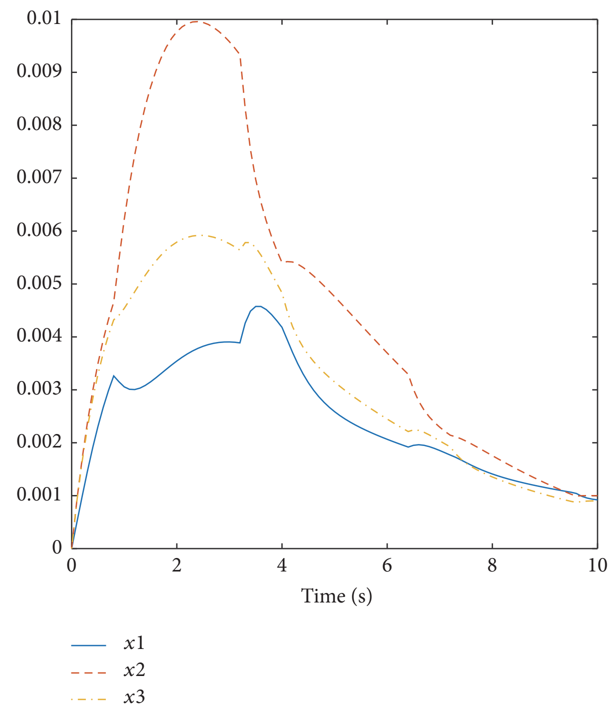

Figure 1: State trajectories of closed-loop system (1).

By $g_p = K_p^T G_p^T \nu_p$, we get

$$\begin{aligned} K_1 &= [-1.8877 \ \ -1.5437 \ \ -0.7742], \\ K_2 &= [-2.2474 \ \ -1.4238 \ \ -0.7837]. \end{aligned} \quad (39)$$

It is easy to verify that $A_p + G_p K_p$ are Metzler matrices. Then, from (19), we can obtain $T_{\alpha 1}^* = 0.7971$, $T_{\alpha 2}^* = 2.3956$. Choose $T_{\alpha 1} = 0.8 > T_{\alpha 1}^*$ and $T_{\alpha 2} = 2.4 > T_{\alpha 2}^*$. The simulation results are shown in Figures 1–3, where $x(0) = [0 \ \ 0 \ \ 0]^T$. The state trajectories of the closed-loop system with MDADT are shown in Figure 1. The switching signal $\sigma(t)$ with MDADT is depicted in Figure 2. Figure 3 plots the evolution of $y^T(t)\delta$ of system (1). From Figure 3, we easily know that $y^T(t)\delta < 1$, which implies that the resulting closed-loop system (1) is IO-FTS.

5. Conclusions

In this paper, the problem of IO-FTS of positive switched systems with time-varying and distributed delays is investigated. The definition of IO-FTS of continuous positive switched systems is proposed. Then, by constructing multiple linear copositive Lyapunov functions, a state feedback controller is designed. Based on the MDADT approach, some sufficient

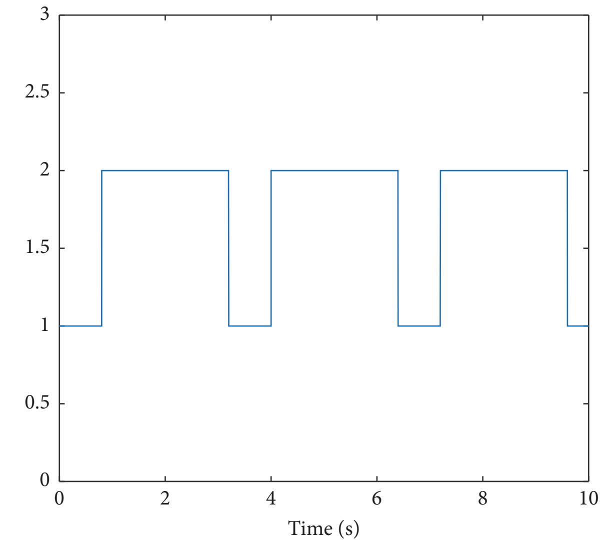

FIGURE 2: Switching signal of system (1) with MDADT.

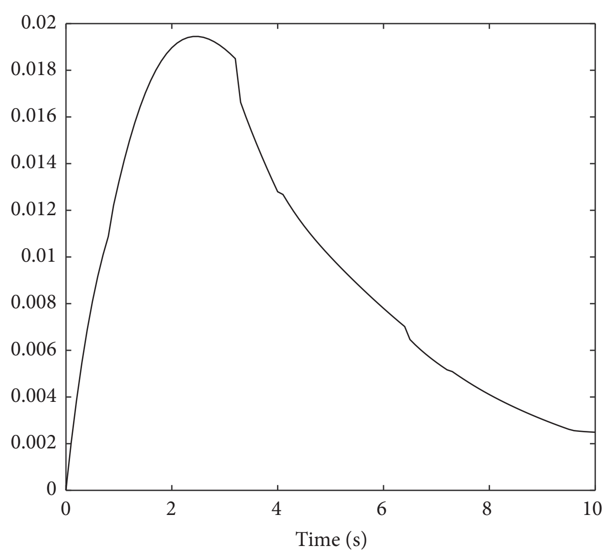

FIGURE 3: The evolution of $y^T(t)\delta$ of system (1).

conditions are obtained to guarantee that the closed-loop system is IO-FTS. Such sufficient conditions can be solved by linear programming. Our further work will focus on the IO-FTS of positive switched nonlinear systems with multiple delays.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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