Hindawi
Journal of Control Science and Engineering
Volume 2017, Article ID 5072308, 13 pages
https://doi.org/10.1155/2017/5072308

Research Article

Finite-Time Synchronization of Complex Dynamical Networks with Time-Varying Delays and Nonidentical Nodes

Ahmadjan Muhammadhaji, Abdujelil Abdurahman, and Haijun Jiang

College of Mathematics and System Sciences, Xinjiang University, Urumqi 830046, China

Correspondence should be addressed to Ahmadjan Muhammadhaji; ahmatjanam@aliyun.com

Received 27 November 2016; Revised 25 February 2017; Accepted 28 February 2017; Published 21 March 2017

Academic Editor: James Lam

In this paper, we investigated the finite-time synchronization (FTS) problem for a class of time-delayed complex networks with nonidentical nodes onto any uniformly smooth state. By employing the finite-time stability theorem and designing two types of novel controllers, we obtained some simple sufficient conditions for the FTS of addressed complex networks. Furthermore, we also analyzed the effects of control variables on synchronization performance. Finally, we showed the effectiveness and feasibility of our methods by giving two numerical examples.

1. Introduction

In the last decades, complex networks have been deeply investigated in various types of subjects, such as biology, engineering, physics, and mathematics [1–3]. In general, a complex network can be depicted as a large set of nodes connected by edges, where every node is a basic unit with specific dynamics. In fact, the structure of many types of natural and artificial systems, such as genetic networks, metabolic pathways, social networks, electrical power distribution networks, and World Wide Web (WWW), can be modeled via complex networks.

Synchronization is unique in nature and can play an extremely vital role in many fields of science including biology, climatology, sociology, and ecology [4, 5]. Over the past two decades, synchronization of complex networks with identical dynamical topology has been broadly studied in different fields of engineering and sciences owing to the fact that they not only can well depict a great number of natural phenomena, but also have many useful applications in biological systems [6], secure communication [7], image processing [8], and so on. For this reason, many useful methods have been introduced for the synchronization of complex networks without control [9–12]. However, sometimes we cannot achieve the synchronization of network without adding any controller to the dynamics of individual node. Thus, how to synchronize the complex networks by designing a suitable controller is seen to be a most significant topic in both theory and application. As a result, many useful approaches have been developed to achieve chaos stabilization and chaos synchronization, such as adaptive control [13, 14], pinning control [15], impulsive control [16, 17], sliding mode control [18], and intermittent control [19, 20].

Time delays inevitably exist in natural and man-made systems and cannot be neglected; for instance, delay effects cannot be ignored in the communication systems due to the limited switching speed of the hardware [21]. So far, in most of the existing works the networks with coupling time delays were considered. However, the time delays in the dynamical nodes [22–28], which can be more complicated, are still neglected in most of the existing works. In addition, it maybe unpractical to always use the hypothesis that all of the network nodes are the same since some real-world complex networks can be modeled by using different dynamical nodes [25]. Taking a software community network or metabolic network as an example, the dynamics of any two nodes in different communities are totally different, while the dynamics of each individual node in every community can be described by the same functional units. When the dynamics of nodes in a complex network are allowed to be nonidentical, the synchronization approaches for networks consisting of identical nodes will not work anymore. Thus, it is of great importance to develop new synchronization approaches for time-delayed complex networks with nonidentical nodes [25–30]. In [26], some exponential synchronization criteria for a class of complex networks with nonidentical nodes were established via combining the local intermittent controller with the open-loop controller. In [27], the outer synchronization of two complex networks with nonidentical dynamical nodes and coupling time delays was investigated by employing the adaptive control technique. In [28], by using pinning control scheme, the cluster synchronization of complex dynamical networks with time-delayed coupling and dynamic nodes was studied.

Many of the results mentioned above have been employed to ensure the asymptotical or exponential stability of error dynamics, which means that the synchronization between the controlled system and the desired system can only be achieved in the infinite horizon. In practical application, however, it is more desirable that the synchronization aim is achieved in a finite-time. As is known to all, the finite-time control technique has been used as an efficient approach to realize the synchronization in a given time due to the fact that the finite-time control schemes may lead to better system performance such as better robustness and disturbance rejection properties [31–35]. In [36], the authors investigated the FTS problem for a class of complex networks with stochastic noise perturbations. In [37], the FTS between two complex networks with general coupling was studied by employing two different types of controllers, that is, periodically intermittent control and impulsive control. In [38], based on the finite-time stability theory and nonlinear control theory, the authors were concerned with FTS synchronization of class of nonidentical drive-response chaotic systems with time-varying delay. In [39], FTS problem of a class of fractional-order memristor-based neural networks with time delays was studied by using Laplace transform method and generalized Gronwall's inequality technique. However, it is worth pointing out that most of the existing results on the FTS of complex networks are concerned with those identical nodes and delay-independent or constant delays, and very little attention has been paid to solving the problem which emerged from complex networks with nonidentical dynamical nodes and time-varying delays.

Inspired by the above analysis, in the paper, we deal with the problem of FTS for a class of complex networks with nonidentical nodes and time-varying delays. By employing the well-known finite-time stability theorem, designing two types of novel controllers, and using some inequality techniques, we develop some simple but useful sufficient criteria for the FTS of the addressed network.

Notations. In this paper, I_N denotes the $N \times N$ identity matrix and sgn represents the sign function. For a vector $x = (x_1, x_2, \ldots, x_n)^T$, $|x|$ is the vector of the form $|x| = (|x_1|, |x_2|, \ldots, |x_n|)^T$, and its Euclidean norm $\|\cdot\|_2$ is defined as $\|x\|_2^2 = x^T x$, where T denotes transposition. $A = (a_{ij})_{n\times n}$ denotes a matrix of n-dimension, $A^s = (1/2)(A + A^T)$, and $\|A\|_2 = \sqrt{\lambda_{\max}(A^T A)}$, where $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue of a symmetric matrix. The notation $A \le 0$ represents A is symmetric and seminegative definite matrix.

The rest of the paper is organized as follows. In Section 2, the FTS problem of a complex network with nonidentical nodes and time-varying delays is introduced, and some related definitions and preliminary lemmas are given. Next section is devoted to investigating the FTS of the addressed networks. In Section 4, two numerical examples with their simulations are given to demonstrate the effectiveness of the obtained theoretical results. Finally, we conclude the paper with some general conclusions in Section 5.

2. Preliminaries

In the paper, we consider the following controlled complex networks model consisting of N nonidentical nodes, in which every node is an n-dimensional dynamical system described by

$$\dot{x}_i(t) = f_i(t, x_i(t), x_i(t - \tau_i(t))) + c\sum_{j=1}^{N} a_{ij}\Gamma x_j(t) + u_i(t), \quad i \in \mathcal{I} \triangleq \{1, 2, \ldots, N\}, \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T$ is state variable of the ith individual node, $f_i : R^+ \times R^n \times R^n \to R^n$ is a vector-valued continuous function, $\tau_i(t)$ is a time-varying delay and $0 \le \tau_i(t) \le \tau_i$ for each $i \in \mathcal{I}$, the positive constant c is the coupling strength, the $\Gamma \in R^{n\times n}$ is the inner connecting matrix between nodes, and the constant matrix $A = (a_{ij})_{N\times N}$ denotes the linear coupling configuration of the network, where a_{ij} is given as follows: if there is a connection between node i and node j $(i \ne j)$, then $a_{ij} > 0$; otherwise, $a_{ij} = 0$, and the diagonal elements of matrix A are defined by $a_{ii} = -\sum_{j=1, j\ne i}^{N} a_{ij}$, and $u_i(t)$ is the control input.

The initial values of system (1) are given by

$$x_i(s) = \phi_i(s), \quad s \in [-\tau, 0], \tag{2}$$

where $\tau = \max_{i\in\mathcal{I}}\{\tau_i\}$, $\phi_i(s) = (\phi_{i1}(s), \phi_{i2}(s), \ldots, \phi_{in}(s))^T \in C([-\tau, 0], R^n)$, which denotes the Banach space of all continuous functions mapping $[-\tau, 0]$ into R^n with norm defined by

$$\|\varphi\|_2 = \sup_{s\in[-\tau,0]} \left(\sum_{i=1}^{n} |\varphi_i(s)|^2\right)^{1/2}, \quad \text{for } \varphi \in C([-\tau, 0], R^n). \tag{3}$$

The main aim of this paper is to finite-timely synchronize the states of networks (1) onto any smooth dynamics $s(t)$, where $s(t) \in R^n$ can be any desired state: equilibrium point, a nontrivial periodic orbit, or even a chaotic orbit. That is, by designing a suitable feedback controller in system (1), there exists time t^* such that

$$x_1(t) = x_2(t) = \cdots = x_2(t) = s(t), \quad \text{for any } t \ge t^*. \tag{4}$$

Throughout this paper, for system (1), we assume that the following hypothesis is satisfied.

(H_1) For every vector-valued function f_i, there exist positive constants L_i^1 and L_i^2 such that

$$\begin{aligned}&\left[x_i(t)-y_i(t)\right]^T\left[f_i\left(t,x_i(t),x_i\left(t-\tau_i(t)\right)\right)\right.\\&\quad\left.-f_i\left(t,y_i(t),y_i\left(t-\tau_i(t)\right)\right)\right]\le L_i^1\left[\left|x_i(t)-y_i(t)\right|\right]^T\\&\quad\cdot\left[\left|x_i\left(t-\tau_i(t)\right)-y_i\left(t-\tau_i(t)\right)\right|\right]+L_i^2\left[x_i(t)\right.\\&\quad\left.-y_i(t)\right]^T\left[x_i(t)-y_i(t)\right],\\&\qquad\forall x_i(t),y_i(t)\in R^n,\ i\in\mathcal{I}.\end{aligned}\tag{5}$$

Definition 1. Assume that $s(t)\in R^n$ is any smooth dynamics. The complex network (1) is said to be synchronized onto the homogeneous state $s(t)$ in a finite-time if, for a suitable designed feedback controller $u_i(t)$, there exists a constant $t^*>0$ such that

$$\lim_{t\to t^*}\left\|x_i(t)-s(t)\right\|_2=0,\tag{6}$$

and $\|x_i(t)-s(t)\|_2=0$ for $t>t^*$, $i\in\mathcal{I}$. The constant t^* is called the settling time of system (1).

Lemma 2 (see [36]). *Assume that $V(t)$ is a positive definite continuous function. If $V(t)$ satisfies the differential inequality*

$$\dot V(t)\le-\varpi V^{\varrho}(t),\quad\forall t\ge t_0,\ V(t_0)\ge0,\tag{7}$$

where $\varpi>0$, $0<\varrho<1$ are two constants, then, for any given t_0, $V(t)$ satisfies the following inequality:

$$\begin{aligned}&V^{1-\varrho}(t)\le V^{1-\varrho}(t_0)-\varpi(1-\varrho)(t-t_0),\quad t_0\le t\le t^*,\\&V(t)\equiv0,\quad\forall t\ge t^*,\end{aligned}\tag{8}$$

with t^* given by

$$t^*=t^0+\frac{V^{1-\varrho}(t_0)}{\varpi(1-\varrho)}.\tag{9}$$

Lemma 3 (Jesen inequality [34]). *If $b_1,b_2,\dots,b_p$, $p\in N^+$ are positive numbers and $0<m<n$, then*

$$\left(\sum_{i=1}^{p}b_i^n\right)^{1/n}\le\left(\sum_{i=1}^{p}b_i^m\right)^{1/m}.\tag{10}$$

3. Main Results

In this section, some effective control schemes are developed to finite-timely synchronize the complex network (1) to any smooth dynamics $s(t)$. First, we design controller $u_i(t)$ of system (1) as the following form:

$$\begin{aligned}u_i(t)&=F_i(t)-\eta_ie_i(t)-L_i^1\mathrm{sgn}\left(e_i(t)\right)\left|e_i\left(t-\tau_i(t)\right)\right|\\&\quad-k\,\mathrm{sgn}\left(e_i(t)\right)\left|e_i(t)\right|^{\beta},\end{aligned}\tag{11}$$

where $F_i(t)=\dot s(t)-f_i(t,s(t),s(t-\tau_i(t)))$, $|e_i(t)|^{\beta}=(|e_{i1}(t)|^{\beta},|e_{i2}(t)|^{\beta},\dots,|e_{in}(t)|^{\beta})^T$, $\mathrm{sgn}(e_i(t))=\mathrm{diag}(\mathrm{sgn}(e_{i1}(t)),\mathrm{sgn}(e_{i2}(t)),\dots,\mathrm{sgn}(e_{in})(t))$, $\eta_i>0$ is a positive constant determined later, k is a tunable constant, and the real number β satisfies $0<\beta<1$.

Let synchronization error $e_i(t)=x_i(t)-s(t)$ for $i\in\mathcal{I}$. Then, according to system (1) and the control law (11), the error dynamical system can be derived as

$$\begin{aligned}\dot e_i(t)&=\overline{f}_i\left(t,e_i(t),e_i\left(t-\tau_i(t)\right)\right)+c\sum_{j=1}^{N}a_{ij}\Gamma e_j(t)\\&\quad+\overline{u}_i(t),\quad i\in\mathcal{I},\end{aligned}\tag{12}$$

where

$$\begin{aligned}&\overline{f}_i\left(t,e_i(t),e_i\left(t-\tau_i(t)\right)\right)\\&\quad=f_i\left(t,x_i(t),x_i\left(t-\tau_i(t)\right)\right)\\&\qquad-f_i\left(t,s(t),s\left(t-\tau_i(t)\right)\right),\\&\overline{u}_i(t)\\&\quad=-\eta_ie_i(t)-L_i^1\mathrm{sgn}\left(e_i(t)\right)\left|e_i\left(t-\tau_i(t)\right)\right|\\&\qquad-k\,\mathrm{sgn}\left(e_i(t)\right)\left|e_i(t)\right|^{\beta},\\&\qquad\qquad i\in\mathcal{I}.\end{aligned}\tag{13}$$

It is not difficult to see that the global FTS of the controlled complex network (1) is realized when the zero solution of the error system (12) is globally finite-time stable and this can be ensured by the following theorems.

Theorem 4. *Under hypothesis (H_1), assume that the following inequality is satisfied:*

$$\lambda_L+\alpha\lambda_{\widehat{A}^s}-\eta\le0,\tag{14}$$

where $\lambda_L=\lambda_{\max}(L)$, $\lambda_{\widehat{A}^s}=\lambda_{\max}(\widehat{A}^s)$, $\alpha=c\|\Gamma\|_2$, $\eta=\min_{i\in\mathcal{I}}\{\eta_i\}$, $L=\mathrm{diag}(L_1^2,L_2^2,\dots,L_N^2)$, $\widehat{A}=(\widehat{a}_{ij})_{N\times N}$, $\widehat{a}_{ij}=a_{ij}$ for $i\ne j$ and $\widehat{a}_{ii}=(\rho_{\min}/\|\Gamma\|_2)a_{ii}$, $\rho_{\min}$ is minimum eigenvalue of Γ^s. Then, the controlled complex network (1) is synchronized to the given smooth dynamics $s(t)$ in a finite-time

$$t^*=\frac{V(0)^{1-\delta}}{k(1-\delta)},\tag{15}$$

where $V(0)=(1/2)\sum_{i=1}^{N}e_i^T(0)e_i(0)$ and $\delta=(1+\beta)/2$.

Proof. Let $e(t)=(e_1^T(t),e_2^T(t),\dots,e_N^T(t))^T$ and construct the following Lyapunov function:

$$V(t)=\frac{1}{2}\sum_{i=1}^{N}e_i^T(t)e_i(t).\tag{16}$$

Calculating the time derivative of $V(t)$ along the trajectory of system (12) leads to

$$\begin{aligned}\frac{dV(t)}{dt} &= \sum_{i=1}^{N} e_i^T(t)\dot{e}_i(t) = \sum_{i=1}^{N} e_i^T(t) \\ &\cdot \left(\overline{f}_i(t, e_i(t), e_i(t-\tau_i(t))) + c\sum_{j=1}^{N} a_{ij}\Gamma e_j(t)\right. \\ &\left. + \overline{u}_i(t)\right) = \sum_{i=1}^{N} e_i^T(t)\overline{f}_i(t, e_i(t), e_i(t-\tau_i(t))) \\ &+ c\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}e_i^T(t)\Gamma e_j(t) + \sum_{i=1}^{N} e_i^T(t)\overline{u}_i(t).\end{aligned} \tag{17}$$

In view of the definition of $\overline{u}_i(t)$, we have

$$\begin{aligned}\frac{dV(t)}{dt} &= \sum_{i=1}^{N} e_i^T(t)\overline{f}_i(t, e_i(t), e_i(t-\tau_i(t))) \\ &+ c\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}e_i^T(t)\Gamma e_j(t) - \sum_{i=1}^{N}\eta_i e_i^T(t)e_i(t) \\ &- \sum_{i=1}^{N} L_i^1 e_i^T(t)\operatorname{sgn}(e_i(t))\left|e_i(t-\tau_i(t))\right| \\ &- k\sum_{i=1}^{N} e_i^T(t)\operatorname{sgn}(e_i(t))\left|e_i(t)\right|^{\beta}.\end{aligned} \tag{18}$$

From the hypothesis (H_1), we get

$$\begin{aligned}\frac{dV(t)}{dt} &\le \sum_{i=1}^{N} L_i^1\left|e_i^T(t)\right|\left|e_i(t-\tau_i(t))\right| \\ &+ \sum_{i=1}^{N} L_i^2 e_i^T(t)e_i(t) + c\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}e_i^T(t)\Gamma e_j(t) \\ &- \sum_{i=1}^{N}\eta_i e_i^T(t)e_i(t) \\ &- \sum_{i=1}^{N} L_i^1\left|e_i^T(t)\right|\left|e_i(t-\tau_i(t))\right| \\ &- k\sum_{i=1}^{N}\sum_{j=1}^{N}\left|e_{ij}(t)\right|^{1+\beta} \\ &\le \sum_{i=1}^{N} L_i^2 e_i^T(t)e_i(t) + c\sum_{i=1}^{N}\rho_{\min} a_{ii}e_i^T(t)e_i(t) \\ &+ \alpha\sum_{i,j=1,i\neq j}^{N} a_{ij}\left\|e_i(t)\right\|_2\left\|e_j(t)\right\|_2 \\ &- \sum_{i=1}^{N}\eta_i e_i^T(t)e_i(t) - k\sum_{i=1}^{N}\sum_{j=1}^{N}\left|e_{ij}(t)\right|^{1+\beta} \\ &= \overline{e}^T(t)\left(L + \alpha\widehat{A}^s - \Xi\right)\overline{e}(t) \\ &- k\sum_{i=1}^{N}\sum_{j=1}^{N}\left|e_{ij}(t)\right|^{1+\beta},\end{aligned} \tag{19}$$

where $\Xi = \operatorname{diag}(\eta_1, \eta_2, \dots, \eta_N)$ and $\overline{e}(t) = (\|e_1(t)\|_2, \|e_2(t)\|_2, \dots, \|e_N(t)\|_2)^T$. Since $\widehat{A}^s$ is the symmetric matrix, from inequality (14), we have

$$\begin{aligned}\frac{dV(t)}{dt} &\le \overline{e}^T(t)\left(\lambda_L + \alpha\lambda_{\widehat{A}^s} - \eta\right)\overline{e}(t) \\ &- k\sum_{i=1}^{N}\sum_{j=1}^{N}\left|e_{ij}(t)\right|^{1+\beta} \le -k\sum_{i=1}^{N}\sum_{j=1}^{N}\left|e_{ij}(t)\right|^{1+\beta}.\end{aligned} \tag{20}$$

Using Lemma 3, one has

$$\left(\sum_{i=1}^{N}\sum_{j=1}^{N}\left|e_{ij}(t)\right|^{1+\beta}\right)^{1/(1+\beta)} \ge \left(\sum_{i=1}^{N}\sum_{j=1}^{N}\left|e_{ij}(t)\right|^{2}\right)^{1/2}. \tag{21}$$

Hence,

$$\begin{aligned}\sum_{i=1}^{N}\sum_{j=1}^{N}\left|e_{ij}(t)\right|^{1+\beta} &\ge \left(\sum_{i=1}^{N}\sum_{j=1}^{N}\left|e_{ij}(t)\right|^{2}\right)^{(1+\beta)/2} \\ &= \left(\sum_{i=1}^{N} e_i^T(t)e_i(t)\right)^{(1+\beta)/2}.\end{aligned} \tag{22}$$

Thus, from (20) and the above inequality, we get

$$\begin{aligned}\frac{dV(t)}{dt} &\le -k\sum_{i=1}^{N}\sum_{j=1}^{N}\left|e_{ij}(t)\right|^{1+\beta} \\ &\le -k\left(\sum_{i=1}^{N} e_i^T(t)e_i(t)\right)^{(1+\beta)/2} \\ &= -kV^{(1+\beta)/2}(t).\end{aligned} \tag{23}$$

Then, from Lemma 2, $V(t)$ converges to zero in a finite-time t^*, where t^* is given by

$$t^* = \frac{V(0)^{1-\delta}}{k(1-\delta)}, \tag{24}$$

where $\delta = (1+\beta)/2$. Therefore, according to Definition 1, the controlled complex network (1) is finite-timely synchronized in the settling time t^*. The proof of Theorem 4 is now completed. □

Theorem 5. *Suppose that the hypothesis (H_1) holds and Γ is a positive definite diagonal matrix with diagonal elements $\gamma_1, \gamma_2, \ldots, \gamma_N$; if the inequality is satisfied*

$$\left(L_j^2 - \eta_j\right) I_N + c\gamma_j A \leq 0, \quad \forall j \in \mathcal{I}, \tag{25}$$

then the controlled complex network (1) is synchronized to the given smooth dynamics $s(t)$ in a finite-time

$$t^* = \frac{V(0)^{1-\delta}}{k(1-\delta)}, \tag{26}$$

where $V(0)$ and δ are the same as defined in Theorem 4.

Proof. Let $e(t) = (e_1^T(t), e_2^T(t), \ldots, e_N^T(t))^T$ and consider the following Lyapunov function:

$$V(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)\, e_i(t). \tag{27}$$

The time derivative of $V(t)$ along the trajectory of (12) is

$$\begin{aligned}\frac{dV(t)}{dt} &= \sum_{i=1}^{N} e_i^T(t)\, \dot{e}_i(t) \\ &= \sum_{i=1}^{N} e_i^T(t)\, \overline{f}_i\left(t, e_i(t), e_i(t-\tau_i(t))\right) \\ &\quad + c\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij} e_i^T(t)\, \Gamma e_j(t) + \sum_{i=1}^{N} e_i^T(t)\, \overline{u}_i(t).\end{aligned} \tag{28}$$

From the hypothesis (H_1) and the definition of $\overline{u}_i(t)$, we get

$$\begin{aligned}\frac{dV(t)}{dt} &\leq \sum_{i=1}^{N} L_i^1 \left|e_i^T(t)\right| \left|e_i(t-\tau_i(t))\right| \\ &\quad + \sum_{i=1}^{N} L_i^2 e_i^T(t)\, e_i(t) + c\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij} e_i^T(t)\, \Gamma e_j(t) \\ &\quad - \sum_{i=1}^{N} \eta_i e_i^T(t)\, e_i(t) \\ &\quad - \sum_{i=1}^{N} L_i^1 \left|e_i^T(t)\right| \left|e_i(t-\tau_i(t))\right| \\ &\quad - k\sum_{i=1}^{N}\sum_{j=1}^{N} \left|e_{ij}(t)\right|^{1+\beta} \\ &\leq \sum_{j=1}^{N} L_j^2 \tilde{e}_j^T(t)\, \tilde{e}_j(t) + c\sum_{j=1}^{N} \gamma_j \tilde{e}_j^T(t)\, A \tilde{e}_j(t) \\ &\quad - \sum_{j=1}^{N} \eta_j \tilde{e}_j^T(t)\, \tilde{e}_j(t) - k\sum_{i=1}^{N}\sum_{j=1}^{N} \left|e_{ij}(t)\right|^{1+\beta} \\ &= \sum_{j=1}^{N} \tilde{e}_j^T(t) \left[\left(L_j^2 - \eta_j\right) I_N + c\gamma_j A\right] \tilde{e}_j(t) \\ &\quad - k\sum_{i=1}^{N}\sum_{j=1}^{N} \left|e_{ij}(t)\right|^{1+\beta},\end{aligned} \tag{29}$$

where I_N is the identity matrix and $\tilde{e}_j(t) = [\tilde{e}_{1j}(t), \tilde{e}_{2j}(t), \ldots, \tilde{e}_{Nj}(t)]^T$ is the jth column vector of the $e(t)$. From inequality (25), we have

$$\frac{dV(t)}{dt} \leq -k\sum_{i=1}^{N}\sum_{j=1}^{N} \left|e_{ij}(t)\right|^{1+\beta}. \tag{30}$$

From the analysis in Theorem 4, we have

$$\sum_{i=1}^{N}\sum_{j=1}^{N} \left|e_{ij}(t)\right|^{1+\beta} \geq \left(\sum_{i=1}^{N} e_i^T(t)\, e_i(t)\right)^{(1+\beta)/2}. \tag{31}$$

Thus, together with (20), Lemma 3, and the above inequality, we get

$$\begin{aligned}\frac{dV(t)}{dt} &\leq -k\left(\sum_{i=1}^{N} e_i^T(t)\, e_i(t)\right)^{(1+\beta)/2} \\ &= -kV^{(1+\beta)/2}(t).\end{aligned} \tag{32}$$

By Lemma 2, $V(t)$ converges to zero in a finite-time t^*, where t^* is given by

$$t^* = \frac{V(0)^{1-\delta}}{k(1-\delta)}. \tag{33}$$

Therefore, according to Definition 1, the controlled complex network (1) is finite-timely synchronized in the settling time t^*. The proof of Theorem 5 is now completed. □

In the controlled network (1), if the evolution function $f_i = f_i(t, x_i(t))$ for $i \in \mathcal{I}$, then system (1) is reduced to the following complex network:

$$\dot{x}_i(t) = f_i(t, x_i(t)) + c\sum_{j=1}^{N} a_{ij}\Gamma x_j(t) + u_i(t), \quad i \in \mathcal{I}. \tag{34}$$

Simultaneously assumption (H_1) can be replaced by the following inequality.

(H_1') For all $x_i, y_i \in R^n$ and $i \in \mathcal{I}$, there exist constants L_i such that

$$\begin{aligned}&\left[x_i(t) - y_i(t)\right]^T \left[f_i(t, x_i(t)) - f_i(t, y_i(t))\right] \\ &\quad \leq L_i \left[x_i(t) - y_i(t)\right]^T \left[x_i(t) - y_i(t)\right].\end{aligned} \tag{35}$$

If we let

$$u_i(t) = \dot{s}(t) - f_i(t, s(t)) - R_i(t), \qquad (36)$$

where $R_i(t) = \eta_i e_i(t) - k\,\mathrm{sgn}(e_i(t))|e_i(t)|^{\beta}$, then, the error system is the following form:

$$\dot{e}_i(t) = \overline{f}_i(t, e_i(t)) + c\sum_{j=1}^{N} a_{ij}\Gamma e_j(t) + R_i(t), \quad i \in \mathcal{I}. \qquad (37)$$

Thus, for the FTS of controlled complex network (34), we have following corollaries.

Corollary 6. *Assume that hypothesis* (H_1') *holds. If* $\eta_1, \eta_2, \ldots, \eta_N$ *satisfy inequality (14), then the controlled complex network (34) is finite-timely synchronized to the given smooth dynamics* $s(t)$ *under controller (36). Moreover, the synchronization time is estimated by*

$$t^* = \frac{V(0)^{1-\delta}}{k(1-\delta)}, \qquad (38)$$

where $V(0)$ *and* δ *are the same as defined in Theorem 4.*

Corollary 7. *Under the hypothesis* (H_1')*, assume that* Γ *is a positive definite diagonal matrix. If* $\eta_1, \eta_2, \ldots, \eta_N$ *satisfy inequality (25), then the controlled complex network (34) is finite-timely synchronized to the given smooth dynamics* $s(t)$ *under controller (36). Moreover, the synchronization time is estimated by*

$$t^* = \frac{V(0)^{1-\delta}}{k(1-\delta)}. \qquad (39)$$

Remark 8. In Theorems 4 and 5, by using special finite-time controller, we achieved the FTS of nonidentical complex network (1) onto any smooth goal dynamics $s(t)$. However, the used control law $u_i(t)$ is somehow expensive and not easily applicable, especially if the governing functions f_i of system (1) satisfy some special conditions. Below, we will modify the adaptive laws $u_i(t)$ to improve the applicability of our results.

Suppose that the evolution functions f_i are bounded and let

$$\begin{aligned}\overline{u}_i(t) = {} & -\eta_i e_i(t) - \mathrm{sgn}(e_i(t)) M_i \\ & - k\,\mathrm{sgn}(e_i(t)) |e_i(t)|^{\beta}, \quad i \in \mathcal{I}.\end{aligned} \qquad (40)$$

Then, for the FTS of complex system (1), we have following results.

Theorem 9. *Assume that evolution functions* f_i *in system (1) satisfy*

$$\begin{aligned}&\left|f_i(t, u(t)), u(t - \tau_i(t))\right| \le M_i, \\ &\qquad \forall u(t) \in R^n, \ i \in \mathcal{I},\end{aligned} \qquad (41)$$

where $M_i = [M_{i1}, M_{i2}, \ldots, M_{in}]^T$*. If the error system (12) is controlled with controller (40), and its control strengths* η_i *satisfy*

$$\alpha\widehat{A}^s - \Xi \le 0, \qquad (42)$$

where α, $\widehat{A}^s$ *and* Ξ *are the same as in Theorem 4, then the controlled complex network (1) is finite-timely synchronized to the given smooth dynamics* $s(t)$ *in settling time* t^**, where* t^* *is the same as in (15).*

Proof. Consider the following Lyapunov function:

$$V(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^T(t) e_i(t). \qquad (43)$$

Taking the time derivative of $V(t)$ along the solutions of (12), from (40) and (42), we have

$$\begin{aligned}\frac{dV(t)}{dt} = {} & \sum_{i=1}^{N} e_i^T(t)\overline{f}_i(t, e_i(t), e_i(t - \tau_i(t))) \\ & + c\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij} e_i^T(t)\Gamma e_j(t) + \sum_{i=1}^{N} e_i^T(t)\overline{u}_i(t) \\ \le {} & \sum_{i=1}^{N}\left|e_i^T(t)\right| M_i + c\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij} e_i^T(t)\Gamma e_j(t) \\ & - \sum_{i=1}^{N}\eta_i e_i^T(t) e_i(t) - \sum_{i=1}^{N} e_i^T(t)\,\mathrm{sgn}(e_i(t)) M_i \\ & - k\sum_{i=1}^{N} e_i^T(t)\,\mathrm{sgn}(e_i(t))|e_i(t)|^{\beta} \\ \le {} & c\sum_{i=1}^{N}\rho_{\min} a_{ii} e_i^T(t) e_i(t) \\ & + \alpha\sum_{i,j=1, i\ne j}^{N} a_{ij}\left\|e_i(t)\right\|_2\left\|e_j(t)\right\|_2 \\ & - \sum_{i=1}^{N}\eta_i e_i^T(t) e_i(t) \\ & - k\sum_{i=1}^{N} e_i^T(t)\,\mathrm{sgn}(e_i(t))|e_i(t)|^{\beta} \\ = {} & \overline{e}^T(t)\left(\alpha\widehat{A}^s - \Xi\right)\overline{e}(t) - k\sum_{i=1}^{N}\sum_{j=1}^{N}\left|e_{ij}(t)\right|^{1+\beta} \\ \le {} & -kV^{(1+\beta)/2}(t).\end{aligned} \qquad (44)$$

Thus, from Lemma 2, $V(t)$ converges to zero in a finite-time t^*, where t^* is given by

$$t^* = \frac{V(0)^{1-\delta}}{k(1-\delta)}. \qquad (45)$$

Therefore, according to Definition 1, the controlled complex network (1) is finite-timely synchronized in the settling time t^*. The proof of Theorem 9 is now completed. □

Theorem 10. *Suppose that Γ is a positive definite diagonal matrix and the evolution functions f_i in system (1) satisfy inequality (41). If the error system (12) is controlled with controller (40), and its control strengths η_j satisfy*

$$c\gamma_j A - \eta_j I_N \le 0, \quad j \in \mathcal{I}, \tag{46}$$

then the controlled complex network (1) is finite-timely synchronized to the given smooth dynamics $s(t)$. Moreover, the synchronization time is estimated by

$$t^* = \frac{V(0)^{1-\delta}}{k(1-\delta)}, \tag{47}$$

where $V(0)$ and δ are the same as defined in Theorem 4.

Proof. Consider the following Lyapunov function:

$$V(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)\, e_i(t). \tag{48}$$

Calculating the time derivative of $V(t)$ along the trajectory of system (12) and using (40) and (46) lead to

$$\begin{aligned}
\frac{dV(t)}{dt} &\le \sum_{i=1}^{N} \left|e_i^T(t)\right| M_i + c\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij} e_i^T(t)\,\Gamma e_j(t) \\
&\quad - \sum_{i=1}^{N} \eta_i e_i^T(t)\, e_i(t) - \sum_{i=1}^{N} e_i^T(t)\,\operatorname{sgn}(e_i(t))\, M_i \\
&\quad - k\sum_{i=1}^{N} e_i^T(t)\,\operatorname{sgn}(e_i(t))\left|e_i(t)\right|^{\beta} \\
&\le c\sum_{j=1}^{N} \gamma_j \tilde{e}_j^T(t)\, A\tilde{e}_j(t) - \sum_{j=1}^{N} \eta_j \tilde{e}_j^T(t)\,\tilde{e}_j(t) \\
&\quad - k\sum_{i=1}^{N}\sum_{j=1}^{N}\left|e_{ij}(t)\right|^{1+\beta} \\
&= \sum_{j=1}^{N} \tilde{e}_j^T(t)\left[c\gamma_j A - \eta_j I_N\right]\tilde{e}_j(t) \\
&\quad - k\sum_{i=1}^{N}\sum_{j=1}^{N}\left|e_{ij}(t)\right|^{1+\beta} \le -kV^{(1+\beta)/2}(t).
\end{aligned} \tag{49}$$

Thus, from Lemma 2, $V(t)$ converges to zero in a finite-time t^*, where t^* is given by

$$t^* = \frac{V(0)^{1-\delta}}{k(1-\delta)}. \tag{50}$$

Therefore, according to Definition 1, the controlled complex network (1) is finite-timely synchronized in the settling time t^*. The proof of Theorem 10 is now completed. □

In the nonidentical complex network (1), if all the functions f_i are the same, that is, $f_i = f$ for all $i \in \mathcal{I}$, then system (1) is degenerated to the following identical complex network:

$$\begin{aligned}
\dot{x}_i(t) &= f\left(t, x_i(t), x_i(t-\tau_i(t))\right) + c\sum_{j=1}^{N} a_{ij}\Gamma x_j(t) \\
&\quad + u_i(t), \quad i \in \mathcal{I}.
\end{aligned} \tag{51}$$

Also, the synchronization state $s(t)$ is chosen as a solution of a decoupled state satisfying

$$\dot{s}(t) = f\left(s, s(t), s(t-\tau_i(t))\right); \tag{52}$$

then the FTS problem of the complex network (1) becomes the FTS problem of a complex network with identical nodes studied in much of the literatures, such as [34–37]. Additionally, the finite-time control scheme (11) is degenerated to

$$\begin{aligned}
u_i(t) &= -\eta_i e_i(t) - L_i^1 \operatorname{sgn}(e_i(t))\left|e_i(t-\tau_i(t))\right| \\
&\quad - k\operatorname{sgn}(e_i(t))\left|e_i(t)\right|^{\beta}.
\end{aligned} \tag{53}$$

Evidently, the following criteria can be directly derived to ensure the FTS of the complex network with identical nodes (51).

Corollary 11. *Assume that hypothesis (H_1) holds and $\eta_1, \eta_2, \ldots, \eta_N$ satisfy inequality (14); then the controlled complex network (51) is globally finite-timely synchronized to the decoupled state (52) under controller (53). Moreover, the synchronization time is estimated by*

$$t^* = \frac{V(0)^{1-\delta}}{k(1-\delta)}, \tag{54}$$

where $V(0)$ and δ are the same as defined in Theorem 4.

In the controlled network (51), if the evolution function f is not relevant to delay, that is, $f = f(t, x_i(t))$ for $i \in \mathcal{I}$, then system (51) is reduced to following complex network:

$$\dot{x}_i(t) = f\left(t, x_i(t)\right) + c\sum_{j=1}^{N} a_{ij}\Gamma x_j(t) + u_i(t), \quad i \in \mathcal{I}. \tag{55}$$

In addition, the synchronization state $s(t)$ is chosen as a solution of a decoupled state satisfying

$$\dot{s}_i(t) = f\left(s, s(t)\right). \tag{56}$$

Let

$$u_i(t) = \eta_i e_i(t) - k\operatorname{sgn}(e_i(t))\left|e_i(t)\right|^{\beta}. \tag{57}$$

Then, the error system is the following form:

$$\dot{e}_i(t) = \overline{f}_i\left(t, e_i(t)\right) + c\sum_{j=1}^{N} a_{ij}\Gamma e_j(t) + u_i(t), \quad i \in \mathcal{I}. \tag{58}$$

Thus, for the FTS of controlled complex network (55), we have following result.

Corollary 12. *Suppose that hypothesis* (H_1') *holds. If* $\eta_1, \eta_2, \dots, \eta_N$ *satisfy inequality (14), then the controlled complex network (55) is finite-timely synchronized to the given decoupled state (56) under controller (57). Moreover, the synchronization time is estimated by*

$$t^* = \frac{V(0)^{1-\delta}}{k(1-\delta)}, \tag{59}$$

where $V(0)$ *and* δ *are the same as defined in Theorem 4.*

Remark 13. From the proofs of Theorems 4, 5, 9, and 10, we can see that the parameters $\eta_1, \eta_2, \dots, \eta_N$, k, and β in controllers (11), (40), (53), and (57) play a central role in FTS of controlled complex networks. Inequality (20) indicates that the synchronization rate increases when $\eta_1, \eta_2, \dots, \eta_N$, k, or β increases. On the other hand, whether network (1) can be synchronized or not relies on the value of $\eta_1, \eta_2, \dots, \eta_N$, whereas the synchronization time depends on values of k and β and has nothing to do with the value of $\eta_1, \eta_2, \dots, \eta_N$.

Remark 14. Since control inputs (11), (40), (53), and (57) contain the discontinuous sign function, an undesirable chattering may appear as a hard switcher. To avoid the chattering, the continuous tanh function can be used for discontinuous sign function to remove discontinuity. For example, controller (11) can be modified to

$$\begin{aligned} u_i(t) = {} & F_i(t) - \eta_i e_i(t) \\ & - L_i^1 \tanh(\theta_i e_i(t)) \left| e_i(t - \tau_i(t)) \right| \\ & - k \tanh(\theta_i e_i(t)) \left| e_i(t) \right|^{\beta}, \end{aligned} \tag{60}$$

where $\theta_i > 0$ for $i \in \mathcal{I}$.

Remark 15. In [37], the FTS between two identical complex networks with delayed and nondelayed coupling was investigated by using two different types of controller, that is, periodically intermittent controller and impulsive controller. In [40], the FTS of complex dynamical networks with or without coupling delay was studied by employing the Lyapunov function method and well-known finite-time stability theorem. However, the proposed controllers $u_i(t)$ in [37, 40] are very complex and $u_i(t) \to \infty$ as $\|e_i(t)\| \to 0$. Thus, the designed control laws in [37, 40] are too expensive and not easily applicable. Evidently, the improper technique somewhat affects the validity of results obtained in [37, 40]. To avoid this, a different technique is used in this paper to achieve the FTS of considered nonidentical complex system.

Remark 16. In the past few years, FTS has drawn an increasing attention due to the fact that it requires considered chaotic networks to be synchronized in finite-time rather than merely asymptotically. In the literature, so far, there are many excellent results on the TNS of complex networks, neural networks, and multiagent systems based on the continuous or discontinuous control approaches [41–44]. However, there are very few results to address the problem of the FTS for complex networks with delays and nonidentical nodes. Generally, different nodes in a complex network have different features including dynamic behaviors and initial conditions, which may result in their different control protocols to optimize synchronization time. In this paper, we employ the finite-time control technique to achieve the synchronization of a class of complex networks with nonidentical dynamical nodes and time-varying delays. Obviously, the results obtained in this paper can fasten the synchronization speed in great extent than the methods of asymptotical or exponential synchronization [22, 28]. Therefore, our results are more conducive and the extension and improvement of the existing results can be seen.

4. Numerical Simulations

In this section, two numerical examples and their simulations are given to demonstrate the feasibility of the proposed FTS schemes.

Example 1. Consider the nonidentical complex dynamical network described by following equation:

$$\begin{aligned} \dot{x}_i(t) = {} & f_i(x_i(t), x_i(t - \tau(t))) + c\sum_{j=1}^{4} a_{ij}\Gamma x_j(t) \\ & + u_i(t), \quad i \in \mathcal{I} = \{1, 2, 3, 4\} \end{aligned} \tag{61}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), x_{i3}(t))^T$ and

$$\begin{aligned} & f_1(x_i(t), x_i(t - \tau(t))) = f_2(x_i(t), x_i(t - \tau(t))) \\ & \quad = Bx_i(t) + g_{11}(x_i(t)) + g_{12}(x_i(t - \tau_1(t))), \\ & f_3(x_i(t), x_i(t - \tau(t))) = f_4(x_i(t), x_i(t - \tau(t))) \\ & \quad = Dx_i(t) + g_{21}(x_i(t)) + g_{22}(x_i(t - \tau_2(t))). \end{aligned} \tag{62}$$

Here, $g_{11}(x_i(t)) = (0, -x_{i1}x_{i3}, x_{i1}x_{i2})^T$, $g_{12}(x_i(t)) = (0, 0.2x_{i2}, 0)^T$, $g_{21}(x_i(t)) = (-2.95(|x_{i1}+1| - |x_{i1}-1|), 0, 0)^T$, $g_{22}(x_i(t)) = (0, 0, -0.5\sin(0.5x_{i1}))^T$, $\tau_i(t) = 1$, $i \in \mathcal{I}$, and

$$\begin{aligned} B &= \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & 0 \\ 0 & 0 & -\dfrac{8}{3} \end{bmatrix}, \\ D &= \begin{bmatrix} -16.8 & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -14.87 & 0 \end{bmatrix}. \end{aligned} \tag{63}$$

The coupling configuration matric A is chosen as

$$A = \begin{bmatrix} -1 & 0.3 & 0.2 & 0.5 \\ 0.3 & -0.8 & 0.4 & 0.1 \\ 0.2 & 0.4 & -1.2 & 0.6 \\ 0.5 & 0.1 & 0.6 & -1.2 \end{bmatrix}. \tag{64}$$

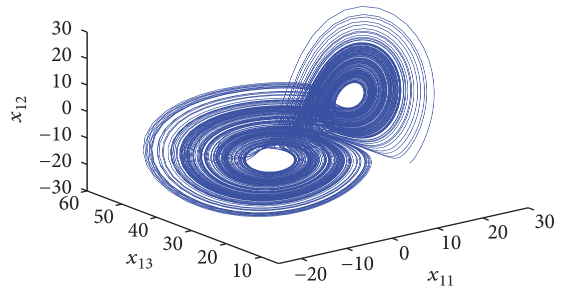


Figure 1: The chaotic behavior of time-delayed Lorenz system.

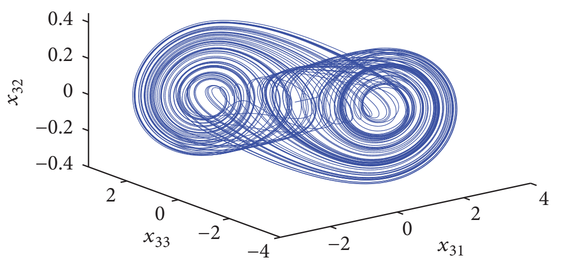


Figure 2: The chaotic behavior of time-delayed Chua oscillator.

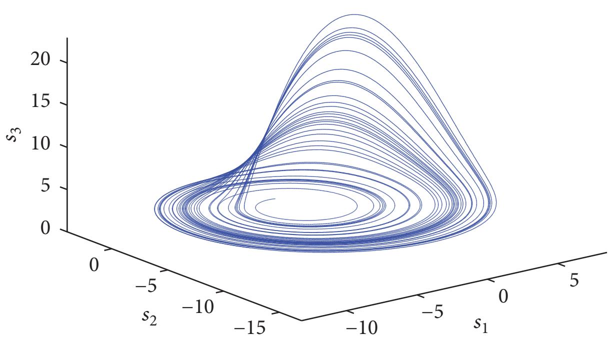


Figure 3: The chaotic behavior of Rössler system.

The time evolutions of delayed chaotic Lorenz system and chaotic Chua oscillator with initial conditions $x_1(\theta) = [0,1,0]^T$ and $x_3(\theta) = [0.7,0,0]^T$ for $\theta \in [-1,0]$ are shown in Figures 1 and 2.

Now we select Rössler system as the synchronized aim; it is given by

$$\begin{aligned} \dot{s}_1(t) &= -(s_2(t) + s_3(t)), \\ \dot{s}_2(t) &= s_1(t) + 0.2 s_2(t), \\ \dot{s}_3(t) &= s_3(t)(s_1(t) - 5.7) + 0.2. \end{aligned} \tag{65}$$

The chaotic behavior of Rössler system (65) with initial values $s(0) = [0,2,0]^T$ is depicted in Figure 3.

By easy computation, we get $\lambda_{\max}(B^s) = 14.0256$, $\lambda_{\max}(D^s) = 7.3184$, and $\lambda_{\max}(A^s) = 0$, $|x_{11}| < m_1 = 27.87$, $|x_{12}| < m_2 = 38.39$, and $|x_{13}| < m_3 = 73.85$ for Lorenz system and $|x_{31}| < r_1 = 2.32$, $|x_{32}| < r_2 = 0.49$, and $|x_{33}| < r_3 = 3.91$ for Chua oscillator. Letting $M = [0, m_3, m_2; m_3, 0, 0; m_2, 0, 0]$ and $R = [0, r_3, r_2; r_3, 0, 0; r_2, 0, 0]$ (see [40]), then we have $\lambda_{\max}(M) = 83.2323$ and $\lambda_{\max}(R) = 3.9406$. Thus, take $L_1^2 = L_2^2 = \lambda_{\max}(B^s) + \lambda_{\max}(M) = 97.2579$, $\text{Ł}_3^2 = \text{Ł}_4^2 = \lambda_{\max}(D^s) + \lambda_{\max}(R) = 11.2590$, $\text{Ł}_1^1 = \text{Ł}_2^1 = 0.2$, and $L_3^1 = L_4^1 = 0.5$. Then, the hypothesis (H_1) is satisfied for system (61). Using finite-time controller (11) and letting $c = 1$, $k = 5$, and $\beta = 0.5$, it is not difficult to check that inequality (14) is satisfied with $\eta_1 = \eta_2 = 97.3$ and $\eta_3 = \eta_4 = 11.3$. Therefore, according to Theorem 4, the controlled coupled network (61) is synchronized to the chosen orbit (65) in a finite-time. The synchronization errors are given in Figure 4.

Example 2. Consider a complex dynamic network consisting of two communities, in which each dynamical node is a chaotic neural network given by

$$\begin{aligned} \dot{x}_i(t) &= -A_1 x_i(t) + B_1 g(x_i(t)) \\ &\quad + D_1 g(x_i(t - \tau_1(t))), \quad i = 1,2, \\ \dot{x}_i(t) &= -A_2 x_i(t) + B_2 h(x_i(t)) \\ &\quad + D_2 h(x_i(t - \tau_2(t))), \quad i = 3,4, \end{aligned} \tag{66}$$

where $x_i(t) = (x_{i1}, x_{i2})^T$, $g(x_i) = (0.5(|x_{i1}+1| - |x_{i1}-1|))^T$, $0.5(|x_{i2}+1| - |x_{i2}-1|))^T$, $h(x_i) = (\tanh(x_{i1}), \tanh(x_{i2}))^T$, $\tau_1(t) = \tau_2(t) = e^t/(1+e^t)$, $A_1 = A_2 = \operatorname{diag}(1.2, 1.2)$, and

$$\begin{aligned} B_1 &= \begin{bmatrix} 1 + \dfrac{\pi}{4} & 20 \\ 0.1 & 1 + \dfrac{\pi}{4} \end{bmatrix}, \\ B_2 &= \begin{bmatrix} 2 & -0.1 \\ -5 & 3 \end{bmatrix}, \\ D_1 &= \begin{bmatrix} \dfrac{1.3\sqrt{2}\pi}{4} & 0.1 \\ 0.1 & \dfrac{1.3\sqrt{2}\pi}{4} \end{bmatrix}, \\ D_2 &= \begin{bmatrix} 1.5 & 0.1 \\ 0.2 & 2.5 \end{bmatrix}. \end{aligned} \tag{67}$$

The time evolutions of each dynamical node in (66) are depicted in Figures 5 and 6.

Now, we consider the following controlled complex networks given by

$$\begin{aligned} \dot{x}_i(t) &= -A_1 x_i(t) + B_1 g(x_i(t)) \\ &\quad + D_1 g(x_i(t - \tau_1(t))) + c \sum_{j=1}^{4} a_{ij} \Gamma x_j(t) \\ &\quad + u_i(t), \quad i = 1,2, \end{aligned}$$

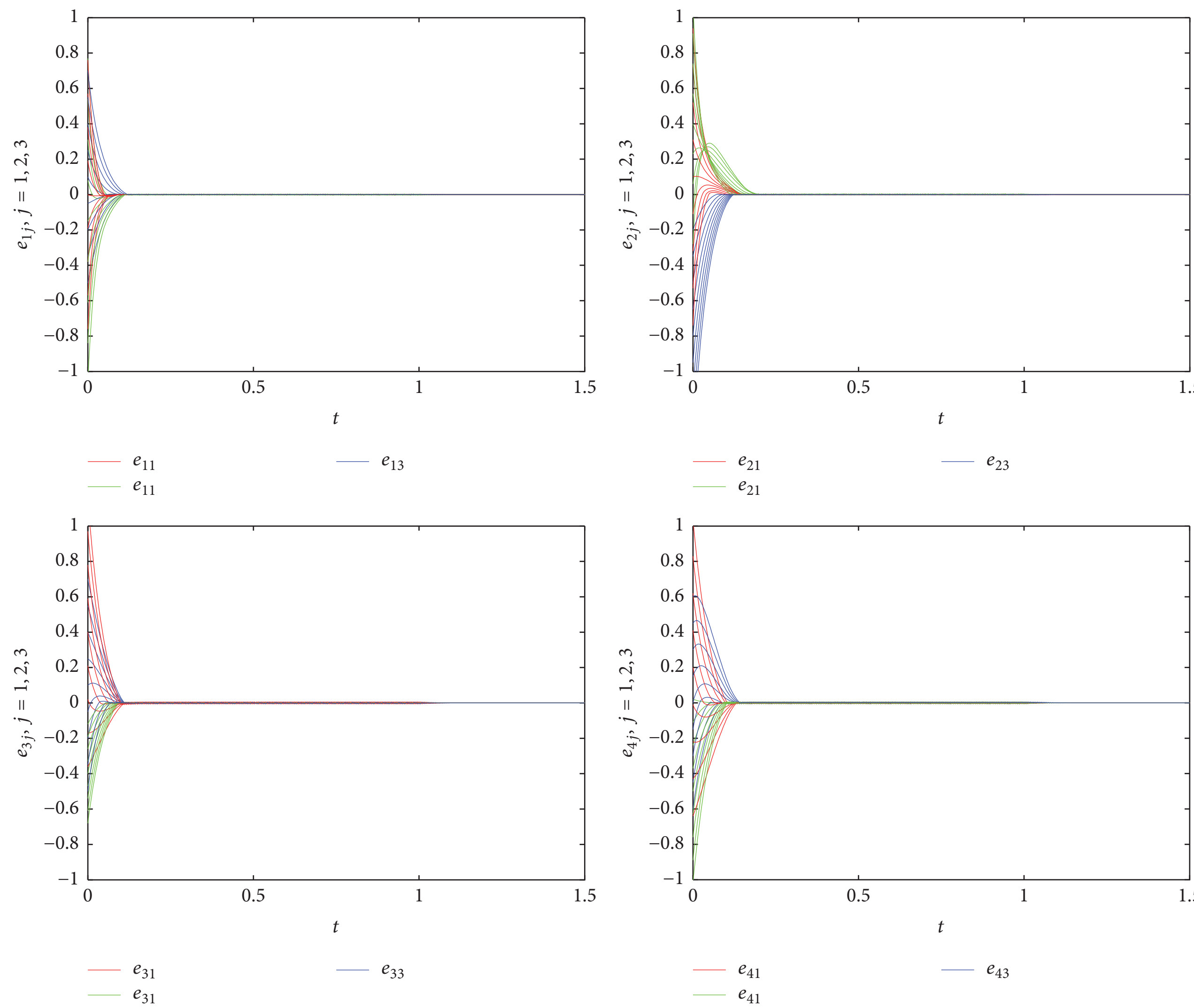


Figure 4: Time evolution of synchronization errors for (61) with different initial values.

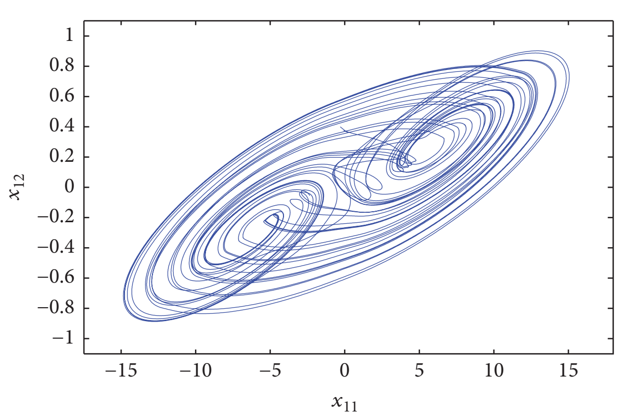


Figure 5: The chaotic behavior of each node in the first cluster in (66).

$$\begin{aligned}\dot{x}_i(t) &= -A_2 x_i(t) + B_2 h(x_i(t)) \\ &\quad + D_2 h(x_i(t - \tau_2(t))) + c\sum_{j=1}^{4} a_{ij}\Gamma x_j(t) \\ &\quad + u_i(t), \quad i = 3, 4.\end{aligned} \tag{68}$$

The inner connecting matrix $\Gamma = \mathrm{diag}(1.5, 2)^T$ and the matrix A is selected as

$$\begin{bmatrix} -0.6 & 0 & 0.3 & 0.3 \\ 0.2 & -0.8 & 0.4 & 0.2 \\ 0.4 & 0.5 & -1.2 & 0.3 \\ 0.3 & 0.4 & 0.2 & -0.9 \end{bmatrix}. \tag{69}$$

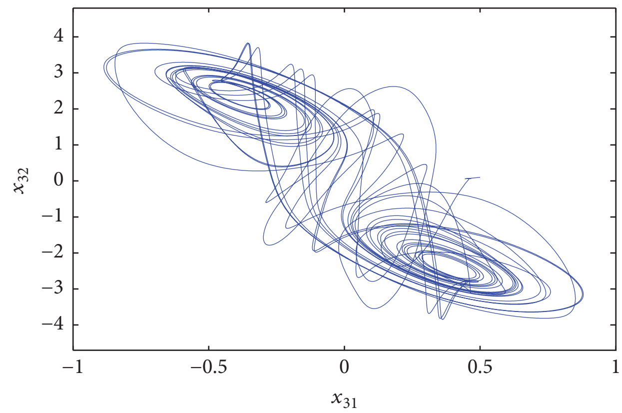


FIGURE 6: The chaotic behavior of each node in the second cluster in (66).

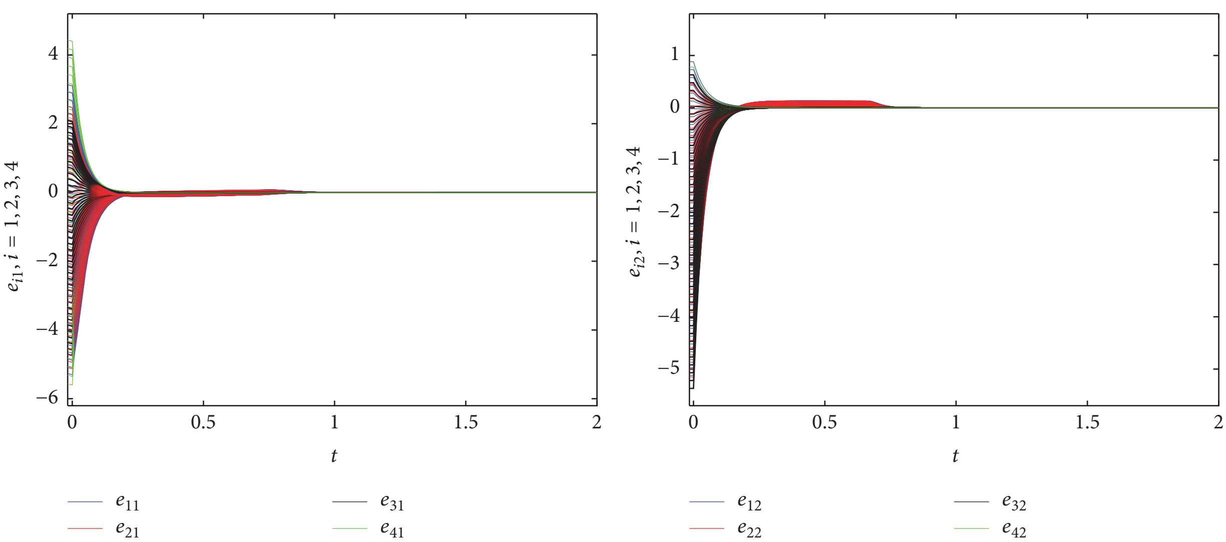


FIGURE 7: Time evolution of synchronization errors for (26) with different initial values.

The following periodic orbit is chosen as a synchronization aim:

$$s(t) = (4\sin(0.5t), 3\cos(t))^T. \tag{70}$$

It is not difficult to check that the hypothesis (H_1) is satisfied for system (66) with $L_1^2 = L_2^2 = 13.04$, $L_3^2 = L_4^2 = 6.30$, $L_1^1 = L_2^1 = 1.56$, and $L_3^1 = L_4^1 = 2.53$. Choosing $c = 1$, $k = 8$, $\eta_1 = \eta_2 = 13.1$, $\eta_3 = \eta_4 = 6.40$, and $\beta = 0.6$, then inequality (14) also holds. Use the following finite-time controller (see Remark 14):

$$\begin{aligned} u_i(t) = F_i(t) - \eta_i e_i(t) \\ - L_i^1 \tanh(\theta_i e_i(t)) \left|e_i(t - \tau_i(t))\right| \\ - k \tanh(\theta_i e_i(t)) \left|e_i(t)\right|^\beta, \end{aligned} \tag{71}$$

where $\theta_i = 0.1$, $F_i(t) = s(t) - f_i(t, s(t), s(t-\tau_i(t)))$, $i = 1, 2, 3, 4$. Then, from Theorem 4, the controlled coupled network (68) is synchronized to the chosen periodic orbit (70) in a finite-time. The synchronization errors are given in Figure 7, and the synchronization curves are depicted in Figure 8.

5. Conclusion

In this paper, we investigate the FTS of nonidentical complex networks with time-varying delays. By using finite-time stability theorem, inequality techniques, and designing suitable controllers, we develop some simple but useful sufficient criteria on the FTS of the addressed model. Our results of the FTS technique have optimal convergence time than the asymptotical and exponential synchronization methods. Finally, we gave two examples to demonstrate the effectiveness and feasibility of the developed FTS schemes.

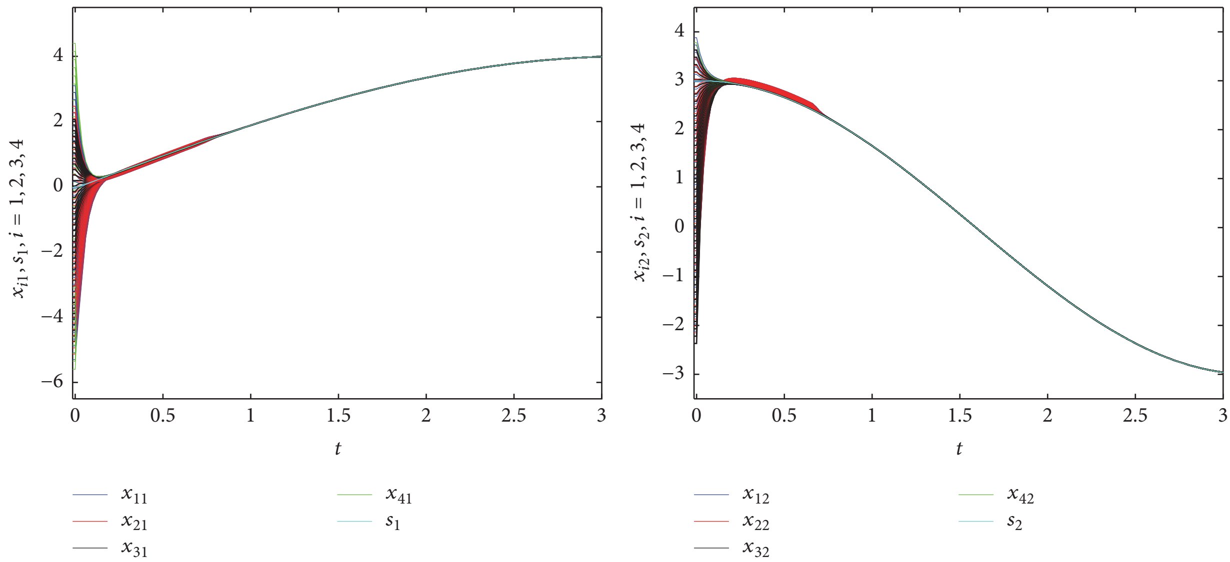


FIGURE 8: Finite-time synchronization curves of x_{ij} for (68) with different initial values.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by the National Natural Science Foundations of China (Grant nos. 11601464 and 61473244) and the Natural Science Foundation of Xinjiang University (Starting Research Fund for the Xinjiang University Doctoral Graduates, Grant no. BS150202).

References

[1] S. H. Strogatz, "Exploring complex networks," *Nature*, vol. 410, no. 6825, pp. 268–276, 2001.

[2] X. F. Wang and G. Chen, "Complex networks: small-world, scale-free and beyond," *IEEE Circuits and Systems Magazine*, vol. 3, no. 1, pp. 6–20, 2003.

[3] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D.-U. Hwang, "Complex networks: structure and dynamics," *Physics Reports*, vol. 424, no. 4-5, pp. 175–308, 2006.

[4] A. Pikovsky, M. Rosenblum, and J. Kurths, *Synchronization: A Universal Concept in Nonlinear Sciences*, vol. 12 of *Cambridge Nonlinear Science Series*, Cambridge University Press, Cambridge, UK, 2001.

[5] G. V. Osipov, J. Kurths, and C. Zhou, *Synchronization in oscillatory networks*, Springer Series in Synergetics, Springer, Berlin, Germany, 2007.

[6] S. H. Strogatz and I. Stewart, "Coupled oscillators and biological synchronization," *Scientific American*, vol. 269, no. 6, pp. 102–109, 1993.

[7] Y. Q. Zhang and Z.-R. He, "A secure communication scheme based on cellular neural networks," in *Proceedings of the IEEE International Conference on Intelligent Process Systems*, vol. 1, pp. 521–524, 1997.

[8] V. Perez-Munuzuri, V. Perez-Villar, and L. O. Chua, "Autowaves for image processing on a two-dimensional CNN array of excitable nonlinear circuits: flat and wrinkled labyrinths," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 40, no. 3, pp. 174–181, 1993.

[9] G. Chen and Z. Duan, "Network synchronizability analysis: a graph-theoretic approach," *Chaos. An Interdisciplinary Journal of Nonlinear Science*, vol. 18, no. 3, 2008.

[10] C. W. Wu and L. O. Chua, "Synchronization in an array of linearly coupled dynamical systems," *IEEE Transactions on Circuits and Systems. I. Fundamental Theory and Applications*, vol. 42, no. 8, pp. 430–447, 1995.

[11] W. Lu and T. Chen, "New approach to synchronization analysis of linearly coupled ordinary differential systems," *Physica D. Nonlinear Phenomena*, vol. 213, no. 2, pp. 214–230, 2006.

[12] Z. Li and J.-J. Lee, "New eigenvalue based approach to synchronization in asymmetrically coupled networks," *Chaos. An Interdisciplinary Journal of Nonlinear Science*, vol. 17, no. 4, Article ID 043117, 2007.

[13] X. Huang and J. Cao, "Generalized synchronization for delayed chaotic neural networks: a novel coupling scheme," *Nonlinearity*, vol. 19, no. 12, pp. 2797–2811, 2006.

[14] J. Cao and J. Lu, "Adaptive synchronization of neural networks with or without time-varying delay," *Chaos*, vol. 16, no. 1, Article ID 013133, 2006.

[15] L. Y. Xiang, Z. X. Liu, Z. Q. Chen, F. Chen, and Z. Z. Yuan, "Pinning control of complex dynamical networks with general topology," *Physica A: Statistical Mechanics and its Applications*, vol. 379, no. 1, pp. 298–306, 2007.

[16] R. Luo, "Impulsive control and synchronization of a new chaotic system," *Physics Letters A*, vol. 372, pp. 648–653, 2008.

[17] Y. Yang and J. Cao, "Exponential lag synchronization of a class of chaotic delayed neural networks with impulsive effects," *Physica A: Statistical Mechanics and its Applications*, vol. 386, no. 1, pp. 492–502, 2007.

[18] H. Wang, Z.-Z. Han, Q.-Y. Xie, and W. Zhang, "Finite-time chaos control via nonsingular terminal sliding mode control," *Communications in Nonlinear Science and Numerical Simulation*, vol. 14, no. 6, pp. 2728–2733, 2009.

[19] S. Cai, Z. Liu, F. Xu, and J. Shen, "Periodically intermittent controlling complex dynamical networks with time-varying delays to a desired orbit," *Physics Letters. A*, vol. 373, no. 42, pp. 3846–3854, 2009.

[20] A. Abdurahman, H. Jiang, and Z. Teng, "Lag synchronization for Cohen-Grossberg neural networks with mixed time-delays via periodically intermittent control," *International Journal of Computer Mathematics*, vol. 94, no. 2, pp. 275–295, 2017.

[21] Z. Yang and D. Xu, "Stability analysis and design of impulsive control systems with time delay," *IEEE Transactions on Automatic Control*, vol. 52, no. 8, pp. 1448–1454, 2007.

[22] L. Huang, Z. Wang, Y. Wang, and Y. Zuo, "Synchronization analysis of delayed complex networks via adaptive time-varying coupling strengths," *Physics Letters A*, vol. 373, no. 43, pp. 3952–3958, 2009.

[23] Q. Wang, Z. Duan, G. Chen, and Z. Feng, "Synchronization in a class of weighted complex networks with coupling delays," *Physica A: Statistical Mechanics and Its Applications*, vol. 387, no. 22, pp. 5616–5622, 2008.

[24] S. Zheng, Q. Bi, and G. Cai, "Adaptive projective synchronization in complex networks with time-varying coupling delay," *Physics Letters. A*, vol. 373, no. 17, pp. 1553–1559, 2009.

[25] S. Cai, Q. He, J. Hao, and Z. Liu, "Exponential synchronization of complex networks with nonidentical time-delayed dynamical nodes," *Physics Letters A*, vol. 374, no. 25, pp. 2539–2550, 2010.

[26] Q. Song, J. Cao, and F. Liu, "Synchronization of complex dynamical networks with nonidentical nodes," *Physics Letters A*, vol. 374, no. 4, pp. 544–551, 2010.

[27] S. Zheng, S. Wang, G. Dong, and Q. Bi, "Adaptive synchronization of two nonlinearly coupled complex dynamical networks with delayed coupling," *Communications in Nonlinear Science and Numerical Simulation*, vol. 17, no. 1, pp. 284–291, 2012.

[28] S. Wang, H. Yao, S. Zheng, and Y. Xie, "A novel criterion for cluster synchronization of complex dynamical networks with coupling time-varying delays," *Communications in Nonlinear Science and Numerical Simulation*, vol. 17, no. 7, pp. 2997–3004, 2012.

[29] X. Wu and H. Lu, "Cluster synchronization in the adaptive complex dynamical networks via a novel approach," *Physics Letters A*, vol. 375, pp. 1559–1565, 2011.

[30] C. Hu, J. Yu, H. Jiang, and Z. Teng, "Synchronization of complex community networks with nonidentical nodes and adaptive coupling strength," *Physics Letters A*, vol. 375, no. 5, pp. 873–879, 2011.

[31] G. Wang, Z. Li, Q. Zhang, and C. Yang, "Robust finite-time stability and stabilization of uncertain Markovian jump systems with time-varying delay," *Applied Mathematics and Computation*, vol. 293, pp. 377–393, 2017.

[32] M. P. Aghababa and H. P. Aghababa, "Finite-time stabilization of a non-autonomous chaotic rotating mechanical system," *Journal of the Franklin Institute*, vol. 349, no. 9, pp. 2875–2888, 2012.

[33] A. Abdurahman, H. Jiang, and Z. Teng, "Finite-time synchronization for memristor-based neural networks with time-varying delays," *Neural Networks*, vol. 69, pp. 20–28, 2015.

[34] A. Abdurahman, H. Jiang, and Z. Teng, "Finite-time synchronization for fuzzy cellular neural networks with time-varying delays," *Fuzzy Sets and Systems*, vol. 297, pp. 96–111, 2016.

[35] J. D. Cao and R. X. Li, "Fixed-time synchronization of delayed memristor-based recurrent neural networks," *Science China Information Sciences*, vol. 60, no. 3, Article ID 032201, 2017.

[36] X. Yang and J. Cao, "Finite-time stochastic synchronization of complex networks," *Applied Mathematical Modelling*, vol. 34, no. 11, pp. 3631–3641, 2010.

[37] J. Mei, M. Jiang, W. Xu, and B. Wang, "Finite-time synchronization control of complex dynamical networks with time delay," *Communications in Nonlinear Science and Numerical Simulation*, vol. 18, no. 9, pp. 2462–2478, 2013.

[38] H. Bao and J. Cao, "Finite-time generalized synchronization of nonidentical delayed chaotic systems," *Nonlinear Analysis: Modeling and Control*, vol. 21, no. 3, pp. 306–324, 2016.

[39] G. Velmurugan, R. Rakkiyappan, and J. Cao, "Finite-time synchronization of fractional-order memristor-based neural networks with time delays," *Neural Networks*, vol. 73, pp. 36–46, 2016.

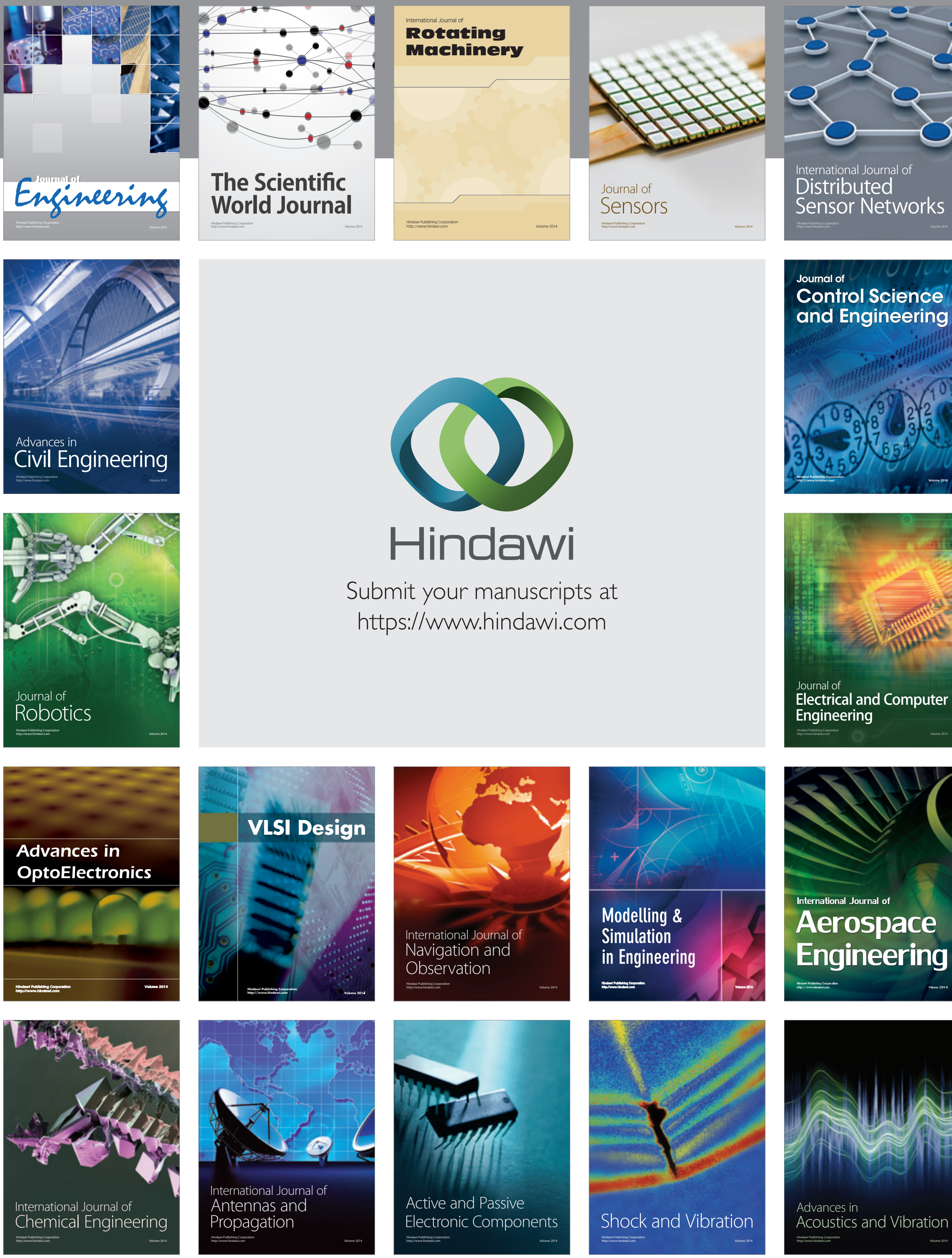
[40] Z. Wu, Q. Ye, and D. Liu, "Finite-time synchronization of dynamical networks coupled with complex-variable chaotic systems," *International Journal of Modern Physics C. Computational Physics and Physical Computation*, vol. 24, no. 9, Article ID 1350058, 10 pages, 2013.

[41] X. Liu, W. Yu, J. Cao, and F. Alsaadi, "Finite-time synchronisation control of complex networks via non-smooth analysis," *IET Control Theory and Applications*, vol. 9, no. 8, pp. 1245–1253, 2015.

[42] X. Liu, J. H. Park, N. Jiang, and J. Cao, "Nonsmooth finite-time stabilization of neural networks with discontinuous activations," *Neural Networks*, vol. 52, no. 4, pp. 25–32, 2014.

[43] M. P. Aghababa, S. Khanmohammadi, and G. Alizadeh, "Finite-time synchronization of two different chaotic systems with unknown parameters via sliding mode technique," *Applied Mathematical Modelling*, vol. 35, no. 6, pp. 3080–3091, 2011.

[44] H. Du, Y. He, and Y. Cheng, "Finite-time synchronization of a class of second-order nonlinear multi-agent systems using output feedback control," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 61, no. 6, pp. 1778–1788, 2014.


Journal of Engineering
The Scientific World Journal
International Journal of Rotating Machinery
Journal of Sensors
International Journal of Distributed Sensor Networks
Advances in Civil Engineering
Hindawi
Submit your manuscripts at
https://www.hindawi.com
Journal of Control Science and Engineering
Journal of Robotics
Journal of Electrical and Computer Engineering
Advances in OptoElectronics
VLSI Design
International Journal of Navigation and Observation
Modelling & Simulation in Engineering
International Journal of Aerospace Engineering
International Journal of Chemical Engineering
International Journal of Antennas and Propagation
Active and Passive Electronic Components
Shock and Vibration
Advances in Acoustics and Vibration