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Research Article

Exponential Synchronization for Second-Order Nodes in Complex Dynamical Network with Communication Time Delays and Switching Topologies

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This paper is devoted to the study of exponential synchronization problem for second-order nodes in dynamical network with time-varying communication delays and switching communication topologies. Firstly, a decomposition approach is employed to incorporate the nodes' inertial effects into the distributed control design. Secondly, the sufficient conditions are provided to guarantee the exponential synchronization of second-order nodes in the case that the information transmission is delayed and the communication topology is balanced and arbitrarily switched. Finally, to demonstrate the effectiveness of the proposed theoretical results, it is applied to the typical second-order nodes in dynamical network, as a case study. Simulation results indicate that the proposed method has a high performance in synchronization of such network.

1. Introduction

A complex dynamical network consists of a number of nodes and links between them. Complex networks exist in many fields of science, engineering, and society and have attracted much attention in recent years [1, 2]. As one of the most important collective behaviors, synchronization phenomena have been a topic of research. This topic occurs in physical science field and mathematics field for quite some time [3–5]. Several books and reviews [6, 7] which deal with this topic have also appeared. Such applications are pervasive and include clock synchronization in complex networks [8–10], coordination of unmanned air vehicles [11], and allocation of network resources fairly [12].

During the past decades, there are lots of results about synchronization of dynamical networks using graph theory and matrix theory. In [11] a Vicsek model for synchronization of dynamical network has been proposed. Also synchronization between multiagent systems with first-order integral plants when the communication topologies are balanced graphs has been studied in [5]. Extensions to the works of [5, 11] have been presented in [13] where the control conditions for synchronization have been broadened. Using the Lyapunov control theory, the synchronization problem of multiagent systems with time-varying communication topologies has been investigated in [14]. A number of research results about synchronization in complex networks have been put forward in [15, 16]. When the nodes in the dynamical network are high order integral plants, the synchronization problem becomes more challenging. A general way to synchronize the dynamical network has been presented and applied to multiple second-order integral plants in complex dynamical network in [17–19]. Recognition issue for unknown system parameters and topology of uncertain general complex dynamical networks with nonlinear couplings and time-varying delay is investigated through generalized outer synchronization [20]. A pinning controller is designed for cluster synchronization of complex dynamical networks with semi-Markovian jump topology [21]. Improved delay-dependent stability criteria for continuous systems with two additive time-varying delay components are analyzed [22]. It is well known that inertia is an important parameter in the controller design. In all the work mentioned above, each inertial node is considered to be the same. So far, there are few research works about synchronization of different inertial nodes in dynamical network. However, in real world, when there are communication time delays between inertial nodes, the synchronization problem will become more and more complicated. Synchronization of dynamical network with fixed topologies and undirected time delays has been studied in [23]. Further study of synchronization of dynamical network with switching topologies has been investigated in [24]. The time delay problem of directed communication network has been also considered in [14, 17, 24, 25].

In real complex dynamical network, time-varying communication topologies, communication time delays, and inertial nodes are three important factors to achieve synchronization. The existing literature fails to consider time-varying communication topologies, communication time delays, and inertial nodes at the same time. This paper takes these three factors into account simultaneously and explores the problem of synchronization between different second-order inertial nodes. The objective of this paper is to analyze the synchronization properties for a complex dynamical network with second-order nodes, time-varying communication delays, and switching communication topologies. Based on this model, several sufficient conditions for exponential synchronization stability are obtained by employing the Lyapunov functional method.

The remainder of this paper is organized as follows. A generalized model of complex dynamical network with second-order nodes, time-varying communication delays, and switching communication topologies is introduced and some useful mathematical preliminaries are given in Section 2. The method of decomposition of dynamical network is studied in Section 3. Section 4 deals with several criteria for synchronization stability. In this section, some sufficient conditions are obtained to achieve exponential synchronization for second-order nodes in dynamical network. To illustrate the theories obtained in Section 4, numerical examples with specific communication delays and switching topologies are used in Section 5. Finally, in Section 6, we summarize our results.

2. Model Formulation and Mathematical Preliminaries

2.1. Model Description. Consider a dynamical network consisting of diffusively coupled identical nodes, with each node being a second-order dynamical unit. The state equations of each node are described by

$$M_i \ddot{x}_i = u_i; \tag{1}$$

that is,

$$\begin{aligned} \dot{x}_i &= v_i, \\ M_i \dot{v}_i &= u_i, \end{aligned} \tag{2}$$

where $x_i \in \mathbb{R}^m$ are the position vectors; $v_i \in \mathbb{R}^m$ stand for the velocity vectors; $M_i \in \mathbb{R}^{m\times m}$ are symmetric positive definite matrices; and $u_i \in \mathbb{R}^m$ are the control inputs. Let $M = \operatorname{diag}\{M_1, M_2, \dots, M_n\} \in \mathbb{R}^{nm\times nm}$, $x = [x_1^{\mathrm{T}}, x_2^{\mathrm{T}}, \dots, x_n^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{nm}$, $v = [v_1^{\mathrm{T}}, v_2^{\mathrm{T}}, \dots, v_n^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{nm}$, $u = [u_1^{\mathrm{T}}, u_2^{\mathrm{T}}, \dots, u_n^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{nm}$.

According to (2), for nth, there is

$$\begin{aligned} \dot{x} &= v, \\ M\dot{v} &= u. \end{aligned} \tag{3}$$

Suppose that there exists a communication time delay $\tau(t)$ between nodes i and j; the time-varying delay $\tau(t)$ satisfies any one assumption of the following:

(A1) $0 \le \tau(t) \le h$, $\dot{\tau}(t) \le r < 1$;

(A2) $0 \le \tau(t) \le h$,

where $h > 0$. For each node i in the dynamical network, the control law based on neighbor's messages is used. The control law can be expressed as follows:

$$\begin{aligned} u_i(t) = \sum_{j\in\aleph_i(\sigma(t))} &\Big[ba_{ij}\Lambda\left(v_j(t-\tau(t)) - v_i(t-\tau(t))\right) \\ &+ ka_{ij}\Lambda\left(x_j(t-\tau(t)) - x_i(t-\tau(t))\right)\Big], \end{aligned} \tag{4}$$

where $b > 0$, $k > 0$; $\Lambda \in \mathbb{R}^{m\times m}$ is a positive definite diagonal matrix; $\sigma : [0, +\infty) \to \wp = \{1, \dots, N\}$ (N denotes the total number of all possible directed graphs) is a switching signal of communication topology; and $\aleph_i(\sigma(t))$ denotes the neighbor net of node i in graph $G_{\sigma(t)}$. In the following, in order to discuss conveniently, we abbreviate $\sigma(t)$ as σ.

According to (3) and (4), the dynamical network model can be acquired as follows:

$$\begin{aligned} \dot{x} &= v, \\ M\dot{v} &= -b\left(L^{\sigma}\otimes\Lambda\right)v(t-\tau(t)) \\ &\quad - k\left(L^{\sigma}\otimes\Lambda\right)x(t-\tau(t)), \end{aligned} \tag{5}$$

where $L^{\sigma} \in \mathbb{R}^{n\times n}$ is Laplacian matrix of graph G_{σ}. Suppose that L^{σ} has the following properties:

(A3) $rank(L^{\sigma}) = n - 1$;

(A4) $L^{\sigma}\mathbf{1}_n = 0$.

Remark 1. If (A3) holds, then graph G_{σ} is a strongly connected graph. And if graph is equilibrium diagram, then (A4) holds [5]. Let $\mathbb{C}([-\tau(t), 0], \mathbb{R}^{nm})$ be a continuous vector-valued function in Banach space. For any given

$$\begin{aligned} &\Phi = \left[\Phi_1^{\mathrm{T}}, \Phi_2^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{C}^2, \\ &\qquad \text{where } \Phi_i \in \mathbb{C}\ \ (i = 1, 2),\ \ t_0 \in \mathbb{R}, \end{aligned} \tag{6}$$

when $t > t_0 - \tau(t)$, there exists an only solution $x(t; t_0, \Phi_1)$, $v(t; t_0, \Phi_2)$ of (5). We define two manifolds: $\Gamma_1 = \{[x_1^{\mathrm{T}}, x_2^{\mathrm{T}}, \dots, x_n^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{nm};\ x_i = x_j,\ i, j \in H\}$, $\Gamma_2 = \{[v_1^{\mathrm{T}}, v_2^{\mathrm{T}}, \dots, v_n^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{nm};\ v_i = v_j,\ i, j \in H\}$.

Definition 2. For $\Phi_i \in \mathbb{C}$ $(i = 1, 2)$ and $t_0 \in \mathbb{R}$, if there are some constants $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\eta_1 > 0$, and $\eta_2 > 0$, such that

$$\begin{aligned} \left\| x_i - x_j \right\| &\le \eta_1 e^{-\varepsilon_1 (t - t_0)}, \quad i, j \in H, \\ \left\| v_i - v_j \right\| &\le \eta_2 e^{-\varepsilon_2 (t - t_0)}, \quad i, j \in H \end{aligned} \tag{7}$$

for all $t \ge t_0$ and $\tau(t) \in [0, h]$ hold; then manifold Γ_1 and manifold Γ_2 are exponential stable.

Remark 3. To discuss in a simple way, we consider how to achieve the synchronization problem of $x_i - x_j \to 0, \dot{x}_i - \dot{x}_j \to 0$, $(i, j \in H)$. By the rational selection of state information, the results in this paper can be applied to many practical problems, such as synchronization problem and formation control. By altering the control law (4), we can achieve $\dot{x}_i - \dot{x}_j \to 0$, $x_i - x_j \to \delta_{ij}$, $(i, j \in H)$, where $\delta_{ij} \in \mathbb{R}^m$ denotes the mutual distance between node i and node j. Let $\delta_i \in \mathbb{R}^m$ be a constant and the control input is

$$\begin{aligned} u_i(t) = \sum_{j \in \aleph_i(\sigma)} & \left[b a_{ij} \Lambda \left(v_j (t - \tau(t)) - v_i (t - \tau(t)) \right) \right. \\ & + k a_{ij} \Lambda \left(\left(x_j (t - \tau(t)) - \delta_j \right) \right. \\ & \left. \left. - \left(x_i (t - \tau(t)) - \delta_i \right) \right) \right]; \end{aligned} \tag{8}$$

then we can get $x_i(t - \tau(t)) - \delta_i \to x_j(t - \tau(t)) - \delta_j$, $(\delta_{ij} = \delta_i - \delta_j)$, $v_i \to v_j$.

2.2. Preliminaries. The interaction topology of a dynamical network of nodes with second-order nodes is represented using a directed graph $G = (V, E, A)$ with the set of nodes $V = \{v_1, v_2, \dots, v_n\}$, the set of directed edges is $E \subseteq V \times V$, and the adjacency matrix is $A = [a_{ij}] \in \mathbb{R}^{n \times n}$. $H \in \{1, 2, \dots, n\}$ is the set of node subscripts; $e_{ij} = (i, j)$ denotes the directed edge from node i to node j. The elements in adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{n \times n}$ satisfy $a_{ii} = 0$, $a_{ij} > 0$ (if and only if $e_{ij} \in E$). The set of neighbors of node i is $\aleph_i = \{j \in H : (i, j) \in E\}$. If there is $d_i = \sum_{j \in \aleph_i} a_{ij}$, $i \in H$, then the degree matrix of a directed graph G can be expressed as $D = \operatorname{diag}\{d_1, d_2, \dots, d_n\}$. The Laplacian matrix L is defined by $L \triangleq D - A$. It can be shown that, using the Gershgorin disc theorem [12], all of the eigenvalues of L have a nonnegative real part. Furthermore, if G is undirected, then the Laplacian matrix of G is symmetric and there is $L = BB^{\mathrm{T}}$, which means that L is positive semidefinite. The in-degree and out-degree of node V_i can be defined as $d_{\text{in}}(v_i) = \sum_{j=1}^{n} a_{ji}$ and $d_{\text{out}}(v_i) = \sum_{j=1}^{n} a_{ij}$, respectively. If $d_{\text{in}}(v_i) = \sum_{j=1}^{n} a_{ji} = d_{\text{out}}(v_i) = \sum_{j=1}^{n} a_{ij}$, then node v_i is an equilibrium point. If there exists a direct graph between nodes in graph G, then G is a strongly connected graph. In addition, I_n denotes n-order identity matrix. $\mathbf{1}_n$ denotes n-dimensional column vector of which all the elements are 1. $\lambda_{\max}(S)$ and $\lambda_{\min}(S)$ denote the maximum and minimum eigenvalue of real symmetric matrix S, respectively. $\| \cdot \|$ denotes the Euclidean norm and $\otimes$ denotes Kronecker product. The Dini time derivative of continuous function $V : \mathbb{R} \to \mathbb{R}$ can be defined as $D^{+}V = \lim_{h \to 0^{+}} \sup((V(t + h) - V(t))/h)$.

3. Decomposition of Dynamical Network

We can decompose the dynamical network (5) into cluster subsystem and formation subsystem according to the method mentioned in [15]. Take the decomposition transformation:

$$z = Sx. \tag{9}$$

$S \in \mathbb{R}^{nm \times nm}$ is the transformation matrix, which is defined by

$$S = \begin{bmatrix} \varphi_1 & \varphi_2 & \varphi_3 & \cdots & \varphi_n \\ I_m & -I_m & 0 & \cdots & 0 \\ 0 & I_m & -I_m & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & I_m & -I_m \end{bmatrix}, \tag{10}$$

where

$$\begin{aligned} \varphi_i &= \left(\sum_{j=1}^{n} M_j \right)^{-1} M_i \in \mathbb{R}^{m \times m}, \\ z &= \left[z_1^{\mathrm{T}}, z_e^{\mathrm{T}} \right]^{\mathrm{T}} \in \mathbb{R}^{nm} \end{aligned} \tag{11}$$

is a new variable, in which

$$\begin{aligned} z_1 &= \left(\sum_{j=1}^{n} M_j \right)^{-1} \sum_{i=1}^{n} M_i x_i \in \mathbb{R}^{m}, \\ z_e &= \left[x_1^{\mathrm{T}} - x_2^{\mathrm{T}}, x_2^{\mathrm{T}} - x_3^{\mathrm{T}}, \dots, x_{n-1}^{\mathrm{T}} - x_n^{\mathrm{T}} \right]^{\mathrm{T}} \in \mathbb{R}^{m(n-1)}. \end{aligned} \tag{12}$$

According to (9), the dynamical network model (5) can be written as

$$\begin{aligned} S^{-\mathrm{T}} M S^{-1} \ddot{z} = & -b S^{-\mathrm{T}} \left(L^{\sigma} \otimes \Lambda \right) S^{-1} \dot{z} (t - \tau(t)) \\ & - k S^{-\mathrm{T}} \left(L^{\sigma} \otimes \Lambda \right) S^{-1} z (t - \tau(t)), \end{aligned} \tag{13}$$

where $S^{-1} \in \mathbb{R}^{nm \times nm}$ has the following form:

$$S^{-1} = \begin{bmatrix} I_m & \phi_2 & \phi_3 & \cdots & \phi_n \\ I_m & \phi_2 - I_m & \phi_3 & \cdots & \phi_n \\ I_m & \phi_2 - I_m & \phi_3 - I_m & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \phi_n \\ I_m & \phi_2 - I_m & \cdots & \phi_{n-1} - I_m & \phi_n - I_m \end{bmatrix}, \tag{14}$$

where $\phi_i = \sum_{j=i}^{n} \varphi_j$. From (14), we have

$$S^{-\mathrm{T}} M S^{-1} = \begin{bmatrix} M_c & 0 \\ 0 & M_e \end{bmatrix}, \tag{15}$$

where $M_c = \sum_{j=1}^{n} M_j \in \mathbb{R}^{m \times m}$ and $M_e \in \mathbb{R}^{(n-1)m \times (n-1)m}$.

Because the row sum of matrix L^σ is zero, then we have

$$S^{-\mathrm{T}}\left(L^\sigma \otimes \Lambda\right) S^{-1} = \begin{bmatrix} 0 & H^{\mathrm{T}} \\ 0 & L_e^\sigma \end{bmatrix}, \tag{16}$$

where $H \in \mathbb{R}^{(n-1)m\times m}$; the jth element of H is

$$H_j = -\left(\sum_{k=j+1}^{n}\sum_{i=1}^{n} L_{ik}^\sigma\right)\Lambda \in \mathbb{R}^{m\times m}, \quad j \in \{1,2,\ldots,n-1\}. \tag{17}$$

(i, j) Jordan of $L_e^\sigma \in \mathbb{R}^{(n-1)m\times(n-1)m}$ is

$$L_{e,ij}^\sigma = \phi_{i+1}^{\mathrm{T}} H_j + \sum_{p=i+1}^{n}\sum_{q=j+1}^{n} L_{ik}^\sigma \Lambda \in \mathbb{R}^{m\times m}. \tag{18}$$

It follows from (13), (15), and (16) that the dynamical network model (5) can be decomposed into

$$M_c\ddot{z}_1 = -bH^{\mathrm{T}}\dot{z}_e\left(t-\tau(t)\right) - kH^{\mathrm{T}}z_e\left(t-\tau(t)\right), \tag{19}$$

$$M_e\ddot{z}_e = -bL^{\mathrm{T}}\dot{z}_e\left(t-\tau(t)\right) - kL^{\mathrm{T}}z_e\left(t-\tau(t)\right). \tag{20}$$

Theorem 4. *Consider the dynamical network (5); its decomposition transformation models are (19) and (20). Suppose that (A3) and (A4) hold; then we have the following.*

(1) *For any initial condition and given $\{b, k\}$, the centre-of-mass velocity $\dot{z}_1(t)$ remains unchanged; that is,*

$$\dot{z}_1(t) = \left(\sum_{j=1}^{n} M_j\right)^{-1}\sum_{i=1}^{n} M_i\dot{x}_i(t_0), \quad \forall t \geq t_0. \tag{21}$$

(2) *If the delay differential equation*

$$\dot{\varepsilon}(t) = A\varepsilon(t) + B^\sigma\varepsilon\left(t-\tau(t)\right) \tag{22}$$

is exponential stable for zero solution, then manifold Γ_1 and manifold Γ_2 are exponential stable, where

$$\varepsilon = \begin{bmatrix} z_e \\ \dot{z}_e \end{bmatrix},$$
$$A = \begin{bmatrix} 0 & I_{(n-1)m} \\ 0 & 0 \end{bmatrix}, \tag{23}$$
$$B^\sigma = \begin{bmatrix} 0 & 0 \\ -kM_e^{-1}L_e^\sigma & -bM_e^{-1}L_e^\sigma \end{bmatrix}.$$

Proof. (1) According to (A4), we have $H_j = -(\sum_{k=j+1}^{n}\sum_{i=1}^{n} L_{ik}^\sigma)\Lambda = 0$. Hence, there is $H = 0$. From (19), we can get the conclusion that $M_c\ddot{z}_1 = 0$. It is obvious that for any $t \geq t_0$, $\dot{z}_1$ is invariable; that is, (21) holds.

(2) According to (20), we have

$$\ddot{z}_e = -bM_e^{-1}L_e^\sigma\dot{z}_e\left(t-\tau(t)\right) - kM_e^{-1}L_e^\sigma z_e\left(t-\tau(t)\right). \tag{24}$$

Furthermore, for the zero solution, if (22) is exponential stable (i.e., there exist constants $\eta_0 > 0$ and $\lambda_0 > 0$), then the solution $\varepsilon(t_0, \Phi)(t)$ of (22) on $(t_0, \Phi) \in \mathbb{R}\times\mathbb{C}^2$ satisfies $\|\varepsilon\| \leq \eta_0 e^{-\lambda_0(t-t_0)}$. It is clear that manifold Γ_1 and manifold Γ_2 are exponential stable. □

4. Synchronization for Second-Order Nodes in Complex Dynamical Network with Communication Time Delays and Switching Topologies

Theorem 5. *Suppose that (A1) holds. The communication topology G_σ satisfies (A3) and (A4). Then for given constants $\alpha > 0$, $h > 0$, and $r \geq 0$, if there exist positive definite matrices P, R, U in the corresponding dimensions and arbitrary matrices H_1, H_2 satisfying*

$$\begin{bmatrix} \Delta_{11} & \Delta_{12} & q_0H_1 & hA^{\mathrm{T}}R \\ \Delta_{12}^{\mathrm{T}} & \Delta_{22} & q_0H_2 & h\left(B^\sigma\right)^{\mathrm{T}}R \\ q_0H_1^{\mathrm{T}} & q_0H_2^{\mathrm{T}} & -q_0R & 0 \\ hRA & hRB^\sigma & 0 & -hR \end{bmatrix} < 0, \tag{25}$$

where $\Delta_{11} = PA + A^{\mathrm{T}}P + \alpha P + U + H_1^{\mathrm{T}} + H_1$, $\Delta_{12} = PB^\sigma - H_1 + H_2^{\mathrm{T}}$, $\Delta_{22} = -(1-r)e^{\alpha h}U - H_2 - H_2^{\mathrm{T}}$, $q_0 = (e^{\alpha h} - 1)/\alpha$, then system (5), manifold Γ_1, and manifold Γ_2 are exponential stable. The solution of (22) satisfies

$$\|\varepsilon(t)\| \leq \sqrt{\frac{b_0}{a_0}e^{-\alpha(t-t_0)/2}}\left\|\varepsilon_{t_0}\right\|_c, \tag{26}$$

where

$$a_0 = \lambda_{min}(P),$$
$$b_0 = \lambda_{max}(P) + h\lambda_{max}(U) + \frac{h^2}{2}\lambda_{max}(R), \tag{27}$$
$$\left\|\varepsilon_{t_0}\right\|_c = \sup_{-\tau(t)\leq\theta\leq 0}\left\{\left\|\varepsilon(t_0+\theta)\right\|, \left\|\dot{\varepsilon}(t_0+\theta)\right\|\right\},$$

$$\dot{x}_i(t) \longrightarrow \dot{z}_1(t) = \left(\sum_{j=1}^{n} M_j\right)^{-1}\sum_{i=1}^{n} M_i\dot{x}_i(t_0), \quad \forall i \in H. \tag{28}$$

Proof. Choose the following Lyapunov function:

$$V(t) = V_1(t) + V_2(t) + V_3(t), \tag{29}$$

where

$$V_1(t) = \varepsilon^{\mathrm{T}}(t)P\varepsilon(t),$$
$$V_2(t) = \int_{-h}^{0}\int_{t+\theta}^{t}\dot{\varepsilon}^{\mathrm{T}}(s)e^{\alpha(s-t)}R\dot{\varepsilon}(s)\,ds\,d\theta, \tag{30}$$
$$V_3(t) = \int_{t-\tau(t)}^{t}\varepsilon^{\mathrm{T}}(s)e^{\alpha(s-t)}U\varepsilon(s)\,ds.$$

According to (22), we have

$$\begin{aligned} D^{+}V_2(t) &\le -\alpha\int_{-h}^{0}\int_{t+\theta}^{t}\dot\varepsilon^{\mathrm T}(s)e^{\alpha(s-t)}R\dot\varepsilon(s)\,ds\,d\theta \\ &\quad + h\dot\varepsilon^{\mathrm T}(t)R\dot\varepsilon(t) - \int_{t-\tau(t)}^{t}\dot\varepsilon^{\mathrm T}(s)e^{\alpha(s-t)}R\dot\varepsilon(s)\,ds, \\ D^{+}V_3(t) &= -\alpha\int_{t-\tau(t)}^{t}\varepsilon^{\mathrm T}(s)e^{\alpha(s-t)}U\varepsilon(s)\,ds + \varepsilon^{\mathrm T}(t)U\varepsilon(t) \\ &\quad - (1-\dot\tau(t))\varepsilon^{\mathrm T}(t-\tau(t))e^{-\alpha\tau}U\varepsilon(t-\tau(t)). \end{aligned} \tag{31}$$

From Leibniz-Newton formula, we know that

$$\begin{aligned} &(1-\dot\tau(t))\varepsilon^{\mathrm T}(t-\tau(t))e^{-\alpha\tau}U\varepsilon(t-\tau(t)) \\ &\quad \ge (1-r)\varepsilon^{\mathrm T}(t-\tau(t))e^{-\alpha h}U\varepsilon(t-\tau(t)). \end{aligned} \tag{32}$$

According to (A1) and (32), we have

$$\begin{aligned} &(1-\dot\tau(t))\varepsilon^{\mathrm T}(t-\tau(t))e^{-\alpha\tau}U\varepsilon(t-\tau(t)) \ge (1-r) \\ &\quad \cdot \varepsilon^{\mathrm T}(t-\tau(t))e^{-\alpha h}U\varepsilon(t-\tau(t)), \\ &-\int_{t-\tau(t)}^{t}\dot\varepsilon^{\mathrm T}(s)e^{\alpha(s-t)}R\dot\varepsilon(s)\,ds - 2\left[\varepsilon^{\mathrm T}(t)H_1\right. \\ &\quad \left. + \varepsilon^{\mathrm T}(t-\tau(t))H_2\right]\int_{t-\tau(t)}^{t}\dot\varepsilon(s)\,ds \\ &\quad \le -\int_{t-\tau(t)}^{t}\left(\xi^{\mathrm T}(t)H + \dot\varepsilon^{\mathrm T}(s)e^{\alpha(s-t)}R\right)\left(e^{\alpha(s-t)}R\right)^{-1} \\ &\quad \cdot \left(H^{\mathrm T}\xi(t) + e^{\alpha(s-t)}R\dot\varepsilon(s)\right)ds + q_0\xi^{\mathrm T}(t) \\ &\quad \cdot HR^{-1}H^{\mathrm T}\xi(t), \end{aligned} \tag{33}$$

where

$$\xi(t) = \begin{bmatrix}\varepsilon(t)\\ \varepsilon(t-\tau(t))\end{bmatrix}, \qquad H = \begin{bmatrix}H_1\\ H_2\end{bmatrix}. \tag{34}$$

It follows from (31) and (33) that

$$\begin{aligned} \dot V(t) + \alpha V(t) &\le \xi^{\mathrm T}(t)\Xi\xi(t) \\ &\quad - \int_{t-\tau(t)}^{t}\left(\xi^{\mathrm T}(t)H + \dot\varepsilon^{\mathrm T}(s)e^{\alpha(s-t)}R\right)\left(e^{\alpha(s-t)}R\right)^{-1} \\ &\quad \cdot \left(H^{\mathrm T}\xi(t) + e^{\alpha(s-t)}R\dot\varepsilon(s)\right)ds, \end{aligned} \tag{35}$$

where

$$\begin{aligned} \Xi &= \begin{bmatrix} \Delta_{11} + hA^{\mathrm T}RA & \Delta_{12} + hA^{\mathrm T}RB^{\sigma} \\ \Delta_{12}^{\mathrm T} + h(B^{\sigma})^{\mathrm T}RA & \Delta_{22} + h(B^{\sigma})^{\mathrm T}RB^{\sigma} \end{bmatrix} \\ &\quad + q_0HR^{-1}H^{\mathrm T}. \end{aligned} \tag{36}$$

According to Schur complement lemma, (25) guarantees $\Xi < 0$. It can be obtained from (35) that $D^{+}V(t) + \alpha V(t) \le 0$. Hence, we have

$$V(t) \le \sup_{-\tau(t)\le s\le 0} V(t_0+s)e^{-\alpha(t-t_0)}. \tag{37}$$

From (27) to (29), it can be obtained that

$$\begin{aligned} a_0\|\varepsilon(t)\|^2 &\le V(t), \\ \sup_{-\tau(t)\le s\le 0} V(t_0+s) &\le b_0\left\|\varepsilon_{t_0}\right\|_c^2. \end{aligned} \tag{38}$$

Then, according to (37) and (38), we can get (26). It is known from (26) that (22) is exponential stable for zero solution. According to Theorem 4, when the time delay $\tau(t)$ satisfies (A1), for the dynamical network (5), manifold Γ_1 and manifold Γ_2 are exponential stable. From $x_i(t)-x_j(t)\to 0$, we can obtain $z_1(t)\to x_i(t)$, $\forall i\in H$. We can also get (28) from (21). □

Remark 6. Suppose that (25) is available at $h = h_0$. For any $0 \le r < 1$, we can obtain the maximum h by the following steps: *Step 1*: let $h = h_0$; *Step 2*: to find the matrices $\{P, R, U\}$ satisfy (25). If we find matrices $\{P, R, U\}$, then let $h = h + g_0$ (g_0 is the step length). Repeat *Step 2*; otherwise, quit the whole procedure. h is the permitted maximal time delay.

Theorem 7. *Suppose that (A2) holds. The communication topology G_σ satisfies (A3) and (A4). If $U = 0$ and (25) holds, then, for the dynamical network (5), manifold Γ_1 and manifold Γ_2 are exponential stable. The solution of (22) satisfies*

$$\|\varepsilon(t)\| \le \sqrt{\frac{b_1}{a_0}}e^{-\alpha(t-t_0)/2}\left\|\varepsilon_{t_0}\right\|_c, \tag{39}$$

where

$$\begin{aligned} a_0 &= \lambda_{min}(P), \\ b_1 &= \lambda_{max}(P) + \frac{h^2}{2}\lambda_{max}(R). \end{aligned} \tag{40}$$

Proof. Choose the following Lyapunov function: $V(t) = V_1(t) + V_2(t)$; we can easily get the conclusion by the similar method which is used in Theorem 5. □

The following lemma can help us obtain the further results.

Lemma 8 (see [16]). *Let $\Upsilon(t) > 0$ ($t \in \mathbb{R}$), $\tau(t) \in [0,\infty)$, $t_0 \in \mathbb{R}$. Suppose that $D^{+}\Upsilon(t) \le -\alpha\Upsilon(t) + c(\sup_{t-\tau(t)\le s\le t}\Upsilon(s))$, $\forall t > t_0$. If $a > c > 0$, then $\Upsilon(t) \le \sup_{-\tau(t)\le s\le 0}\Upsilon(t_0+s)e^{-\varsigma(t-t_0)}$, $\forall t > t_0$, in which $0 < \varsigma < a$ is determined by the equation $\varsigma - a + ce^{\varsigma\tau(t)} = 0$.*

Corollary 9. *Suppose that (A2) holds; the communication topology G_σ satisfies (A3) and (A4). If there exist positive definite matrices P, Q and a constant $\beta > 0$ such that*

$$P\left(A + B^\sigma\right) + \left(A + B^\sigma\right)^{\mathrm{T}} P \le -\beta I_n, \quad \sigma \in \wp, \tag{41}$$

$$\begin{aligned} &2h^2 \frac{\lambda_{max}(P)}{\lambda_{min}(P)} \max_{\sigma\in\wp} \left\| \left(PB^\sigma\right)^{\mathrm{T}} Q^{-1} PB^\sigma \right\| \\ &\quad \cdot \left(\left\| A^{\mathrm{T}} A \right\| + \max_{\sigma\in\wp} \left\| \left(B^\sigma\right)^{\mathrm{T}} B^\sigma \right\| \right) + \lambda_{max}(Q) < \beta \end{aligned} \tag{42}$$

then, for the dynamical network (5), manifold Γ_1 and manifold Γ_2 are exponential stable.

Proof. Define a Lyapunov-Razumikhin function $V(t) = V_1(t)$. According to Leibniz-Newton formula, we have

$$\varepsilon(t) + \varepsilon(t - \tau(t)) = \int_{t-\tau(t)}^{t} \dot{\varepsilon}(s)\, ds. \tag{43}$$

Then, (22) can be written as

$$\dot{\varepsilon}(t) = \left(A + B^\sigma\right)\varepsilon(t) - B^\sigma \int_{t-\tau(t)}^{t} \dot{\varepsilon}(s)\, ds. \tag{44}$$

According to (44), we have

$$\begin{aligned} D^{+}V(t) &= \varepsilon^{\mathrm{T}}(t) \left(P\left(A + B^\sigma\right) + \left(A + B^\sigma\right)^{\mathrm{T}} P \right) \varepsilon(t) \\ &\quad - 2\int_{t-\tau(t)}^{t} \varepsilon^{\mathrm{T}}(t)\, PB^\sigma \dot{\varepsilon}(s)\, ds. \end{aligned} \tag{45}$$

It is noted that, for any positive definite matrix χ, there is

$$2a^{\mathrm{T}} b \le a^{\mathrm{T}} \chi a + b^{\mathrm{T}} \chi^{-1} b. \tag{46}$$

Let

$$\begin{aligned} D^{+}V(t) &\le -\beta\varepsilon^{\mathrm{T}}(t)\,\varepsilon(t) + \varepsilon^{\mathrm{T}}(t)\, Q\varepsilon(t) \\ &\quad + \left(PB^\sigma \int_{t-\tau(t)}^{t} \dot{\varepsilon}(s)\, ds \right)^{\mathrm{T}} \\ &\quad \cdot Q^{-1} \left(PB^\sigma \int_{t-\tau(t)}^{t} \dot{\varepsilon}(s)\, ds \right). \end{aligned} \tag{47}$$

From (41), we have

$$\begin{aligned} &D^{+}V(t) \le -\beta\varepsilon^{\mathrm{T}}(t)\,\varepsilon(t) + \varepsilon^{\mathrm{T}}(t)\, Q\varepsilon(t) \\ &\quad + \left(PB^\sigma \int_{t-\tau(t)}^{t} \dot{\varepsilon}(s)\, ds \right)^{\mathrm{T}} Q^{-1} \left(PB^\sigma \int_{t-\tau(t)}^{t} \dot{\varepsilon}(s)\, ds \right) \\ &\quad \le \left(-\beta + \lambda_{\max}(Q)\right) \varepsilon^{\mathrm{T}}(t)\,\varepsilon(t) \\ &\quad + \max_{\sigma\in\wp} \left\| \left(PB^\sigma\right)^{\mathrm{T}} Q^{-1} PB^\sigma \right\| \int_{t-\tau(t)}^{t} \int_{t-\tau(t)}^{t} \dot{\varepsilon}(s) \\ &\quad \cdot \dot{\varepsilon}(r)\, ds\, dr \le \left(-\beta + \lambda_{\max}(Q)\right) \varepsilon^{\mathrm{T}}(t)\,\varepsilon(t) + 2h^2 \\ &\quad \cdot \max_{\sigma\in\wp} \left\| \left(PB^\sigma\right)^{\mathrm{T}} Q^{-1} PB^\sigma \right\| \\ &\quad \cdot \left(\left\| A^{\mathrm{T}} A \right\| + \max_{\sigma\in\wp} \left\| \left(B^\sigma\right)^{\mathrm{T}} B^\sigma \right\| \right) \times \sup_{t-2\tau(t)\le s\le t} \varepsilon^{\mathrm{T}}(s) \\ &\quad \cdot \varepsilon(s) \le -\frac{\beta - \lambda_{\max}(Q)}{\lambda_{\max}(P)} V(t) + 2h^2 \\ &\quad \cdot \max_{\sigma\in\wp} \left\| \left(PB^\sigma\right)^{\mathrm{T}} Q^{-1} PB^\sigma \right\| \\ &\quad \cdot \left(\left\| A^{\mathrm{T}} A \right\| + \max_{\sigma\in\wp} \left\| \left(B^\sigma\right)^{\mathrm{T}} B^\sigma \right\| \right) \\ &\quad \times \frac{\sup_{t-2\tau(t)\le s\le t} V(s)}{\lambda_{\min}(P)}. \end{aligned} \tag{48}$$

According to Lemma 8, there exists $\eta > 0$, such that $V(t) \le \sup_{-2\tau(t)\le s\le 0} V(\varepsilon(t_0 + s)) e^{-\eta(t-t_0)}$, $t \ge t_0$. It is obvious that

$$\begin{aligned} &\|\varepsilon(t)\| \\ &\quad \le \left(\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)} \right)^{1/2} \sup_{-2\tau(t)\le s\le 0} \left\| \varepsilon(t_0 + s) \right\| e^{-(1/2)\eta(t-t_0)}, \\ &\qquad t \ge t_0. \end{aligned} \tag{49}$$

That is, (22) is exponential stable for zero solution. Therefore, for the dynamical network (5), manifold Γ_1 and manifold Γ_2 are exponential stable.

If the Laplacian matrix L^σ satisfies (A3) and (A4), then L^σ has a zero eigenvalue; other eigenvalues have positive real parts. Let $L_*^\sigma = (L^\sigma + (L^\sigma)^{\mathrm{T}})/2$, where L_*^σ is the Laplacian matrix of mirror strongly connected graph of G_σ [5]. Therefore, L_*^σ has one zero eigenvalue; other eigenvalues have positive real parts. Let $S^{-\mathrm{T}} L_*^\sigma S^{-1} = \mathrm{diag}[0, \overline{L}_*^\sigma]$; then $\overline{L}_*^\sigma$ is a positive definite matrix. □

Corollary 10. *Suppose that (A2) holds, the communication topology G_σ satisfies (A3) and (A4). If $b, k > 0$, such that*

$$\frac{k^2}{b^2} \lambda_{max}\left(M_e\right) + \frac{1}{4k \min_{\sigma\in\wp} \lambda_{min}\left(\overline{L}_*^\sigma\right)} < 1, \tag{50}$$

where h is small enough, then, for the dynamical network (5), manifold Γ_1 and manifold Γ_2 are exponential stable.

Proof. Define a Lyapunov-Razumikhin function $V(t) = V_1(t)$, where

$$P = \begin{bmatrix} I_{(n-1)m} & \frac{k}{b} M_e \\ \frac{k}{b} M_e & M_e \end{bmatrix}. \tag{51}$$

According to Schur complement lemma and (50), we know that matrix P is positive definite. We use the method that is similar to Corollary 9; then we have

$$\begin{aligned} D^{+}V(t) &= -\varepsilon^{\mathrm{T}}(t) Q \varepsilon(t) - 2\int_{t-\tau}^{t} \left(\varepsilon^{\mathrm{T}}(t) P B^{\sigma} A \varepsilon(s)\right. \\ &\quad \left. + \varepsilon^{\mathrm{T}}(t) P B^{\sigma} B^{\sigma} \varepsilon(s - \tau(t))\right) ds, \end{aligned} \tag{52}$$

where

$$Q = \begin{bmatrix} \frac{2k^2}{b} \overline{L}_{*}^{\sigma} & 2k\overline{L}_{*}^{\sigma} - I_{(n-1)m} \\ 2k\overline{L}_{*}^{\sigma} - I_{(n-1)m} & 2b\overline{L}_{*}^{\sigma} - \frac{2k}{b} M_e \end{bmatrix}. \tag{53}$$

According to Schur complement lemma and (50), we know that matrix Q is positive definite. After a simple calculation, we have

$$\begin{aligned} & D^{+}V(t) \\ & \leq \left(-\min_{\sigma\in\wp} \lambda_{\min}(Q) + h \max_{\sigma\in\wp}\left(\left\|PB^{\sigma}\right\|\left(\|A\| + \left\|B^{\sigma}\right\|\right)\right)\right) \\ & \cdot \varepsilon^{\mathrm{T}}(t)\varepsilon(t) + h \max_{\sigma\in\wp}\left(\left\|PB^{\sigma}\right\|\left(\|A\| + \left\|B^{\sigma}\right\|\right)\right) \\ & \cdot \sup_{t-2\tau(t)\leq s\leq t} \varepsilon^{\mathrm{T}}(s)\varepsilon(s) \leq -V(t) \\ & \cdot \frac{\min_{\sigma\in\wp}\lambda_{\min}(Q) - h\max_{\sigma\in\wp}\left(\left\|PB^{\sigma}\right\|\left(\|A\| + \left\|B^{\sigma}\right\|\right)\right)}{\lambda_{\max}(P)} \\ & + \frac{h\max_{\sigma\in\wp}\left(\left\|PB^{\sigma}\right\|\left(\|A\| + \left\|B^{\sigma}\right\|\right)\right)}{\lambda_{\min}(P)} \sup_{t-2\tau(t)\leq s\leq t} V(s); \end{aligned} \tag{54}$$

we take

$$\begin{aligned} & h \\ & < \frac{\min_{\sigma\in\wp}\lambda_{\min}(Q)}{\max_{\sigma\in\wp}\left(\|PB^{\sigma}\|\left(\|A\| + \|B^{\sigma}\|\right)\right)\left(1 + \lambda_{\max}(P)/\lambda_{\min}(P)\right)}. \end{aligned} \tag{55}$$

According to Lemma 8, we can get the conclusion. □

Remark 11. Learning from Corollaries 9 and 10, the upper bound of permitted time delay is given by (42) and (55). Based on Corollary 9, we can use a special P matrix to get Corollary 10. Therefore, Corollary 10 is a special case of Corollary 9.

5. Simulation Results

In this section the proposed theorems have been used for synchronizing the second-order nodes in the dynamical network. Considering twelve second-order nodes in the dynamical network with switching topologies $\{G_a, G_b, G_c\}$, Figure 1 gives three strongly connected and balanced graphs with 0–2 weights. The initial values of x_i and v_i are selected randomly in the regions $[0, 800] \times [0, 800]$ and $[0, 800] \times [0, 800]$, respectively.

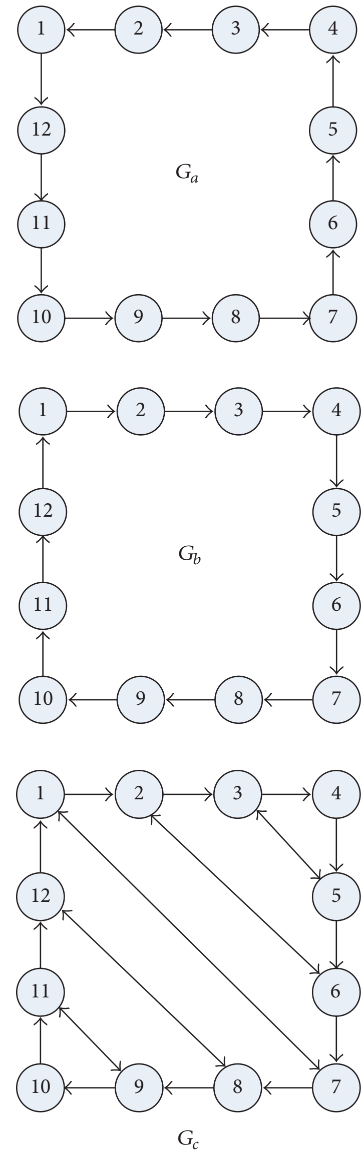

Figure 1: Strongly connected and balanced graphs.

The inertia of twelve second-order nodes are $M_1 = \mathrm{diag}\{1, 1, 1, 1\}, M_2 = \mathrm{diag}\{2, 2, 2, 2\}, \ldots, M_{12} = \mathrm{diag}\{12, 12, 12, 12\}$. It is known from Theorem 5 that there exists a proper $h > 0$ for any switching signal $\sigma(t)$ such that (22) is exponential stable for zero solution. Let $b = 1$, $k = 0.6$, $\alpha = 0.3$, and $r = 0.2$. Through the use of Theorem 5, the maximum upper bound of time delay $h = 0.12$ can be acquired; the corresponding solutions are as follows.

$$P=\begin{bmatrix}
0.0432 & 0 & 0.0095 & 0 & 0.0265 & 0 & 0.0179 & 0 & 0.0009 & 0 & 0.0245 & 0\\
0 & 0.0432 & 0 & 0.0095 & 0 & 0.0265 & 0 & 0.0179 & 0 & 0.0009 & 0 & 0.0245\\
0.0095 & 0 & 0.0239 & 0 & 0.0108 & 0 & 0.0091 & 0 & 0.0182 & 0 & 0.009 & 0\\
0 & 0.0095 & 0 & 0.0239 & 0 & 0.0108 & 0 & 0.0091 & 0 & 0.0182 & 0 & 0.009\\
0.0265 & 0 & 0.0108 & 0 & 0.0398 & 0 & 0.0068 & 0 & 0.007 & 0 & 0.0312 & 0\\
0 & 0.0265 & 0 & 0.0108 & 0 & 0.0398 & 0 & 0.0068 & 0 & 0.007 & 0 & 0.0312\\
0.0179 & 0 & 0.0091 & 0 & 0.0068 & 0 & 0.0188 & 0 & 0.0115 & 0 & 0.0123 & 0\\
0 & 0.0179 & 0 & 0.0091 & 0 & 0.0068 & 0 & 0.0188 & 0 & 0.0115 & 0 & 0.0123\\
0.0009 & 0 & 0.0182 & 0 & 0.007 & 0 & 0.0115 & 0 & 0.0462 & 0 & 0.0217 & 0\\
0 & 0.0009 & 0 & 0.0182 & 0 & 0.007 & 0 & 0.0115 & 0 & 0.0462 & 0 & 0.0217\\
0.0245 & 0 & 0.009 & 0 & 0.0312 & 0 & 0.0123 & 0 & 0.0217 & 0 & 0.0703 & 0\\
0 & 0.0245 & 0 & 0.009 & 0 & 0.0312 & 0 & 0.0123 & 0 & 0.0217 & 0 & 0.0703
\end{bmatrix},$$

$$R=\begin{bmatrix}
0.1033 & 0 & 0.0044 & 0 & 0.0741 & 0 & 0.026 & 0 & -0.0063 & 0 & 0.053 & 0\\
0 & 0.1033 & 0 & 0.0044 & 0 & 0.0741 & 0 & 0.026 & 0 & -0.0063 & 0 & 0.053\\
0.0044 & 0 & 0.0213 & 0 & 0.003 & 0 & 0.0075 & 0 & 0.0209 & 0 & 0.0091 & 0\\
0 & 0.0044 & 0 & 0.0213 & 0 & 0.003 & 0 & 0.0075 & 0 & 0.0209 & 0 & 0.0091\\
0.0741 & 0 & 0.003 & 0 & 0.073 & 0 & 0.0146 & 0 & 0.0005 & 0 & 0.0593 & 0\\
0 & 0.0741 & 0 & 0.003 & 0 & 0.073 & 0 & 0.0146 & 0 & 0.0005 & 0 & 0.0593\\
0.026 & 0 & 0.0075 & 0 & 0.0146 & 0 & 0.023 & 0 & 0.0076 & 0 & 0.0191 & 0\\
0 & 0.026 & 0 & 0.0075 & 0 & 0.0146 & 0 & 0.023 & 0 & 0.0076 & 0 & 0.0191\\
-0.0063 & 0 & 0.0209 & 0 & 0.0005 & 0 & 0.0076 & 0 & 0.0469 & 0 & 0.007 & 0\\
0 & -0.0063 & 0 & 0.0209 & 0 & 0.0005 & 0 & 0.0076 & 0 & 0.0469 & 0 & 0.007\\
0.053 & 0 & 0.0091 & 0 & 0.0593 & 0 & 0.0191 & 0 & 0.007 & 0 & 0.0808 & 0\\
0 & 0.053 & 0 & 0.0091 & 0 & 0.0593 & 0 & 0.0191 & 0 & 0.007 & 0 & 0.0808
\end{bmatrix}, \tag{56}$$

$$U=\begin{bmatrix}
0.016 & 0 & 0.0013 & 0 & 0.011 & 0 & 0.006 & 0 & -0.0013 & 0 & 0.0091 & 0\\
0 & 0.016 & 0 & 0.0013 & 0 & 0.011 & 0 & 0.006 & 0 & -0.0013 & 0 & 0.0091\\
0.0013 & 0 & 0.001 & 0 & 0.0002 & 0 & 0.0028 & 0 & 0.0001 & 0 & 0.0022 & 0\\
0 & 0.0013 & 0 & 0.001 & 0 & 0.0002 & 0 & 0.0028 & 0 & 0.0001 & 0 & 0.0022\\
0.011 & 0 & 0.0002 & 0 & 0.0084 & 0 & 0.0009 & 0 & -0.0011 & 0 & 0.0042 & 0\\
0 & 0.011 & 0 & 0.0002 & 0 & 0.0084 & 0 & 0.0009 & 0 & -0.0011 & 0 & 0.0042\\
0.006 & 0 & 0.0028 & 0 & 0.0009 & 0 & 0.0174 & 0 & 0.0004 & 0 & 0.0134 & 0\\
0 & 0.006 & 0 & 0.0028 & 0 & 0.0009 & 0 & 0.0174 & 0 & 0.0004 & 0 & 0.0134\\
-0.0013 & 0 & 0.0001 & 0 & -0.0011 & 0 & 0.0004 & 0 & 0.0005 & 0 & -0.003 & 0\\
0 & -0.0013 & 0 & 0.0001 & 0 & -0.0011 & 0 & 0.0004 & 0 & 0.0005 & 0 & -0.003\\
0.0091 & 0 & 0.0022 & 0 & 0.0042 & 0 & 0.0134 & 0 & -0.0003 & 0 & 0.0121 & 0\\
0 & 0.0091 & 0 & 0.0022 & 0 & 0.0042 & 0 & 0.0134 & 0 & -0.0003 & 0 & 0.0121
\end{bmatrix}.$$

We set $\tau(t) = 0.2\sin(3t) + 0.05$; Figures 2 and 3 show the numerical simulation results of twelve second-order nodes in the dynamical network with arbitrary switching signal. Figure 2 gives the inertial nodes' position error curves. Figure 3

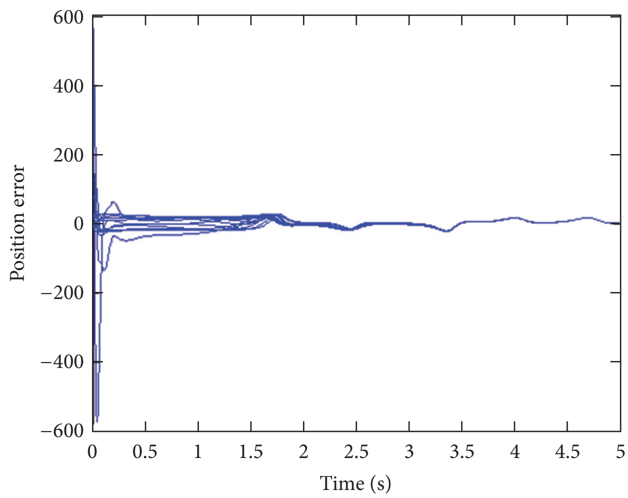

Figure 2: The inertial nodes' position error curves.

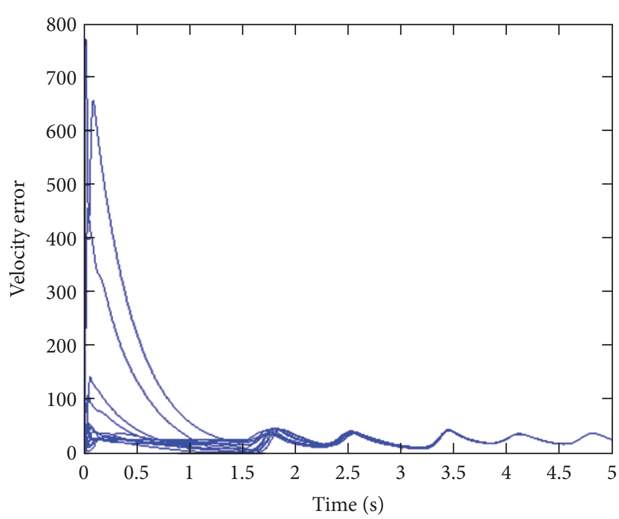

Figure 3: The inertial nodes' velocity error curves.

shows the inertial nodes' velocity error curves. It is obvious that for the dynamical network with communication time delay and switching topology, the control strategy achieves the exponential stability.

6. Conclusions

In this paper the exponential synchronization problem for second-order nodes in complex dynamical network with time-varying communication delays and switching communication topologies is investigated. Using the decomposition approach, a distributed control law has been designed such that the second-order nodes are exponential synchronized. The proposed method has been applied for synchronization of twelve second-order nodes, as a case study. Simulation results show the effective performance of the proposed control scheme. The results are expected to be more constructive in some practical control problems.

Competing Interests

This article content has no conflict of interests.

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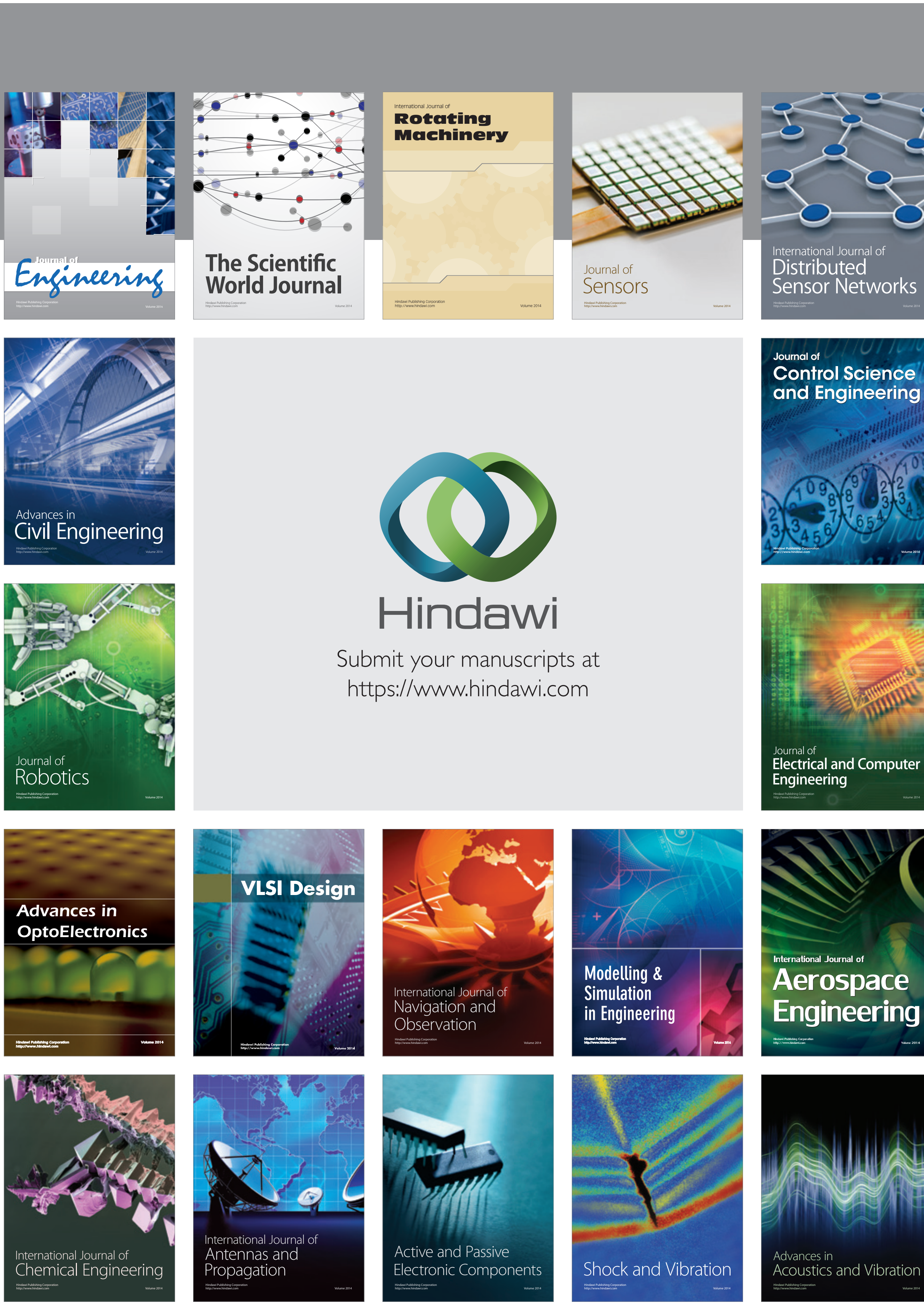

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