Research Article

Robust Command Filtered Adaptive Backstepping Control for a Quadrotor Aircraft

Yicheng Liu, Junhui Ma, and Haiyan Tu

College of Electrical & Information Engineering, Sichuan University, Chengdu 610065, China

Correspondence should be addressed to Haiyan Tu; haiyantu@163.com

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This paper addresses a robust trajectory tracking controller for an underactuated quadrotor with external bounded disturbances and unknown inertia parameters. Different from most of the existing control algorithms, the proposed method does not adopt the dual-loop scheme in which the design is divided into position control and attitude control. Instead, command filter backstepping is employed to design the controller based on the integrated motion model such that the stability can be guaranteed strictly for the flight control system. Furthermore, adaptive compensation and robust compensation are introduced to deal with the uncertainty of the inertia parameters and the external bounded disturbances, respectively. Finally, a similar skew symmetric structure is chosen as the desired structure of the closed-loop system to facilitate the analysis of the stability of the integrated system. Stability and robust performance of the designed controller are verified by Lyapunov stability theorem. Simulations are provided to validate the proposed controller.

1. Introduction

Recently, the unmanned aerial vehicles (UAV) have attracted the extensive attention from researchers in the field of control. As a popular UAV, a quadrotor can implement vertical take-off, landing, and autonomous hover, and it has stable control performance. In addition, it has advantages of small size, simple structure, and low cost, and it is safer than ordinary helicopters. Based on the above advantages, the research of UAV is of great significance in search, monitoring, and rescue [1–3].

Many control methods have been proposed for flight control of a quadrotor. The typical linear control methods, such as linear quadratic regulator control and proportional integral derivative control, can be used to design the controller [4, 5]. However, the stability of these methods cannot be guaranteed when the quadrotor moves away from its equilibrium configuration too much. Compared to linear control, nonlinear control can implement global stable flight for a quadrotor. Typical nonlinear control methods, such as sliding model control, backstepping control, and adaptive control, have been applied to quadrotor flight control. For example, a global fast dynamic terminal sliding mode control method

was proposed for the position and attitude tracking control [6]; a backstepping approach based on a full state cascaded dynamics has been introduced in [7]; an observer-based adaptive fuzzy backstepping controller was designed for trajectory tracking of a quadrotor under wind gust conditions and parametric uncertainties [8]. It is noteworthy that most of the existing controller design methods adopt a multiloop scheme. For example, a nonlinear controller was proposed in [9] by using a backstepping-like feedback linearization method. The control system is composed of 3 loops, named attitude loop, altitude loop, and position loop. In [10], the dynamic system of the quadrotor includes two loops as follows: the inner loop for attitude control and the outer loop for position control. However, the stability of these methods is proved, respectively, for the related control loop; respectively, the stability of the integrated flight control system cannot be guaranteed strictly. To overcome this drawback, a command filtered implementation approach for adaptive backstepping has been introduced in [11], and the stability of the closed-loop system is proved via Lyapunov direct method. In fact, an adaptive command filtered backstepping flight control law has been applied to the trajectory tracking problem of a nonlinear quadrotor model [12]. Unlike in [11], online update

FIGURE 1: Schematic of the quadrotor.

laws were established for the uncertain parameters which represent the mass, inertia, actuator efficiency, and thruster misalignment. In [13], a robust adaptive attitude tracking control for a spacecraft with unknown inertial parameters and bounded external disturbances is proposed. Its robust compensation and closed-loop system design ideas have given this great inspiration.

In this paper, a trajectory tracking controller is designed for a quadrotor by employing a robust command filtered adaptive backstepping control method based on the integrated flight motion model. Thus, the stability is strictly guaranteed for the closed-loop system in contrast to most of the existing multiloop control methods. During the controller design, the similar skew symmetric structure is chosen as the desired structure for the closed-loop system. On this basis, a backstepping design procedure is adopted to obtain the stabilizing functions of the system. A command filter scheme is proposed to bypass the difficulty of analytic calculation of the partial derivatives of stabilizing functions. Accordingly, controller design is simplified significantly compared with the conventional backstepping procedure. Furthermore, adaptive compensation and robust compensation are adopted to deal with the uncertainty of the inertia parameters and the external bounded disturbances, respectively.

This paper is organized as follows: Section 2 addressed the flight movement modeling for a quadrotor. Section 3 is dedicated to robust command filtered adaptive backstepping controller design and the stability analysis of the closed-loop system. In Section 4, simulation results are provided to demonstrate the effectiveness and robustness of the proposed methods. Finally, some conclusion remarks are included in Section 5.

2. Quadrotor Flight Model

This section addresses quadrotor flight model. To facilitate the modeling process, as shown in Figure 1, some notations are introduced as follows:

E (x_e, y_e, z_e): the earth fixed frame;

B (x_b, y_b, z_b): the body fixed frame;

R: the rotation matrix of **B** with respect to **E**, and it can be expressed as

$$\mathbf{R} = \begin{bmatrix} c\theta c\psi & s\phi s\theta c\psi - c\phi s\psi & c\phi s\theta c\psi + s\phi s\psi \\ c\theta s\psi & s\phi s\theta s\psi + c\phi c\psi & c\phi s\theta s\psi - s\phi c\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{bmatrix}; \quad (1)$$

$\xi = [x \ y \ z]^T$: the position of the quadrotor with respect to the frame **E**;

$\mathbf{v} = [\dot{x} \ \dot{y} \ \dot{z}]^T$: the velocity of the quadrotor with respect to the frame **E**;

$s(\cdot) = \sin(\cdot)$, $c(\cdot) = \cos(\cdot)$;

ϕ, θ, ψ : the roll angle, the pitch angle and the yaw angle, respectively;

$\boldsymbol{\eta} = [\phi \ \theta \ \psi]^T$: the vector of Euler angle;

$\mathbf{w} = [p \ q \ r]^T$: the angular velocity of the quadrotor expressed in frame **B**;

m : the mass of the quadrotor;

g : the gravity acceleration;

k_b : the thrust factor of the motor;

Ω_i : the angular velocity of the motor i ;

$U_1 = k_b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2)$: the height control input of the quadrotor;

$T_f = U_1$: the thrust force produced by the four propellers;

k_d : the drag factor of the motor;

$U_2 = k_b(\Omega_4^2 - \Omega_2^2)$, $U_3 = k_b(\Omega_3^2 - \Omega_1^2)$, $U_4 = k_d(\Omega_1^2 + \Omega_3^2 - \Omega_2^2 - \Omega_4^2)$: the control inputs for pitch angle, roll angle, and yaw angle, respectively;

l : the distance from the motor to the center of gravity;

$\boldsymbol{\tau}_f = [\tau_{f\phi} \ \tau_{f\theta} \ \tau_{f\psi}]^T = [lU_2 \ lU_3 \ U_4]^T$: the control torque obtained by varying the rotor speeds;

$\mathbf{E}_z = [0 \ 0 \ 1]^T$: the unit vector along the z -axis in the frame **E**;

$\mathbf{J} = \text{diag}\{J_x, J_y, J_z\}$: the inertia matrix of the quadrotor.

The flight model of the quadrotor consists of earth fixed frame **E** and body fixed frame **B**. The Kinematics and dynamics models of quadrotor can be described as

$$\dot{\xi} = \mathbf{v} \quad (2a)$$

$$m\dot{\mathbf{v}} = T_f \mathbf{R} \mathbf{E}_z - mg \mathbf{E}_z + \mathbf{d}_1 \quad (2b)$$

$$\dot{\boldsymbol{\eta}} = \mathbf{W}(\boldsymbol{\eta}) \mathbf{w} \quad (2c)$$

$$\mathbf{J}\dot{\mathbf{w}} = -\mathbf{w} \times \mathbf{J}\mathbf{w} + \boldsymbol{\tau}_f + \mathbf{d}_2, \quad (2d)$$

where $\mathbf{d}_1, \mathbf{d}_2$ represent the aerodynamic forces and moments acting on the quadrotor. $\mathbf{W}(\boldsymbol{\eta})$ can be expressed as

$$\mathbf{W}(\boldsymbol{\eta}) = \begin{bmatrix} 1 & s\phi t\theta & c\phi t\theta \\ 0 & c\phi & -s\phi \\ 0 & \frac{s\phi}{c\theta} & \frac{c\phi}{c\theta} \end{bmatrix}. \quad (3)$$

Define the states vector to be $\boldsymbol{\chi} = [\xi \ \mathbf{v} \ \boldsymbol{\eta} \ \mathbf{w}]^T$.

To facilitate the controller design, we introduce the inertia parameters vector

$$\lambda = \left[\frac{J_y - J_z}{J_x} \frac{l}{J_x} \frac{J_z - J_x}{J_y} \frac{l}{J_y} \frac{J_x - J_y}{J_z} \frac{1}{J_z} \right]^T, \quad (4)$$

where the elements of λ are assumed to be unknown.

For simplifying the derivation, some denotations are introduced as follows: λ_0 stands for the nominal value of λ , which belongs to prior information for attitude controller design, and the relative error is denoted as $\Delta\lambda = \lambda - \lambda_0$; $\hat{\lambda}$ is an estimate of the unknown parameter vector λ ; estimation error vector is expressed by $\tilde{\lambda} = \lambda - \hat{\lambda}$.

3. Controller Design and Stability Analysis

This section addresses robust command filtered adaptive backstepping implementation approach with an integrated stability analysis. The control objective for the quadrotor is to track a desired trajectory and a desired yaw angle. The composition of this method is as follows: First, a backstepping procedure is used to derive the stabilizing functions. Then, the command filter is to ensure that the desired state of the system is closer to the value of the stabilizing function. Finally, adaptive compensation and robust compensation are introduced to deal with the uncertainty of the inertia parameters and the external bounded disturbances, respectively.

We give the desired position and the desired yaw angle and its derivative $\{\xi^c, \dot{\xi}^c, \psi^c(t), \dot{\psi}^c(t)\}$. Define the tracking error as

$$\begin{aligned} \tilde{\xi} &= \xi - \xi^c, \\ \tilde{\nu} &= \nu - \nu^c, \\ \tilde{\eta} &= \eta - \eta^c, \\ \tilde{\mathbf{w}} &= \mathbf{w} - \mathbf{w}^c, \end{aligned} \quad (5)$$

where $[\xi^{cT} \ \nu^{cT} \ \eta^{cT} \ \mathbf{w}^{cT}]^T$ represent the desired state.

Assumption 1. The desired trajectory ξ^c and its derivative $\dot{\xi}^c$ are bounded, continuous, and known.

Assumption 2. The model uncertainties of the quadrotor are subject to constraints as follows:

$$\begin{aligned} \|\mathbf{d}_1\| &\leq \delta, \\ \|\mathbf{J}^{-1}\mathbf{d}_1\| &\leq \gamma, \\ \|\Delta\lambda\| &\leq \varepsilon, \end{aligned} \quad (6)$$

where $\delta \in R^+$, $\gamma \in R^+$, $\varepsilon \in R^+$.

3.1. Controller Design. The design can be carried out with the backstepping procedure as follows.

Step 1. Consider ν as the control of (2a), and the virtual control can be chosen as

$$\nu_0^c = \alpha_1 = -\mathbf{k}_1 \tilde{\xi} + \dot{\xi}^c, \quad (7)$$

where $\mathbf{k}_1 \in R^{3 \times 3}$ is a diagonal matrix and each diagonal element of the matrix is a positive constant. The error equation (8) can be obtained by combining (2a). Taking time derive of $\tilde{\xi}$ yields

$$\dot{\tilde{\xi}} = -\mathbf{k}_1 \tilde{\xi} + \nu - \nu_0^c. \quad (8)$$

The command filter compensation for the virtual control ν_0^c is as follows:

$$\dot{\nu}^c = \mathbf{k}_{C1} (\nu_0^c - \nu^c), \quad (9)$$

where $\mathbf{k}_{C1} \in R^{3 \times 3}$ is a diagonal matrix and each diagonal element of the matrix is a positive constant. Denote $\alpha_1^c = \nu^c$, and then from (8), we can get

$$\begin{aligned} \dot{\tilde{\xi}} &= -\mathbf{k}_1 \tilde{\xi} + (\nu^c - \nu_0^c) + (\nu - \nu^c) \\ &= -\mathbf{k}_1 \tilde{\xi} + (\alpha_1^c - \alpha_1) + \mathbf{a}\tilde{\nu}, \end{aligned} \quad (10)$$

where

$$\mathbf{a} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (11)$$

Step 2. Since the yaw angle has been given, consider (ϕ, θ, U_1) as the control of (2b), and we define the stabilizing function as follows:

$$\alpha_2 = -\mathbf{k}_2 \tilde{\nu} + \dot{\nu}^c - \mathbf{a}^T \tilde{\xi} - \rho_1 \tilde{\nu}, \quad (12)$$

where $\mathbf{k}_2 \in R^{3 \times 3}$ is a diagonal matrix and each diagonal element of the matrix is a positive constant. ρ_1 is introduced to compensate the external disturbances $\mathbf{d}_1$.

According to (2b), α_2 will be achieved if we have the values

$$\begin{aligned} \eta_0^c(1:2) &= \begin{bmatrix} \arctan\left(\frac{\alpha_{21}c\psi + \alpha_{22}s\psi}{\alpha_{23} + g}\right) \\ \arctan\left(\frac{(\alpha_{21}s\psi - \alpha_{22}c\psi)c\theta_0^c}{\alpha_{23} + g}\right) \end{bmatrix} \\ U_1 &= m \frac{\alpha_{23} + g}{c\theta_0^c c\phi_0^c}, \end{aligned} \quad (13)$$

where $\alpha_2 = [\alpha_{21} \ \alpha_{22} \ \alpha_{23}]^T$.

The command filter compensation for the virtual control $\eta_0^c(1:2)$ is as follows:

$$\dot{\eta}^c(1:2) = \mathbf{k}_{C2} (\eta_0^c(1:2) - \eta^c(1:2)), \quad (14)$$

where $\mathbf{k}_{C2} \in R^{2 \times 2}$ is a diagonal matrix and each diagonal element of the matrix is a positive constant. $\eta(1:2)$ represents the first two elements in vector η .

According to (14), we can compute $\dot{\eta}^c$. α_2^c is constructed to facilitate the subsequent acquisition of the quadrotor error dynamic equation.

$$\alpha_2^c = \begin{bmatrix} (c\phi^c s\theta^c c\psi + s\phi^c s\psi) \frac{U_1}{m} \\ (c\phi^c s\theta^c s\psi - s\phi^c c\psi) \frac{U_1}{m} \\ c\phi^c c\theta^c \frac{U_1}{m} - g \end{bmatrix}. \quad (15)$$

The error equation (16) can be obtained by combining (2b).

$$\begin{aligned} \dot{\tilde{\mathbf{y}}} = & -\mathbf{k}_2 \tilde{\mathbf{y}} - \mathbf{a}^T \tilde{\xi} + (\alpha_2^c - \alpha_2) + \mathbf{b} \tilde{\eta} (1:2) + \frac{1}{m} \mathbf{d}_1 \\ & - \rho_1 \tilde{\mathbf{y}}, \end{aligned} \quad (16)$$

where

$$\mathbf{b} = \frac{U_1}{m} \begin{bmatrix} \frac{c\phi - c\phi^c}{2\tilde{\phi}} (s\theta^c + s\theta) c\psi + \frac{s\phi - s\phi^c}{\tilde{\phi}} s\psi & \frac{s\theta - s\theta^c}{2\tilde{\theta}} (c\phi^c + c\phi) c\psi \\ \frac{c\phi - c\phi^c}{2\tilde{\phi}} (s\theta^c + s\theta) s\psi - \frac{s\phi - s\phi^c}{\tilde{\phi}} c\psi & \frac{s\theta - s\theta^c}{2\tilde{\theta}} (c\phi^c + c\phi) s\psi \\ \frac{c\phi - c\phi^c}{2\tilde{\phi}} (c\theta^c + c\theta) & \frac{c\theta - c\theta^c}{2\tilde{\theta}} (c\phi^c + c\phi) \end{bmatrix}. \quad (17)$$

when $\tilde{\phi}, \tilde{\theta}$ approach zero and the vector function $\mathbf{b}$ can be written as

$$\mathbf{b} = \frac{U_1}{m} \begin{bmatrix} \frac{-s\phi^c}{2} (s\theta^c + s\theta) c\psi + c\phi^c s\psi & \frac{c\theta^c}{2} (c\phi^c + c\phi) c\psi \\ \frac{-s\phi^c}{2} (s\theta^c + s\theta) s\psi - c\phi^c c\psi & \frac{c\theta^c}{2} (c\phi^c + c\phi) s\psi \\ \frac{-s\phi^c}{2} (c\theta^c + c\theta) & \frac{-s\theta^c}{2} (c\phi^c + c\phi) \end{bmatrix}. \quad (18)$$

Step 3. Consider $\mathbf{w}$ as the control of (2c), and we define the stabilizing function as follows:

$$\alpha_3 = -\mathbf{k}_3 \tilde{\eta} + \dot{\eta}^c - [\mathbf{b}^T \tilde{\mathbf{y}}; \mathbf{0}], \quad (19)$$

where $\mathbf{k}_3 \in R^{3 \times 3}$ is a diagonal matrix and each diagonal element of the matrix is a positive constant.

According to (2c), α_3 can be obtained if $\mathbf{w}$ is chosen as

$$\mathbf{w}_0^c = \begin{bmatrix} \alpha_{31} - \alpha_{33} s\theta \\ \alpha_{32} c\phi + \alpha_{33} c\theta s\phi \\ \alpha_{33} c\theta c\phi - \alpha_{32} s\phi \end{bmatrix}, \quad (20)$$

where $\alpha_3 [\alpha_{31} \ \alpha_{32} \ \alpha_{33}]^T$.

The command filter of $\mathbf{w}_0^c$ can be described as

$$\dot{\mathbf{w}}^c = \mathbf{k}_{C3} (\mathbf{w}_0^c - \mathbf{w}^c), \quad (21)$$

where $\mathbf{k}_{C3} \in R^{3 \times 3}$ is a diagonal matrix and each diagonal element of the matrix is a positive constant. According to (21), we can compute $\mathbf{w}^c$. α_3^c can be chosen as

$$\alpha_3^c = \begin{bmatrix} p^c + q^c s\phi \tan \theta + r^c c\phi \tan \theta \\ q^c c\phi - r^c s\phi \\ \frac{q^c s\phi + r^c c\phi}{c\theta} \end{bmatrix}. \quad (22)$$

Thus we can obtain

$$\dot{\tilde{\eta}} = -\mathbf{k}_3 \tilde{\eta} + (\alpha_3^c - \alpha_3) + \mathbf{c} \tilde{\mathbf{w}} - [\mathbf{b}^T \tilde{\mathbf{y}}; \mathbf{0}], \quad (23)$$

where

$$\mathbf{c} = \begin{bmatrix} 1 & s\phi t\theta & c\phi t\theta \\ 0 & c\phi & -s\phi \\ 0 & s\phi \sec \theta & c\phi \sec \theta \end{bmatrix}. \quad (24)$$

Step 4. Now the control can be chosen as

$$\begin{aligned} \boldsymbol{\tau} = \alpha_4 = & \text{diag} \left\{ \frac{1}{\hat{\lambda}_2}, \frac{1}{\hat{\lambda}_4}, \frac{1}{\hat{\lambda}_6} \right\} \left(-\mathbf{k}_4 \tilde{\mathbf{w}} + \dot{\mathbf{w}}^c - \mathbf{c}^T \tilde{\eta} \right. \\ & \left. - [qr\hat{\lambda}_1, pr\hat{\lambda}_3, pq\hat{\lambda}_5]^T - \rho_2 \tilde{\mathbf{w}} \right), \end{aligned} \quad (25)$$

where $\mathbf{k}_4 \in R^{3 \times 3}$ is a diagonal matrix and each diagonal element of the matrix is a positive constant. ρ_2 is introduced to compensate the external disturbances $\mathbf{d}_2$. Thus combining (25) and (2d) yields

$$\dot{\tilde{\mathbf{w}}} = -\mathbf{k}_4 \tilde{\mathbf{w}} - \mathbf{c}^T \tilde{\eta} + \Phi^T \tilde{\lambda} + \mathbf{J}^{-1} \mathbf{d}_2 - \rho_2 \tilde{\mathbf{w}}, \quad (26)$$

where Φ can be expressed as

$$\Phi = \begin{bmatrix} qr & U_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & pr & U_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & pq & U_4 \end{bmatrix}^T. \quad (27)$$

The adaptive algorithm is introduced to compensate for the interference caused by the uncertain model parameters. The updating law of $\hat{\lambda}$ can be given by

$$\begin{aligned}\dot{\hat{\lambda}} &= \Gamma \Phi \tilde{w} - \sigma (\hat{\lambda} - \lambda_0) \\ &= \Gamma \Phi \tilde{w} + \sigma \tilde{\lambda} - \sigma \Delta \lambda,\end{aligned}\quad (28)$$

where $\Gamma \in R^{6 \times 6}$ is a diagonal matrix and each diagonal element of the matrix is a positive constant. Since λ is a constant vector, we obtain that

$$\dot{\tilde{\lambda}} = \dot{\lambda} - \dot{\hat{\lambda}} = -\Gamma \Phi \tilde{w} - \sigma \tilde{\lambda} + \sigma \Delta \lambda. \quad (29)$$

The command filter described in (9), (14), and (21) can be summarized as follows:

$$\begin{aligned}\dot{v}^c &= k_{C1} (\alpha_1 - v^c) \\ \dot{\eta}^c (1:2) &= k_{C2} (\eta_0^c (1:2) - \eta^c (1:2)) \\ \dot{w}^c &= k_{C3} (w_0^c - w^c).\end{aligned}\quad (30)$$

Equation (30) is a first-order, low-pass filter, and bandwidth parameterized by k_{ci} . The purpose of this command filter is to generate $|\chi_i^c - \alpha_{i-1}|$, $i = 2, 3, 4$ is small. The initial values of $(v^c, \eta^c (1:2), w^c)$ can be chosen as

$$\begin{aligned}v^c(0) &= v(0), \\ \eta^c(0) &= \eta(0), \\ w^c(0) &= w(0).\end{aligned}\quad (31)$$

3.2. Error Dynamic Equation. According to the error equations (10), (16), (23), (26), and (29), the closed-loop system can be described as

$$\begin{aligned}\begin{bmatrix} \dot{\tilde{\xi}} \\ \dot{\tilde{v}} \\ \dot{\tilde{\eta}} \\ \dot{\tilde{w}} \\ \dot{\tilde{\lambda}} \end{bmatrix} &= \begin{bmatrix} -k_1 & \mathbf{a} & 0 & 0 & 0 \\ -\mathbf{a}^T & -k_2 - \rho_1 & [\mathbf{b}, \mathbf{0}] & 0 & 0 \\ 0 & -[\mathbf{b}, \mathbf{0}]^T & -k_3 & \mathbf{c} & 0 \\ 0 & 0 & -\mathbf{c}^T & -k_4 - \rho_2 & \Phi^T \\ 0 & 0 & 0 & -\Gamma \Phi & -\sigma \mathbf{I}_6 \end{bmatrix} \begin{bmatrix} \tilde{\xi} \\ \tilde{v} \\ \tilde{\eta} \\ \tilde{w} \\ \tilde{\lambda} \end{bmatrix} \\ &+ \begin{bmatrix} (\alpha_1^c - \alpha_1) \\ (\alpha_2^c - \alpha_2) + \frac{1}{m} \mathbf{d}_1 \\ (\alpha_3^c - \alpha_3) \\ \mathbf{J}^{-1} \mathbf{d}_2 \\ \sigma \Delta \lambda \end{bmatrix},\end{aligned}\quad (32)$$

where we can easily analyze the stability of the closed-loop system when the system interference is 0. In this case, it is

clear to see that the closed-loop system has the similar skew symmetric structure of system in [13].

The compensating signal is introduced to eliminate the virtual control error $\alpha_i^c - \alpha_i$ for $i = 1, 2, 3$. According to the error dynamic system (32), the compensating system dynamic equation is constructed as follows:

$$\begin{aligned}\begin{bmatrix} \dot{\zeta}_1 \\ \dot{\zeta}_2 \\ \dot{\zeta}_3 \\ \dot{\zeta}_4 \\ \dot{\zeta}_5 \end{bmatrix} &= \begin{bmatrix} -k_1 & \mathbf{a} & 0 & 0 & 0 \\ -\mathbf{a}^T & -k_2 - \rho_1 & [\mathbf{b}, \mathbf{0}] & 0 & 0 \\ 0 & -[\mathbf{b}, \mathbf{0}]^T & -k_3 & \mathbf{c} & 0 \\ 0 & 0 & -\mathbf{c}^T & -k_4 - \rho_2 & \Phi^T \\ 0 & 0 & 0 & -\Gamma \Phi & -\sigma \mathbf{I}_6 \end{bmatrix} \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \\ \zeta_5 \end{bmatrix} \\ &+ \begin{bmatrix} (\alpha_1^c - \alpha_1) \\ (\alpha_2^c - \alpha_2) \\ (\alpha_3^c - \alpha_3) \\ 0 \\ 0 \end{bmatrix},\end{aligned}\quad (33)$$

where $\zeta_1, \zeta_2, \zeta_3, \zeta_4 \in R^3$ are the state vectors of compensating system and $\zeta = [\zeta_1^T \ \zeta_2^T \ \zeta_3^T \ \zeta_4^T]^T$, $\alpha_i^c - \alpha_i$ for $i = 1, 2, 3$ are the input signal of compensating system. The initial condition can be chosen as $\zeta = \mathbf{0}$.

Assumption 3 ($|\alpha_i^c(t) - \alpha_i(t)| \leq \mu$, $t \geq T_0$ for $i = 1, 2, 3$). Combining (16) and (19) yields, constructing the error dynamic equations (35) for quadrotor compensating system. New state variables are defined as follows:

$$\begin{aligned}\mathbf{v}_1 &= \tilde{\xi} - \zeta_1 \\ \mathbf{v}_2 &= \tilde{v} - \zeta_2 \\ \mathbf{v}_3 &= \tilde{\eta} - \zeta_3 \\ \mathbf{v}_4 &= \tilde{w} - \zeta_4 \\ \mathbf{v}_5 &= \tilde{\lambda}.\end{aligned}\quad (34)$$

The error dynamic equations of quadrotor compensating system can be described as

$$\begin{aligned}
& \begin{bmatrix} \dot{\mathbf{v}}_1 \\ \dot{\mathbf{v}}_2 \\ \dot{\mathbf{v}}_3 \\ \dot{\mathbf{v}}_4 \\ \dot{\mathbf{v}}_5 \end{bmatrix} \\
&= \begin{bmatrix} -\mathbf{k}_1 & \mathbf{a} & 0 & 0 & 0 \\ -\mathbf{a}^T & -\mathbf{k}_2 - \rho_1 & [\mathbf{b}, \mathbf{0}] & 0 & 0 \\ 0 & -[\mathbf{b}, \mathbf{0}]^T & -\mathbf{k}_3 & \mathbf{c} & 0 \\ 0 & 0 & -\mathbf{c}^T & -\mathbf{k}_4 - \rho_2 & \Phi^T \\ 0 & 0 & 0 & -\Gamma\Phi & -\sigma\mathbf{I}_6 \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \mathbf{v}_3 \\ \mathbf{v}_4 \\ \mathbf{v}_5 \end{bmatrix} \quad (35) \\
&+ \begin{bmatrix} 0 \\ \frac{1}{m}\mathbf{d}_1 \\ 0 \\ \mathbf{J}^{-1}\mathbf{d}_2 \\ \sigma\Delta\lambda \end{bmatrix}.
\end{aligned}$$

3.3. Stability Analysis

Theorem 4. Under Assumption 3, the states of system (33) are bounded.

Proof. For the compensating system (33), we can construct the Lyapunov function V_c as

$$V_c = \frac{1}{2} \left(\zeta_1^T \zeta_1 + \zeta_2^T \zeta_2 + \zeta_3^T \zeta_3 + \zeta_4^T \zeta_4 + \frac{1}{\Gamma} \zeta_5^T \zeta_5 \right). \quad (36)$$

Substituting (33) and Assumption 3 yields

$$\begin{aligned}
\dot{V}_c &= -\zeta_1^T \mathbf{k}_1 \zeta_1 - \zeta_2^T \mathbf{k}_2 \zeta_2 - \zeta_3^T \mathbf{k}_3 \zeta_3 - \zeta_4^T \mathbf{k}_4 \zeta_4 - \frac{\sigma}{\Gamma} \zeta_5^T \zeta_5 \\
&+ \sum_i^3 \zeta_i^T (\alpha_i^c - \alpha_i) - \rho_1 \zeta_2^T \zeta_2 - \rho_2 \zeta_4^T \zeta_4 \quad (37) \\
&\leq -2k_0 \|\zeta\|^2 + \mu \|\zeta\|,
\end{aligned}$$

where $k_0 = 0.5 \min(\|\mathbf{k}_1\|, \|\mathbf{k}_1\| + \rho_1, \|\mathbf{k}_3\|, \|\mathbf{k}_4\| + \rho_2, \sigma/\Gamma)$.

Since $(1/2)k_V \|\zeta\|^2 \leq V_c$, $k_V = \min(1, 1/\Gamma)$, we obtain

$$\dot{V}_c \leq -\frac{4k_0}{k_V} V_c + \mu \sqrt{\frac{2}{k_V}} \sqrt{V_c}. \quad (38)$$

Then, from (38) we can get

$$\sqrt{V_c} \leq \frac{\mu \sqrt{2k_V}}{4k_0} \implies \|\zeta\| \leq \frac{\mu}{2k_0}. \quad (39)$$

Thus, the state of compensating system (33) is bounded convergence for bounded inputs satisfying $|\alpha_i^c - \alpha_i| \leq \mu$. $\square$

Theorem 5. Under Assumptions 1 and 2, the states of system (35) are uniformly bounded.

Proof. For the error dynamic equations of quadrotor compensating system (35), we can construct the Lyapunov function V_{ce} as

$$V_{ce} = \frac{1}{2} \left(\mathbf{v}_1^T \mathbf{v}_1 + \mathbf{v}_2^T \mathbf{v}_2 + \mathbf{v}_3^T \mathbf{v}_3 + \mathbf{v}_4^T \mathbf{v}_4 + \frac{1}{\Gamma} \mathbf{v}_5^T \mathbf{v}_5 \right). \quad (40)$$

Differentiating V_{ce} with respect to time, we get

$$\begin{aligned}
\dot{V}_{ce} &= \mathbf{v}_1^T (-\mathbf{k}_1 \mathbf{v}_1 + \mathbf{a} \mathbf{v}_2) \\
&+ \mathbf{v}_2^T \left(-\mathbf{k}_2 \mathbf{v}_2 + [\mathbf{b}, \mathbf{0}] \mathbf{v}_3 - \mathbf{a}^T \mathbf{v}_1 + \frac{1}{m} \mathbf{d}_1 - \rho_1 \mathbf{v}_2 \right) \\
&+ \mathbf{v}_3^T \left(-\mathbf{k}_3 \mathbf{v}_3 + \mathbf{c} \mathbf{v}_4 - [\mathbf{b}, \mathbf{0}]^T \mathbf{v}_2 \right) \\
&+ \mathbf{v}_4^T \left(-\mathbf{k}_4 \mathbf{v}_4 - \mathbf{c}^T \mathbf{v}_3 + \Phi^T \mathbf{v}_5 + \mathbf{J}^{-1} \mathbf{d}_2 - \rho_2 \mathbf{v}_4 \right) \\
&+ \frac{1}{\Gamma} \mathbf{v}_5^T (-\Gamma \Phi \mathbf{v}_4 - \sigma \mathbf{v}_5 + \sigma \Delta \lambda) \\
&= -\mathbf{v}_1^T \mathbf{k}_1 \mathbf{v}_1 - \mathbf{v}_2^T \mathbf{k}_2 \mathbf{v}_2 - \mathbf{v}_3^T \mathbf{k}_3 \mathbf{v}_3 - \mathbf{v}_4^T \mathbf{k}_4 \mathbf{v}_4 - \frac{\sigma}{\Gamma} \mathbf{v}_5^T \mathbf{v}_5 \\
&- \rho_1 \mathbf{v}_2^T \mathbf{v}_2 - \rho_2 \mathbf{v}_4^T \mathbf{v}_4 + \frac{1}{m} \mathbf{v}_2^T \mathbf{d}_1 + \mathbf{v}_4^T \mathbf{J}^{-1} \mathbf{d}_2 \\
&+ \frac{\sigma}{\Gamma} \mathbf{v}_5^T \Delta \lambda \quad (41) \\
&= -\mathbf{v}_1^T \mathbf{k}_1 \mathbf{v}_1 - \mathbf{v}_2^T \mathbf{k}_2 \mathbf{v}_2 - \mathbf{v}_3^T \mathbf{k}_3 \mathbf{v}_3 - \mathbf{v}_4^T \mathbf{k}_4 \mathbf{v}_4 - \frac{\sigma}{\Gamma} \mathbf{v}_5^T \mathbf{v}_5 \\
&+ \frac{\sigma}{2\Gamma} (\mathbf{v}_5^T \mathbf{v}_5 + \Delta \lambda^T \Delta \lambda) + \frac{1}{4\rho_2} (\mathbf{J}^{-1} \mathbf{d}_2)^T \mathbf{J}^{-1} \mathbf{d}_2 \\
&- \rho_1 \left(\mathbf{v}_2 - \frac{1}{2m\rho_1} \mathbf{d}_1 \right)^T \left(\mathbf{v}_2 - \frac{1}{2m\rho_1} \mathbf{d}_1 \right) \\
&+ \frac{1}{4m^2\rho_1} \mathbf{d}_1^T \mathbf{d}_1 \\
&- \rho_2 \left(\mathbf{v}_4 - \frac{1}{2\rho_2} \mathbf{J}^{-1} \mathbf{d}_2 \right)^T \left(\mathbf{v}_4 - \frac{1}{2\rho_2} \mathbf{J}^{-1} \mathbf{d}_2 \right).
\end{aligned}$$

From (41) and Assumption 2, we can get

$$\begin{aligned}
\dot{V}_{ce} &\leq -\mathbf{v}_1^T \mathbf{k}_1 \mathbf{v}_1 - \mathbf{v}_2^T \mathbf{k}_2 \mathbf{v}_2 - \mathbf{v}_3^T \mathbf{k}_3 \mathbf{v}_3 - \mathbf{v}_4^T \mathbf{k}_4 \mathbf{v}_4 \\
&- \frac{\sigma}{2\Gamma} \mathbf{v}_5^T \mathbf{v}_5 + \frac{\delta^2}{4m^2\rho_1} + \frac{\gamma^2}{4\rho_2} + \frac{\sigma\epsilon^2}{2\Gamma} \quad (42) \\
&\leq -\kappa V_{ce} + M,
\end{aligned}$$

where

$$\begin{aligned}
\kappa &= \min \left(\frac{1}{2} \|\mathbf{k}_1\|, \frac{1}{2} \|\mathbf{k}_2\|, \frac{1}{2} \|\mathbf{k}_3\|, \frac{1}{2} \|\mathbf{k}_4\|, \frac{\sigma}{2} \right) \\
M &= \frac{\delta^2}{4m^2\rho_1} + \frac{\gamma^2}{4\rho_2} + \frac{\sigma\epsilon^2}{2\Gamma}. \quad (43)
\end{aligned}$$

Therefore, V_{ce} decreases monotonically until $\mathbf{v}$ reaches the bounded set $\{\mathbf{v} : V_{ce} \leq \kappa^{-1}M\}$. This means that $\mathbf{v}$ is uniformly bounded. $\square$

TABLE 1: Quadrotor model parameters.

Parameter	Value
m (kg)	1.336
g (m/s ²)	9.807
l (m)	0.285
k_b	$1.23e^{-4}$
I_x (kg·m ²)	0.0259
I_y (kg·m ²)	0.0260
I_z (kg·m ²)	0.0397
k_d	$6.48e^{-6}$

According to Theorems 4 and 5, we can conclude that $\tilde{\chi}$ bounded by

$$\|\tilde{\chi}(t)\| \leq \frac{\mu}{2k_0} + \|\mathbf{v}(t)\|. \quad (44)$$

Thus, when t approaches infinity, $\tilde{\chi}$ is uniformly bounded. Apparently, κ represents the response speed of the closed system; thus the response speed can be arbitrarily adjusted by tuning the parameters $\mathbf{k}_i$ ($i = 0, \dots, 4$) and σ appropriately. $\kappa^{-1}M$ reflects the residual error bound of the closed-loop system, and the external disturbances and parameter uncertainties can be mitigated by tuning the parameters ρ_1, ρ_2 , and Γ .

4. Simulation

In this section, we design two simulation examples to illustrate the effectiveness of the proposed control algorithm. Sustained disturbances and parametric uncertainties have been imposed on the quadrotor to indicate the ability of antidisturbance and robustness. The model parameters of the quadrotor were obtained from a real quadrotor platform in paper [9] as Table 1. For the purpose of verifying the superiority of the algorithm, the proposed controller is simulated compared with that of [9]. The two simulations are carried out under the same conditions as follows:

(a) The external bounded disturbances are described by

$$\mathbf{d}_1 = 0.2 \times \begin{bmatrix} \sin(20\pi t) + \cos(200\pi t) \\ \cos(20\pi t) + \sin(200\pi t) \\ \sin(20\pi t) - \cos(200\pi t) \end{bmatrix} \mathbf{N} \quad (45)$$

$$\mathbf{d}_2 = 2 \times \text{randn}(3) \mathbf{N} \cdot \mathbf{m},$$

where $\text{randn}(3)$ represents a 3-dimensional random noise vector.

(b) The true inertia parameters are given by $\boldsymbol{\lambda} = [-0.529 \ 11 \ 0.53 \ 10.96 \ 0.0025 \ 7.18]^T$. The initial value of $\hat{\boldsymbol{\lambda}}$ is set as $\hat{\boldsymbol{\lambda}}(0) = [-1 \ 14.25 \ 1 \ 28.5 \ 0.333 \ 9.5]^T$, and $\boldsymbol{\lambda}_0 = [-0.35 \ 12.39 \ 0.32 \ 12.95 \ 0.033 \ 9.5]^T$.

Case 1 (hovering). The initial position and yaw angle of the quadrotor are set to $\boldsymbol{\xi}(0) = [0 \ 0 \ 0]^T$ m, $\psi(0) = 5$ deg. The desired position and yaw angle are set to $\boldsymbol{\xi}^c = [3 \ 3 \ 2]^T$ m, $\psi^c = 0$ deg. Figures 2 and 3 show the position response

FIGURE 2: Comparison result of position response.

and attitude response. Figure 4 represents the transformation process of adaptive estimation parameters.

Figures 2 and 3 show that the proposed controller can drive the quadrotor to the target position in a short time. When reaching the target position, the quadrotor is then hovering around it within a small range. From Figure 2, it can be seen that the position error is $\|\tilde{\mathbf{r}}\| \leq 2$ cm when using the proposed controller, while the position error is $\|\tilde{\mathbf{r}}\| \leq 5$ cm when using that of [9]. Therefore, the proposed controller leads to high position tracking precision. Figure 3 shows that the proposed controller results in higher attitude control precision than that when using the controller of [9]. In addition, Figure 4 shows that $\hat{\boldsymbol{\lambda}}$ converges to a constant value, instead of its real value. The above analysis shows that much better performance can be obtained by the proposed controller compared with the controller of [9].

Case 2 (trajectory tracking). The initial position of the quadrotor is set to $\boldsymbol{\xi}(0) = [0 \ 2 \ 0]^T$ m, $\psi(0) = 5$ deg. The reference path is given as

$$\boldsymbol{\xi}^c(t) = \begin{bmatrix} 2 \cos\left(\frac{\pi t}{20}\right) \\ 2 \sin\left(\frac{\pi t}{20}\right) \\ 3 \left(1 - \cos\left(\frac{\pi t}{20}\right)\right) \end{bmatrix} \text{m}, \quad (46)$$

$$\psi^c = 0 \text{ deg.}$$

Figure 5 depicts the trajectory tracking results of the aircraft along the given path in 3D space. Figures 6 and 7 show the position response and position error response. Figure 8 presents the comparative response of attitude angles.

FIGURE 3: Comparison result of attitude response.

FIGURE 4: Adaptive parameter $\hat{\lambda}$.

Figures 5–8 show that the aircraft position is moving closer to the set path. It is proved that the proposed controller can ensure that the error dynamic system of quadrotor converges the desired path. Figures 6 and 8 show the system performance of tracking the reference trajectory. It can be seen that both methods track the trajectory soon and remain stable. In Figures 7 and 8, it can be seen that the quadrotor takes 5 seconds to approach the desired path, while

FIGURE 5: Path following.

FIGURE 6: Comparison result of position response.

ensuring that the position tracking error is maintained within ± 0.5 m and the attitude tracking error is maintained within ± 1 deg. However, the proposed controller leads to a smoother position response and attitude response than that when the controller of [9] is adopted. Thus, it can be observed that the proposed controller has stronger robustness against external disturbances and uncertain inertia matrix.

5. Conclusion

A robust command filtered adaptive backstepping controller for a quadrotor has been presented under the unknown inertia parameter and unknown external bounded disturbance. The proposed method is designed based on the integrated model of the quadrotor, and it is strict to analyze the stability of the integrated system than the double-loop

FIGURE 7: Comparison result of position error response.

FIGURE 8: Comparison result of attitude response.

control method. The similar skew symmetric structure is constructed to facilitate the analysis of the stability of the integrated system. Theoretical analysis proves that the high precision trajectory tracking of the quadrotor system can be obtained by choosing appropriate control parameters, and the flight control system has an inhibitory effect on the external disturbances. Simulation results show that satisfactory performance can be achieved by using the proposed controller. The robustness of the proposed controller is better than that of the conventional block controller in [9].

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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