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*Research Article*

# An Optimal Hybrid Control Method for Energy-Saving of Chilled Water System in Central Air Conditioning

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Chilled water system of central air conditioning is a typical hybrid system; variable frequency behavior with amplitude limited of pumps reflects continuous and discrete dynamic characteristics. The way of energy-saving is variable water volume, via variable frequency behavior of pumps to gain adjustment of power consumption. Facing the situation of the variable frequency pumps with parallel operation, single continuous or discrete modeling cannot reflect the hybrid features. Thus, the control method will have some questions, such as bad energy-saving effect, difficult accurate adjustment of cold capacity, and low running energy efficiency. However, hybrid system modeling can reflect hybrid dynamic behavior of pumps, which is combining continuous and discrete features. The questions of nonlinear and multiparameters can be solved by control method based on hybrid system. Here, an optimum control method is proposed with the principle of the minimum, by setting the minimum power consumption as the performance function in fixed time, which realizes variable control of pumps and accurate adjustment of temperature inside room. At last, it shows the system characteristics and energy-saving affection by hybrid system modeling and the optimum control method.

## 1. Introduction

Nowadays China is a country with severe energy shortages. Energy conservation and emission reduction, as an important basic national policy of China, have far-reaching guiding significance for the development of industrial technology. At the national level, the "13th Five-Year Plan for Comprehensive Energy Conservation and Emission Reduction" clearly states that vigorously developing energy-saving products and technologies promoting the transformation of high-energy-consuming enterprises or industries and fully implementing energy-saving and emission reduction are the key tasks in the technical field. In building energy consumption, central air-conditioning accounts for 50-70% of total energy consumption [1], followed by lighting and other types of electrical equipment. Energy-saving of air-conditioning systems are not only the key to the success of building energy-saving work, but also the realization of a conservation-oriented society [2].

In addition to the chiller, the energy-saving control of the central air-conditioning is mainly for pumps and fans. The chilled water system is the research object, which is mainly reflected in the variable flow control. At present, the variable flow control objects of the central air conditioning water system mainly include return water temperature [3], supply and return water temperature difference [4, 5], and supply and return water pressure difference [6], as well as temperature difference and differential pressure cascade control [7]. The control methods mainly include switch control, PID control, fuzzy control, adaptive control, optimization control, and decoupling control. J. Mohammed et al. [8] proposed a dual-proportional PID hybrid controller to improve cooling performance and achieve energy saving of chilled water flow. J. Li et al. [9] adopted neural network adaptive control method to monitor the system's work. The state adjusts the parameters of the neural network controller in real time, realizes the intelligent control, and verifies the dynamic performance and steady state performance of the

method through simulation. G. Liu et al. [10] combines the technical advantages of fuzzy control and PID algorithm to propose a parameter adaptive fuzzy PID. The algorithm has achieved good test results; M. Koor [11] proposed a predictive control method for finding the best energy efficiency point for the variable frequency pump set with the same performance parameters, considering the pressure and flow factors. Luo et al. [12] designed a system based on PLC fuzzy prediction combined with expert practical experience and control rules, which embodies better control effects, T.Zhao et al. [6]. The number of parallel hydraulic pumps for chilled water systems. Compared with the rate, an online optimization control strategy is proposed, and the performance improvement is demonstrated through experiments. Zhang Yan et al. [13] proposed a kind of central air-conditioning cooling based on hybrid system model. The model of energy saving in the frozen water system, and set the action of the pump group for frequency conversion and switching, and demonstrate the applicability of the model and control strategy through computer simulation; H. Wang et al. [14] put various energy saving such as variable flow, temperature difference and differential pressure. The technology is compared with the corresponding control strategy to analyze the energy saving effect of different methods. J.M. Gordon et al. used a simple thermodynamic model that captures the universal aspects of chiller behavior [15]. Next, they used the simple thermodynamic model to predict the fundamental relation between coefficient of performance and cooling rate for the centrifugal chiller [16].

The traditional model and control method is based on a simple continuous or discrete dynamic model, which cannot accurately reflect the mixed dynamic behavior of continuous features and discrete features. Hybrid systems contain both continuous and discrete characteristics, and have significant advantages in dealing with power system and appliance modeling issues. The central air-conditioning chilled water system is a typical hybrid system. Its energy-saving control is mainly used to change the flow of the water system. It is realized by the frequency conversion or switching action of the pumps group, which reflects the hybridization with the goal of minimum energy consumption. According to the change of the cooling demand, real-time adjustment of the flow of the chilled water system can effectively realize the power saving of the pumps group. Therefore, the modeling and energy-saving control strategies of chilled water systems based on hybrid systems are highly targeted. The nonlinearity, large time lag, and multiparameters of chilled water systems can be modeled and effectively solved by hybrid systems. Thus, it has extremely important theoretical significance.

## 2. Mathematical Description of Piecewise Affine System

Throughout, the following notation is adopted: $\mathbb{R}$ and $\mathbb{C}$ denote the fields of real and complex number, respectively; $\mathbb{R}^n$, $\mathbb{R}^m$, $\mathbb{R}^l$, and $\mathbb{R}^s$ denote the $n$, $m$, $l$, and $s$-dimensional real Euclidean spaces; $\mathbb{R}^{n\times n}$ denotes the space of $n \times n$ matrices with real entries; $x_i$ denotes the $i^{th}$ component of the vector $x$ in $\mathbb{R}^n$, respectively; $a_{ij}$ denotes the entry in the $(i, j)$ position of the matrices $A$ or $E$ in $\mathbb{R}^{n\times n}$.

The Piecewise Affine System (PWA) [17] is a class in hybrid systems. The continuous-time PWA model can be described as follows:

$$\begin{aligned} \dot{x}(\tau) &= A_i x(\tau) + B_i u(\tau) + f_i \\ y(\tau) &= C_i x(\tau) + D_i u(\tau) + g_i \end{aligned} \tag{1}$$

which satisfies $\begin{bmatrix} x(\tau) \\ u(\tau) \end{bmatrix} \in \Omega_i$, where $x(\tau) \in \mathbb{R}^n$ and $u(\tau) \in \mathbb{R}^m$, $y(\tau) \in \mathbb{R}^l$, respectively, represent the system's state variables, input variables, and output variables. $\tau$ represents a time scalar. $f_i$ and $g_i$ are constant vectors. $A_i$ and $C_i$ represent the state matrices of the system; $B_i$ and $D_i$ are the input matrices of the system. $\Omega_i$ denotes a convex polyhedron that represents the state variable of the system and the input variable space, which is a convex partition, namely, $X \times U = \cup\Omega_i$, $\Omega_i \cap \Omega_j = \varnothing$, $i \neq j$. The system switches when the state of the system crosses the boundary. The system consists of $N$ subsystems, $\cup_{i=1}^{N}\Omega_i = \Omega$, and the system switches between subsystems. The switching rules can be state dependent, time dependent, and so on.

## 3. Mathematical Model of Room Temperature Changes in Air Conditioning Systems

The mathematical model of air-conditioned rooms adopts a common method: based on the dynamic change of the heat capacity in the air-conditioned room, the cold sending air is the control input, and the temperature in the air-conditioned room is the state variable. The heat dissipation device and human constitute a disturbance of heat and the heat transfer medium such as wall surface, ceiling, window, etc. can also affect the indoor temperature to a certain extent, and can be considered as a disturbance or a fixed value. Then this dynamic process conforms to the law of conservation of energy and, according to the adjustment of appropriate parameters, the temperature in the air-conditioned room can be stabilized at the set value, as shown in Figure 1. The dynamic heat balance equation can be expressed as

$$C_z \frac{dt}{d\tau} = q_{sa} c_a (t_{sa} - t) + \sum K_i A_i (t_i - t) + Q \tag{2}$$

where $C_z$ is the heat capacity in the air-conditioned room, J/K; $t$ is the real-time temperature of the air-conditioned room, °C; $\tau$ is the time, s; $q_{sa}$ is the air supply volume to the air-conditioned room, kg/s; $c_a$ is the specific heat capacity of the air, J/(kg·K); $t_{sa}$ is the air supply temperature of the air conditioning, °C; $K_i$ is the heat transfer coefficient of the heat transfer medium (wall, window, and roof), W/(m2·K); $A_i$ is the heat transfer medium The area of the wall (wall, window, and roof), $m^2$; $t_i$ is the temperature of the inner surface of the envelope structure (wall, window and roof), °C; $Q$ is the heat dissipation of the human body and equipment in the air-conditioned room, W.

The cooling capacity produced by the cold units is transferred from the chilled water system to the air system, and the

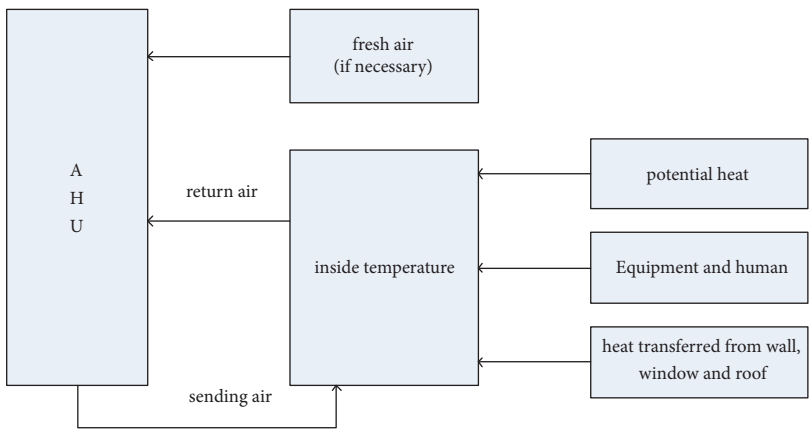



Figure 1: Schematic diagram of dynamic thermal equilibrium in air conditioning room.

heat exchange is realized by the cooler in the Air Handling Unit (AHU), and the heat exchange efficiency is set to $\eta$. The AHU is also responsible for mixing the indoor return air with the outdoor fresh air, and sending the filtered clean air into the air-conditioned room through the cooler. Among them, the ratio of the fresh air volume $q_n$ to the return air volume $q_r$ is *1-w*:*w*. According to the requirements of Chinese "Public Building Energy Efficiency Design Standards," there should generally be a fresh air ratio of not less than 15% to ensure indoor air quality. Then the expression of the cooling between the water system and the air system is

$$\gamma\eta Q_{\mathrm{w}} = Q_{\mathrm{a}} = \left(Q_{\mathrm{xr}} + Q_{\mathrm{qr}}\right) \tag{3}$$

By introducing the expressions of the water system carrying the cold capacity and the cooling capacity carried by the air system, the formula (3) can be rewritten as

$$\begin{aligned}\gamma\eta q_{\mathrm{w}} c_{\mathrm{w}} \Delta t = {} & (1-w)\, q_{\mathrm{sa}} c_{\mathrm{a}} \left(t_{\mathrm{out}} - t_{\mathrm{sa}}\right) + w q_{\mathrm{sa}} c_{\mathrm{a}} \left(t - t_{\mathrm{sa}}\right) \\ & + Q_{\mathrm{qr}}\end{aligned} \tag{4}$$

where $\gamma$ is the percentage of cold capacity assigned to an air-conditioned room, $0 \le \gamma \le 100\%$; $Q_{\mathrm{w}}$ is the cold capacity carried by the water system, W; $Q_{\mathrm{a}}$ is the cold capacity that is sent to the air-conditioned room by the AHU, W; $Q_{\mathrm{xr}}$ is the sensible heat in the air-conditioned room, W; $Q_{\mathrm{qr}}$ is the potential heat in the room, W; $q_w$ is the flow rate of the water system, kg/s; $c_w$ is the specific heat capacity of water, J/(kg·K); $\Delta t$ is a constant temperature difference between the supply and return water, °C; $t_{out}$ is the fresh air temperature introduced from the outside, °C.

Bringing (4) into the heat balance equation (2) of the room, we can get

$$\begin{aligned}C_{\mathrm{z}} \frac{dt}{d\tau} = {} & -\gamma\eta q_{\mathrm{w}} c_{\mathrm{w}} \Delta t - \left[(1-w)\, q_{\mathrm{sa}} c_{\mathrm{a}} + \sum K_i A_i\right] t \\ & + (1-w)\, q_{\mathrm{sa}} c_{\mathrm{a}} t_{\mathrm{out}} + \sum K_i A_i T_i + Q + Q_{\mathrm{qr}}\end{aligned} \tag{5}$$

Set the state variable $x(\tau) = t(\tau)$ and the input variable $u(\tau) = q_{\mathrm{w}}(\tau)$, use the variable water volume of water system and the constant air volume of air system transfer modes [3, 13], and use the differential equation to describe the system model:

$$\begin{aligned}\dot{x}(\tau) &= Ax(\tau) + Bu(\tau) + H \\ y(\tau) &= x(\tau)\end{aligned} \tag{6}$$

The system parameters are

$$\begin{aligned}A &= -\frac{(1-w)\, q_{\mathrm{sa}} c_a + \sum K_i A_i}{C_z} \\ B &= -\frac{\gamma\eta c_{\mathrm{w}} \Delta t}{C_z} \\ H &= \frac{(1-w)\, q_{\mathrm{sa}} c_{\mathrm{a}} T_{\mathrm{out}} + \sum K_i A_i T_i + Q + Q_{\mathrm{qr}}}{C_z}\end{aligned} \tag{7}$$

where the behaviors of the pumps group show the hybrid characteristics. First, the water system of the air conditioning is dynamic and random, and the factors affecting the system characteristics also show multivariate, dynamic, strong coupling, and nonlinear relationship. Characteristics [18]; second, in variable flow control method, the water flow is generally not less than 50% of the rated value, otherwise the Coefficient of Performance (COP) of the pump will be abruptly attenuated, so the range of variable flow is limited to [50% rated, 100% rated], showing continuous characteristics within this range. If the cold capacity is reduced, the flow control is out of the frequency conversion range, and the amplitude truncation occurs, showing a discrete characteristic. The flow constraints of a single pump and the entire water

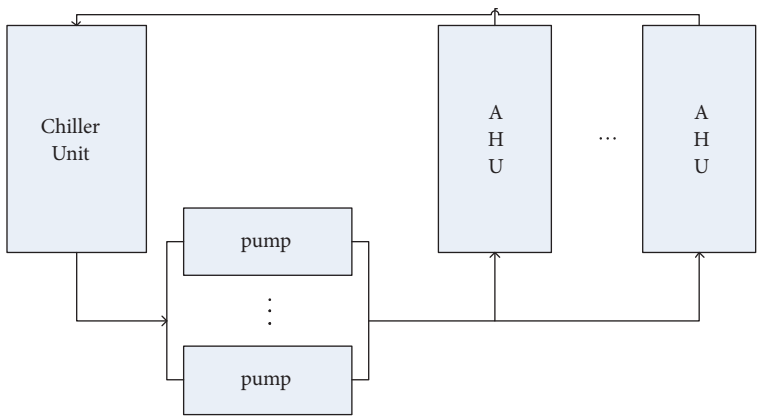



Figure 2: Schematic diagram of cold capacity transfer process in chilled water system.

system can be expressed by mathematical descriptions as

$$u_i = \begin{cases} 0.5u_{i0}, & u < 0.5u_{i0} \\ u_{i0}, & u > u_{i0} \\ u_i, & 0.5u_{i0} \le u \le u_{i0} \end{cases}, \quad i = 1, 2, \ldots, N_{\mathrm{p}} \tag{8a}$$

$$u_{\mathrm{s}} = \sum_{i=1}^{N_{\mathrm{p}}} u_i \tag{8b}$$

$$u = \begin{cases} 0.5u_{\mathrm{s}0}, & u < 0.5u_{\mathrm{s}0} \\ u_{\mathrm{s}0}, & u > u_{\mathrm{s}0} \\ u, & 0.5u_{\mathrm{s}0} \le u \le u_{\mathrm{s}0} \end{cases} \tag{9}$$

where $u_i$ is the flow rate of the *i-th* pump as the *i-th* input variable; $u_{i0}$ is the rated flow of the *i-th* pump, which is the constant; $N_p$ is the number of variable-frequency pumps; $u_s$ is the flow of the whole water system; $u_{s0}$ is the rated flow rate of the whole water system. The schematic diagram of the cold capacity of the water system is shown in Figure 2.

## 4. Hybrid Optimization Control Strategy Based on the Minimum Principle

According to the minimum principle proposed by the former Soviet scholar Pontiac King, the water consumption of the water system in the finite time is set to the performance functional $J$, and the optimal allowable control $u$ is sought to minimize the performance function. $u$ is a multivariate parameter, which is limited to a closed set and satisfies the inequality constraint $g[x(\tau), u(\tau), \tau] \ge 0$, and $g$ is a $r$-dimensional continuous vector variable function, $r \le m$.

Because there are many ways to control the frequency of the pumps group, the energy saving effect is also quite different. The method adopts a fixed terminal pressure difference control mode, which sets an electric regulating valve and a differential pressure transmitter on the most unfavorable loop, so that the power of the pump is between one power and three powers [19]. In order to facilitate the calculation, it is approximated that the power ($P$, W) of the pump and the rotational speed ($n$, rad/s) are squared, and the flow rate of the pump is proportional to the rotational speed, i.e., $n_i/n_{i0} = q_{\mathrm{w}i}/q_{\mathrm{w}i0}$, where $n_i$ is the rotational speed of a pump, rad/s, $n_{i0}$ is the rated speed of the pump, rad/s; $q_{wi}$ is the flow of a pump, kg/s; $q_{wi0}$ is the rated flow of the pump, kg/s. Let $P_0/q_0^2 = \xi$. Then the performance function is the sum of the energy consumption of the variable frequency pumps, which can be expressed as

$$J = \int_{\tau_o}^{\tau_f} L\left[x(\tau), u(\tau), \tau\right] d\tau = \int_{\tau_o}^{\tau_f} \left( \sum_{i=1}^{N_{\mathrm{p}}} \xi_i {u_i}^2 \right) d\tau \tag{10}$$

where $L$ is a continuous differentiable quantity function; $\tau_0$ and $\tau_f$ are initial and terminal moments, respectively, s.

Select the Hamilton function as

$$\begin{aligned} H &= L\left[x(\tau), u(\tau), \tau\right] + \lambda^T f\left[x, u, \tau\right] \\ &= \sum_{i=1}^{N_{\mathrm{p}}} \xi_i {u_i}^2 + \lambda^T \left( Ax + \sum_{i=1}^{N_{\mathrm{p}}} Bu_i + H \right) \end{aligned} \tag{11}$$

where $\lambda$ is the costate vector and $f[\cdot]$ is the state function of the system formula (6), $f[\cdot] = \dot{x}$.

First, suppose the extremum condition and the costate equation are analyzed under the condition of unconstrained control; then the optimal control is discussed according to the actual constraints.

Let

$$\frac{\partial H}{\partial u} = 2\sum_{i=1}^{N_{\mathrm{p}}} \xi_i u_i + \sum_{i=1}^{N_{\mathrm{p}}} B\lambda = 0 \tag{12}$$

We get

$$u_i^*(\tau) = -\frac{B\lambda(\tau)}{2\xi_i} \tag{13}$$

which is

$$u^*(\tau) = \sum_{i=1}^{N_{\mathrm{p}}} u_i(\tau) \tag{14}$$

Then the equation of state can be rewritten as

$$\dot{x} = Ax - \frac{B^2\lambda}{2}\sum_{i=1}^{N_p}\frac{1}{\xi_i} + H \tag{15}$$

The coordination equation is

$$\dot{\lambda} = -\frac{\partial H}{\partial x} = -A\lambda \tag{16}$$

Find the second derivative of the state equation:

$$\begin{aligned}\ddot{x} &= A\dot{x} - \dot{\lambda}\frac{B^2}{2}\sum_{i=1}^{N_p}\frac{1}{\xi_i} + \dot{H} \\ &= A\left(Ax - \frac{B^2\lambda}{2}\sum_{i=1}^{N_p}\frac{1}{\xi_i} + H\right) + \frac{AB^2\lambda}{2}\sum_{i=1}^{N_p}\frac{1}{\xi_i}\end{aligned} \tag{17}$$

We get

$$\ddot{x} = A^2x + AH \tag{18}$$

Integral can be obtained

$$x(\tau) = C_1e^{-A\tau} + C_2e^{A\tau} - \frac{H}{A} \tag{19}$$

where $C_1$ and $C_2$ are undetermined coefficients and (19) is substituted into the equation of state (15) and the coordination equation can be obtained as

$$\lambda(\tau) = \frac{4A}{B^2\sum_{i=1}^{N_p}(1/\xi_i)}C_1e^{-A\tau} \tag{20}$$

Furthermore, substituting (20) into (14) for optimal control,

$$u_i^*(\tau) = -\frac{2A}{\xi_iB\sum_{i=1}^{N_p}(1/\xi_i)}C_1e^{-A\tau} \tag{21}$$

Substituting the initial condition $x(\tau_0) = x_0$ and the terminal constraint $N[x(\tau_f), \tau_f] = 0$ (or terminal condition $x(\tau_f) = x_f$), the optimal state trajectory $x^*$ and the optimal performance function $J^*$ are obtained together, where $N$ is a $s$-dimensional continuous differentiable vector function, $s \leq n$; $\tau_0$ and $\tau_f$ are the initial and terminal moments, respectively, s; $x_0$ and $x_f$ are the initial and terminal values of the state variable, respectively.

The goal proposed in this paper is that the energy consumption is minimum, and the boundary conditions are degenerated into fixed initial and terminal constant values, that is, $x(\tau_0) = x_0$ and $x(\tau_f) = x_f$, respectively. In order to obtain the optimal state trajectory $x^*$ and the optimal performance function $J^*$, substitute the initial condition $x(\tau_0) = x_0$ and the terminal condition $x(\tau_f) = x_f$ into (19) and determine the values of the undetermined coefficients $C_1$ and $C_2$ in the equation of state (19), the coordination equation (20), and the control equation (21).

Considering the limitation of (9) for water system flow, when $u_s$ is used as a single input variable, the optimal control based on the minimum principle is essentially the form of limiting amplifier. However, considering the multi-input variable of multipump and the hybrid dynamic behavior of frequency conversion and truncation, the flow rate of water system is only the sum of the flow rate of multipump, as shown in equation (8a) and (8b), as another constraint condition for input variable. Therefore, in order to achieve the necessary conditions for optimal control, the optimal control $u^*$ must be required to take values from the allowed finite closed set (8a), which is

$$u_i^* = \begin{cases} 0.5u_{i0}, & u_i^* < 0.5u_{i0} \\ u_{i0}, & u_i^* > u_{i0} \\ u_i, & 0.5u_{i0} \leq u_i^* \leq u_{i0} \end{cases} \tag{22}$$

## 5. Case Analysis

Assume that the building area of air-conditioned room is 120 $m^2$ and the height is 3 m. The central air-conditioning sends cold air to the indoor temperature and the indoor temperature is required to be within a limited time (3 minutes), and the initial temperature ($t_0$, 30°C) is reduced to the setting temperature ($t_{set}$, 26°C), and the goal of power consumption in accordance with the control strategy is minimal. The chilled water system has four variable frequency water pumps with different parameters, and they never leave the work. They transport the cold capacity produced by the cold unit to the AHU for heat exchange and send it into the room in the form of cold air. Assume that the thermodynamic parameters in the air-conditioned room and the parameters and values of the water system pump are shown in Table 1.

Then, (6) can be numerically

$$\begin{aligned}\dot{x}(\tau) &= -0.0002098x(\tau) - 0.003374u(\tau) + 0.005651 \\ y(\tau) &= x(\tau)\end{aligned} \tag{23}$$

The performance functional (10) can be rewritten as

$$\begin{aligned}J = \int_0^{180} &\left(609.6{u_1}^2 + 1567{u_2}^2 + 3858{u_3}^2 \right. \\ &\left. + 4542{u_4}^2\right)d\tau\end{aligned} \tag{24}$$

The Hamilton function (11) is updated to

$$\begin{aligned}H &= L[x(\tau), u(\tau), \tau] + \lambda^T f[x, u, \tau] = 609.6{u_1}^2 \\ &+ 1567{u_2}^2 + 3858{u_3}^2 + 4542{u_4}^2 \\ &+ \lambda^T\{-0.0002098x(\tau) \\ &- 0.003374[u_1(\tau) + u_2(\tau) + u_3(\tau) + u_4(\tau)] \\ &+ 0.005651\}\end{aligned} \tag{25}$$

After the solution of the equation of state (formula (19)) is digitized, substitute the initial value (30°C, 0s) and

Table 1: Symbols and values of parameters in cold capacity transfer process.

| Parameter (Description) | Value |
|---|---|
| indoor hear capacity, $C_z$ (KJ/K) | 469 |
| specific heat of air, $c_a$ (kJ/kg∗k) | 1.01 |
| specific heat of water, $c_w$ (kJ/kg∗k) | 4.186 |
| temperature of fresh air, $t_{out}$ (°C) | 26 |
| setting temperature, $t_{set}$ (°C) | 25 |
| indoor initial temperature, $t_0$ (°C) | 30 |
| volume of sending air, $q_{sa}$ (kg/s) | 0.623 |
| temperature of walls, windows, and roof, respectively, $t_i$ (°C) | 35,36,35 |
| area of walls, windows, and roof, respectively, $A_i$, ($m^2$) | 80,5,0 |
| heat transfer coefficient of walls, windows, and roof, respectively, $K_i$, ($W/m^2$∗k) | 0.047,0.054,0.051 |
| latent heat load, $Q_{qr}$ (W) | 25 |
| random heat load, $Q$ (W) | 30 |
| return air rate, $w$ | 0.85 |
| transfer efficiency from water system to air system, $\eta$ | 0.9 |
| ratio of cooling capacity allocation, $\gamma$ | 7% |
| temperature difference, $\triangle t$ (°C) | 6 |
| rated power, respectively, $P_0$ (kW) | 1.69;2.17;4.91;10.6 |
| rated flow, respectively, $q_{w0}$ (L/s) | 0.61;0.75;1.77;4.17 |

Table 2: Square ratio of rated power and rated flow.

| Number | Rated volume L/s | Rated power kW | $\xi$ |
|---|---|---|---|
| 1 | 4.17 | 10.6 | 609.6 |
| 2 | 1.77 | 4.91 | 1567 |
| 3 | 0.75 | 2.17 | 3858 |
| 4 | 0.61 | 1.69 | 4542 |

the terminal value (26°C, 180s) and find the undetermined coefficients $C_1$ and $C_2$ as -51.751 and 54.811. Then the solution to the equation of state can be updated to

$$x(\tau) = -51.751e^{0.0002098\tau} + 54.811e^{-0.0002098\tau} + 26.94 \quad (26)$$

From the costate equation (20), the square ratio of rated power to rated flow (Table 2), the numerical solution of the costate equation is obtained

$$\lambda(\tau) = 1.383300e^{0.0002098\tau} \quad (27)$$

Then the optimal governing equation can be obtained as

$$u_i^*(\tau) = \begin{cases} 3.828e^{0.0002098\tau}, & i = 1 \\ 1.489e^{0.0002098\tau}, & i = 2 \\ 0.605e^{0.0002098\tau}, & i = 3 \\ 0.514e^{0.0002098\tau}, & i = 4 \end{cases} \quad (28a)$$

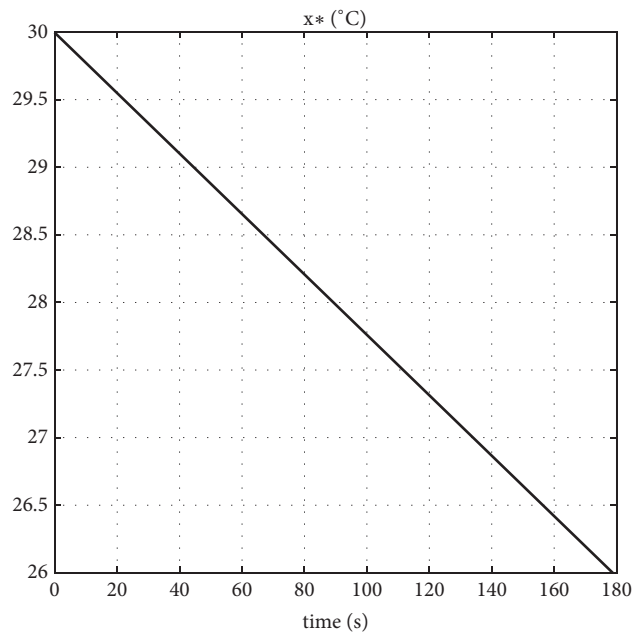



Figure 3: Optimal state curve.

The optimal equation for the total flow of the chilled water system is

$$u^*(\tau) = 6.436e^{0.0002098\tau} \quad (28b)$$

The optimal governing equation is verified for a finite time (0-180 s) to satisfy the finite closed set (8a). Then the performance is

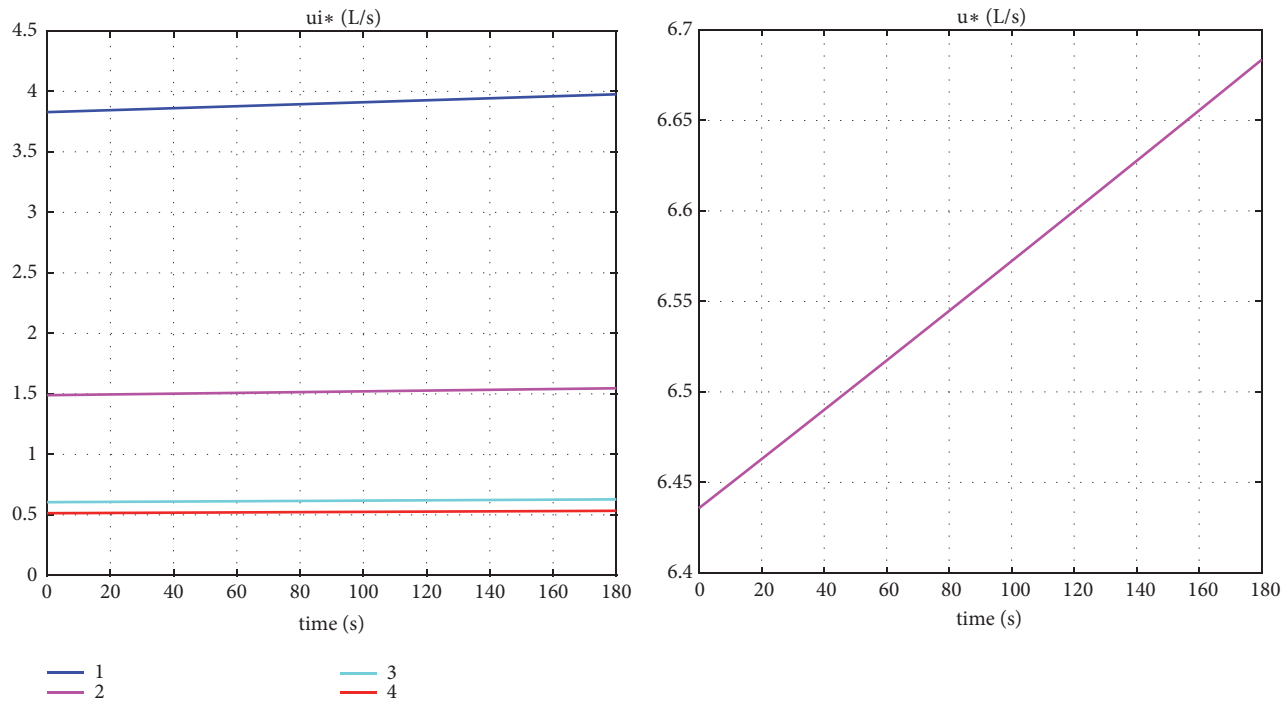
$$J^* = 2.808 \times 10^6 (W \cdot s) = 0.78 (kW \cdot h) \quad (29)$$

The optimal curve of this example can be seen in Figures 3–6.

Figure 3 is the optimal state curve showing the requirement that the temperature in the air-conditioned room drops from the initial value (30°C) to the terminal value (26°C) within a limited time (180 seconds) under the action of the optimal control law.

The optimal control equations (28a) and (28b) and Figure 4 show the flow changes of the four pumps under the optimal control law and the change of the total flow of the chilled water system. Under the condition of considering the frequency conversion range, the above flow rate meets the requirements of the limited interval. The total flow of the chilled water system varies from 6.4 to 6.7 L/s in 180 seconds, meeting the amplitude limit requirements of [3.65, 7.3], as shown in Figure 4(b). The flow rate of the pumps (No. 1-4) also meets the variable flow range requirements, as shown in Figure 4(a).

Figure 5 is the curve of the coordination equation, and Figure 6 is the performance function curve, which is the calculation of the total energy consumption of the pumps




(a) Optimal control curve ($u_i$)

(b) Optimal control curve (u)

Figure 4

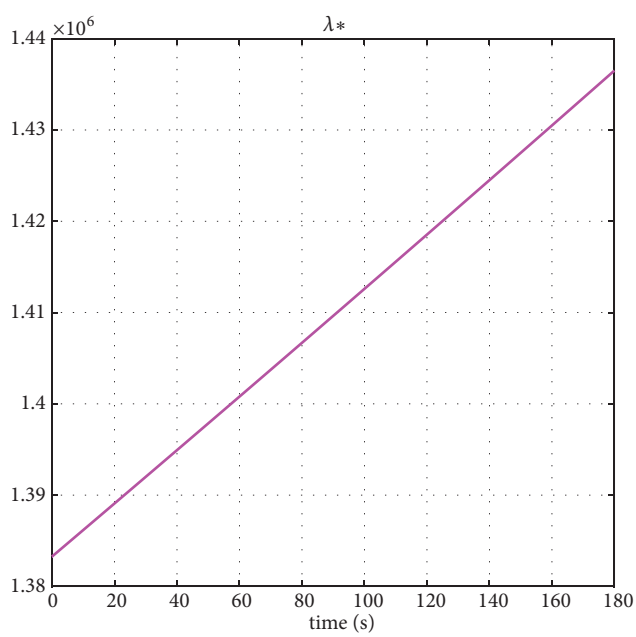



Figure 5: Optimal costate curve.

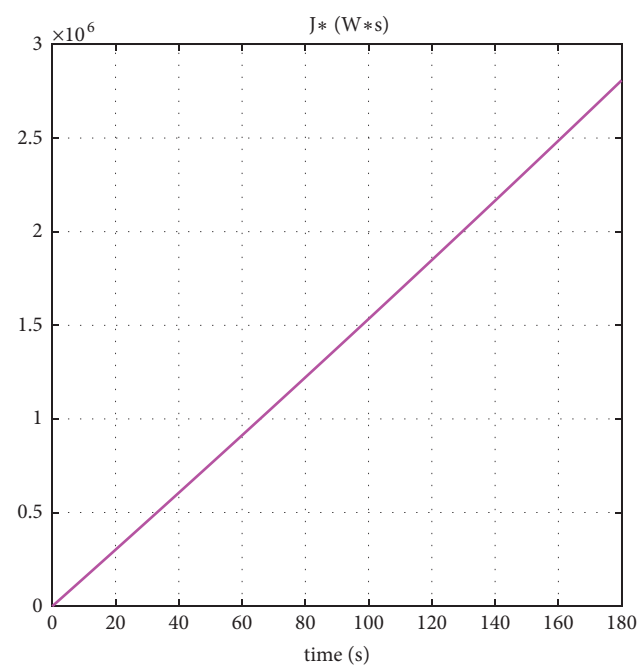



Figure 6: Optimal performance curve.

group. Compare the fixed frequency operation of the pumps group, the energy saving situation can be calculated as

$$\varepsilon = 1 - \frac{J^*}{\int_0^{180} \left(\sum_{i=1}^{4} P_{0i}\right) d\tau} = 1 - \frac{2.808 \times 10^6}{3.4866 \times 10^6} = 19.46\% \quad (30)$$

According to the calculation of (30), the optimal control strategy based on the minimum principle of the energy saving rate of 19.46% for the pumps group of the chilled water system within 3 minutes when the indoor temperature drops to the set temperature.

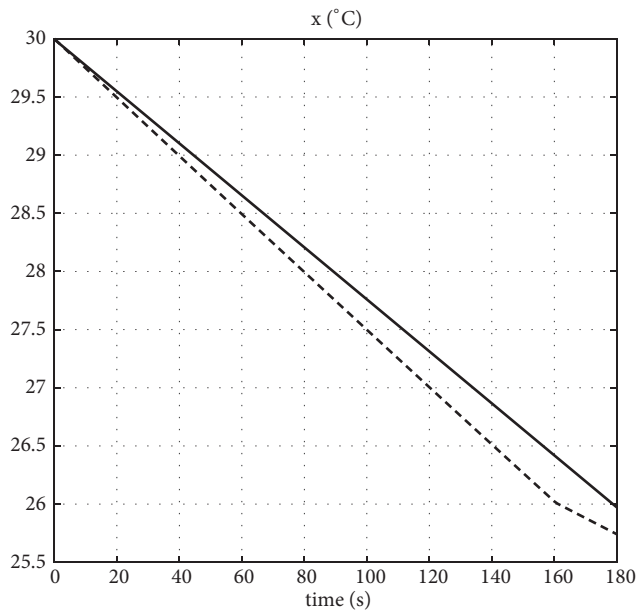



Figure 7: Temperature variation (compare with group control strategy).

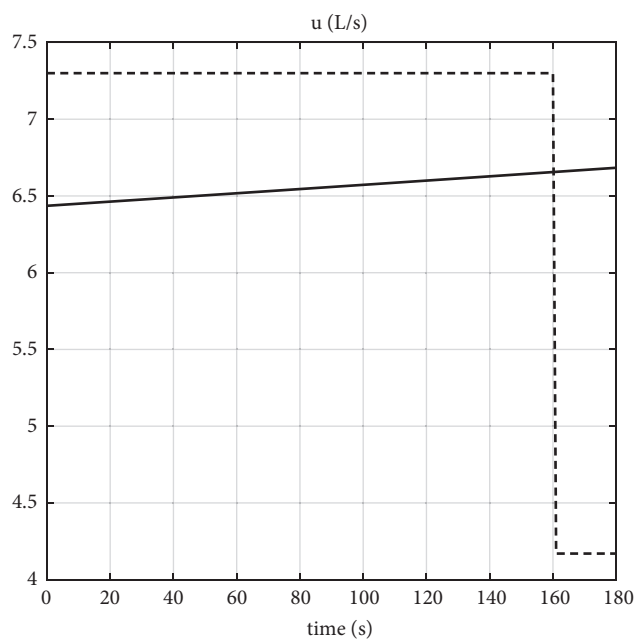



Figure 8: Volume variation (compare with group control strategy).

Compare the above method with the pumps group control strategy. The group control strategy is set as follows: 4 pumps are all fixed frequency operation, which are put into use at the initial time. After reaching the set temperature range, the pumps are sequentially discharged according to the switching rules. Let the water pump state stable range be 26 ± 0.5°C. If the temperature is more than 26.5°C for several minutes, put one pump; if the temperature is less than 25.5°C for several minutes, exit one pump. Considering the different pump power, for the sake of energy saving, the high-power

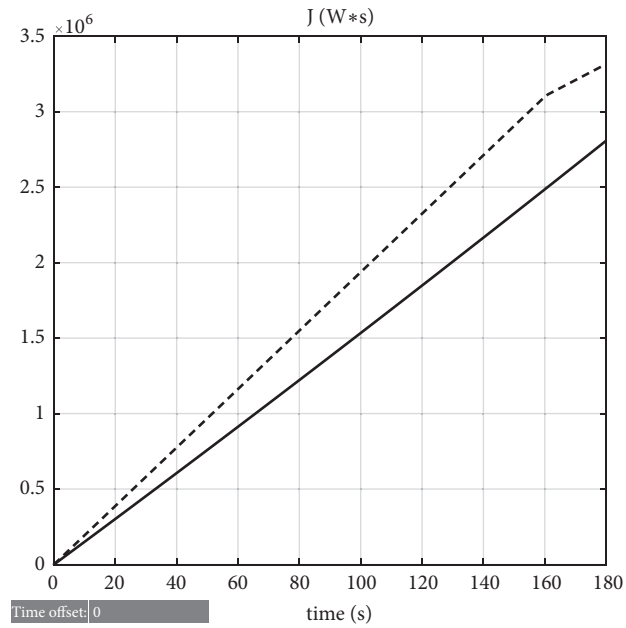



Figure 9: Energy consumption (compare with group control strategy).

pump is preferred when withdrawing, and the low-power pump is preferred when cutting in.

The flow of the chilled water system shall not be less than 50% of the rated flow rate, then, the No.1 pump must be fixed and not withdrawn. Other pumps exit in turn, or in turn.

Considering that this numerical case takes 180 seconds as the simulation time, it is assumed that the group control strategy is set to all the pumps Nos. 2, 3, and 4 out of work when the indoor temperature reaches the set value, to meet the constraint minimum flow operation. It can be seen that, in Figure 7, the group control strategy (dashed line) reaches the set temperature of 26°C at about 160 seconds, and then the temperature continues to drop in the stable range. In Figure 8, under the group control strategy, the flow of the water system has changed drastically due to the withdrawal of multiple pumps (dotted line) in the vicinity of 160 seconds. In comparison, the control method (solid line) of this paper is more accurate than the group control strategy, and the curve is smoother.

Figure 9 shows the comparison of energy consumption during the simulation time. It can be seen that the solid line is the energy consumption of the control method in this paper. The value is 2.808 × 106W.s, and the energy consumption value (dashed line) under the group control strategy is 3.316 × 106W.s, and the energy saving rate increased by 15.32% compared with the group control strategy. Compared with the group control strategy, the energy saving effect is obviously superior.

## 6. Conclusion

The chilled water system of central air conditioning is a multiparameter complex structural system. The continuous

and discrete dynamic behaviors of the pumps group determine the mixed characteristics of mathematical modeling. The minimum energy consumption is the goal for the energy-saving control strategy based on hybrid system. The method based on the minimum principle, the optimal control law is established, the frequency conversion regulation of the pumps group is realized, and the optimal state trajectory and optimal association are obtained. The trajectory of the state and the applicability of the method are further verified by numerical examples and computer simulation. The method shows a more accurate indoor temperature regulation effect, a clearer pump group flow control process, and an objective function calculation result. Due to the variety of energy-saving control methods for chilled water systems, the adjustment effects are not the same, and there are also large differences in energy-saving effects. The control strategy of this paper adopts idealized approximation results in water pipe network resistance and pump energy efficiency. Therefore, there are certain limitations. In actual engineering cases, it is still necessary to correct and improve through a large number of measured data.

## Data Availability

The data used to support the findings of this study are included within the article.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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