Hindawi
Journal of Control Science and Engineering
Volume 2018, Article ID 3927108, 9 pages
https://doi.org/10.1155/2018/3927108

Research Article

Nonlinear Uncertainties Canceling in Multi-Agent Systems Enabled by Cooperative Adaptation

Cyuansi Shih and Chengyu Cao

Department of Mechanical Engineering, The University of Connecticut, Storrs, CT 06269, USA

Correspondence should be addressed to Cyuansi Shih; cyuan-si.shih@uconn.edu

Received 3 May 2018; Accepted 17 July 2018; Published 2 September 2018

Academic Editor: Radek Matušů

This paper deals with uncertainties problem in multi-agent systems with novel cooperative adaptation approach. Since uncertainties in multi-agent systems are interconnected, local agent often faces uncertainties not only from itself but also from neighbors. The proposed approach is that a local agent estimates uncertainties from itself and neighboring agents and then changes control strategy. The uncertainties or the equivalences of neighbors can be estimated based on their available outputs; thus, the local agent can adapt to them to cancel out these effects. Stability analysis is also derived that characterizes the transient and steady state performance of multi-agent system. The simulation presents the details of the proposed cooperative adaptation mechanism by compared typical cooperative control.

1. Introduction

The cooperative control in multi-agent system has attracted compelling attention in recent years from various applications including scheduling satellite formation, cooperative unmanned air vehicles, connected autonomous vehicles, and multiple underwater vehicle coordination. In these applications, the agents are required to collaborate with each other in order to finish cooperative mission. Consensus in multi-agent systems is a typical example that agents have common interest and then make negotiations to reach consensus by appropriate algorithmic methods. Consensus problems with various dynamic systems have already been successfully addressed in literature. The consensus in multi-agent systems with single-integrator and double-integrator kinematics is studied in [1–3]. The general linear dynamics of consensus in multi-agent systems is also discussed in [4–10]. In [11–16], the authors further extended to nonlinear dynamics for multi-agent systems.

In practical applications, the models of agents are difficult to obtain because of sensing or measurement limitation. Also, disturbances often exist in interconnected network systems, such as channel noise and communication noise. Thus, it is necessary to consider uncertainty effects in multi-agent systems. The typical direct and indirect adaptive control [17] are two nonlinear schemes explicitly designed for signal system uncertainties. In [18], the model reference adaptive control approach is used to ensure asymptotic tracking of a defined reference model for a class of multi-agent systems with constant unknown parameters. Utilizing the universal approximation capability of artificial neural network [19], adaptive control could deal with nonlinear functions instead of linear parameters in multi-agent systems, and this leads to the neural network controller design. Consensus problem of second-order systems with Lipschitz nonlinearity is addressed [16]. The uncertainties of nonlinear multi-agent systems under jointly connected directed switching networks are also addressed in [20, 21]. The results in [9, 18] deal with high-order system with Lipschitz nonlinearity. In these literatures, the results are restricted to local stability and certain type of network structure.

This paper proposed a novel cooperative adaptation approach to deal with uncertainties issue of network consensus. The cooperative law drives agents to collaborate and then cooperative adaptation law eliminates uncertainties from network. The idea is that the local agent adjusts his own control strategies based on the uncertainty estimation of itself and neighbors. The uncertainties from other agents can be treated as external disturbances and are not available to the local agent. Thus, other agents uncertainties can be estimated based on their available output and then local agent cancels out these uncertainties by adapting to them. This paper also provided transient and steady state performance analysis in the presence of uncertainties for multi-agent systems.

The article is organized as follows. The problem formulation is given in Section 2. The adaptive cooperation control scheme is introduced in Section 3. In Section 4, the stability analysis and performance bounds for multi-agent systems are presented in Section 4. The simulation results are presented in Section 5, while Section 6 concludes this paper.

2. Problem Formulation

There are N agents moving in m dimensional Euclidean space, where the dynamics of each agent is described by

$$\dot{x}_i(t) = Ax_i(t) + B\left(u_i(t) + \sigma_i(x_i, t)\right) \tag{1}$$

where $x_i \in \mathbb{R}^m$ is the state vector, such as position and velocity vector of agent i, $u_i \in \mathbb{R}^m$ are the control input, and $\sigma_i : \mathbb{R}^m \times \mathbb{R}^m \times \mathbb{R} \longmapsto \mathbb{R}^m$ represents the unknown, nonlinear, and time-varying uncertainties.

The objective is to design decentralized control inputs that ensure multi-agent systems with uncertainties achieve consensus. Consensus in this paper means all agents converge to common attitude or speed. Each agent is subjected to individual uncertainties, such as dynamics uncertainty, affected from neighborhood agents. The decentralized control input plays the role not only cooperating but also canceling uncertainty among agents. The network connectivity has a fairly general assumption that creates a spanning tree in the network connection. The uncertainties are restricted to being Lipschitz and allow less restriction on structure of nonlinearity.

The connection among agents are specified by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ which consists of a set of vertices denoted by $\mathcal{V} = \{1, 2, \ldots, N\}$ and a set of edges denoted by $\mathcal{E} \subseteq \{(i, j) : i, j \in \mathcal{V},\ i \neq j\}$. A vertex represents an agent, and each edge represents a connection. The adjacency matrix $\mathcal{A}$ with elements a_{ij} from the graph denotes the connections, where $a_{ij} = 1$ means a path from agent j to agent i; otherwise $a_{ij} = 0$. The Laplacian matrix $\mathcal{L}$ is defined as $l_{ii} = \sum_{j=1}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$ when $i \neq j$. M is defined as the normalized Laplacian matrix, and thus $M = I_N - \mathbf{1}r^T$. The rank of M is $N-1$, and M has $N-1$ eigenvalues at 1 and one eigenvalue at 0. The connection graph is undirected if $a_{ij} = a_{ji}$ and thus the adjacency matrix is symmetric; otherwise the graph is directed. We consider the undirected graph among agents communication in this paper.

Assumption 1. The uncertainties depended on agent's state and its changes. For local agent $x_i > 0$, there exist $L > 0$, $\beta_0 > 0$, and $\beta > 0$ such that

$$\left\|\sigma_i\left(\xi_i(t_1)\right) - \sigma_i\left(\xi_i(t_2)\right)\right\| < L\left\|\xi_i(t_1) - \xi_i(t_2)\right\|_\infty + \beta_0 \tag{2}$$

where $\xi_i = x_i - \sum_{j\in N_i} r_j x_j$ and r_j is normalized edge of graph. For overall network, we have

$$\left\|\sigma\left(\xi(t_1)\right) - \sigma\left(\xi(t_2)\right)\right\| < LM\left\|\xi(t_1) - \xi(t_2)\right\|_\infty + \beta \tag{3}$$

Assumption 2. The nonlinear uncertainties $\sigma_i(\xi_i, t)$ are bounded with respect to state ξ_i. This means the uncertainties do not become unbounded even system state keep changing. The partial derivatives of $\sigma_i(\xi_i, t)$ with respect to ξ_i and t are bounded, i.e., $\|\partial\sigma_i(\xi_i, t)/\partial\xi_i\| \leq d_{\sigma_{i_{\xi_i}}}$ and $\|\partial\sigma_i(\xi_i, t)/\partial t\| \leq d_{\sigma_{i_t}}$. We also further have $\|\partial\sigma(\xi, t)/\partial\xi\| \leq d_{\sigma_\xi}$ and $\|\partial\sigma(\xi, t)/\partial t\| \leq d_{\sigma_t}$.

Assumption 3. The undirected graph $\mathcal{G}$ is connected and thus creates a spanning tree in the graph. The Laplacian matrix $\mathcal{L}$ has a single eigenvalue at 0 and all other eigenvalues are with positive real part.

3. Uncertainties Canceling Algorithm

In this section, we introduce a novel cooperative adaptation law to drive multi-agent systems as (1) achieve global objective, x_i toward neighbor agents x_j. The algorithm first deals with the agent's collaboration by the cooperative law that is based on local neighbors' relative information. The algorithm further handles uncertainties issue among network through cooperative adaptation law. The overall control law is represented as below:

$$u_i = u_{i_c} + u_{i_a} \tag{4}$$

where u_{i_c} is cooperative control law and u_{i_a} is cooperative adaptation law.

First, we introduced the state transformation to describe the relation between individual agent and network as follows:

$$\xi_i = x_i - \sum_{j\in N} r_j x_j \tag{5}$$

The network dynamics between local agent and neighbors is described as

$$\begin{aligned}\dot{\xi}_i = {} & A\xi_i + B\left(u_{i_c} - \sum_{j\in N} r_j u_{j_c}\right) + B\left(u_{i_a} - \sum_{j\in N} r_j u_{j_a}\right) \\ & + B\left(\sigma_i - \sum_{j\in N} r_j \sigma_j\right)\end{aligned} \tag{6}$$

The overall network transformation follows

$$\xi = x - \left(\mathbf{1}r^T \otimes I_n\right)x = M \otimes BI_n x \tag{7}$$

Cooperative Law. In order to achieve the network consensus, the cooperative control law design is moving to the average state of neighbors. In other words, it uses neighbors' relative information as feedback and is presented as

$$u_{i_c} = -k\left(x_i - \sum_{j\in N} r_j x_j\right) \tag{8}$$

where the selection of k will make sure to stabilize the overall network system that is described as

$$\dot{\xi} = \left(I_N \otimes A - \mathscr{L} \otimes Bk\right)\xi + (M \otimes B)\left(u_a + \sigma\right) \quad (9)$$

where $\xi = [\xi_1 \dots \xi_n]$, $u_a = [u_1 \dots u_n]$, $\sigma = [\sigma_1 \dots \sigma_n]$, $A_m = (I_N \otimes A - \mathscr{L} \otimes Bk)$, and $B_m = M \otimes B\mathbf{1}_n$. Thus, A_m is Hurwitz matrix by k selection.

Cooperative Adaptation Law. We further design the decentralized cooperative adaptation law to deal with uncertainty issue among network. This disturbance rejection includes state predictor, adaptive law, and control as below. Since uncertainty for dynamical systems is made in real time, all uncertainties from network can be estimated based on their available output. Hence, a local agent can adjust its own control law based on the estimation of network uncertainty. Thus, the local agent estimates these uncertainties through transferred dynamics. With network dynamics from (9), we consider the following state predictor:

$$\dot{\widehat{\xi}} = A_m\widehat{\xi} + B_m\left(u_a + \widehat{\sigma}\right) \quad (10)$$

where $\widehat{\xi} \in \mathbb{R}^m$ is the estimated state vector of ξ in (9) and $\widehat{\sigma}$ is the estimated uncertainty of σ. These parameters vectors are updated by adaptive law below.

Fast adaptation mechanism is applied in here as a high-gain feedback for state predictor. It is noted that both state predictor and adaptation work together and there is no time-delay or at most one integration time-step in the loop. Here, the predicted state, $\widehat{\xi}_i$, is used to generate the adaptive parameter, $\widehat{\sigma}_i$, based on the network error dynamics. The piece-wise constant adaptive law is used in here. In order to drive $\widehat{\xi}$ to track $\widehat{\xi}$. The update law is given by

$$\widehat{\sigma}(t) = -\Phi^{-1}(T)\, \mathrm{e}^{A_m T}\widetilde{\xi}(kT), \quad t \in [kT, (k+1)T] \quad (11)$$

where $\Phi(T) = \int_0^T e^{A_m(T-\tau)}d\tau$, $T > 0$ is the adaptation step size, and $\widetilde{\xi} = \widehat{\xi} - \xi$. The adaptive law drives $\widetilde{\xi}$ to be small and as a result, $\widehat{\sigma}$ will approach σ. The information of $\widehat{\sigma}$ will be used in the control law design. The error dynamics for network system is described as

$$\dot{\widetilde{\xi}} = A_m\widetilde{\xi} + B_m\left(\widehat{\sigma} - \sigma\right) \quad (12)$$

To prevent aggressive signals in real application, a low-pass filtering mechanism is added in control law, which filters away aggressive signals and recovers robustness. The adaptive control law design is

$$u_a(t) = -C(s)\,\widehat{\sigma}(t) \quad (13)$$

where $C(s)$ is low-pass filter to filter the control signal. The introduction of low-pass filter at this point in the control loop allows us to decouple control and estimation by canceling the overall network uncertainty. Furthermore, the bandwidth of filter can be chosen to attenuate frequencies based on the variation of uncertainties. The closed-loop network system from (1) and (13) becomes

$$\dot{\xi} = A_m\xi - C(s)\,B_m\widehat{\sigma} + B_m\sigma \quad (14)$$

Remark 4. The objective of cooperative adaptation law is to make the adaptive parameter $\widehat{\sigma}(t)$ approach the network disturbance $\sigma(t)$. The effects of the control law $u_a(t)$ cancel the effects of $\sigma(t)$ and as a result $\widetilde{\xi}(t)$ in (12) will become very small. Thus, the agents can achieve consensus without uncertainties.

4. Network Stability Analysis

The analysis first verifies the network stability by introducing a reference network system. The reference system will asymptotically track desired system when the bandwidth of low-pass filter is large enough. The network convergence analysis also proved the stability conditions. Further, the real network system is proven to track the reference system and its stability condition is provided. Finally, the estimation errors between real network and predict network system are shown to be bounded and the uncertainties canceling is achieved by decreasing the size of time-step.

4.1. Nonadaptive Reference System. We consider a reference network dynamics for (9) as

$$\dot{\xi}_r = A_m\xi_r + B_m\left(u_{r_a} + \sigma_r\right) \quad (15)$$

where $\xi_r = [\xi_{r_1}, \dots, \xi_{r_n}]^T$, $\xi_{r_i} = x_{r_i} - \sum_{j\in N_i} r_j x_{r_j}$, $\sigma_r = [\sigma_{r_1}, \dots, \sigma_{r_n}]^T$ and the reference control law

$$u_{r_a} = -C(s)\,\sigma_r \quad (16)$$

The closed-loop reference network system from (15) and (16) becomes

$$\dot{\xi}_r = A_m\xi_r + (1 - C(s))\,B_m\sigma_r \quad (17)$$

Lemma 5. *For the closed-loop network reference system in (15) and (16) subject to the $\mathscr{L}_1$ gain upper bound*

$$\frac{\Theta_{r_1}\beta + \left\|\xi_{r_0}\right\|_\infty}{1 - \Theta_{r_1}L\left(\rho_r + \gamma_\xi\right)} < \rho_r \quad (18)$$

$$\Theta_{r_1}L\left(\rho_r + \gamma_\xi\right) < 1 \quad (19)$$

where $\Theta_{r_1} = \|(sI_N - A_m)^{-1}(I_N - C(s)I_N)B_m\|_{\mathscr{L}_1}$.

$$\left\|\xi_{r_0}\right\| < \rho_r \quad (20)$$

Then

$$\left\|\xi_r\right\|_\infty < \rho_r \quad (21)$$

$$\left\|u_{r_a}\right\|_\infty < \rho_{u_r} \quad (22)$$

where ρ_r is introduced in (18) and (19)

$$\rho_{u_r} = \|C(s)\,M\|\left(L\left(\rho_r\right)\rho_r + \beta\right) \quad (23)$$

Proof. Because $\|\xi_{r_0}\|_\infty < \rho_r$ and $\xi_r(t)$ is continuous, to prove $\|\xi_r\|_\infty < \rho_r$, assuming the opposite is true, it implies there exists a time t' such that

$$\xi_r(t') = \rho_r, \qquad \left\|\xi_{r_{t'}}\right\|_\infty \leq \rho_r \tag{24}$$

Take Laplace transform for (17); we have

$$\xi_r(s) = (1 - C(s))(sI_N - A_m)^{-1} B_m \sigma_r(s) + \xi_{r_0} \tag{25}$$

and its upper bound has

$$\left\|\xi_{r_{t'}}\right\|_\infty \leq \left\|(1 - C(s))(sI_N - A_m)^{-1} B_m\right\|_{\mathscr{L}_1} \left\|\sigma_r(s)\right\|_\infty + \left\|\xi_{r_0}\right\|_\infty \tag{26}$$

Using Assumption 1 and upper bound in (24), we arrived at

$$\left\|\sigma_{r_{t'}}\right\|_\infty \leq L(\rho_r)\left\|\xi_{r_{t'}}\right\|_\infty + \beta \tag{27}$$

Substituting (27) into (26), we obtain

$$\left\|\xi_{r_{t'}}\right\|_\infty \leq \frac{\Theta_{r_1}\beta + \left\|\xi_{r_0}\right\|_\infty}{1 - \Theta_{r_1} L(\rho_r)} \tag{28}$$

where $\Theta_{r_1} = \|(sI_N - A_m)^{-1}(I_N - C(s)I_N)B_m\|_{\mathscr{L}_1}$. The stability condition in (18), together with (28), implies that $\|\xi_{r_{t'}}\|_\infty < \rho_r$, which clearly contradicts assumption in (24) since there is no t' to make assumption true. This proves (21). From (16), the network control law is

$$u_{r_a t'} = -C(s) M \sigma_r \tag{29}$$

It follows from (29), (21), and (27) that we have

$$\left\|u_{r_a t'}\right\|_\infty < \|C(s) M\|_{\mathscr{L}_1} (L(\rho_r)\rho_r + \beta) < \rho_{u_r} \tag{30}$$

which proves (22) and concludes the proof. □

Lemma 6. *Consider the reference system in (17). If the bandwidth of low-pass filter $C(s)$ is large enough, the control law in (16) can make ξ_r asymptotically track the desired network system described by*

$$\dot{\xi}_d = A_m \xi_d \tag{31}$$

where $\xi_d = [\xi_{d_1}, \ldots, \xi_{d_n}]$ and $\xi_{d_i} = x_{d_i} - \sum_{j \in N_i} r_j x_{d_j}$ are desired network state, $A_m = I_N \otimes A - \mathscr{L} \otimes Bk$.

Proof. For a given network, if the bandwidth of low-pass filter $C(s) = a/(s + a)$ can be chosen large enough, the disturbance effect in (17) will be canceled and (17) become as

$$\dot{\xi}_r = A_m \xi_r \tag{32}$$

This further implies

$$\xi_r(s) = (sI_N - A_m)^{-1} \xi_{r_0} \tag{33}$$

We further obtain that

$$\lim_{s \to 0} \xi_r(s) = \lim_{s \to 0} \xi_d(s) \tag{34}$$

Thus, the reference system will track the desired network system, which concludes the proof. □

4.2. Network Convergence Analysis. Since network $\mathscr{G}$ is connected, it follows from Assumption 3 that 0 is a simple eigenvalue of $\mathscr{L}$ and the other eigenvalues have positive real part. Thus, there exists a nonsingular matrix T such that $T^{-1}\mathscr{L}T$ is in the Jordan canonical form [9]. Because the left and right eigenvectors of $\mathscr{L}$ corresponding to zero eigenvalue are r and $\mathbf{1}$, respectively, the following can be chosen:

$$T = \begin{bmatrix} \mathbf{1} & U \end{bmatrix}, \qquad T^{-1} = \begin{bmatrix} r^T \\ V \end{bmatrix} \tag{35}$$

where $U \in \mathbf{C}^{N\times(N-1)}$ and $V \in \mathbf{C}^{N\times(N-1)}$ such that

$$T^{-1}\mathscr{L}T = J = \begin{bmatrix} 0 & 0 \\ 0 & \Delta \end{bmatrix} \tag{36}$$

where $\Delta \in \mathbf{C}^{N\times(N-1)}$ is an upper-triangular matrix, where the diagonal entries are the nonzero eigenvalues of $\mathscr{L}$. Let $\zeta_d = (T^{-1} \otimes I_n)\xi_d$. Then, (31) can be presented as

$$\dot{\zeta}_d = (I_N \otimes A - J \otimes Bk)\zeta_d \tag{37}$$

Lemma 7. *Suppose Assumption 3 holds. The distributed control law in (16) with agents in (15) solves consensus problem if and only if all the matrices $A - \lambda_i B$, $i = 2, \ldots, N$, are Hurwitz, where λ_i are the nonzero eigenvalues of Laplacian matrix $\mathscr{L}$.*

Proof. The elements of the state matrix in (37) are either block diagonal or block upper-triangular. Thus, ξ_i, $i = 2, \ldots, N$, converge asymptotically to zero if and only if the $N - 1$ subsystems along the diagonal as

$$\dot{\zeta}_{d_i} = (A - \lambda_i B)\zeta_{d_i}, \quad i = 2, \ldots, N \tag{38}$$

are asymptotically stable. The stability of closed-loop system in (32) and (31) is equal to the system in (37) that converges asymptotically to zero. The system consensus is achieved. This concludes the proof. □

Lemma 8. *Suppose Assumption 3 holds and the cooperative control law (8) satisfies Lemma 7; then*

$$\lim_{t \to \infty} x_{d_i} = (r^T \otimes e^{At}) r \begin{bmatrix} x_{d_1}(0) \\ \vdots \\ x_{d_N}(0) \end{bmatrix} \tag{39}$$

where $r \in \mathbf{R}^N$ such that $r^T\mathscr{L} = 0$ and $r^T\mathbf{1} = 1$.

Proof. The desired network dynamics (31) can transform as

$$\dot{x}_d = (I_N \otimes A - \mathscr{L} \otimes Bk) x_d \tag{40}$$

where $\xi_d = \mathscr{L} x_d$. The solution of (40) with Lemma 7 can be obtained as

$$\begin{aligned} x_d(t) &= e^{(I_N \otimes A - \mathscr{L} \otimes Bk)t} x_d(0) \\ &= (T \otimes I_n) e^{I_N \otimes A - J \otimes Bk} (T^{-1} \otimes I_n) x_d(0) \\ &= (T \otimes I_n) \begin{bmatrix} e^{At} & 0 \\ 0 & e^{(I_{N-1} \otimes A - \Delta \otimes Bk)t} \end{bmatrix} (T^{-1} \otimes I_n) x_d(0) \end{aligned} \tag{41}$$

From Lemma 7, $I_N \otimes A - \Delta \otimes Bk$ is Hurwitz. Thus,

$$\begin{aligned} \lim_{t \longrightarrow \infty} e^{(I_N \otimes A - \mathscr{L} \otimes Bk)t} &= (\mathbf{1} \otimes I_n) e^{At} (r^T \otimes I_n) \\ &= (\mathbf{1} r^T) \otimes e^{At} \end{aligned} \tag{42}$$

Thus, the final consensus value is

$$\lim_{t \longrightarrow \infty} x_d(t) = (\mathbf{1} r^T \otimes e^{At}) x_d(0) \tag{43}$$

This concludes the proof. □

4.3. Network Transient Performance. We discussed the relation between reference network system and desired network system and then perform the network convergence analysis in last section. We further prove the real network system can track reference network system in this section. Before this proof, we will establish the error bounds between the network state predictor and real network system and between the adaptive law and the signal it estimates.

Lemma 9. *Given the network system in (9) with $\mathscr{L}_1$ adaptive control law in (13) and the reference system in (15) with (16) subject to stability conditions in (18) and (19), if $\|\xi_{r_0}\| < \rho_r$ and we choose T in (11) and low-pass filter C(s) in (13) to ensure*

$$\gamma_0(T) < \gamma_{\tilde{\xi}} \tag{44}$$

$$\gamma_{\xi_e}(T) < \gamma_{\xi} \tag{45}$$

where $\gamma_{\tilde{\xi}}$ is arbitrary positive constant and γ_ξ is introduced in (19); then we have

$$\|\tilde{\xi}\|_\infty < \gamma_{\tilde{\xi}} \tag{46}$$

$$\|\xi - \xi_r\|_\infty < \gamma_\xi \tag{47}$$

$$\|u - u_{r_a}\|_\infty < \gamma_u \tag{48}$$

Suppose there exist upper bounds for the truncated $\mathscr{L}_\infty$ norms of $\xi(t)$ and $\sigma(t) - \hat{\sigma}(t)$ at some time, t_1

$$\|\xi_{t1}\|_\infty < \rho \tag{49}$$

Define

$$\beta_1 = \sqrt{\lambda_{max}(A_m^T A_m)} \tag{50}$$

$$\begin{aligned} \beta_2 = &\int_0^T \sqrt{\lambda_{max} \Phi(T-\tau)^T \Phi(T-\tau) B_m} d\tau \\ &\cdot \left\|\Phi^{-1}(T) e^{AT}\right\| \end{aligned} \tag{51}$$

$$\beta_3 = \int_0^T \sqrt{\lambda_{max} \Phi(T-\tau)^T \Phi(T-\tau) B_m} d\tau \tag{52}$$

Proof. Because $\xi(t), \tilde{\xi}(t)$ is continuous, to prove $\|\tilde{\xi}\|_\infty < \gamma_{\tilde{\xi}}$, assume that the opposite is true assuming $\|\tilde{\xi}\|_\infty \geq \gamma_{\tilde{\xi}}$. This implies that there exists a time t' such that

$$\left\|\tilde{\xi}(t')\right\|_\infty = \gamma_{\tilde{\xi}} \tag{53}$$

$$\left\|\xi(t') - \xi_r(t')\right\|_\infty = \gamma_\xi, \quad or \tag{54}$$

$$\left\|u_a(t') - u_{r_a}(t')\right\| = \gamma_u, \quad or \tag{55}$$

The solution of network error dynamics (12) over the interval $[jT, (j+1)T]$ is described as

$$\begin{aligned} \tilde{\xi}(t) = &\Phi(T) \tilde{\xi}(jT) + \int_{jT}^t \Phi(t-\tau) B_m \hat{\sigma}(t) d\tau \\ &- \int_{jT}^t \Phi(t-\tau) B_m \sigma(t) d\tau, \quad j = 0, 1, 2, \cdots \end{aligned} \tag{56}$$

where $\Phi(T) = e^{A_m T}$, $(j+1)T < t'$ and τ is a dummy variable. At $t = (j+1)T$, it becomes

$$\begin{aligned} \tilde{\xi}((j+1)T) = &\Phi(T) \tilde{\xi}(jT) \\ &+ \int_0^T \Phi(T-\tau) B_m \hat{\sigma}(jT) d\tau \\ &- \int_0^T \Phi(T-\tau) B_m \sigma(jT+\tau) d\tau \end{aligned} \tag{57}$$

We can choose $\hat{\sigma}$ such that its effects on the system derive $\tilde{x}$ to zero at the next time-step. Then, we have

$$\tilde{\xi}((j+1)T) = -\int_0^T \Phi(T-\tau) B_m \sigma(jT+\tau) d\tau \tag{58}$$

It follows from Assumption 1, Lemma 5, and (54) that

$$\|\sigma(t)\|_\infty \leq L(\rho_r + \gamma_\xi) + \beta \leq b_\sigma \tag{59}$$

Moreover, following Lemma 5 and (55), we have

$$\|u_a(t)\|_\infty \leq \rho_{ur} + \gamma_u \leq b_u \tag{60}$$

The upper bound in (59) and (60) allows for the following upper bound:

$$\left\|\tilde{\xi}(j+1)T\right\|_\infty \leq \beta_3 b_\sigma \leq \kappa(T) \tag{61}$$

for all j such that $(j+1)T < t'$. From (56), we arrived at the following upper bound:

$$\left\|\tilde{\xi}(jT+t)\right\|_{\infty} \leq \beta_1\kappa + \beta_2\kappa + \beta_3 b_{\sigma} \leq \gamma_0(T) \tag{62}$$

where β_1, β_2, and β_3 are defined in (50), (51), and (52), respectively. Then, for all $t \in [0, t']$, following from (44), we have

$$\left\|\tilde{\xi}(t)\right\| \leq \gamma_0(T) < \gamma_{\tilde{\xi}} \tag{63}$$

The results clearly contradict assumption in (53) since there is no t' to make assumption true. This proves (46).

Next, we will drive the bounds for the estimation error of adaptive law, $\tilde{\sigma} = \hat{\sigma} - \sigma$, which are used to further obtain the bounds of $\|\xi - \xi_r\|$. We derive the bound for $\dot{\sigma}(x,t)$ first and then get

$$\dot{\sigma}(t) = \frac{\partial\sigma}{\partial\xi}^T \dot{\xi}(t) \tag{64}$$

From (14), we have

$$\dot{\xi} = A_m\xi - C(s)B_m\hat{\sigma} + B_m\sigma \tag{65}$$

To establish the bound for this signal, we need to derive a bound for $\xi(s)$. Using (11) and (53) for all $t < t_1$, then we have

$$\left\|\hat{\sigma}_{t1}\right\|_{\infty} \leq \left\|\Phi^{-1}(T)e^{A_m T}\right\|\gamma_{\tilde{\xi}} \leq b_{\hat{\sigma}} \tag{66}$$

Using (64), we can now write the

$$\begin{aligned}\|\dot{\sigma}(t)\|_{\infty} &\leq L\left\{\left\|A_m\right\|_{\mathscr{L}_1}\left(\rho_r + \gamma_{\xi}\right) + \left\|C(s)B_m\right\|_{\mathscr{L}_1} b_{\hat{\sigma}}\right. \\ &\quad \left. + \left\|B_m\right\|_{\mathscr{L}_1}\left[L\left(\rho_r + \gamma_{\xi}\right) + \beta\right]\right\} \leq b_{d\sigma}\end{aligned} \tag{67}$$

Then, control signal upper bound

$$\|\dot{u}\|_{\infty} \leq \left\|sC(s)M \otimes \mathbf{1}_n\right\| b_{\hat{\sigma}} \leq b_{du} \tag{68}$$

Note that the adaptive law of (11) is designed such that

$$\Phi(T)\tilde{\xi}(jT) + \int_0^T \Phi(T-\tau)\,d\tau B_m\hat{\sigma}(jT) = 0 \tag{69}$$

For the time $t \in [(j-1)T, jT]$, we also have

$$\tilde{\xi}(jT) = -\int_0^T \Phi(T-\tau)B_m\sigma((j-1)T+\tau)\,d\tau \tag{70}$$

Following from the definition of adaptive law, we obtain

$$\begin{aligned}\tilde{\xi}(jT) &= -\int_0^T \Phi(T-\tau)A_m\tilde{\xi}(jT)\,d\tau \\ &\quad - \int_0^T \Phi(T-\tau)B_m\hat{\sigma}(jT)\,d\tau\end{aligned} \tag{71}$$

Thus, (70) and (71) give us

$$\begin{aligned}&\int_0^T \Phi(T-\tau)B_m\sigma((j-1)T+\tau)\,d\tau \\ &\quad = \int_0^T \Phi(T-\tau)\left(A_m\tilde{\xi}(jT) + B_m\hat{\sigma}(jT)\right)d\tau\end{aligned} \tag{72}$$

which imply there exist a time, $t_p \in [(j-1)T, jT]$, such that

$$B_m\hat{\sigma}(jT) + A_m\tilde{\xi}(jT) = B_m\sigma(t_p) \tag{73}$$

This imply that for any $t \in [jT, (j+1)T]$

$$\begin{aligned}&\left\|B_m(\hat{\sigma}(t) - \sigma(t))\right\| \\ &\quad \leq \left\|B_m\right\|\left(\left\|\hat{\sigma}(jT) - \sigma(t_p)\right\| + \left\|\sigma(t) - \sigma(t_p)\right\|\right) \\ &\quad \leq \beta_1\left\|\tilde{\xi}(jT)\right\| + \int_{t_p}^t \|\dot{\sigma}(\tau)\|\,d\tau \leq \beta_1\gamma_0 + 2(b_{d\sigma})T \\ &\quad \leq b_{\tilde{\sigma}}\end{aligned} \tag{74}$$

Let $\tilde{\sigma} = \hat{\sigma} - \sigma$; then we have

$$\left\|B_m\tilde{\sigma}\right\|_{\infty} \leq b_{\tilde{\sigma}}\left(T, \rho_b, \rho_{\xi}\right) \tag{75}$$

Next, we derive the bounds of $\|u_a - u_{r_a}\|$. From network control law in (8) and (13), we have

$$u_a(s) = -C(s)\hat{\sigma} \tag{76}$$

From (29), we have

$$u_{r_a}(s) = -C(s)\sigma_r \tag{77}$$

Thus, apply (76) and (77), we arrived

$$u_a(s) - u_{r_a}(s) = -C(s)(\sigma - \sigma_r) - C(s)\tilde{\sigma} \tag{78}$$

where $\hat{\sigma} = \sigma + \tilde{\sigma}$. Let $\xi_e(t) = \xi(t) - \xi_r(t)$; from Assumption 1 and (45), we can arrive at the following upper bound:

$$\left\|(\sigma - \sigma_r)\right\|_{\infty} \leq L\left\|\xi_e\right\|_{\infty} + \beta \tag{79}$$

Now, we derive the bounds of $\|\xi - \xi_r\|$ and then we have

$$\dot{\xi}_e(t) = A_m\xi_e + B_m((1 - C(s))(\sigma - \sigma_r) - C(s)\tilde{\sigma}) \tag{80}$$

The solution of (80) is

$$\begin{aligned}&\xi_e(s) \\ &\quad = (sI_N - A_m)^{-1}B_m((1 - C(s))(\sigma - \sigma_r) - C(s)\tilde{\sigma})\end{aligned} \tag{81}$$

Substituting (79) and (55) to (81), we further get

$$\gamma_{\xi_e} = \frac{\Theta_{e_1}\beta + \Theta_{e_2}b_{\tilde{\sigma}}}{1 - \Theta_{e_1}L\left(\rho_r + \gamma_{\xi}\right)} \tag{82}$$

and

$$\left\|\xi_e(t)\right\|_{\infty} \leq \Theta_{e_1}\left(L\left\|\xi_e\right\|_{\infty} + \beta\right) + \Theta_{e_2}b_{\tilde{\sigma}} \leq \gamma_{\xi_e} \tag{83}$$

where $\Theta_{e_1} = \|(sI_N - A_m)^{-1}B_m(1 - C(s))\|_{\mathscr{L}_1}$ $\Theta_{e_2} = (sI_N - A_m)^{-1}C(s)$.

$$\left\|\xi_e(t)\right\|_{\infty} \leq \gamma_{\xi_e} \leq \gamma_{\xi} \tag{84}$$

Substituting (75) and (79) to (78), the upper bound is

$$\left\| u_a(s) - u_{r_a}(s) \right\| \le \|C(s)\| \left(L\gamma_{\xi_e} + \beta + b_{\tilde{\sigma}} \right) < \gamma_u \tag{85}$$

The stability condition in (18), together with (84), implies that $\|\xi - \xi_r\|_\infty < \gamma_\xi$, which clearly contradicts assumption in (55) since there is no t' to make assumption true. This proves (47). If we choose $T = 0$ and the bandwidth of low-pass filters $C(s)$ large enough, (84) with the condition become $\|\xi_{e_t}\|_\infty < \gamma_\xi$. Thus, it always exists γ_ξ to satisfy (84). This completes the proof of Lemma 9. □

Lemma 10. *The error bounds from Lemma 9 depend on the time-step T. When T approaches 0, the error approaches 0 as well.*

$$\lim_{T \to 0} \gamma_{\tilde{\xi}}(T, \rho_b) \longrightarrow 0 \tag{86}$$

$$\lim_{T \to 0} \gamma_{\xi}(T, \rho_b) \longrightarrow 0 \tag{87}$$

$$\lim_{T \to 0} \gamma_{u}(T, \rho_b) \longrightarrow 0 \tag{88}$$

Proof. The inside of integration in (61) is a bounded function; so when $T \longrightarrow 0$, then $\gamma_{\tilde{\xi}} \longrightarrow 0$. Since (61) holds, the first term in (74) goes to 0 as $T \longrightarrow 0$ and the second term clearly goes to 0 as well. Thus, $\gamma_\xi \longrightarrow 0$ when $T \longrightarrow 0$. The last statement is γ_u. Because (74) holds, we can clearly see from (48) that when $T \longrightarrow 0$, $\gamma_u \longrightarrow 0$. This completes the proof. □

5. Simulation

The simulation will show the details of proposed adaptive cooperation algorithm by compared with typical cooperative control. The network systems are included uncertainties and disturbances in these examples. The agent dynamics under consideration is a vertical motion of a quadrotor. There are 10 quadrotors that try to maintain in the specified altitude and velocity for cooperation observation in simulation scenarios. The dynamics described by a second-order state space as

$$\dot{x}_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_i + \begin{bmatrix} 0 \\ \dfrac{1}{m} \end{bmatrix} (u_i + \sigma_i) \tag{89}$$

where $x_i = [z_i \;\; w_i]^T$ are vertical position and velocity, $m = 1kg$ is the mass of quadrotor, u_i is the thrust generated by rotors, and $\sigma_i(x_i, t)$ is the force produced by wind on quadrotor.

The first example shows the cooperative control algorithm from (8). The integration step in simulation is set as $T = 10^{-2}$ seconds; the initial altitude and velocity $x(0)_i$ are chosen randomly within the range $0 \sim 10m$ and $-1 \sim 1m/s$ individually. The control gain is chosen as $k = [1 \;\; 1]$. The communication links are considered as undirected ring network topology as Figure 1. The Laplacian matrix is given by

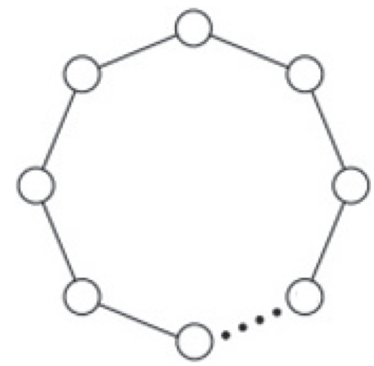

Figure 1: Undirected ring network.

$$L = \begin{bmatrix} 2 & -1 & 0 & & 0 & -1 \\ -1 & & & \ddots & 0 & 0 \\ 0 & & \ddots & \ddots & \ddots & \\ & \ddots & \ddots & \ddots & & 0 \\ 0 & 0 & \ddots & & 2 & -1 \\ -1 & 0 & & 0 & -1 & 2 \end{bmatrix}. \tag{90}$$

The eigenvalues of L only have one zero eigenvalues and other eigenvalues are positive that means Assumption 3 is satisfied. The uncertainties considered in agent dynamics are globally Lipschitz as

$$\sigma_i = c * e^{-x_i} \left(\sin(x_i) + \cos(x_i) \right) \tag{91}$$

Although the cooperative control tries to drive quadrotors to common position, the uncertainties affect the effectiveness of algorithm (8). The cooperative control signals from Figure 3 show the fluctuation and diverge gradually. Thus, the consensus from Figure 2 cannot be achieved by cooperative control (8) for the group of quadrotors.

The second example shows the proposed adaptive cooperation algorithm from (8) and (13). The integration step in simulation is set as $T = 10^{-2}$ seconds. The initial altitude and velocity $x(0)_i$ are chosen randomly within the range $0 \sim 10m$ and $-1 \sim 1m/s$ individually. The filter of adaptive cooperation controller for agents is chosen as $C(s) = 10/(s + 10)$. The communication links are also considered as undirected ring network topology as Figure 1.

Figure 4 shows the consensus achieved with the same position and velocity for the group of quadrotors, and the uncertainties in network are also canceled in Figure 6. Figure 5 shows the adaptive cooperation control signal of quadrotors. Although network systems create uncertainties, the transient performance is still maintained well by (8) and (13). The simulation results with Figures 4 and 5 demonstrate Lemmas 6–8. Figure 6 shows the mismatched dynamics that means the state predictor tracks the real state well and adapts to uncertainty quickly. The results in Figure 6 thus demonstrate Lemma 10. Further, the results in Figures 4–6

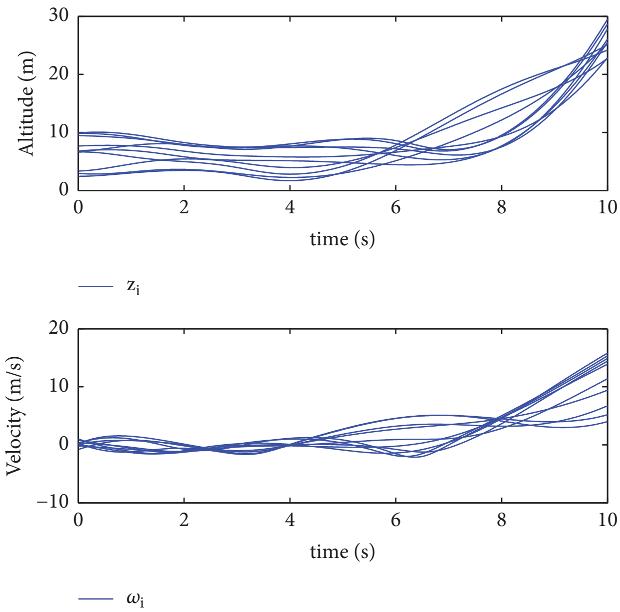

FIGURE 2: The trajectory of quadrotors.

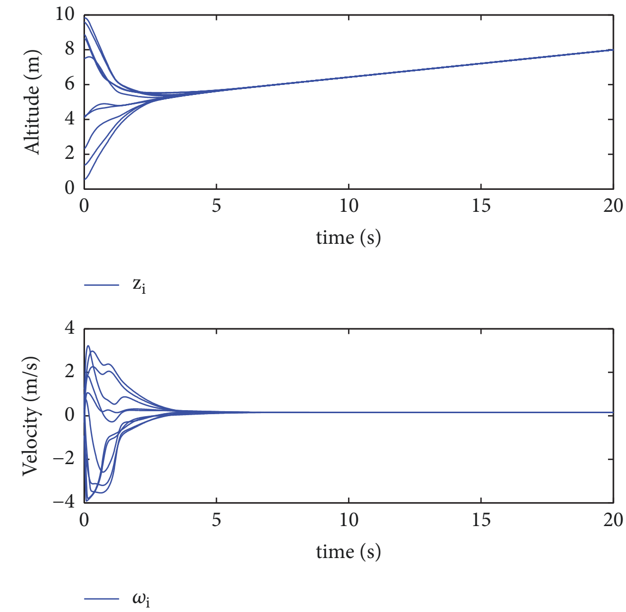

FIGURE 4: The trajectory of quadrotors.

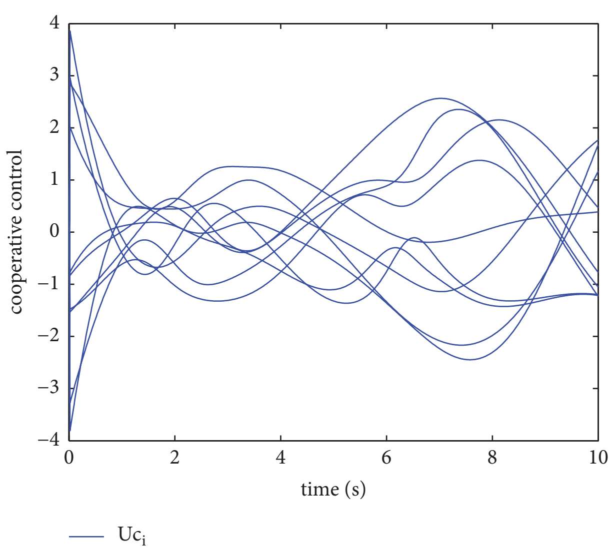

FIGURE 3: Cooperative control input.

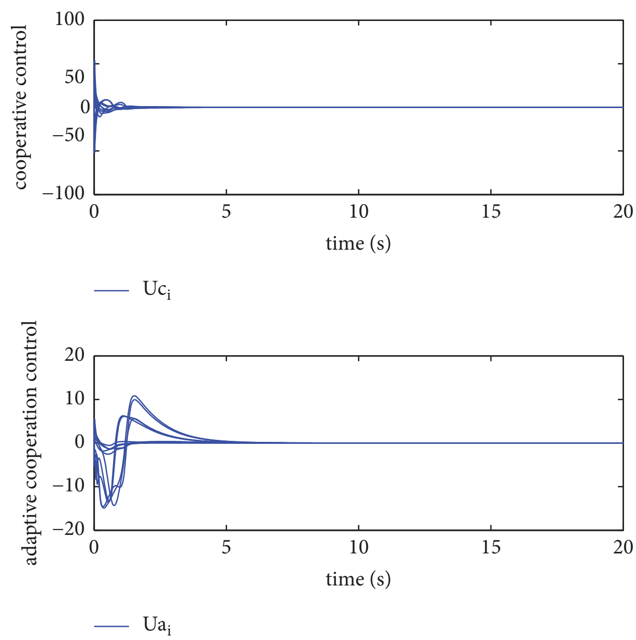

FIGURE 5: Adaptive cooperation control input.

also show the robustness of proposed adaptive cooperation control design.

6. Conclusion

This paper presents a novel cooperative adaptation approach to deal with uncertainty of multi-agent systems. The typical cooperative control was used to achieve collaboration in multi-agent systems but it also causes interconnected uncertainties among network. Then, the cooperative adaptation approach was proposed to cancel uncertainties in multi-agent systems. The cooperative adaptation algorithm features as fast adaptation scheme and can be treated as high-gain approach. To prevent aggressive changing in real control signal, a low-pass filtering mechanism is added in control law, which filters away aggressive signals and recovers robustness. The stability condition and tracking performance for closed-loop network system are presented. The estimation errors between predicted states and real state are proved to be arbitrarily small. Then, the network stability and performance bounds between real system and reference system can be made arbitrarily small by decreasing the integration time-step. The simulation results show the details of proposed scheme. This approach can be further extended to deal with uncertainties issue in multi-unmanned vehicle systems.

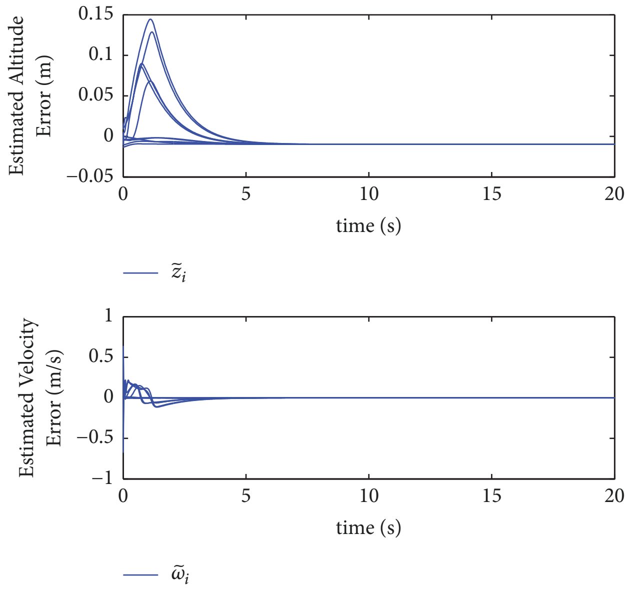

FIGURE 6: Estimation error of states.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by NSF Grant IIS-1208499.

References

[1] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1520–1533, 2004.

[2] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 215–233, 2007.

[3] W. Ren and R. W. Beard, "Consensus seeking in multiagent systems under dynamically changing interaction topologies," *IEEE Transactions on Automatic Control*, vol. 50, no. 5, pp. 655–661, 2005.

[4] C.-Q. Ma and J.-F. Zhang, "Necessary and sufficient conditions for consensusability of linear multi-agent systems," *IEEE Transactions on Automatic Control*, vol. 55, no. 5, pp. 1263–1268, 2010.

[5] H. Kim, H. Shim, and J. H. Seo, "Output consensus of heterogeneous uncertain linear multi-agent systems," *IEEE Transactions on Automatic Control*, vol. 56, no. 1, pp. 200–206, 2011.

[6] S. E. Tuna, "Conditions for synchronizability in arrays of coupled linear systems," *IEEE Transactions on Automatic Control*, vol. 54, no. 10, pp. 2416–2420, 2009.

[7] L. Scardovi and R. Sepulchre, "Synchronization in networks of identical linear systems," *Automatica*, vol. 45, no. 11, pp. 2557–2562, 2009.

[8] J. H. Seo, H. Shim, and J. Back, "Consensus of high-order linear systems using dynamic output feedback compensator: low gain approach," *Automatica*, vol. 45, no. 11, pp. 2659–2664, 2009.

[9] Z. Li, Z. Duan, G. Chen, and L. Huang, "Consensus of multiagent systems and synchronization of complex networks: a unified viewpoint," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 57, no. 1, pp. 213–224, 2010.

[10] T. Yang, S. Roy, Y. Wan, and A. Saberi, "Constructing consensus controllers for networks with identical general linear agents," *International Journal of Robust and Nonlinear Control*, vol. 21, no. 11, pp. 1237–1256, 2011.

[11] K. Liu, G. Xie, W. Ren, and L. Wang, "Consensus for multi-agent systems with inherent nonlinear dynamics under directed topologies," *Systems & Control Letters*, vol. 62, no. 2, pp. 152–162, 2013.

[12] G. Wen, Z. Duan, G. Chen, and W. Yu, "Consensus tracking of multi-agent systems with Lipschitz-type node dynamics and switching topologies," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 61, no. 2, pp. 499–511, 2014.

[13] E. Nuno, R. Ortega, L. Basanez, and D. Hill, "Synchronization of networks of nonidentical Euler-Lagrange systems with uncertain parameters and communication delays," *Institute of Electrical and Electronics Engineers Transactions on Automatic Control*, vol. 56, no. 4, pp. 935–941, 2011.

[14] A. Das and F. L. Lewis, "Distributed adaptive control for synchronization of unknown nonlinear networked systems," *Automatica*, vol. 46, no. 12, pp. 2014–2021, 2010.

[15] H. Su, G. Chen, X. Wang, and Z. Lin, "Adaptive second-order consensus of networked mobile agents with nonlinear dynamics," *Automatica*, vol. 47, no. 2, pp. 368–375, 2011.

[16] W. Yu, G. Chen, and M. Cao, "Consensus in directed networks of agents with nonlinear dynamics," *IEEE Transactions on Automatic Control*, vol. 56, no. 6, pp. 1436–1441, 2011.

[17] I. D. Landau, R. Lozano, M. M'Saad, and A. Karimi, *Adaptive Control*, vol. 51, Springer, New York, NY, USA, 1998.

[18] Y. Liu and Y. Jia, "Adaptive leader-following consensus control of multi-agent systems using model reference adaptive control approach," *IET Control Theory & Applications*, vol. 6, no. 13, pp. 2002–2008, 2012.

[19] G.-X. Wen, C. L. Chen, Y.-J. Liu, and Z. Liu, "Neural-network-based adaptive leader-following consensus control for second-order non-linear multi-agent systems," *IET Control Theory & Applications*, vol. 9, no. 13, pp. 1927–1934, 2015.

[20] W. Liu and J. Huang, "Adaptive leader-following consensus for a class of higher-order nonlinear multi-agent systems with directed switching networks," *Automatica*, vol. 79, pp. 84–92, 2017.

[21] W. Liu and J. Huang, "Cooperative global robust output regulation for nonlinear output feedback multiagent systems under directed switching networks," *Institute of Electrical and Electronics Engineers Transactions on Automatic Control*, vol. 62, no. 12, pp. 6339–6352, 2017.