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Research Article

Dynamic Output Feedback Control of Discrete Markov Jump Systems based on Event-Triggered Mechanism

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This paper is devoted to the co-design strategy of event-triggered scheme and dynamic output feedback controller for a class of discrete-time networked control systems (NCSs) with random time delay. An event-triggered mechanism is given to ease the information transmission. Both the sensor and controller are set with mode-dependent quantizers in the system. A Markov process is used to model the time delay which is used to describe the quantization density. By employing the Lyapunov-Krasovskii functional and linear matrix inequality (LMI), sufficient conditions are obtained for the system. A specific example is given to demonstrate the proposed approach.

1. Introduction

In recent years, a boom of computer science and technology has led to the increasing role of network control in industry. Nevertheless, main problems in the network such as time delay, disorder, packet loss, and the occupation of network bandwidth cannot be ignored.

The issue of time delay is usually treated as the major cause of deterioration of system performance which has been dealt with by quite a few researchers, and they proposed a variety of approaches to these problems; see [1–5]. In [6], the authors propose sequential measurement fusion and state fusion estimation methods to deal with delayed data for clustered sensor networks, since they can handle the data that are available sequentially. In order to save network resources, some literatures have proposed time-triggered mechanism (such as [7–9]); this scheme transfers each sample data which reduces the utilization of network bandwidth. The event-triggering mechanism proposed in the 1990s is more advantageous than the time-triggered scheme (see [10–13]), which means that the data will not be transmitted unless condition set in advance is satisfied, and then it will reduce the time of accessing network and save the network resources. In [14], the event-triggered scheme is applied to the fault detection to guarantee the fault detection accuracy. Furthermore, an event-triggered scheme with an adaptive threshold would subserve the quality of the control; see [15, 16].

Quantization also has become a research hotspot these years, paper [17] notes that quantization process plays an important role in the networked control systems, an adaptive quantizer can help to alleviate network congestion and maintain system stability, paper [18] proposes a quantized state feedback strategy for global and asymptotical stabilization of the networked control systems, papers [19, 20] address the problem of output feedback control for networked control systems with limited information, it should be noted that the impact of quantification cannot be ignored, and, in [18, 19, 21], the quantization deviation is disposed as an uncertainty. Papers [20, 22, 23] design dynamic quantizer to reduce the impact of the quantization error.

Markov jump system is a random system with multimodes which has attracted much attention in the past few decades (see [24–26]); the jump transition of the system in each mode is determined by a Markov chain so it can be applied to the multimodes in many kinds of systems. In [4, 27, 28], the Markov process is applied to modeling the time delay, in [10, 20], the time delay is used to describe the quantization density in the form of function and the authors investigate the dynamic quantization output feedback controller; in [10], the author also takes into account an event-triggered scheme to

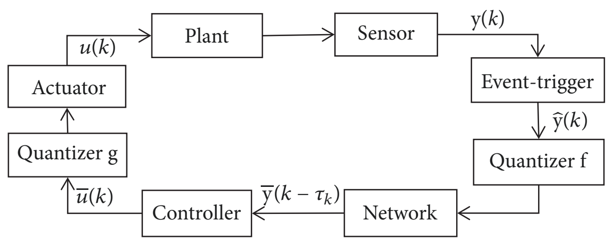

Figure 1: The structure of NCS with event-trigger and quantizers.

the aforementioned systems. Nevertheless, no article has considered a system with a dynamic output feedback controller and two quantizers based on event-triggered scheme.

In the view of the aforementioned analysis, in this paper, we aim to investigate the co-design strategy of the event-triggered scheme and dynamic output feedback controller. An adaptive event-triggered mechanism is presented to reduce the information transmission. A Markov chain is applied to modeling the time delay which is used to describe quantization density as a function to ease bandwidth usage. Methods utilizing Lyapunov-Krasovskii functional and linear matrix inequality (LMI) are presented to develop the sufficient conditions of stochastic stability. An event-triggered scheme based dynamic output feedback controller is adopted to guarantee the performance of the closed-loop system. Finally, an example demonstrates the proposed method in detail.

The remainder of this paper is organized as follows. The system description and formulation are provided in **Section 2**. The sufficient condition of stochastic stability and the co-design strategy are presented in **Section 3**. In **Section 4**, a simulation example is presented to demonstrate the feasibility of the proposed approach. The conclusion is given in **Section 5**.

2. Problem Formulation

The plant of NCS is considered as a discrete-time model

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) \end{aligned} \tag{1}$$

where $x(k) \in R^n$ is the system state, $u(k) \in R^m$ is the control input, and $y(k) \in R^n$ is the output, A, B, and C are constant matrices of appropriate dimensions, and the structure of our concerned NCS is shown as in Figure 1.

Throughout the paper, some assumptions as follows are needed for the considered NCS.

Assumption 1. Both the controller and actuator are event-driven, while the sensor is time-driven with a sampling period h.

Assumption 2. The data communication in the network is assumed to be single packet transmission without packet dropout.

Assumption 3. The network-induced delay is smaller than one sampling period and bounded by $0 \leq \underline{\tau} \leq \tau_k \leq \overline{\tau}$.

The event-triggered scheme is shown as follows:

$$e^{\mathrm{T}}(k)\, e(k) \geq \sigma(k)\, y^{\mathrm{T}}(k)\, y(k) \tag{2}$$

where k is the sampling instant and $k = 0, 1, 2, \cdots$. $e(k)$ is the quantization error and defined as

$$e(k) = y(k) - \hat{y}(k-1) \tag{3}$$

where $\hat{y}(k-1)$ is the latest transmitted signal and $y(k)$ is the signal which is to be transmitted. Based on [10], $\sigma(k)$ is the adaptive threshold and defined as

$$\sigma(k) = \begin{cases} \sigma_1, & if\ e^{\mathrm{T}}(k-1)\, e(k-1) \geq \Delta \\ \sigma_2, & if\ e^{\mathrm{T}}(k-1)\, e(k-1) \leq \Delta \end{cases} \tag{4}$$

and $\alpha(k)$ is defined as

$$\alpha(k) = \begin{cases} 1, & if\ \ (2)\ \ is\ satisfied \\ 0, & otherwise \end{cases} \tag{5}$$

and

$$\hat{y}(k) = \alpha(k)\, y(k) + (1-\alpha(k))\, \hat{y}(k-1) \tag{6}$$

The quantization density in this paper is designed as a function of time delay $\tau_k = \tau(k)$, which is modeled as a discrete homogeneous Markov chain $\{r_k, k\}$, with a finite number of states $\omega = \{1, 2, \ldots, s\}$, and the transition probability of this Markov chain from mode i to mode j at time $k+1$ is

$$p_{ij} = Prob\,\{r_{k+1} = j \mid r_k = i\} \tag{7}$$

where $i, j \in \omega$ and $0 \leq p_{ij} \leq 1$.

The quantizer f is described as follows:

$$f(y_j, i) = \begin{cases} \rho^h(i), & if\ \dfrac{\rho^h(i)}{1+\delta(i)} < y_j < \dfrac{\rho^h(i)}{1-\delta(i)}, \\ & y_j > 0,\ h = 0, \pm 1, \pm 2, \cdots \\ 0, & if\ \ y_j = 0 \\ -f(-y_j, i), & if\ \ y_j < 0 \end{cases} \tag{8}$$

where $j = 1, 2, \ldots, s$, and the relation between $\delta_f(i)$ and $\rho_f(i)$ is

$$\delta_f(i) = \frac{1-\rho_f(i)}{1+\rho_f(i)} \tag{9}$$

and the quantized set is

$$\zeta = \{\pm\rho^h(i), h = 0, \pm 1, \pm 2, \cdots\} \tag{10}$$

Based on [21], $\Delta_f \in [-\delta_f(i), \delta_f(i)]$, and then

$$f\left(y_j, i\right) = \left(1 + \Delta_f\right) \hat{y}(k) \tag{11}$$

The quantized density and quantized set of the quantizer g is similar to the quantizer f, and it can be expressed as follows:

$$g\left(u_r, i\right) = \left(1 + \Delta_g\right) \overline{u}(k) \tag{12}$$

where $r = 1, 2, \cdots, m$ and $\Delta_g \in [-\delta_r(i), \delta_r(i)]$.

According to the above statement, the dynamic output feedback controller is designed as

$$\begin{aligned} \hat{x}(k+1) &= A_c(i)\hat{x}(k) + B_c(i)\overline{y}(k-\tau_k) \\ \overline{u}(k) &= C_c(i)\hat{x}(k) \end{aligned} \tag{13}$$

Combining (11) and (12), the closed-loop system of (1) can be represented as follows:

$$\begin{aligned} x(k+1) &= Ax(k) + B\left(1+\Delta_g\right)C_c(i)\hat{x}(k) \\ \hat{x}(k+1) &= A_c(i)\hat{x}(k) + B_c(i)\left(1+\Delta_f\right)Cx(k-\tau_k) \\ &\quad - B_c(i)\left(1+\Delta_f\right)(1-\alpha(k))e(k) \\ e(k+1) &= (CA-C)x(k) + CBC_c(i)\left(1+\Delta_g\right)\hat{x}(k) \\ &\quad + (1-\alpha(k))e(k) \end{aligned} \tag{14}$$

Define an augmented vector

$$\eta(k) = \left[x^{\mathrm{T}}(k) \;\; \hat{x}^{\mathrm{T}}(k) \;\; e^{\mathrm{T}}(k)\right]^{\mathrm{T}} \tag{15}$$

and then

$$\begin{aligned} &\eta(k+1) \\ &= \left(\Phi_{1\alpha(k)}(i) + \Phi_2(i) + \Phi_3(i)\left(1+\Delta_g\right)\right)\eta(k) \\ &\quad + \Phi_{4\alpha(k)}(i)\left(1+\Delta_f\right)\eta(k-\tau_k) \end{aligned} \tag{16}$$

where

$$\Phi_{1\alpha(k)}(i) = \begin{bmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ CA-C & 0 & (1-\alpha(k))I \end{bmatrix}$$

$$\Phi_2(i) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & A_c(i) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Phi_3(i) = \begin{bmatrix} 0 & BC_c(i) & 0 \\ 0 & 0 & 0 \\ 0 & CBC_c(i) & 0 \end{bmatrix}$$

$$\Phi_{4\alpha(k)}(i) = \begin{bmatrix} 0 & 0 & 0 \\ B_c(i)C & 0 & -(1-\alpha(k))B_c(i) \\ 0 & 0 & 0 \end{bmatrix} \tag{17}$$

We define

$$J = \sum_{k=0}^{\infty} \begin{pmatrix} x(k) \\ u(k) \end{pmatrix}^{\mathrm{T}} Q' \begin{pmatrix} x(k) \\ u(k) \end{pmatrix} \tag{18}$$

as the cost function for system (16), where $Q' = \operatorname{diag}\{Q_1', Q_2'\} \in R^{2n\times 2n}$, $Q_1' \in R^{n\times n}$, and $Q_2' \in R^{n\times n}$.

3. Main Results

The stabilization of the closed-loop system (16) will be considered in this section, and, before going any further, a lemma and a definition are introduced, which will be helpful for deriving the following results.

Lemma 4 (see [10]). *Let* $\overline{x}(k) = x(k+1) - x(k)$,

$$\begin{aligned} &\tilde{\eta}(k) \\ &= \left[\eta^{\mathrm{T}}(k) \;\; \eta^{\mathrm{T}}(k)\Delta_g^{\mathrm{T}} \;\; \eta^{\mathrm{T}}(k-\tau_k) \;\; \eta^{\mathrm{T}}(k-\tau_k)\Delta_f^{\mathrm{T}}\right]^{\mathrm{T}}, \end{aligned} \tag{19}$$

and $\tilde{\eta}(k) \in R^l$, *for any matrices* $E \in R^{n\times n}$, $M \in R^{n\times l}$, *and* $Z \in R^{l\times l}$ *satisfying*

$$\begin{bmatrix} E & M \\ M^{\mathrm{T}} & Z \end{bmatrix} \geq 0 \tag{20}$$

Then the following inequality holds:

$$-\sum_{i=k-\tau}^{k-1} \overline{x}(i)^{\mathrm{T}} E\overline{x}(i) \leq \tilde{\eta}^{\mathrm{T}}(k)\left\{\gamma + \gamma^T + \overline{\tau}Z\right\}\tilde{\eta}(k) \tag{21}$$

where

$$\gamma = M^{\mathrm{T}}\left[\operatorname{diag}\{I,0,0\} \;\; 0 \;\; \operatorname{diag}\{-I,0,0\} \;\; 0\right] \tag{22}$$

Proof is given in the Appendix.

Definition 5 (see [20]). The closed-loop systems given in (23) is stochastically stable; i.e., there exists a constant $0 \leq \alpha \leq \infty$, such that $E\{\sum_{l=0}^{\infty}\eta^{\mathrm{T}}(l)\eta(l)\} < \alpha$ for any initial states $x(0)$ and r_0.

Theorem 6. *For given matrices* $A_c(i)$, $B_c(i)$, *and* $C_c(i)$, *positive constants* $\overline{\tau}$, $\underline{\tau}$, *and quantization density values* $\rho_f(i)$ *and* $\rho_g(i)$, *if there exists a set of positive definite matrices* $E_{\alpha(k)}(i)$, $Z_{\alpha(k)}(i)$, Q', $P_0(i)$, $P_1(i)$, $W_0(i)$, $W_1(i)$, $Q_0(i)$, $Q_1(i)$, $T(i)$, $N(i)$, *and* $M(i)$ *of appropriate dimensions the following inequalities hold:*

$$min \quad tr\left(P_1\left(r_0\right)\right) \tag{23}$$

$$s.t. \quad \begin{bmatrix} E_{\alpha(i)}(i) & M(i) \\ * & Z_{\alpha(i)} \end{bmatrix} \geq 0 \tag{24}$$

$$\Lambda_0(i) + \Gamma_{10}^{\mathrm{T}}(i)\widetilde{P}_0(i)\Gamma_{10}(i) + \overline{\tau}Z_0(i) + \Gamma_2^{\mathrm{T}}(i)\overline{\tau}E_0(i)\Gamma_2(i) + \gamma(i) + \gamma^{\mathrm{T}}(i) + \sigma(i)I_1^{\mathrm{T}}C^{\mathrm{T}}CI_1 - I_2^{\mathrm{T}}I_2 < 0 \tag{25}$$

$$\Lambda_1(i) + \Gamma_{11}^{\mathrm{T}}(i)\widetilde{P}_1(i)\Gamma_{11}(i) + \overline{\tau}Z_1(i) + \Gamma_2^{\mathrm{T}}(i)\overline{\tau}E_1(i)\Gamma_2(i) + \gamma(i) + \gamma^{\mathrm{T}}(i) + I_2^{\mathrm{T}}I_2 - \sigma(i)I_1^{\mathrm{T}}C^{\mathrm{T}}CI_1 < 0 \tag{26}$$

$$\Gamma_{1\alpha(k)} = \left[\Phi_{1\alpha}(i) + \Phi_2(i) + \Phi_3(i) \quad \Phi_3(i) \quad \Phi_{4\alpha(k)}(i)\,\Phi_{4\alpha(k)}(i)\right]$$

$$\Gamma_2 = \left[\overline{A} \quad \overline{B} \quad 0 \quad 0\right]$$

$$\overline{A} = \left[A - I \quad BC_c(i) \quad 0\right]$$

$$I_1 = \left[\operatorname{diag}\{I, 0, 0\} \quad 0 \quad 0 \quad 0\right]$$

$$I_2 = \left[\operatorname{diag}\{0, 0, I\} \quad 0 \quad 0 \quad 0\right]$$

$$\gamma = M^{\mathrm{T}}\left[\operatorname{diag}\{I, 0, 0\} \quad 0 \quad \operatorname{diag}\{-I, 0, 0\} \quad 0\right]$$

$$\begin{aligned} \Lambda_0(i) &= \operatorname{diag}\left\{\left(\overline{\tau} - \underline{\tau} + 1\right)Q - P_0(i) - T(i) - N(i) + \Psi^{\mathrm{T}}(i)Q'\Psi(i), -W_0(i), \delta^2(i)W_1(i) - Q_0, -W_1(i)\right\} \\ \Lambda_1(i) &= \operatorname{diag}\left\{\left(\overline{\tau} - \underline{\tau} + 1\right)Q - P_1(i) - T(i) - N(i) + \Psi^{\mathrm{T}}(i)Q'\Psi(i), -W_2(i), \delta^2(i)W_3(i) - Q_1, -W_3(i)\right\} \end{aligned} \tag{27}$$

$$\widetilde{P}_0(i) = \sum_{j=1}^{s} p_{ij}P_0(i)(j)$$

$$\widetilde{P}_1(i) = \sum_{j=1}^{s} p_{ij}P_1(i)(j)$$

$$\Psi(i) = \begin{bmatrix} I & 0 & 0 \\ 0 & C_c(i) & 0 \end{bmatrix}$$

Then the closed-loop system (16) is stochastically stable with the dynamic feedback controller.

Proof. Based on **Lemma 4**, (16) can be represented as follows:

$$\widetilde{\eta}(k+1) = \Gamma_{1\alpha(k)}\left(r_k\right)\widetilde{\eta}(k) \tag{28}$$

where

$$\begin{aligned} \Gamma_{1\alpha(k)} = {} & \left[\Phi_{1\alpha}\left(r_k\right) + \Phi_2\left(r_k\right)\right. \\ & \left. + \Phi_3\left(r_k\right) \quad \Phi_3\left(r_k\right) \quad \Phi_{4\alpha(k)}\left(r_k\right)\Phi_{4\alpha(k)}\left(r_k\right)\right] \end{aligned} \tag{29}$$

Then select Lyapunov-Krasovskii functional as

$$\begin{aligned} V\left(k, r_k\right) = {} & V_1\left(k, r_k\right) + V_2\left(k, r_k\right) + V_3\left(k, r_k\right) \\ & + V_4\left(k, r_k\right) + V_5\left(k, r_k\right) \end{aligned} \tag{30}$$

where

$$\begin{aligned} V_1\left(k, r_k\right) &= \eta_k^{\mathrm{T}}P\left(r_k\right)\eta_k, \\ V_2\left(k, r_k\right) &= \sum_{l=-\overline{\tau}}^{-1}\sum_{j=k+l}^{k-1}\overline{x}_j^{\mathrm{T}}E\left(r_k\right)\overline{x}_j, \\ V_3\left(k, r_k\right) &= \eta_k^{\mathrm{T}}T\left(r_k\right)\eta_k, \\ V_4\left(k, r_k\right) &= \sum_{l=k-\tau_k}^{k-1}\eta_k^{\mathrm{T}}Q\left(r_k\right)\eta_k \\ &\quad + \sum_{l=-\overline{\tau}+2}^{-\underline{\tau}+1}\sum_{j=k+l-1}^{k-1}\eta_j^{\mathrm{T}}Q\left(r_k\right)\eta_j, \\ V_5\left(k, r_k\right) &= \eta_k^{\mathrm{T}}T\left(r_k\right)\eta_k, \end{aligned} \tag{31}$$

Let

$$\begin{aligned} E_{\alpha(k+1)}\left(r_{k+1}\right) &= E_0\left(r_k\right) \\ Q_{\alpha(k+1)}\left(r_{k+1}\right) &= Q_0\left(r_k\right) \end{aligned} \tag{32}$$

Then we can obtain the following equalities and inequalities:

$$\begin{aligned}
\Delta V_1(k,r_k) &= \tilde{\eta}_k^{\mathrm{T}}\Gamma_{1\alpha(k)}^{\mathrm{T}}\tilde{P}(r_k)\Gamma_{1\alpha(k)}\tilde{\eta}_k - \eta_k^{\mathrm{T}}P(r_k)\eta_k,\\
\Delta V_2(k,r_k) &= \overline{x}_k^{\mathrm{T}}\overline{\tau}E_0(r_k)\overline{x}_k - \sum_{l=k-\overline{\tau}}^{k-1}\overline{x}_k^{\mathrm{T}}E_0(r_k)\overline{x}_k\\
&\le \tilde{\eta}(k)\left[\Gamma_2^{\mathrm{T}}(r_k)\overline{\tau}E_0(r_k)\Gamma_2(r_k)+\gamma(r_k)+\gamma^{\mathrm{T}}(r_k)\right.\\
&\quad\left.+\overline{\tau}Z(r_k)\right]\tilde{\eta},\\
\Delta V_3(k,r_k) &= \tilde{\eta}_k^{\mathrm{T}}\Gamma_{1\alpha(k)}^{\mathrm{T}}(r_k)T_0(r_k)\Gamma_{1\alpha(k)}\tilde{\eta}(k)\\
&\quad-\eta_k^{\mathrm{T}}T_0(r_k)\eta_k,\\
\Delta V_4(k,r_k) &\le (\overline{\tau}-\underline{\tau}+1)\eta_k^{\mathrm{T}}Q_0(r_k)\eta_k - \eta_{k-\tau_k}^{\mathrm{T}}Q_0(r_k)\\
&\quad\cdot\eta_{k-\tau_k},\\
\Delta V_5(k,r_k) &= \tilde{\eta}_k^{\mathrm{T}}\Gamma_{1\alpha(k)}^{\mathrm{T}}(r_k)N_0(r_k)\Gamma_{1\alpha(k)}\tilde{\eta}(k)\\
&\quad-\eta_k^{\mathrm{T}}N_0(r_k)\eta_k,\\
\Delta V(k,r_k)&\\
&\le \tilde{\eta}_k^{\mathrm{T}}\left[\Gamma_{1\alpha(k)}^{\mathrm{T}}\left(\tilde{P}(r_k)+T_0(r_k)+N_0(r_k)\right)\Gamma_{1\alpha(k)}\right.\\
&\quad+\Gamma_2^{\mathrm{T}}(r_k)\overline{\tau}E_0(r_k)\Gamma_2(r_k)+\gamma(r_k)+\gamma^{\mathrm{T}}(r_k)\\
&\quad\left.+\overline{\tau}Z(r_k)\right]\tilde{\eta}_k+\eta_k^{\mathrm{T}}\left((\overline{\tau}-\underline{\tau}+1)Q_0(r_k)-P(r_k)\right.\\
&\quad\left.-T_0(r_k)-N_0(r_k)\right)\eta_k-\eta_{k-\tau_k}^{\mathrm{T}}Q_0(r_k)\eta_{k-\tau_k}
\end{aligned}\tag{33}$$

Let

$$\begin{aligned}
J' &= \begin{pmatrix}x(k)\\u(k)\end{pmatrix}^{\mathrm{T}}Q'x\begin{pmatrix}x(k)\\u(k)\end{pmatrix}\\
&=\eta_k^{\mathrm{T}}\Psi^{\mathrm{T}}(r_k)Q'\Psi(r_k)\eta_k
\end{aligned}\tag{34}$$

according to (6), we know $\alpha(k)=0$ means that $e^{\mathrm{T}}(k)e(k)<\sigma(k)y^{\mathrm{T}}(k)y(k)$, then, adding $\eta_k^{\mathrm{T}}\Delta_g^{\mathrm{T}}W_0(r_k)\Delta_g\eta_k$ and subtracting $\eta_{k-\tau_k}^{\mathrm{T}}\Delta_f^{\mathrm{T}}W_1(r_k)\Delta_f\eta_{k-\tau_k}$ to (33),

$$\begin{aligned}
\Delta V(k,r_k)+J' &\le \tilde{\eta}_k^{\mathrm{T}}\left[\Lambda_0(i)\right.\\
&\quad+\Gamma_{10}^{\mathrm{T}}(i)\left(\tilde{P}(r_k)+T_0(r_k)+N_0(r_k)\right)\Gamma_{10}(i)\\
&\quad\left.+\Gamma_2^{\mathrm{T}}(i)\overline{\tau}E\Gamma_2(i)+\gamma(i)+\gamma^{\mathrm{T}}(i)+\overline{\tau}Z_0(i)\right]\tilde{\eta}_k
\end{aligned}\tag{35}$$

and, according to (25), $\Delta V(k,r_k)+J'\le 0$. When $\alpha(k)=1$, namely, (2) is satisfied, then, adding $\eta_k^{\mathrm{T}}\Delta_g^{\mathrm{T}}W_2(r_k)\Delta_g\eta_k$ and subtracting $\eta_{k-\tau_k}^{\mathrm{T}}\Delta_f^{\mathrm{T}}W_3(r_k)\Delta_f\eta_{k-\tau_k}$ to (35), in a similar way, $\Delta V(k,r_k)+J'\le 0$ is obtained, and then

$$\Delta V(k,r_k)\le -J' = -\begin{pmatrix}x(k)\\u(k)\end{pmatrix}^{\mathrm{T}}Q'x\begin{pmatrix}x(k)\\u(k)\end{pmatrix}\tag{36}$$

Then take expectation on both sides of it, and we obtain

$$\begin{aligned}
&E\{V(\infty,r_\infty)\}-E\{V(0,r_0)\}\le-\beta_0E\left\{\sum_{k=0}^{\infty}\eta_k^{\mathrm{T}}\eta_k\right\}\\
&E\left\{\sum_{k=0}^{\infty}\eta_k^{\mathrm{T}}\eta_k\right\}<\frac{1}{\beta_0}E\{V(0,r_k)\}-\frac{1}{\beta_0}E\{V(\infty,r_\infty)\}\\
&\quad<\beta
\end{aligned}\tag{37}$$

where $\beta_0=\inf\{\lambda_{min}(-\Psi^T(r_k)Q'\Psi(r_k))\}$. It can be concluded from **Definition 5** that the system is stochastically stable, and the proof is completed. □

Theorem 7. *For given matrices $A_c(i)$, $B_c(i)$, and $C_c(i)$, positive constants $\overline{\tau}$, $\underline{\tau}$, and quantization density values $\rho_f(i)$ and $\rho_g(i)$, if there exists a set of positive definite matrices $E_{\alpha(k)}(i)$, $Z_{\alpha(k)}(i)$, $\overline{Q}$, $P_0(i)$, $P_1(i)$, $W_0(i)$, $W_1(i)$, $Q_0(i)$, $Q_1(i)$, $T(i)$, $N(i)$, and $M(i)$ of appropriate dimensions and constant $0<\mu<1$ the following inequalities hold:*

$$min \quad tr(P_1(r_0))\tag{38}$$

$$s.t.\quad\begin{bmatrix}E_{\alpha(i)}(i) & M(i)\\ * & Z_{\alpha(i)}\end{bmatrix}\ge 0\tag{39}$$

$$\Pi_0=\begin{bmatrix}\Pi_{10} & \Gamma_{10}^{\mathrm{T}}(i) & \Gamma_2^{\mathrm{T}}(i) & \Psi^{\mathrm{T}}(i)\\ * & \overline{P_0} & 0 & 0\\ * & * & \overline{E_0} & 0\\ * & * & * & \widetilde{Q}\end{bmatrix}<0\tag{40}$$

$$\Pi_1=\begin{bmatrix}\Pi_{11} & \Gamma_{11}^{\mathrm{T}}(i) & \Gamma_2^{\mathrm{T}}(i) & \Psi^{\mathrm{T}}(i)\\ * & \overline{P_1} & 0 & 0\\ * & * & \overline{E_1} & 0\\ * & * & * & \widetilde{Q}\end{bmatrix}<0\tag{41}$$

where

$$\begin{aligned}
\Pi_{10}&=\Theta_0(i)+\gamma(i)+\gamma^{\mathrm{T}}(i)+\overline{\tau}Z_0(i)+\mu I_1^{\mathrm{T}}C^{\mathrm{T}}CI_1\\
&\quad-I_2^{\mathrm{T}}I_2,\\
\Pi_{11}&=\Theta_1(i)+\gamma(i)+\gamma^{\mathrm{T}}(i)+\overline{\tau}Z_1(i)+I_2^{\mathrm{T}}I_2\\
&\quad-\mu I_1^{\mathrm{T}}C^{\mathrm{T}}CI_1,\\
\Theta_0(i)&=\operatorname{diag}\left\{(\overline{\tau}-\underline{\tau}+1)Q-P_0(i)-T(i)-N(i),\right.\\
&\quad\left.-W_0(i),\delta^2(i)W_1(i)-Q_0,-W_1(i)\right\}\\
\Theta_1(i)&=\operatorname{diag}\left\{(\overline{\tau}-\underline{\tau}+1)Q-P_1(i)-T(i)-N(i),\right.\\
&\quad\left.-W_2(i),\delta^2(i)W_3(i)-Q_1,-W_3(i)\right\}
\end{aligned}$$

$$\Gamma_{1\alpha(k)} = [\Phi_{1\alpha}(i) + \Phi_2(i) + \Phi_3(i) \;\; \Phi_3(i) \;\; \Phi_{4\alpha(k)}(i) \;\; \Phi_{4\alpha(k)}(i)]$$
$$\Gamma_2 = [\overline{A} \;\; \overline{B} \;\; 0 \;\; 0]$$
$$\overline{A} = [A - I \;\; BC_c(i) \;\; 0]$$
$$I_1 = [\operatorname{diag}\{I, 0, 0\} \;\; 0 \;\; 0 \;\; 0]$$
$$I_2 = [\operatorname{diag}\{0, 0, I\} \;\; 0 \;\; 0 \;\; 0]$$
$$\gamma = M^{\mathrm{T}} [\operatorname{diag}\{I, 0, 0\} \;\; 0 \;\; \operatorname{diag}\{-I, 0, 0\} \;\; 0]$$
$$\overline{P}_0(i) = -\left(\sum_{j=1}^{s} p_{ij} P_0(i)(j)\right)^{-1}$$
$$\overline{P}_1(i) = -\left(\sum_{j=1}^{s} p_{ij} P_1(i)(j)\right)^{-1}$$
$$\overline{E}_0(i) = -(\overline{\tau} E_0(i))^{-1}$$
$$\overline{E}_1(i) = -(\overline{\tau} E_1(i))^{-1}$$
$$\Psi(i) = \begin{bmatrix} I & 0 & 0 \\ 0 & C_c(i) & 0 \end{bmatrix}$$
$$\widetilde{Q} = (-Q')^{-1} \tag{42}$$

Proof. $\widetilde{P}_0(i) = \sum_{j=1}^{s} p_{ij} P_0(j)$ and $\widetilde{P}_1(i) = \sum_{j=1}^{s} p_{ij} P_1(j)$ have been proposed in **Theorem 6**, let $\mu = \sigma$, and then the Schur complement lemma to (25) and (40) is obtained. In a similar way, (41) can be obtained from (26), and then the proof is completed. □

4. Simulation

Consider a system as follows:

$$x(k+1) = \begin{bmatrix} 0 & 2 \\ -2 & -3 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} u(k)$$
$$y(k) = [0 \;\; 1] \, x(k) \tag{43}$$

The time delay is bounded as $0.01 \le \tau_k \le 0.09$ and modeled by a Markov chain $\omega = \{1, 2\}$, which is corresponded to mode 1 and mode 2, assume that $\delta_f = \delta_g = 0.023$, $x(0) = [1 \;\; 0]^T$, and the quantization density of each mode is defined as $\delta(1) = 0.02$ and $\delta(2) = 0.03$, $\mu = 0.0195$ is chosen as threshold, and the transition probability matrix is as follows:

$$p = \begin{bmatrix} 0.42 & 0.58 \\ 0.46 & 0.54 \end{bmatrix} \tag{44}$$

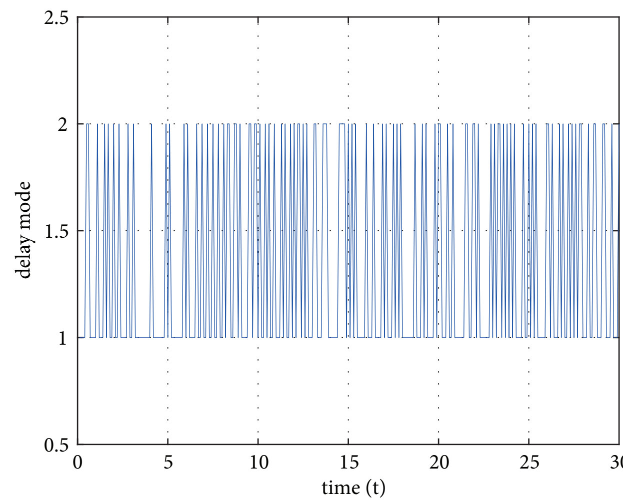

Figure 2: Group 1: the mode of time delay.

The linear matrix inequality is utilized and the matrices of controller are obtained as follows:

$$A_c(1) = \begin{bmatrix} -1.600 & 0.8703 \\ -0.6204 & -1.5032 \end{bmatrix}$$
$$A_c(2) = \begin{bmatrix} -1.4306 & 0.3010 \\ 1.3071 & 0 \end{bmatrix}$$
$$B_c(1) = [0.0012 \;\; 0.1203]^{\mathrm{T}} \tag{45}$$
$$B_2(2) = [-0.0210 \;\; -0.0013]^{\mathrm{T}}$$
$$C_c(1) = [-3.2122 \;\; -0.0013]$$
$$C_c(2) = [-3.5421 \;\; -0.0009]$$

and we take two groups parameters to make a comparison:

Group 1. $\sigma_1 = 0.01$; $\sigma_2 = 0.023$.

Group 2. $\sigma_1 = 0.01$; $\sigma_2 = 0.0195$.

Remark 8. Figures 2 and 5 show the switching of two modes of the time delay in a Markov chain, and the switching is determined by the transition probability. Figures 3 and 6 show the curves of the state response which eventually tend to be stable. The event-triggered release instants and intervals are shown in Figures 4 and 7, and the numbers of triggers in two figures are 79 and 73, which are much less than the number of the traditional sampling, which means that the network resources are greatly saved.

Remark 9. In literature [10], the structure of its system based on event-triggered scheme contains one quantizer, and the event-triggered release instants and interval are 74 and 84 while the sampling time is 10s. Comparing with [10], the method proposed in this article can save the network resources better.

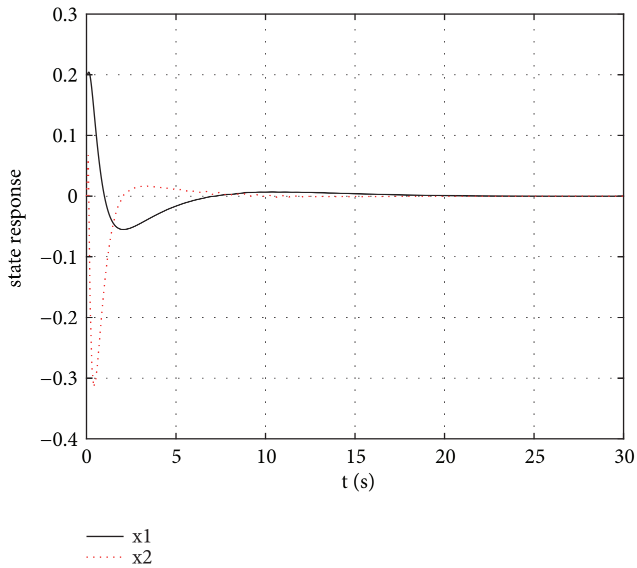

Figure 3: Group 1: the response of state.

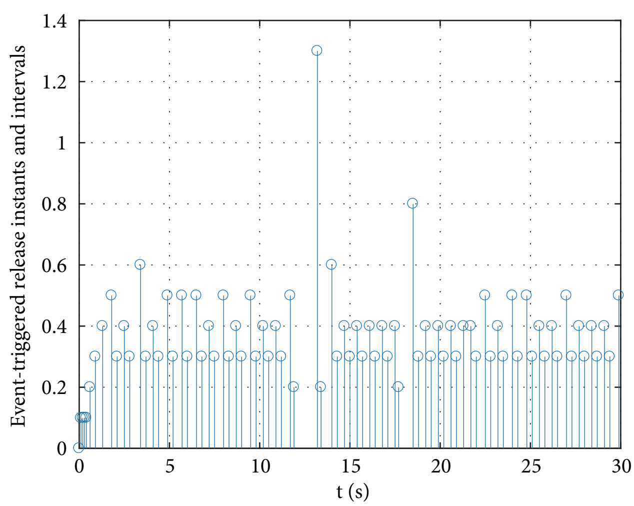

Figure 4: Group 1: the event-triggered release instants and intervals.

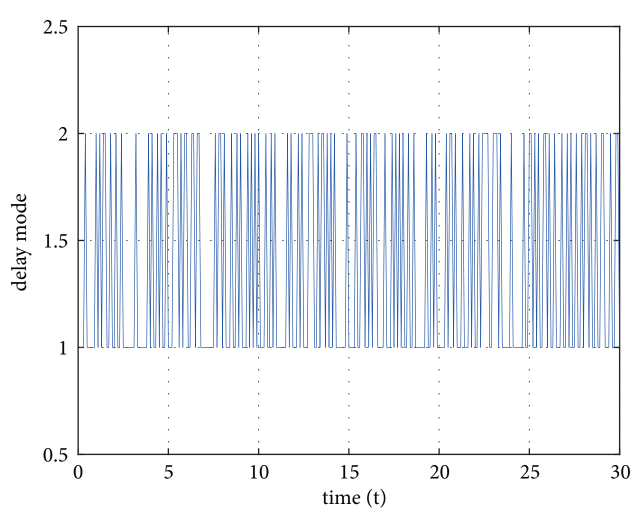

Figure 5: Group 2: the mode of time delay.

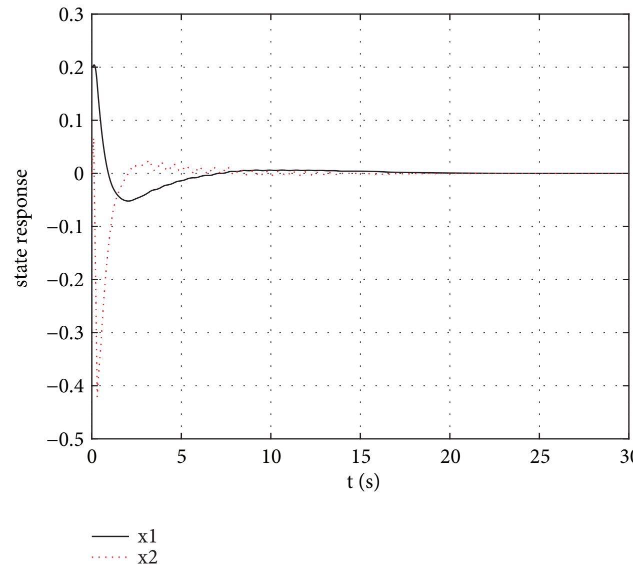

Figure 6: Group 2: the response of state.

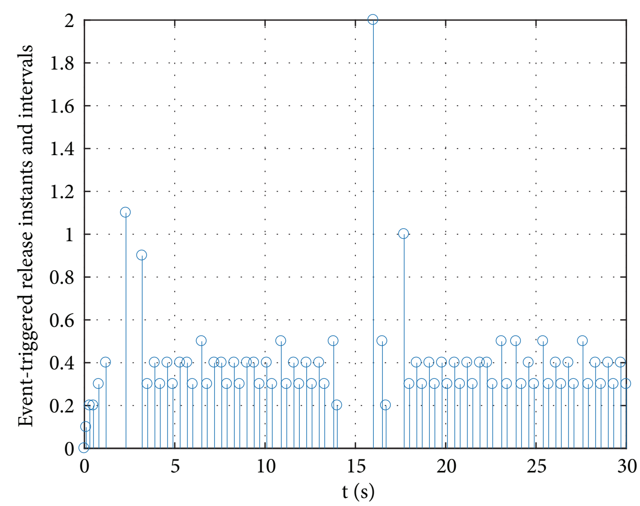

Figure 7: Group 2: the event-triggered release instants and intervals.

5. Conclusion

A co-design of event-triggered scheme and dynamic output feedback controller for a class of NCSs with random time delay is presented. Two mode-dependent quantizers are set to balance the network congestion and quantization error. A Markov process is used to model the time delay which is used to describe the quantization density as a function. Lyapunov-Krasovskii functional and LMI are used to derive the stability sufficient conditions. A numerical example is given to illustrate the proposed approach.

Appendix

Proof of Lemma 4 . From (20), we learn that

$$\sum_{i=k-\overline{\tau}}^{k-1} \begin{bmatrix} \overline{x}(i) \\ \widetilde{\eta}(i) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} E & M \\ M^{\mathrm{T}} & Z \end{bmatrix} \begin{bmatrix} \overline{x}(i) \\ \widetilde{\eta}(i) \end{bmatrix} \geq 0 \qquad \text{(A.1)}$$

Then

$$\sum_{i=k-\overline{\tau}}^{k-1}\left[\overline{x}^{\mathrm{T}}(i)E\overline{x}(i)+\widetilde{\eta}^{\mathrm{T}}(k)M^{\mathrm{T}}\overline{x}(i)+\overline{x}^{\mathrm{T}}(i)M\widetilde{\eta}(k)+\widetilde{\eta}^{\mathrm{T}}(k)Z\widetilde{\eta}(k)\right]\geq 0,$$

$$\sum_{i=k-\overline{\tau}}^{k-1}\widetilde{\eta}^{\mathrm{T}}(k)M^{\mathrm{T}}\overline{x}(i)+\sum_{i=k-\overline{\tau}}^{k-1}\overline{x}^{\mathrm{T}}(i)M\widetilde{\eta}(k)+\sum_{i=k-\overline{\tau}}^{k-1}\widetilde{\eta}^{\mathrm{T}}(k)Z\widetilde{\eta}(k)\geq-\sum_{i=k-\overline{\tau}}^{k-1}\overline{x}^{\mathrm{T}}(i)E\overline{x}(i),\quad (A.2)$$

$$\widetilde{\eta}^{\mathrm{T}}(k)M^{\mathrm{T}}\{x(k)-x(k-\overline{\tau})\}+\{x(k)-x(k-\overline{\tau})\}^{\mathrm{T}}\cdot M\widetilde{\eta}(k)+\overline{\tau}\widetilde{\eta}^{\mathrm{T}}(k)Z\widetilde{\eta}(k)\geq-\sum_{i=k-\overline{\tau}}^{k-1}\overline{x}^{\mathrm{T}}(i)E\overline{x}(i),$$

Let

$$x(k)-x(k-\overline{\tau})=\left[diag\{I,0,0\}\ \ 0\ \ diag\{-I,0,0\}\ \ 0\right]\widetilde{\eta}(k)\quad (A.3)$$

$$\gamma=M^{\mathrm{T}}\left[diag\{I,0,0\}\ \ 0\ \ diag\{-I,0,0\}\ \ 0\right]$$

Then (21) is obtained. □

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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