Research Article

Backstepping Control with Disturbance Observer for Permanent Magnet Synchronous Motor

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For the speed tracking control problem of Permanent Magnet Synchronous Motor (PMSM), a disturbance observer-based (DOB) backstepping speed tracking control method is presented in this paper. First, to reconstruct the load disturbance, a slowly time-varying disturbance observer for PMSM is constructed. Next, by stability theory, the norm of the observation error and disturbance observer design method are obtained. On the basis of these, through the designing of the virtual control input that include the reconstruction disturbances, and using backstepping control strategy, the DOB controller of the closed-loop system is proposed. The obtained controller can achieve high precision speed tracking and disturbance rejection. Finally, some results of evaluative experiments verified the effectiveness of the proposed method for high-performance control and disturbance rejection for the PMSM drive.

1. Introduction

Permanent magnet synchronous motor (PMSM) is receiving increasing attention in high-performance industrial servo applications due to its advantages such as high torque to current ratio, super power density, and fast response, as well as low noise [1, 2]. However, PMSM is a typical high nonlinear, multivariable coupled system. It is sensitive to load disturbance, unmodeled dynamics, nonlinear uncertainties, and parameter uncertainties.

Over the last decades, various design methods have been developed for PMSM control systems, such as adaptive control [3], robust control [4], neural network control [5], predictive control [6], and so on. Recently, to improve the control performances in PMSM systems, much attention has been given to disturbance reject-based control method [7], which are insensitive to parameter variations and external disturbances. In [8, 9], the fractional order sliding-mode control (SMC) scheme has been proposed. The proposed control system not only obtained high control performance but also was robust against to external load disturbance and parameter variations.

To further improve the disturbance rejection performance of SMC, extended sliding-mode disturbance observer was proposed in [10, 11], and the estimated system disturbance is considered as the feed forward compensation part to compensate sliding-mode speed controller. In [12], an improved SMC scheme to solve time-varying parameters and disturbances for PMSM drive system was proposed. The new speed controller was designed by the nonsingular terminal SMC strategy with the disturbance observer. In [13], a new speed controller is designed by the nonsingular terminal SMC strategy with disturbance observer. The controller can make the motor speed reach the reference value in finite time, accompanied with a faster convergence and a better tracking precision. It is worth noting that one obvious disadvantage of SMC method is the chattering phenomenon, which is caused by discontinuous control law and frequent switching action near sliding surface. Besides, as the upper bound of lumped disturbances is not easy to be determined in advance, which could cause a large amount of chattering of SMC strategy in PMSM system.

Recently, disturbance observer-based (DOB) [14] control methods have been applied to PMSM system for better
robustness against system disturbance. In [15], a DOB state feedback controller was designed for PMSM system. By using the same disturbance observer, a sensorless control method for PMSM drive was developed in [16]. The proposed DOB method involved the use of a back electromotive force observer and a torque observer to estimate rotor position and compensate for load torque disturbance, respectively. For the mismatched disturbance, in [17], a DOB integral sliding-mode control approach for linear systems with mismatched disturbances was presented. The disturbance observer is proposed to generate the disturbance estimate, which can be incorporated in the controller to counteract the disturbance.

Backstepping is a well-known recursive and systematic design methodology for the feedback control of uncertain nonlinear system with matched uncertainties [18]. The key point is to use the virtual control variable to make the original high-order system simple; thus the final controller can be derived through Lyapunov stable theorem. In [19], an adaptive backstepping speed controller was proposed for the speed control of PMSM. The controller is robust against stator resistance, viscous friction uncertainties, and load torque disturbance. However, this approach uses the feedback linearization. By means of a nonlinear and adaptive backstepping design method, a speed and current control scheme for PMSM was presented in [20], in which all the parameters in both PMSM and load dynamics were considered unknown. In [21], a new nonlinear and full adaptive backstepping speed tracking control scheme was developed for an uncertain PMSM. Except for the number of pole pairs, all the other parameters in both PMSM and load dynamics were assumed unknown. Taking into account the unobservable of the systems states, a backstepping control method for speed sensorless PMSM based on slide model observer was proposed in [22], in which the slide model observer was designed by using slide model control technique and phase loop lock (PLL) method. In [23], a backstepping control algorithm based on disturbance observer was proposed. The minimum-order observer was established to observe the disturbance value of load inertia. Furthermore, the estimated disturbance value was used to identify the load inertia. Based on the identification results, the backstepping controller was designed. However, the above observer and controller are designed separately.

Motivated by the discussions above, in this paper, we mainly investigate backstepping speed control for PMSM based on disturbance observer. A nonlinear disturbance observer is first constructed to estimate the external slowly time-varying disturbance. Then, based on the backstepping control theory, the PMSM rotor speed and current tracking backstepping controllers are designed. Meanwhile, global asymptotic stability is guaranteed by Lyapunov stability analysis.

The rest of this paper is structured as follows. In Section 2, the mathematic model of PMSM and problem formulation are presented. The nonlinear disturbance observer design and stability analysis are derived in Section 3. In Section 4, the DOB backstepping controller design method is obtained. Section 5 presents the numerical simulation and experimental results. Finally, some conclusions are drawn in Section 6.

2. Mathematic Model of PMSM and Problem Formulation

Assume that magnetic circuit is unsaturated and hysteresis and eddy current loss are ignored. With above standard assumptions, the mathematical model of a conventional surface mounted PMSM with mismatched external disturbances can be given in the $d-q$ frame as follows [20, 22]:

\[
\begin{align*}
    \frac{d\omega}{dt} &= \frac{3p\phi_f}{2J}i_q - \frac{B}{J}\omega - \frac{T_r}{J}, \\
    \frac{di_q}{dt} &= -\frac{R}{L}i_q - \frac{p\phi_f}{L}\omega + \frac{1}{L}u_q + \frac{1}{L}d_1, \\
    \frac{di_d}{dt} &= -\frac{R}{L}i_d + p\omega i_q + \frac{1}{L}u_d + \frac{1}{L}d_2,
\end{align*}
\]

where $\omega$ is the rotor speed, $i_d$ and $i_q$ are the $d-q$ axis currents, $u_d$ and $u_q$ are the $d-q$ axis voltages, and $d_i$ ($i = 1, 2$) are external disturbances. $R$ and $L$ denote the stator resistance and inductance per phase, respectively, $p$ is the number of pole pairs, $\phi_f$ is the permanent magnet flux, $J$ is the rotor moment of inertia, $B$ is the viscous friction factor, and $T_r$ also represents the applied load torque disturbance.

To formulate the design problem, according to system (1), the state space model of the PMSM can be rewritten as the following nonlinear system:

\[
\begin{align*}
    \dot{x}(t) &= Ax(t) + f(x(t)) + Bu(t) + \eta + Bd(t), \\
    y(t) &= Cx(t),
\end{align*}
\]

where $x(t) = [\omega \; i_q \; i_d]^T$, $u = [u_q \; u_d]^T$, and

\[
A = \begin{bmatrix}
    -\frac{B}{J} & \frac{3p\phi_f}{2J} & 0 \\
    -\frac{p\phi_f}{L} & -\frac{R}{L} & 0 \\
    0 & 0 & -\frac{R}{L}
\end{bmatrix},
B = \begin{bmatrix}
    0 & 0 \\
    \frac{1}{L} & 0 \\
    0 & \frac{1}{L}
\end{bmatrix},
C = [1, 0, 0],
\]

\[
f(x(t)) = \begin{bmatrix}
    0 \\
    -p\omega i_d \\
    -p\omega i_d
\end{bmatrix},
\]

\[
\eta = \begin{bmatrix}
    T_r \\
    0 \\
    0
\end{bmatrix}.
\]
The external disturbance \(d(t)\) is assumed that
\[
d(t) \leq \alpha, \quad \dot{d}(t) = 0,
\]
where \(\alpha\) is a positive constant.

**Assumption 2.** For all \(x_1, x_2 \in \Omega\), the nonlinear term \(f(x(t))\) in system (2) satisfies
\[
\|f(x_1) - f(x_2)\| \leq \gamma \|x_1 - x_2\|,
\]
where \(\gamma\) is Lipschitz constant.

The main control objective is to design a DOB backstepping controller to keep all the signals in the closed-loop system bounded and ensures global asymptotic convergence of the desired speed and current tracking errors to zero eventually.

### 3. Nonlinear Disturbance Observer and Stability Analysis

Motivated by the disturbance observers in [14–16], the following disturbance observer for system (2) can be employed:

\[
\dot{V}(t) = \Lambda \ddot{d}(t) + \Lambda B^\top \dot{A} x(t) + f(x(t)) + Bu(t) + \eta,
\]
where \(B^\top = (B^\top B)^{-1}B^\top\), \(\ddot{d}(t)\) is the estimation of \(\ddot{d}(t)\), \(v(t)\) is the internal variable vector of the observer, and \(\Lambda\) is a Hurwitz matrix chosen by designer. For simplicity, we assume \(\ddot{d}(0) = 0\). Define the disturbance estimation error \(\dot{\ddot{d}}(t)\) as
\[
\dot{\ddot{d}}(t) = \ddot{d}(t) - \ddot{d}(t).
\]

To show that the disturbance can be observed asymptotically by observer (5), we first give the following lemma.

**Lemma 3.** For nonlinear system (2), suppose that the disturbance observer holds form (5); then the disturbance can be observed asymptotically.

**Proof.** It can be obtained that
\[
\dot{v}(t) = \Lambda \ddot{d}(t) + \Lambda B^\top \dot{A} x(t) - \ddot{d}(t),
\]
which implies that \(\ddot{d}(t) = -\Lambda \dot{d}(t)\). Therefore, we have
\[
\dot{\ddot{d}}(t) = \Lambda \ddot{d}(t).
\]

Define a monochromatic Lyapunov function as
\[
V_1(t) = \frac{1}{2} \ddot{d}^T(t) \ddot{d}(t).
\]
Clearly,
\[
\dot{V}_1(t) \leq \ddot{d}^T(t) (\Lambda + \Lambda^T) \ddot{d}(t).
\]
Since \(\Lambda\) is Hurwitz, then
\[
\dot{V}_1(t) \leq 2\lambda_{\text{max}}(\Lambda) \|\ddot{d}(t)\|^2 = -\beta \|\ddot{d}(t)\|^2, \quad \beta > 0,
\]
which implies the error dynamics (8) is asymptotically stable. This completes the proof.

**Remark 4.** It follows from Lemma 3 that parameter \(\Lambda\) in the disturbance observer (5) can change the rate of convergence of the observer and should be selected as large enough.

### 4. DOB Backstepping Controller Design

In the backstepping procedure, a virtual control state is firstly defined and then it is forced to become a stabilizing function. By defining a corresponding error variable and using Lyapunov stability theory, the related control input can be obtained to stabilize the error dynamics [21]. In this paper, the overall control design can be established by three steps in the following order.

**Step 1.** In the first step of the backstepping control design, a fictitious control input for the rotor speed \(\omega\) has to be determined. Let \(\omega^*_r\) be the desired trajectory. Furthermore, the trajectory of \(\omega^*_r\) is sketched to be smooth and \(\dot{\omega}^*_r = 0\).

Define the speed tracking error \(e_\omega = \omega^*_r - \omega\). The speed tracking error dynamics can be obtained as
\[
e_\omega(t) = \dot{\omega}^*_r - \omega = \frac{1}{J} \left( B \omega + T_L - \frac{3p\phi_f}{2} i_q \right).
\]

The first positive definite Lyapunov function can be defined as
\[
V_1(t) = \frac{1}{2} e_\omega^2 + \dot{V}_1(t).
\]

Differentiating \(V_1(t)\) with respect to time and using the results of (11) and (12), the time derivative of \(V_1(t)\) is given as
\[
\dot{V}_1(t) = e_\omega \dot{e}_\omega + \dot{V}_1(t) \leq e_\omega \left( B \omega + T_L - \frac{3p\phi_f}{2} i_q \right) - \beta \|\ddot{d}(t)\|^2.
\]

In order to stabilize the speed tracking error dynamics, define the stabilizing virtual control input:
\[
i_q = \frac{2}{3p\phi_f} \left( B \omega + T_L + k_1 e_\omega \right),
\]
where \(k_1 > 0\) is a real constant. Therefore, this definition leads to
\[
\dot{V}_1(t) \leq -k_1 e_\omega^2 - \beta \|\ddot{d}(t)\|^2 \leq 0,
\]
which guarantees that the tracking error for rotor speed will converge asymptotically.
Step 2. In order to realize the complete decoupling and speed tracking of PMSM, the virtual input current can be chosen as

\[
i_q^* = \frac{2}{3} \phi_f (B \omega + T_L + k_1 e_{\omega}), \quad i_q = 0. \tag{17}
\]

Define the tracking error of \( q \) axis current as \( e_q = i_q^* - i_q \). The time derivative of \( e_q \) is

\[
\dot{e}_q = \frac{di_q}{dt} - \frac{di_q}{dt} = \frac{2}{3} \phi_f \left( B \frac{d\omega}{dt} + k_1 \frac{de_{\omega}}{dt} - k_1 \frac{de_{\omega}}{dt} \right)
\]

\[
= \frac{2}{3} \phi_f \left( \frac{3}{2} i_q - B \omega - T_L \right) + \frac{R i_q}{L} + p \omega i_d \tag{18}
\]

where \( \bar{d}_1 + \bar{d}_1 = d_1 \).

Choose the second Lyapunov function to stabilize \( q \) axis current tracking error dynamics as

\[
V_2(t) = V_1(t) + \frac{1}{2} e_q^2. \tag{19}
\]

By some mathematical manipulation, the time derivative of \( V_2(t) \) is given by

\[
\dot{V}_2(t) = V_1(t) + e_q \dot{e}_q \leq -k_2 e_q^2 + e_q \left( \frac{2}{3} \phi_f \left( \frac{3}{2} i_q - B \omega - T_L \right) + \frac{R i_q}{L} + p \omega i_d \right)
\]

\[
+ \frac{R i_q}{L} + p \omega i_d - \frac{u_q}{L} + \frac{p \phi_f}{L} \omega - \frac{\bar{d}_1}{L} - \frac{\bar{d}_1}{L} e_q - \beta \| \bar{d}(t) \|^2. \tag{20}
\]

Setting

\[
\frac{2}{3} \phi_f \left( \frac{3}{2} i_q - B \omega - T_L \right) + \frac{R i_q}{L} + p \omega i_d
\]

\[
- \frac{u_q}{L} + \frac{p \phi_f}{L} \omega - \frac{\bar{d}_1}{L} = -k_2 e_q^2
\]

where \( k_2 > 0 \) and using the generic inequality \( a b \leq \frac{1}{2} a^2 + \frac{1}{4} c^2 \) \((c > 0)\), it yields

\[
\dot{V}_2(t) \leq -k_1 e_{\omega}^2 - k_2 e_q^2 - \frac{\bar{d}_1}{L} e_q - \beta \| \bar{d}(t) \|^2
\]

\[
\leq -k_1 e_{\omega}^2 - k_2 e_q^2 + \left( \frac{1}{2} e_{\omega}^2 + \frac{1}{4} e_q^2 \right) \| \bar{d}_1 \| (t) \|^2
\]

\[
- \beta \| \bar{d}(t) \|^2 = -k_1 e_{\omega}^2 - C_1 e_q^2 - C_2 \| \bar{d}(t) \|^2, \tag{22}
\]

where

\[
C_1 = k_2 - \frac{e_1}{L}, \quad C_2 = \beta - \frac{1}{4L e_1}. \tag{23}
\]

If parameters \( k_2 \) and \( \beta \) are properly selected such that \( C_1 > 0 \) and \( C_2 > 0 \), then \( V_2(t) < 0 \), which indicates the tracking error for \( q \)-axis stator current will converge asymptotically to zero.

On the other hand, from (21), the stabilizing control law \( u_q \) can be designed to stabilize \( q \) axis current tracking error dynamics as follows:

\[
u_q^* = L \left( \frac{B}{J} + R \right) i_q - \frac{2B^2}{3 \phi_f} \omega + \frac{R i_q}{L} + p \omega i_d + \frac{2k_2^2 f}{3 \phi_f} e_{\omega} + k_2 e_q - \bar{d}_1 \tag{24}
\]

Step 3. As to design of command input for \( u_d \), define the tracking error

\[e_d = i_d^* - i_d \tag{25}\]

with \( \bar{d}_2 = 0 \) being the desired stator current of \( d \) axis. The time derivative of \( e_d \) is

\[
\dot{e}_d = \frac{di_d}{dt} - \frac{di_d}{dt} = \frac{R i_d}{L} + p \omega i_d - \frac{1}{L} u_d - \frac{\bar{d}_1}{L} - \frac{\bar{d}_1}{L} e_d + \dot{e}_d \tag{26}
\]

where \( \bar{d}_2 + \bar{d}_2 = d_2 \).

Choose the last Lyapunov function candidate as

\[
V_3(t) = V_2(t) + \frac{1}{2} e_d^2. \tag{27}
\]

which results in

\[
\dot{V}_3(t) = V_2(t) + e_d \dot{e}_d
\]

\[
\leq -k_1 e_{\omega}^2 - C_1 e_q^2 - C_2 \| \bar{d}(t) \|^2
\]

\[
+ e_d \left( \frac{R i_d}{L} + p \omega i_d - \frac{1}{L} u_d - \frac{\bar{d}_1}{L} - \frac{\bar{d}_1}{L} e_d + \dot{e}_d \right) \tag{28}
\]

If the stabilizing control law for \( u_d \) is defined as

\[
u_d = R i_d - p L \omega i_q + L k_3 e_d - \frac{\bar{d}_1}{L}, \quad k_3 > 0, \tag{29}
\]

then the following result is obtained:

\[
V_3(t) \leq -k_1 e_{\omega}^2 - C_1 e_q^2 - C_2 \| \bar{d}(t) \|^2 - k_3 e_d^2 + \frac{\bar{d}_2}{L} e_d \tag{30}
\]

Again, based on the generic inequality, inequality (30) can be rewritten as follows:

\[
V_3(t) \leq -k_1 e_{\omega}^2 - C_1 e_q^2 - C_2 \| \bar{d}(t) \|^2 - k_3 e_d^2 + \frac{\bar{d}_2}{L} e_d
\]

\[
+ \frac{1}{4L e_1} \| \bar{d}_1(t) \|^2 \tag{31}
\]

where

\[
C_1 = k_2 - \frac{e_1}{L}, \quad C_2 = \beta - \frac{1}{4L e_1}, \quad \bar{d}_1 = \frac{\bar{d}_1}{L} e_d
\]

\[
- \left( C_2 - \frac{1}{4L e_1} \right) \| \bar{d}(t) \|^2.
\]
If parameters $k_1, k_2, k_3$ and $\varepsilon_1, \varepsilon_2$ are properly chosen to make the following conditions hold:

$$k_1 > 0,$$

$$\left(k_2 - \frac{\varepsilon_1}{L}\right) = C_1 > 0,$$

$$\left(\beta - \frac{1}{4L\varepsilon_1}\right) = C_2 > 0,$$

$$\left(k_3 - \frac{\varepsilon_2}{L}\right) > 0,$$

$$\left(C_2 - \frac{1}{4L\varepsilon_2}\right) > 0,$$

then $V_3(t) < 0$. Consequently, the tracking error for $d$-axis stator current will tend to zero eventually. The objective of tracking control of PMSM is completed.

5. Numerical Simulation and Experimental Results

In this section, the numerical example and experimental results are presented to demonstrate the validity of the proposed method.

(1) Numerical Simulation Results. The motor parameters used are listed in Table 1. The MATLAB/Simulink model of the proposed DOB backstepping PMSM control system is shown in Figure 1. The initial rotation speed of the motor is 1200 r/min, and the rotation speed is 1000 r/min at 0.6 s.

The initial load torque of the motor is 0N-m and the load disturbance torque is 10N-m at 0.8 s. The parameters of the DOB-backstepping controller are selected as $\Lambda = -180I_2$, $\varepsilon_1 = \varepsilon_2 = 0.153$, $k_1 = 250$, $k_2 = 500$, $k_3 = 160$.

In this simulation, to illustrate the effectiveness of the proposed method, the phase traces by the conventional backstepping (BS) method [22] and the proposed DOB backstepping (DOB-BS) method are simulated and compared. The numerical simulation results are shown in Figures 2–4. Figure 2(a) indicates the actual rotation speed of the motor with BS method and the proposed DOB-BS method in the presence of the above disturbances load, respectively. Figure 2(b) shows the rotation speed response between $[0.008s, 0.012s]$, during which the PMSM motor just started. Figure 2(c) shows the rotation speed response between $[0.585s, 0.625s]$. In this time period, the reference rotation speed has a sudden change at $t = 0.6s$. Figure 2(d) shows the rotation speed response between $[0.585s, 0.625s]$. In this time period, the reference rotation speed has a sudden change at $t = 0.6s$. The reference rotation speed has a sudden change at $t = 0.6s$. From Figure 2, it can be seen that the rotation speed of the proposed DOB-BS method can rapidly track the reference rotation speed with smaller stability error, faster response, and smaller overshoot than that of the conventional BS method.

Figure 3 shows the three-phase current of the stator. The current amplitude is proportional to the rotation torque and changes rapidly as the load torque varies. The current frequency is inversely proportional to the rotation speed. Figure 4 shows the variation of electromagnetic torque as the load torque changes. Obviously, the DOB-BS method gives less load torque fluctuations.

<table>
<thead>
<tr>
<th>Table 1: The parameters of PMSM.</th>
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<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Pole Pairs $p$</td>
</tr>
<tr>
<td>Friction factor $B$ (N·m·s/rad)</td>
</tr>
<tr>
<td>Stator Inductance $L$ (H)</td>
</tr>
<tr>
<td>Rotor moment of inertia $J$ (kg·m²)</td>
</tr>
<tr>
<td>Permanent magnet flux $\phi_f$ ($wb$)</td>
</tr>
<tr>
<td>Stator Resistance $R$ (Ω)</td>
</tr>
</tbody>
</table>
(2) Experimental Results. To evaluate the performance of the proposed method, a three-phase PMSM control system is set up. The experimental platform configuration is shown in Figure 5. The main chip of the inverter adopts the TMS320F28335 digital signal processor (DSP). The initial rotation speed of the motor is 1600 r/min, and the rotation speed is 600 r/min at 0.6s. The initial load torque of the motor is 0N·m and the load disturbance torque is 5N·m at 0.8s.

The results of the experiment are shown in Figures 6–8. Figure 6(a) shows the speed response of the closed-loop system under the BS and DOB-BS schemes. It can be seen
that the settling time of the DOB-BS method is shorter
than that of the BS method. The experimental results of
antiloaddisturbance of the two control schemes are shown in
Figure 6(b). After the speed output of PMSM system (without
load) is in the steady state, a step disturbance load, i.e., a rated
torque of 10N.m at 0.8s, is added suddenly. Clearly, the DOB-
BS method has a less speed drop, i.e., a better disturbance
rejection capacity when the disturbance load is added.

The load torque and current responses of DOB-BS
method is shown in Figures 7 and 8, which illustrates the
quick response and low ripple.

From these results, it can be observed that the proposed
DOB-BS method is effective under different operating

## 6. Conclusion

In this paper, a disturbance observer-based (DOB) backstep-
ning speed tracking control method has been presented for
the speed tracking control PMSM. Through disturbance esti-
mation, the DOB backstepping control strategy can achieve
high precision speed tracking and disturbance rejection
performance. Both simulation and experimental results have
shown the effectiveness of the proposed method.

### Data Availability

The data used to support the findings of this study are
available from the corresponding author upon request.

### Conflicts of Interest

The authors declare that there are no conflicts of interest
regarding the publication of this paper. There have no
financial and personal relationships with other people or
organizations that will influence our work, there is no
professional or other personal interest of any nature or kind in
any product, service, and/or company that could be construct
as influencing the position presented in or the review of the
manuscript.

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mation Processing of Ministry of Education.

### References

pmsm drive with ekf estimation of speed and rotor position,”


