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Research Article

Robust High-Gain Observers Based Liquid Levels and Leakage Flow Rate Estimation

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The paper aims to solve the problem of liquid level and leakage flow rate estimations for a state coupled four-tank process, that is why an UIO is developed to simultaneously estimate the unmeasured state variables and the perturbations considered as unknown inputs. We have proposed a state repartition that allows putting the model of the quadruple tank system to the canonical form for which the design of the observer is more easier. The observation scheme that uses a combination of high-gain observers and sliding mode observers allows improving robustness in the state estimation quality and a perfect reconstruction of the disturbance waveforms.

1. Introduction

The unknown inputs (UI) estimation issue and its mathematical formulation have received considerable interest over the last two decades in several domains (secure communication [1], civil engineering [2], biomedical domain [3], chemistry [4], etc.). In fact, the estimation of the UI is required in many engineering applications and scientific studies, especially where the plants embody unknown disturbances, faults, parameters mismatch, *etc.* Through an unknown input observer (UIO) the aim to simultaneously estimate the unmeasured system state and the UI can be achieved. The methods used in the literature can be classified into two types: decoupling of the UI or the estimation of UI by extending the system state vector.

First alternatives mainly rely on decoupling the UI through nonlinear transformation [5, 6]. Consequently, strict conditions are imposed on the disturbance distribution matrix and the UI structures. The framework given in [7] discusses the UI estimation subject in different conditions. Such assumption is recently released in [8], where the authors propose a systematic design methodology for state observers for a large class of nonlinear systems with bounded exogenous inputs. Sliding mode observers- (SMO-) based UI estimation is investigated in several works [9, 10]. The chattering phenomena which represent the main hindrance of such approach have recently been overcome by the higher-order SMO [11]. The high-gain observer- (HGO-) based approaches have been also successfully used in the conjoint estimation of state variables and UI or faults. In [12], under some global Lipschitz assumptions, a cascade HGO for a large class of nonlinear MIMO systems is designed in such a way that each subobserver provides an estimation of one component of the UI vector except the last one which achieves a reconstruction of the whole state variables.

For the second alternative, the UIs are considered as a part of the system state under the condition that their variations are relatively slow with respect to the state dynamics (constant, time polynomial, etc.) [10]. The corresponding observers are then constructed in such a way to estimate both the state vector and the UIs. Furthermore, in [13, 14], the authors also estimate the states and disturbance using an extended state observer (ESO). Some authors have sought to extend conventional observation algorithms such as Lunberger and EKF, so that the state vector includes the UI ([2, 15, 16]). Others, however, transform the UI identification problem into a constrained optimization problem which can be easily solved by adopting the linear matrix inequalities (LMIs) formalism [17, 18].

To add robustness to the quality of estimation, a combination between the HGO and the SMO algorithms is performed in some recent works. In [19] auxiliary outputs are generated using high-gain approximate differentiators and then used in the design of SMO without the match requirement for linear MIMO systems. For SISO nonlinear Lipschitz systems with nonmatching uncertainty, a hybrid observer structure that combines a HGO with higher-order sliding mode term is proposed in [20]. An extension of the HGO by a sliding mode term that follows the disturbance vector for affine input nonlinear MIMO class is investigated in [21]. Conjoint estimation of state and UI is required in many industrial applications, namely, in the processes where liquid level control intervenes, e.g., in food processing, water treatment systems, breeding, and pharmaceutical and petrochemical industries. The simultaneous observation of the liquid levels and the leakage flow rates is a real problem in these industrial processes where liquids are pumped, stored in tanks, and then pumped to other tanks. Besides the intrinsic nonlinearities shown in the corresponding models and the strong coupling of its states, the presence of UI such as valves perturbations or unknown flow rates present more challenges whether in the control or in diagnostic objectives.

This paper aims to solve the problem of state estimation of a quadruple tank system in presence of the UI (Figure 1). A HGO is used together with the SMO so as to improve the quality of the liquid levels estimation and reconstruct the leakage flow rates of the underlying system. A change of the state coordinates is used for transforming the original system into the canonical observable form. We introduce multiple sliding modes to handle the disturbance inputs. Under a structural assumption for the disturbance distribution matrix, the multiple SMO is designed in order to guarantee the complete observability of the system with respect to the UI. The proposed method relies only on the output estimation error in the sliding surface. The paper is organized as follows: the next section introduces the quadruple tank system. Section 3 gives some preliminaries on the nonlinear systems class of study and the state transformation. The main results on the design and analysis of a robust nonlinear observer that combines the HGO and SMO are presented in Section 4. Section 5 is devoted to the simulation results with a comparison between the Robust High-Gain Observer (R-HGO), our proposed method, and the Extended Kalman Filter (EKF) algorithm. Finally, a conclusion and perspectives are drawn.

2. Quadruple Tank Process Modeling

The so-called quadruple tank system, introduced in [22], has recently attracted much attention as it exhibits characteristics of interest in both control research and education [23, 24]. In our case, the quadruple tank process, shown in Figure 1, is a slightly modified version compared to the design given in [22]. This system consists of a liquid basin, two pumps, four tanks having the same area with orifices, and level sensors at the bottom of each tank. In this experimental setup, Pump 1 and Pump 2 provide, respectively, in-feed to tanks 3 and 4 and the outflows of tank 3 and tank 4 become in-feed to tank 1 and tank 2 as shown in Figure 1. The outflows of tank 1 and tank 2 are emptied into the liquid basin. The dynamic equations for the liquid level in the four tanks issued from the Bernoulli's law are as follows:

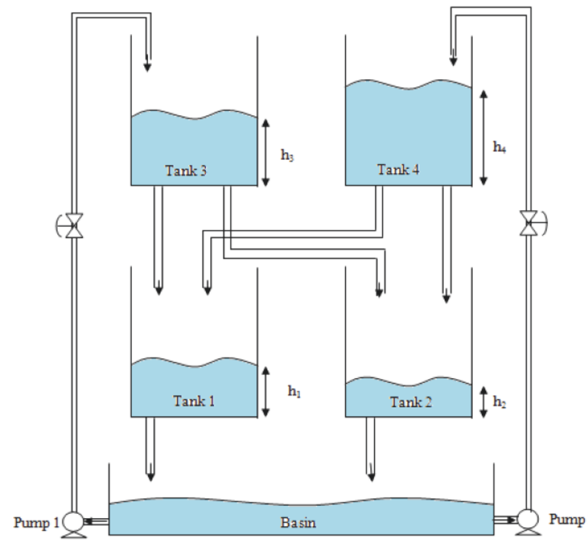


Figure 1: The quadruple tank system.

$$\dot{h}_i(t) = \frac{1}{S_i}\left(Q_i^{in}(t) - Q_i^{out}(t)\right), \quad for\ i = 1,\ldots,4, \tag{1}$$

where $h_i(t), S_i, Q_i^{in}(t)$ *and* $Q_i^{out}(t)$ are respectively the liquid level, the cross-sectional area, the inflow rate, and the outflow rate, for the i^{th} tank. Note that both pumps are identical. So, the inflow rates into the two top tanks 3 and 4 are given by

$$\begin{aligned} Q_3^{in}(t) &= K_p u_1(t) \\ Q_4^{in}(t) &= K_p u_2(t) \end{aligned} \tag{2}$$

where K_p is the pump's constant $(cm^3 s^{-1}/V)$. The outflow rate from the orifice at the bottom of each tank is

$$V_i^{out}(t) = \sqrt{2gh_i(t)}, \quad for\ i = 1,\ldots,4. \tag{3}$$

Then, the outflow rate for each top tank is given by

$$Q_i^{out}(t) = (s_{i1} + s_{i2})\sqrt{2gh_i(t)}, \quad i = 3,4 \tag{4}$$

where g is the gravitational acceleration and s_{ij} denotes the cross-sectional areas of the outflow orifice at the bottom of the i^{th} tank into the j^{th} tank and for each bottom tank.

$$Q_i^{out}(t) = s_i\sqrt{2gh_i(t)}, \quad i = 1,2 \tag{5}$$

where s_i denotes the cross-sectional area of the outflow orifice at the bottom of the i^{th} tank into the basin. Finally, note that for the four-tank system the following equation should be respected:

$$Q_1^{in}(t) + Q_2^{in}(t) = Q_3^{out}(t) + Q_4^{out}(t) \tag{6}$$

We have considered that the four tanks have the same cross-sectional area $S_i = S$ for $i = 1, \ldots, 4$. So, let us define the stationary parameter c_i as follows:

$$\begin{aligned} &\{c_1, \ldots, c_7, c_9\} \\ &\quad = \frac{\sqrt{2g}}{S}\left[s_1, s_{31}, s_{41}, s_2, s_{32}, s_{42}, s_{31} + s_{32}, s_{41} + s_{42}\right] \\ &c_8 = c_{10} = \frac{K_p}{S} \end{aligned} \tag{7}$$

Then, we can rewrite the quadruple tank model as follows:

$$\begin{aligned} \dot{h}_1 &= -c_1\sqrt{h_1} + c_2\sqrt{h_3} + c_3\sqrt{h_4} + d_1 \\ \dot{h}_2 &= -c_4\sqrt{h_2} + c_5\sqrt{h_3} + c_6\sqrt{h_4} + d_2 \\ \dot{h}_3 &= -c_7\sqrt{h_3} + c_8 u_1 \\ \dot{h}_4 &= -c_9\sqrt{h_4} + c_{10} u_2 \\ y &= \left[h_1 \;\; h_2\right]^T \end{aligned} \tag{8}$$

where d_1 and d_2 are the unknown leakage flow rates, respectively, from the bottom tanks 1 and 2.

The main objective of this work is to simultaneously estimate the missing liquid levels in both upper tanks h_3 and h_4 and the UI waveforms d_1 and d_2 with only the measurements of the liquid in both bottom tanks h_1 and h_2.

3. Nonlinear Class of Study and State Transformation

We consider the following class of the affine input nonlinear MIMO systems to which the quadruple tank model belongs:

$$\begin{aligned} \dot{\chi} &= F(\chi) + G(\chi)\cdot u + \sum_{i=1}^{m} P_i(\chi)\, d_i(t) \\ y_j &= h_j(\chi) \quad \text{for } j = 1, \ldots, s \end{aligned} \tag{9}$$

where $\chi \in \mathscr{M} \subset \mathscr{R}^n$, aC^∞ connected manifold of dimension n, and we assume the state space of interest $\mathscr{M}$ to be compact; $F(\chi)$ and $P_i(\chi)$, $i = 1, \ldots, m$, are smooth vector fields on $\mathscr{M}$; $h_j(\chi)$, $j = 1, \ldots, s$ are smooth functions from $\mathscr{M}$ *to* $\mathscr{R}$, $u =\in \mathscr{M} \subset \mathscr{R}^l$, $G(\chi, u)$ is a vector field on $\mathscr{M}$; and the disturbance vector is represented by $d(t) = [d_1(t), \ldots, d_m(t)]^T$ with $d_i(t)$ denoting the disturbance signals that affect the system, and we assume that each $d_i(t)$ is bounded.

The traditional nonlinear transformation uses the structural properties of the system to decouple the known/UI by transforming the system into another domain. In order to design the nonlinear UIO, the system distribution vectors $P_1(\chi), \ldots, P_m(\chi)$ must satisfy the involutive property [5]. The outputs should also have vector relative degree corresponding to $G(\chi, u)$ at each point $\chi_0 \in \mathscr{M}$. These assumptions in general are conserved. Instead of decoupling the UIs, we shall deal with the disturbances directly in our design. In the context of our paper, the relative degree of the system is defined with respect to the UI as follows.

Assumption 1. From the s outputs, there are at least $q \geq s$ outputs with relative degrees $r_j = 1$ with respect to the UI, $j = 1, \ldots, q$.

Remark. Among the methods that allow constructing the state transformation, one can proceed as follows.

For each output y_j, we define the following transformation:

$$\begin{aligned} \phi_j &= \left[h_j(\chi) \;\; L_f h_j(\chi) \;\; \cdots \;\; L_f^{k_j - 1} h_j(\chi)\right]^T, \\ &\text{for } = 1, \ldots q, \text{ where } L_f h(\chi) \triangleq \left(\left[\frac{\partial h(\chi)}{\partial \chi}\right] f\right) \end{aligned} \tag{10}$$

Thereafter, let the transformation matrix be as follows:

$$\mathrm{x} = \Phi(\chi) = \left[\phi_1^T \cdots \phi_q^T\right]^T, \tag{11}$$

such that $\mathrm{x}_j = [x_1^j \; x_2^j \ldots x_{k_j}^j]^T = \phi_j$, For $j = 1, \ldots, q$.

Consequently, model (9) can be transformed into the new coordinates with transformation (10), so that

$$\dot{\mathrm{x}} = \left[\dot{\mathrm{x}}_1^T \cdots \dot{\mathrm{x}}_q^T\right]^T = \left[\frac{\partial \Phi(\chi)}{\partial \chi}\right]\dot{\chi} \tag{12}$$

For the subsystems under the transformations $\phi_1, \ldots, \phi_q$, the following structure can be obtained:

$$\begin{aligned} \dot{\mathrm{x}}_j &= \begin{bmatrix} \dot{x}_1^j \\ \vdots \\ \dot{x}_{k_j-1}^j \\ \dot{x}_{k_j}^j \end{bmatrix} \\ &= \begin{bmatrix} x_2^j \\ \vdots \\ x_{k_j}^j \\ 0 \end{bmatrix} + \begin{bmatrix} \mu_1^j(\mathrm{x}, u) \\ \vdots \\ \mu_{k_j-1}^j(\mathrm{x}, u) \\ \mu_{k_j}^j(\mathrm{x}, u) \end{bmatrix} \\ &\quad + \sum_{i=1}^{m} \begin{bmatrix} z_{1i}^j(\mathrm{x}) \\ \vdots \\ z_{(k_j-1,i)}^j(\mathrm{x}) \\ z_{(k_j,i)}^j(\mathrm{x}) \end{bmatrix} d_i(t) \\ y_j &= x_1^j \end{aligned} \tag{13}$$

with

$$\mu_v^j(\mathrm{x},u)=\begin{cases}\dfrac{\partial L_f^v h_j}{\partial \mathrm{x}}G\left(\Phi^{-1}(\mathrm{x}),u\right) & \text{for } j=1\cdots q,\ v=1\cdots k_{j-1}\\ L_f^{kj}h_j\left(\Phi^{-1}(\mathrm{x})\right)+\dfrac{\partial \mathrm{L_f^v h_j}}{\partial \mathrm{x}}G\left(\Phi^{-1}(\mathrm{x}),u\right) & \text{for } j=1\cdots q,\ v=k_j\end{cases}$$
$$z_{vi}^j(\mathrm{x})=L_{p_i}L_f^v h_j\left(\Phi^{-1}(\mathrm{x})\right)\quad \text{for } j=1\cdots q,\ v=1,\ldots,k_j,\ i=1,\ldots,m \tag{14}$$

Each subsystem in form (13) can be rewritten in a condensed form as follows:

$$\dot{\mathrm{x}}_j=A_j\mathrm{x}_j+\mu_j(\mathrm{x},u)+\sum_{i=1}^m Z_i^j(\mathrm{x})\,d_i(t)$$
$$y_j=x_1^j=C_j\mathrm{x}_j \tag{15}$$

where

$$A_j=\begin{bmatrix}0 & I_{(k_j-1)\times(k_j-1)}\\ & 0_{1\times(k_j-1)}\end{bmatrix},$$
$$C_j=\begin{bmatrix}1 & 0 & \cdots & 0\end{bmatrix}$$
$$\mu_j(\mathrm{x},u)=\left[\mu_1^j\cdots\mu_{k_j-1}^j\mu_{k_j}^j\right]^T, \tag{16}$$
$$Z_i^j(\mathrm{x})=\left[z_{1i}^j\ldots z_{(k_j,i)}^j\right]^T$$
$$\text{for } j=1\cdots q.$$

So, the whole system is given in x coordinates by

$$\dot{\mathrm{x}}=A\mathrm{x}+\mu(\mathrm{x},u)+\sum_{i=1}^m Z_i(\mathrm{x})\,d_i(t)$$
$$y=\left[x_1^j\cdots x_1^q\right]^T=C\mathrm{x} \tag{17}$$

where

$$\mathrm{x}=\begin{bmatrix}\mathrm{x}_1\\ \vdots\\ \mathrm{x}_{q-1}\\ \mathrm{x}_q\end{bmatrix},$$
$$\mu=\begin{bmatrix}\mu_1\\ \vdots\\ \mu_{q-1}\\ \mu_q\end{bmatrix}, \tag{18}$$
$$Z_i(\mathrm{x})=\begin{bmatrix}Z_i^1(\mathrm{x})\\ \vdots\\ Z_i^{q-1}(\mathrm{x})\\ Z_i^q(\mathrm{x})\end{bmatrix}$$

and $A=\mathrm{diag}[A_1,A_2,\ldots,A_q]$, $C=\mathrm{diag}[C_1,C_2,\ldots,C_q]$.

Before the synthesis of the observer, some other assumptions are required as follows.

Assumption 2. The mapping $\Phi(x)$ is a diffeomorphism.

Assumption 3. The norms of $F(x), G(x,u)$ and $P_i(x)$ are bounded. Furthermore, system (9) is assumed to be a stable bounded-input-bounded-state (BIBS).

Assumption 4. The transformed system (15) should have the following structure:

$$\begin{bmatrix}\dot{x}_1^j\\ \vdots\\ \dot{x}_{k_j-1}^j\\ \dot{x}_{k_j}^j\end{bmatrix}=\begin{bmatrix}x_2^j\\ \vdots\\ x_{k_j}^j\\ 0\end{bmatrix}+\begin{bmatrix}\mu_1^j\left(x_1^j,\overline{\mathrm{x}}_j^*,u\right)\\ \mu_2^j\left(x_1^j,x_2^j,\overline{\mathrm{x}}_j,u\right)\\ \vdots\\ \mu_{k_j}^j\left(x_1^j,x_2^j,\ldots,x_{k_j}^j,\overline{\mathrm{x}}_j,u\right)\end{bmatrix}$$
$$+\sum_{i=1}^m\begin{bmatrix}z_{1i}^j\left(x_1^j,\overline{\mathrm{x}}_j^*\right)\\ z_{2i}^j\left(x_1^j,x_2^j,\overline{\mathrm{x}}_j\right)\\ \vdots\\ z_{k_ji}^j\left(x_1^j,x_2^j,\ldots,x_{k_j}^j,\overline{\mathrm{x}}_j\right)\end{bmatrix}d_i(t) \tag{19}$$

for , $j=1,\ldots,q$, where $\overline{\mathrm{x}}_j^*=\{x_1^1,x_1^2,\ldots,x_1^{j-1}\}$ with $\overline{\mathrm{x}}_j=\{\mathrm{x}_1,\mathrm{x}_2,\ldots,\mathrm{x}_{j-1}\}$. According to the [25], each subsystem $\overline{\mathrm{x}}_j$ is uniformly observable with respect to $\overline{\mathrm{x}}_j$.

Assumption 5. The distribution vector $Z_i(\mathrm{x})$ and the function $\mu(\mathrm{x},u)$ are Lipschitz functions with respect to x for all $i=1,\ldots,q$.

The following assumption is the key requirement that guarantees the reconstruction of all the UI from the multiple sliding mode.

Assumption 6. The dynamics of states that are measured as outputs of s subsystems have the following structure:

$$\begin{bmatrix}\dot{y}_1\\ \dot{y}_2\\ \vdots\\ \dot{y}_s\end{bmatrix}=\begin{bmatrix}\dot{x}_1^1\\ \dot{x}_1^2\\ \vdots\\ \dot{x}_1^s\end{bmatrix}$$

$$= \begin{bmatrix} x_2^1 \\ x_2^2 \\ \vdots \\ x_2^s \end{bmatrix} + \begin{bmatrix} \mu_1^1(\mathrm{x}, u) \\ \mu_1^2(\mathrm{x}, u) \\ \vdots \\ \mu_1^s(\mathrm{x}, u) \end{bmatrix}$$

$$+ \begin{bmatrix} 1 & 0 & \cdots & 0 \\ z_{11}^2(\mathrm{x}) & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ z_{11}^m(\mathrm{x}) & z_{12}^m(\mathrm{x}) & \cdots & 0 \end{bmatrix} \begin{bmatrix} d_1(t) \\ d_2(t) \\ \vdots \\ d_m(t) \end{bmatrix} \tag{20}$$

where $z_{1i}^i(\mathrm{x}) \neq 0,\ i = 1, \dots, m$.

4. Robust High-Gain Observer Design

4.1. Observer Synthesis. Our objective consists in synthesizing an observer to simultaneously estimate the unmeasured state and the UI without assuming any model for the latter. According to [21], for the subsystem (11) satisfying Assumptions 4–6, the proposed observer can be designed as follows:

$$\begin{aligned} \dot{\hat{\mathrm{x}}}_j &= A_j \hat{\mathrm{x}}_j + \mu_j\left(\hat{\mathrm{x}}_j, u\right) + L_j\left(y_j - C_j \hat{\mathrm{x}}_j\right) \\ &\quad + \sum_{i=1}^{m} Z_i^j(\hat{\mathrm{x}})\, v_i(t) \end{aligned} \tag{21}$$

where $L_j = [l_1^j l_2^j \cdots l_{n_j-1}^j l_{n_j}^j]^T$ for all $j = 1, \dots, q$.

$v_i(t)$ is a scalar-valued robust term given by the sliding mode estimation:

$$\begin{aligned} v_i(t) &= -\rho_i sat\left(\left(y_i - \hat{x}_1^i\right), \varepsilon_i\right) \\ &= \begin{cases} -\rho_i \dfrac{\left(y_i - \hat{x}_1^i\right)}{\left|\left(y_i - \hat{x}_1^i\right)\right|} & if\ \left|\left(y_i - \hat{x}_1^i\right)\right| > \varepsilon_i \\ -\rho_i \dfrac{\left(y_i - \hat{x}_1^i\right)}{\varepsilon_i} & if\ \left|\left(y_i - \hat{x}_1^i\right)\right| \leq \varepsilon_i \end{cases} \end{aligned} \tag{22}$$

ρ_i (for $i = 1, \dots, m$) is the sliding mode estimation gain and ε_i is the boundary layer design parameter.

In summary, the whole proposed observer of the system (17) is given by

$$\dot{\hat{\mathrm{x}}} = \mathrm{A}\hat{\mathrm{x}} + \mu(\hat{\mathrm{x}}, \mathrm{u}) + \mathrm{L}(\mathrm{y} - \mathrm{C}\hat{\mathrm{x}}) + \sum_{\mathrm{i}=1}^{\mathrm{m}} \mathrm{Z}_{\mathrm{i}}(\hat{\mathrm{x}})\, \mathrm{v}_{\mathrm{i}}(\mathrm{t}) \tag{23}$$

where

$$L = \operatorname{diag}\left[L_1, L_2, \dots, L_q\right] \tag{24}$$

$$L_j = S_{\theta_j}^{-1} C_j^T = \theta_j \Delta_{\theta_j}^{-1} S^{-1} C_j^T \tag{25}$$

with

$$\Delta_{\theta_j} = \operatorname{diag}\left[1, \frac{1}{\theta_j}, \dots, \frac{1}{\theta_j^{n_j-1}}\right] \tag{26}$$

S_θ is a definite positive solution of the following algebraic Lyapunov equation:

$$\theta_j S_{\theta_j} + A^T S_{\theta_j} + S_{\theta_j} A + C^T C = 0 \tag{27}$$

It can be explicitly given as follows:

$$S_\theta^{-1}(m, \nu)\, C^T = \left[C_m^1 \theta, C_m^2 \theta^2, \dots, C_m^\nu \theta^\nu\right] \tag{28}$$

with $C_m^\nu = m!/(m-\nu)!\nu!$ and $\theta > 1$ is the sole design parameter.

The proof of the error convergence is detailed in [21].

4.2. Observer Form in the Original State Coordinates. Under Assumption 1 and using (11), we can write

$$\dot{\chi} = \left[\frac{\partial \Phi(\chi)}{\partial \chi}\right]^{-1} \dot{\mathrm{x}} \tag{29}$$

then the observer can be written in the original state coordinates as follows:

$$\dot{\hat{\chi}} = F(\hat{\chi}) + G(\hat{\chi}, u) + \left[\frac{\partial \Phi(x)}{\partial x}\right]_{\chi=\hat{\chi}}^{-1} L_{trans} \tag{30}$$

where L_{trans} is given by

$$L_{trans} = \begin{bmatrix} L_1\left(y_1 - \hat{x}_1^1\right) - \sum_{i=1}^{m} Z_i^1(\hat{x})\, \rho_i sat\left(y_1 - \hat{x}_1^i\right) \\ \vdots \\ L_q\left(y_q - \hat{x}_1^q\right) - \sum_{i=1}^{m} Z_i^q(\hat{x})\, \rho_i sat\left(y_q - \hat{x}_1^i\right) \\ 0_{c\times 1} \end{bmatrix} \tag{31}$$

The UIs can be reconstructed from their respectively equivalent control signals as follows:

$$\hat{d}_i \approx \left(\rho_i \operatorname{Sign}\left(e_1^i\right)\right) = \rho_i \frac{e_1^i}{\left|e_1^i\right| + \delta} \tag{32}$$

for $i = 1, \dots, m$, where δ is a small positive scalar.

The UI estimation relies on the output estimation error and hence the estimation can be performed online together with state estimation.

5. Application of the UIO on the Four-Tank System

To show the effectiveness of the proposed nonlinear UIO previously described, we consider the following intuitive state transformation:

$$\begin{aligned} z^1 &= \begin{pmatrix} z_1^1 \\ z_2^1 \end{pmatrix} = \begin{pmatrix} h_1 \\ c_2\sqrt{h_3} + c_3\sqrt{h_4} \end{pmatrix} \\ z^2 &= \begin{pmatrix} z_1^2 \\ z_2^2 \end{pmatrix} = \begin{pmatrix} h_2 \\ c_5\sqrt{h_3} + c_6\sqrt{h_4} \end{pmatrix} \end{aligned} \tag{33}$$

In the z coordinate, system (8) can be written as follows:

$$\begin{aligned} \dot{z}_1^1 &= -c_1\sqrt{z_1^1} + z_2^1 + d_1 \\ \dot{z}_2^1 &= c_{12} + a(z)\,u_1 + b(z)\,u_2 \\ \dot{z}_1^2 &= -c_4\sqrt{z_1^2} + z_2^2 + d_2 \\ \dot{z}_2^2 &= c_{22} + c(z)\,u_1 + d(z)\,u_2 \end{aligned} \tag{34}$$

with

$$\begin{aligned} a(z) &= \frac{c_8c_2\left(c_6c_2 - c_5c_3\right)}{2\left(c_6z_2^1 - c_3z_2^2\right)}; \\ b(z) &= \frac{c_3c_{10}\left(c_6c_2 - c_5c_3\right)}{2\left(c_2z_2^2 - c_5z_2^1\right)} \\ c(z) &= \frac{c_8c_5\left(c_6c_2 - c_5c_3\right)}{2\left(c_6z_2^1 - c_3z_2^2\right)}; \\ d(z) &= \frac{c_6c_{10}\left(c_6c_2 - c_5c_3\right)}{2\left(c_2z_2^2 - c_5z_2^1\right)} \\ c_{12} &= -\frac{1}{2}\left(c_7c_2 + c_3c_9\right), \\ c_{22} &= -\frac{1}{2}\left(c_7c_5 + c_6c_9\right) \end{aligned} \tag{35}$$

5.1. Conjoint Estimation of the Liquid Levels and Leakage Flow Rates. The obtained system (34) is in the canonical form as (17) with $q = 2$. The objective is to reconstruct the liquid level of tanks 3 and 4 and the UIs d_1 and d_2. Only the measurements of h_1 and h_2 are considered available. The previous assumptions are not very restrictive and they can be verified for a large class of MIMO nonlinear systems. The appropriate state observer can be rewritten in the following form:

$$\begin{aligned} \dot{\tilde{z}}_1^1 &= -c_1\sqrt{\tilde{z}_1^1} + \tilde{z}_2^1 - 2\theta\tilde{z}_1^1 - \rho_1 sat\left(\tilde{z}_1^1\right) \\ \dot{\tilde{z}}_2^1 &= c_{12} + a(\tilde{z})\,u_1 + b(\tilde{z})\,u_2 - \theta^2\left(\tilde{z}_1^1\right) - \rho_1 sat\left(\tilde{z}_1^1\right) \\ \dot{\tilde{z}}_1^2 &= -c_4\sqrt{\tilde{z}_1^2} + \tilde{z}_2^2 - 2\theta\left(\tilde{z}_1^2\right) - \rho_2 sat\left(\tilde{z}_1^2\right) \\ \dot{\tilde{z}}_2^2 &= c_{22} + c(z)\,u_1 + d(\tilde{z})\,u_2 - \theta^2\left(\tilde{z}_1^2\right) - \rho_2 sat\left(\tilde{z}_1^2\right) \end{aligned} \tag{36}$$

with $\tilde{z}_1^1 = \tilde{z}_1^1 - z_1^1$; and $\tilde{z}_1^2 = \tilde{z}_1^2 - z_1^1 2$.

The UIs can be estimated from the robust term through the multiple sliding modes (32) as follows:

$$\begin{aligned} \hat{d}_1 &= -\rho_1\frac{\left(\tilde{z}_1^1 - z_1^1\right)}{\left|\tilde{z}_1^1 - z_1^1\right| + \delta} \\ \hat{d}_2 &= -\rho_2\frac{\left(\tilde{z}_1^2 - z_1^2\right)}{\left|\tilde{z}_1^2 - z_1^2\right| + \delta} \end{aligned} \tag{37}$$

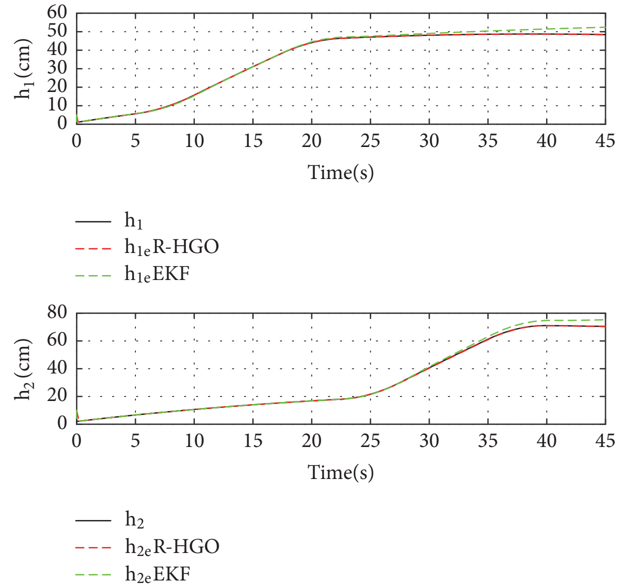


Figure 2: Estimation of the liquid levels h_1 and h_2.

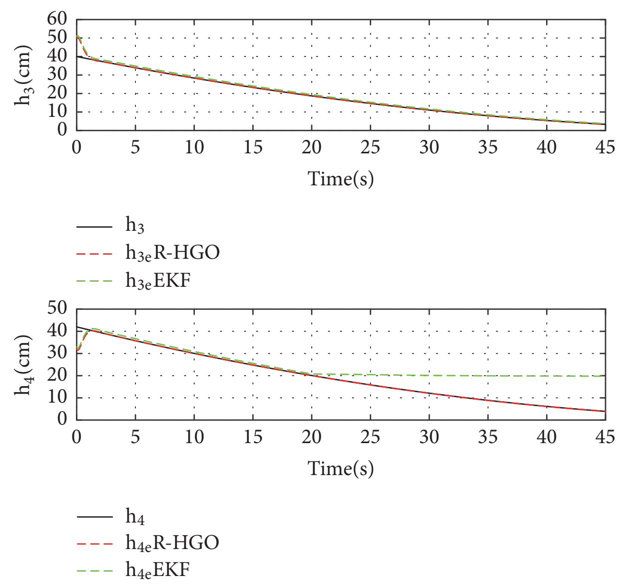


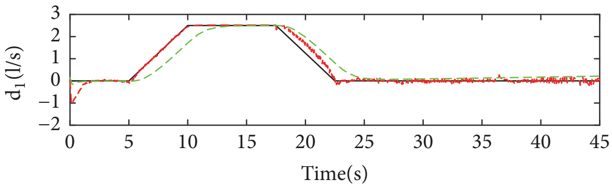
Figure 3: Estimation of the non-measured liquid levels.

By means of inverse transformation (29), the observer can be given the original coordinates as (30).

The parameters of the quadruple tank model used in the numerical simulations are as follows:

$c_1 = c_4 = 0.005$, $c_2 = c_6 = 0.014$, $c_3 = c_5 = 0.02$, $c_7 = c_9 = 0.02$ and $c_8 = c_{10} = 43$.

Both trapezoidal profiles for the leakage flow rates are imposed as disturbance inputs to the plant (Figure 4). The time evolution of the liquid levels h_i for $(i = 1\cdots4)$ issued from the model simulation is compared to their respective estimates provided by the observer h_{ie} for $(i = 1\cdots4)$ (Figures 2 and 3). Notice that, with a choice of the synthesis parameter $\theta = 1$ and $\rho_1 = \rho_2 = 30$, we remark that all the estimates need



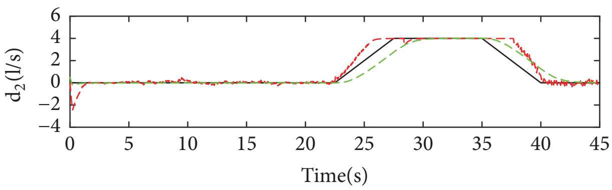


Figure 4: Estimation of the leakage flow rates (UIs).

less than $2s$ to track well their true value at the transient. For both leakage flow rate estimations d_i $(i = 1, 2)$, as shown in Figure 4, their reconstruction is quite precise when they are constant, whereas a little bounded error (less than 10%) is recorded when d_i varies linearly with time.

5.2. Comparison between the Robust High-Gain Observer (R-HGO) and the Extended Kalman Filter (EKF). In order to highlight the features of the R-HGO design, besides its privilege in time computation and in the number of synthesis parameters, it is compared with the standard EKF algorithm which is one of the most industrial diffused observer [26, 27]. After several attempts to adjust the design parameters of this last $(Q(0), R(0), S(0))$, we have choose the following values: $Q(0) = 10^{-9} I_6$;, $R(0) = 5 \cdot 10^{-8} I_2$, $S(0) = 5 \cdot 10^{-6} I_6$. Simulation results for conjoint state and UIs estimation of both techniques are illustrated in Figures 2–4. It is shown that when the abrupt disturbance $d_2(t)$ is applied, only the estimates that arise from the R-HGO remain rallied around the trues states. This is can be explicated by the local nature of the EKF which approximates the nonlinear model only around some small neighborhood of the operating point. Moreover, through the zoom of the $d_1(t)$ estimation, we remark that the EKF induces a biased reconstruction, whereas, in spite of the ripples arising from the sliding mode term in the R-HGO, the mean value of the estimated signal is more near to its truth waveform.

6. Conclusion

A combination of HGO and SMO is used for a conjoint estimation of the state variables and the UIs. The robust terms designed from the sliding surfaces allow preserving a little bound estimation error when the disturbance occurs. Besides, it contributes to the reconstruction of the UI waveforms. As an application, both liquid levels of the upper tanks and leakage flow rates in both bottom tanks are conjointly estimated for a quadruple tank process. Simulation results demonstrate a good estimation performance especially when the UIs are constant or vary relatively slow.

In the majority of engineering applications, measurements are collected only at the sampling instants. So, a logical extension of our work consists in the development of a continuous-discrete time UIO for the quadruple tank process.

Data Availability

No data were used to support this study. Our study is based on simulation results.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

References

[1] S. Sundaram and C. N. Hadjicostis, “Distributed function calculation via linear iterative strategies in the presence of malicious agents,” *Institute of Electrical and Electronics Engineers Transactions on Automatic Control*, vol. 56, no. 7, pp. 1495–1508, 2011.

[2] S. Pan, D. Xiao, S. Xing, S. S. Law, P. Du, and Y. Li, “A general extended Kalman filter for simultaneous estimation of system and unknown inputs,” *Engineering Structures*, vol. 109, pp. 85–98, 2016.

[3] A. Chakrabarty, S. M. Pearce, R. P. Nelson, and A. E. Rundell, “Treating acute myeloid leukemia via HSC transplantation: A preliminary study of multi-objective personalization strategies,” in *Proceedings of the 2013 1st American Control Conference, ACC 2013*, pp. 3790–3795, USA, June 2013.

[4] E. Rocha-Cózatl and A. V. Wouwer, “State and input estimation in phytoplanktonic cultures using quasi-unknown input observers,” *Chemical Engineering Journal*, vol. 175, no. 1, pp. 39–48, 2011.

[5] Y. Xiong and M. Saif, “Sliding mode observer for nonlinear uncertain systems,” *Institute of Electrical and Electronics Engineers Transactions on Automatic Control*, vol. 46, no. 12, pp. 2012–2017, 2001.

[6] T. Floquet, J. P. Barbot, W. Perruquetti, and M. Djemai, “On the robust fault detection via a sliding mode disturbance observer,” *International Journal of Control*, vol. 77, no. 7, pp. 622–629, 2004.

[7] C. Edwards, S. K. Spurgeon, and R. . Patton, “Sliding mode observers for fault detection and isolation,” *Automatica*, vol. 36, no. 4, pp. 541–553, 2000.

[8] A. Chakrabarty, M. J. Corless, G. T. Buzzard, S. H. Zak, and A. E. Rundell, “State and unknown input observers for nonlinear systems with bounded exogenous inputs,” *Institute of Electrical and Electronics Engineers Transactions on Automatic Control*, vol. 62, no. 11, pp. 5497–5510, 2017.

[9] X.-G. Yan and C. Edwards, “Nonlinear robust fault reconstruction and estimation using a sliding mode observer,” *Automatica*, vol. 43, no. 9, pp. 1605–1614, 2007.

[10] Y. Xiong and M. Saif, “Unknown disturbance inputs estimation based on a state functional observer design,” *Automatica*, vol. 39, no. 8, pp. 1389–1398, 2003.

[11] M. Defoort, M. Djemai, T. Floquet, and W. Perruquetti, “Robust finite time observer design for multicellular converters,” *International Journal of Systems Science*, vol. 42, no. 11, pp. 1859–1868, 2011.

[12] S. H. Saïd, F. M’Sahli, and M. Farza, “Simultaneous state and unknown input reconstruction using cascaded high-gain observers,” *International Journal of Systems Science*, vol. 48, no. 15, pp. 3346–3354, 2017.

[13] J. Yao and W. Deng, “Active Disturbance Rejection Adaptive Control of Hydraulic Servo Systems,” *IEEE Transactions on Industrial Electronics*, vol. 64, no. 10, pp. 8023–8032, 2017.

[14] J. Yao and W. Deng, “Active disturbance rejection adaptive control of uncertain nonlinear systems: theory and application,” *Nonlinear Dynamics*, vol. 89, no. 3, pp. 1611–1624, 2017.

[15] Y. Wang, R. Rajamani, and D. M. Bevly, “Observer design for differentiable Lipschitz nonlinear systems with time-varying parameters,” in *Proceedings of the 2014 IEEE 53rd Annual Conference on Decision and Control (CDC)*, pp. 145–152, Los Angeles, CA, USA, December 2014.

[16] M. Xiao, Y. Zhang, and H. Fu, “Three-stage unscented Kalman filter for state and fault estimation of nonlinear system with unknown input,” *Journal of The Franklin Institute*, vol. 354, no. 18, pp. 8421–8443, 2017.

[17] S. Ahmadizadeh, J. Zarei, and H. R. Karimi, “Robust unknown input observer design for linear uncertain time delay systems with application to fault detection,” *Asian Journal of Control*, vol. 16, no. 4, pp. 1006–1019, 2014.

[18] K. Mohamed, M. Chadli, and M. Chaabane, “Unknown inputs observer for a class of nonlinear uncertain systems: An LMI approach,” *International Journal of Automation and Computing*, vol. 9, no. 3, pp. 331–336, 2012.

[19] K. Kalsi, J. Lian, S. Hui, and S. H. Zak, “Sliding-mode observers for systems with unknown inputs: a high-gain approach,” *Automatica*, vol. 46, no. 2, pp. 347–353, 2010.

[20] Y. Zhou, Y. C. Soh, and J. X. Shen, “High-gain observer with higher order sliding mode for state and unknown disturbance estimations,” *International Journal of Robust and Nonlinear Control*, vol. 24, no. 15, pp. 2136–2151, 2014.

[21] K. C. Veluvolu, M. Defoort, and Y. C. Soh, “High-gain observer with sliding mode for nonlinear state estimation and fault reconstruction,” *Journal of The Franklin Institute*, vol. 351, no. 4, pp. 1995–2014, 2014.

[22] K. H. Johansson, “The quadruple-tank process: a multivariable laboratory process with an adjustable zero,” *IEEE Transactions on Control Systems Technology*, vol. 8, no. 3, pp. 456–465, 2000.

[23] T. Raff, S. Huber, Z. Nagy, and F. Allgower, “Nonlinear Model Predictive Control of a Four Tank System: An Experimental Stability Study,” in *Proceedings of the 2006 IEEE International Conference on Control Applications*, pp. 237–242, Munich, Germany, October 2006.

[24] T. Deepa, P. Lakshmi, and S. Vidya, “Level control of quadruple tank process using discrete time model predictive control,” in *Proceedings of the 2011 3rd International Conference on Electronics Computer Technology (ICECT)*, pp. 162–166, Kanyakumari, India, April 2011.

[25] J.-P. Gauthier, H. Hammouri, and S. Othman, “A simple observer for nonlinear systems applications to bioreactors,” *Institute of Electrical and Electronics Engineers Transactions on Automatic Control*, vol. 37, no. 6, pp. 875–880, 1992.

[26] B. E, *Sur les observateurs des systemes non linéaires [Ph.D. thesis]*, Université de Bourgogne Dijon, France, 2004.

[27] S. Hadj Said and F. M. Sahli, “A set of observers design to a quadruple tank process,” in *Proceedings of the 2008 IEEE International Conference on Control Applications (CCA) part of the IEEE Multi-Conference on Systems and Control*, pp. 954–959, San Antonio, TX, USA, September 2008.