Research Article

Improved Absolute Stability for a Class of Nonlinear Switched Delay Systems via Mode-Dependent Average Dwell Time

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In this paper, the absolute stability for a class of switched delay systems with time-varying uncertainties is analyzed. By constructing an appropriate Lyapunov-Krasovskii functional, a new and less conservative criterion is proposed based on the MDADT method. Besides, the idea of N–segmentation is also utilized to make it more flexible to solve the LMIs. Finally, a numerical example is given to show the feasibility and advantages of the method proposed in this paper.

1. Introduction

Switched system is an important class of hybrid system which can be described by a series of continuous or discrete subsystems with a rule organizing the switching among them. As well known, many kinds of practical systems, such as power systems and chemical procedure control systems, may be properly modeled by the switched systems [1]. The main problem of switched systems is the stability analysis. In the last two decades, there have been increases in interests in the stability analysis for switched systems; see, for example, [2–5] and the references cited therein.

Apart from that, in many physical systems, such as chemical process and electric network, time-delay phenomenon is also unavoidable. Time-delays may lead to instability or even poor performance such as chaos [6]. Recently, the stability of systems with delays has aroused extensive research of scholars (see [7–10] and the references cited therein). For systems with time delay, the most important technique is to construct appropriate Lyapunov-Krasovskii functional.

Consequently, the stability of switched systems with time delay is worth considering. In fact, switched delay systems have a wide range of practical backgrounds in network control [11], chemical reactors [12], drilling system [13], and so on. To cope with the stability and design problem of switched delay systems, a good deal of methods have been proposed in terms of state-dependent switching and time-dependent switching. For instance, in [14], a delay-dependent stability criterion is derived under a state-driven law for switched delay systems. With regard to time-dependent switching, the dwell time (DT) method [15] and the average dwell time (ADT) method [16], as well as the mode-dependent average dwell time (MDADT) method [17], are put forward successively. In particular, for impulsive dynamical networks, in order to describe the frequency of occurrence of impulses, in 2010, by referring to the concept of ADT, Jianquan, Daniel W.C. Ho, and Jinde Cao [18] presented the concept of average impulsive interval. Recently, more and more work has been done on some special switched systems. For instance, for a class of impulsive stochastic differential systems, the stability properties are investigated under Markovian switching [19]. For switched cellular neural networks, by the MDADT approach, the exponential stability is considered in two cases where all subsystems are stable or unstable subsystems exist [20]. For a class of genetic regulatory networks (GRNs) with mixed delays, adaptive feedback control scheme is used to obtain the stability criteria [21].

In addition, since it was raised in 1940s, Lur’e systems have been deeply concerned. The main concern of Lur’e systems is absolute stability which covers systems without delays and those with delays [22–24]. To list a few, [25, 26] survey the absolute stability of Lur’e systems with constant delays and
time-varying delays, respectively. From then on, more and more results have appeared to get less conservative stability criteria. Free-matrix-weighing method [27] and improved Lyapunov-Krasovskii functional method [28] are adopted in succession to reduce the conservativeness of previous results.

Integrated with the features of switched systems and Lur’e systems, switched Lur’e systems have also been paid much attention. A number of systems in control communities can be modeled by switched Lur’e systems, such as Hopfield neural network [29] and variable structure system [30]. However, as far as we know, up to now, absolute stability of switched Lur’e systems has not been completely considered.

Recently, [31] studies the absolute stability of switched delay Lur’e systems. By the method of ADT and Lyapunov functional, the efficient delay-dependent condition is obtained in the form of LMI.s. However, the system considered in [31] is linear; meanwhile, the Lyapunov function chosen in [31] is too simple and restrictive, as only quadratic terms and the integration term of state are included. Therefore, the result obtained in [31] is more conservative, which is one motivation of present work. Also, when a system with delay is considered, one chief objective is to get maximum allowed delay upper bounds and the bigger, the better. Considering the complexity of switched systems, it is challenging to design a switching law for switched delay systems to get bigger maximum allowed delay upper bounds and better performance, which is the other driving force of this paper. Besides, sometimes, in previous work, such as [32], it is difficult to get the same positive matrix which satisfies the corresponding LMI.s.

In this paper, the absolute exponential stability of switched Lur’e systems with delays is investigated. The main contribution of this paper is as follows. (1) Employing the integration of state derivative, an appropriate Lyapunov-Krasovskii functional is constructed to get new stability criteria. (2) The switching law is designed based on MDADT method to reduce the conservativeness of previous results. (3) The criteria proposed in this paper are more flexible than the previous results [31], involving those criteria obtained in [25, 31]. On top of that, compared with those in [32], the method derived in this paper can also be used to get less conservative results for general switched delay systems.

The rest of this paper is organized as follows. The problem is stated in Section 2. Section 3 shows the main results and a simulation example is given in Section 4. In the end of the paper, conclusions are presented.

Notations. Throughout this paper, \( R \) denotes the real number set; \( N \) denotes the nonnegative integer set; \( R^n \) is the Euclid space with dimension \( n \); \( R^{m \times n} \) is the real matrices set with dimension \( m \times n \). For the two given symmetric matrices \( P \) and \( Q \), we use \( P > Q \) (\( P \geq Q \)) to denote that \( P - Q \) is a positive definite (positive semidefinite) matrix. 

\[
A^T \text{ denotes the transpose of matrix } A; \ I \text{ denotes the identity matrix with appropriate dimension. For any matrix } W \text{ and two symmetric matrices } P, Q, \text{ we use } * \text{ to denote the symmetric part of the symmetric matrix.}
\]

2. Problem Statement

Consider the following system

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) + C_2 \omega(t) \\
\dot{z}(t) &= D_1 x(t) + E_1 u(t) - \dot{\phi}(t, z(t)) \\
\omega(t) &= -\varphi(t, z(t)) \\
x(t) &= \phi(t), \quad \forall t \in [-\tau, 0]
\end{align*}
\]

where \( x(t) \in R^n, \omega(t) \in R^m, z(t) \in R^m \) are the state vector, input vector, and output vector, respectively; \( \tau > 0 \) is system delay; \( \varphi(t) \) is a continuous vector-valued function and \( A(t), B(t), C_1(t), C_2(t) \) are all constant matrices; \( \sigma(t) : [0, \infty) \rightarrow M = \{1, 2, \ldots, m\} \) is the switching signal; \( \varphi(t, z(t)) : [0, \infty) \times R^m \rightarrow R^m \) is a nonlinear vector-valued function which is continuous about \( t \) and globally Lipschitz at \( z(t) \) with \( \varphi(t, 0) = 0 \). Besides \( \varphi(t, z(t)) \) also satisfies

\[
\begin{align*}
\varphi(t, z(t)) - K_1 z(t) &
\leq
\int_{t-\tau}^{t} \varphi(s, z(s)) ds + \int_{t-\tau}^{t} z(s) ds + E \leq 0 \\
\forall t \geq 0, \forall z(t) \in R^m
\end{align*}
\]

where \( K = K_1 - K_2 \) is a symmetric positive definite matrix with constant matrices \( K_1, K_2 \). The nonlinear function \( \varphi(t, z(t)) \) which satisfies (2) is said to belong to \([K_1, K_2] \). Initially, we will give some definitions and lemmas needed in this paper.

Definition 1 (see [22]). System described by (1) and (2) is called to be absolutely exponentially stable in \([K_1, K_2] \) if the zero solutions of system (1) are all globally exponentially stable under some switching signal \( \sigma(t) \).

Definition 2 (see [15]). For a switching signal \( \sigma(t) \) and any \( T \geq t \geq 0 \), let \( N_{ai}(T, t) \) be the switching numbers, that is, the \( i^{th} \) subsystem is activated over the interval \([T, T] \). For given matrices \( T_i(T, t) \) denote the total running time of the \( i^{th} \) subsystem over \([T, T] \), \( i \in M \). We say that \( \sigma(t) \) has a mode-dependent average dwell time (MDADT) \( \tau_{ai} \) if there exist positive numbers \( N_{ai} \) (chatter bounds) and \( \tau_{ai} \) such that

\[
N_{ai}(T, t) \leq N_{ai} + T_i(T, t) \tau_{ai}, \quad \forall T \geq t \geq 0.
\]

Lemma 3 (see [25]). For any symmetric positive definite matrix \( M > 0 \), scalar \( \gamma > 0 \), and function \( \omega : [0, \gamma] \rightarrow R^n \), if the integrations related are well defined, the following holds.

\[
\gamma \int_0^\gamma \omega^T(s) M \omega(s) ds \geq \left( \int_0^\gamma \omega(s) ds \right)^T M \left( \int_0^\gamma \omega(s) ds \right) \geq \gamma^2 \gamma_0 \int_0^\gamma \omega^T(s) M \omega(s) ds
\]

Lemma 4 (see [26]). For given matrices \( Q = Q^T, H, E \) with appropriate dimensions,

\[
Q + H^T F(t) E + F^T(t) H^T < 0
\]

holds for all \( F(t) \) satisfying \( F(t) F(t) H^T < 1 \) if and only if there exists \( \varepsilon > 0 \) such that

\[
Q + \varepsilon^{-1} HH^T + \varepsilon E^T E < 0.
\]
3. Main Results

In the previous work [31], the Lyapunov function used is of the following form.

\[ V(t, x(t)) = V_{σ(t)}(t, x(t)) = x^T(t) P_{σ(t)} x(t) \]

\[ + \int_{t-\tau}^{t} e^{\alpha(x(s))} x^T(s) Q_{σ(t)} x(s) \, ds \]  

\[ \text{with } P_{σ(t)} = P_{σ(t)}^T > 0, Q_{σ(t)} = Q_{σ(t)}^T > 0, \text{ and } R_{σ(t)} = R_{σ(t)}^T > 0, i = 1, 2, 3, \ldots, n, j = 1, 2, 3, \ldots, n; \text{ are positive definite matrices to be determined.} \]

As in [31], the case when the nonlinear function \( \varphi(t, z(t)) \) \( \in \{0, K\} \) will be firstly considered and we have the following result.

**Theorem 5.** For given \( h > 0, \alpha > 0, \mu_j \geq 1, j \in M \), system (1) is absolutely exponentially stable in sector \( [0, K] \) with MDADT \( \tau_{aj} > \tau_{aj}^* = (\ln \mu_j) / \alpha \) if there exist a real number \( \varepsilon > 0 \) and symmetric matrices \( P_i > 0, Q_i > 0, R_i > 0, i = 1, 2, 3, \ldots, n, j \in M \) such that the following LMIs hold

\[ Y = \begin{pmatrix} Y_{11} & Y_{12} & Y_{13} \\ \ast & Y_{22} & Y_{23} \\ \ast & \ast & Y_{33} \end{pmatrix} < 0 \]  

where

\[ Y_{11} = \begin{pmatrix} Y_{11}^{11} & P_i B_j & P_i C_j - \varepsilon D_j^T K_i^T \\ \ast & Y_{22}^{12} & -\varepsilon E_j^T K_i^T \\ \ast & \ast & -2\varepsilon I \end{pmatrix} \]

\[ Y_{12} = \begin{pmatrix} R_{ij} 0 \cdots 0 0 \\ 0 0 \cdots 0 e^{-\alpha(n-1)h} R_{ij} \end{pmatrix} \]

\[ Y_{13} = h^T \sum_{i=1}^{n} R_{ij}, \quad Y_{22} = \begin{pmatrix} S_1 & R_{ij} & \vdots \\ R_{ij} & S_2 & \ddots \\ \vdots & \ddots & \ddots & \ddots \end{pmatrix} \]

\[ S_i = e^{-\alpha_j (i+1)h} (Q_{(i+1)j} - Q_{ij}) - e^{-\alpha_j (i-1)h} (R_{ij} + R_{(i+1)j}) \]

\[ Y_{33} = -e^{-\alpha_j (i-1)h} \sum_{i=1}^{n} R_{ij}, \quad i = 1, 2, \ldots, n-1, \]

\[ P_i \leq \mu_i P_m, \]

\[ Q_i \leq \mu_i Q_m, \]

\[ R_i \leq \mu_i R_m, \quad \forall i, m \in M. \]

**Proof.** Differentiating function (8) about time \( t \) along the solutions of system (1), we will get the following.

\[ V(t, x(t)) + \alpha_j V(t, x(t)) = x^T(t) \left( A_j^T P_j + P_j A_j + \alpha_j \alpha_j \right) x(t) + 2x^T(t) P_j C_j \omega(t) + 2x^T(t) P_j B_j x(t - \tau) \]

\[ + 2x^T(t) P_j B_j x(t - \tau) + 2x^T(t) x(t) + 2x^T(t) P_j B_j x(t - \tau) \]

\[ + 2x^T(t) P_j B_j x(t - \tau) \]

\[ + \sum_{i=1}^{n} e^{-\alpha_j (i-1)h} x(t) - (i-1)h) \right)^T Q_{ij} x(t - (i-1)h) \]

\[ - \sum_{i=1}^{n} e^{-\alpha_j (i-1)h} x(t) - (i-1)h) \right)^T Q_{ij} x(t - (i-1)h) \]

\[ + \sum_{i=1}^{n} \int_{t-ih}^{t} e^{\alpha_j (i-1)h} \dot{x}(s) \dot{x}(s) ds = x^T(t) \]

By use of Lemma 3, we can get

\[ - \sum_{i=1}^{n} \int_{t-ih}^{t} e^{\alpha_j (i-1)h} \dot{x}(s) \dot{x}(s) ds \]
From (1) and considering $\varphi(t, z(t)) \in [0, K]$, the following holds.

$$-\omega^T(t) \omega(t) - \omega^T K \left[ D_j x(t) + E_j x(t - \gamma(t)) \right] \geq 0$$

(15)

According to (12)-(15), for a scalar $\varepsilon > 0$, it yields that

$$V(t, x(t)) + aV(t, x(t))$$

$$\leq x^T(t) \left( A^TP_j + P_j A_j + \alpha P_j + Q_{ij} \right) x(t)$$

$$+ 2x^T(t) P_j C_j \omega(t) + 2x^T(t) P_j B_j x(t - \tau)$$

$$- e^{-\alpha j h} x^T(t - \tau) Q_{ij} x(t - \tau)$$

$$+ \sum_{i=1}^{n-1} e^{-\alpha j h} x^T(t - ih) \left( Q_{(i+1)j} - Q_{ij} \right) x(t - ih)$$

$$- n e^{-\alpha j h} x^T(t - (i - 1) h) - x(t - ih))^T R_{ij}$$

$$\cdot [x(t - (i - 1) h) - x(t - ih)] - 2\varepsilon \omega^T(t) \omega(t)$$

$$- 2\varepsilon \omega^T K [D_j x(t) + E_j x(t - \gamma(t))]$$

(16)

$$+ h^2 q^T(t) \left( \frac{A^T_j}{B^T_j} \right) \sum_{i=1}^{n} R_{ij} \left( A_j B_j C_j \right)$$

$$= \eta^T(t) \left( \Psi + h^2 \Gamma^T \sum_{i=1}^{n} R_{ij} \Gamma \right) \eta(t)$$

where

$$\Psi = \begin{pmatrix} Y_{11} & Y_{12} \\ * & Y_{22} \end{pmatrix}$$

(17)

$$\eta(t) = \left( x^T(t) \; x^T(t - \tau) \; \omega^T(t) \; \varsigma^T(t) \right)^T$$

with

$$\varsigma(t) = \left( x^T(t - h) \; x^T(t - 2h) \; \cdots \; x^T(t - (n - 1)h) \right)^T.$$  (18)

By Schur complement, it yields that $\Psi + h^2 \Gamma^T \sum_{i=1}^{n} R_{ij} \Gamma < 0$ is equivalent to inequality (9). Then it can be concluded from [17] that system (1) is absolutely exponentially stable.

When $n = 1$, the following corollary can be derived from Theorem 5.

**Corollary 6.** For given $h > 0$, $\alpha_j > 0$, $\mu_j \geq 1$, $j \in M$, system (1) is absolutely exponentially stable in sector $[0, K]$ with MDADT $\tau_{aj} > \tau_{aj}^*$ if there exist a real number $\epsilon > 0$ and symmetric matrices $P_j > 0$, $Q_{ij} > 0$, $R_{ij} > 0$, $j \in M$ such that the following LMIs hold

$$\begin{pmatrix} \Omega_{11} & P_j B_j + R_{ij} & P_j C_j - e^{D_j K^T} \tau A_j R_{1j} \\ * & -e^{-\mu_j \tau_{1j}} & -2\varepsilon I \\ * & * & -R_{1j} \end{pmatrix} < 0$$

(19)

where

$$\Omega_{11} = P_j A_j^T + A_j^T P_j + Q_{1j} - R_{1j}$$

$$P_j \leq \mu_j P_j,$$

$$Q_{1j} \leq \mu_j Q_{1j},$$

$$R_{ij} \leq \mu_j R_{1j},$$

$$\forall i, j \in M.$$ (20)

**Remark 7.** In fact, Corollary 6 is the same as Proposition 3 in [25] when $j = 1$, $m = 1$; also it is Theorem 1 in [31] with $R_{ij} = 0$. So Proposition 3 in [25] and Theorem 1 in [31] are included in Theorem 5 in this paper.

**Remark 8.** In fact, if we set $Q_{ij} = Q_j$, $R_{ij} = 0$, the Lyapunov-Krasovskii functional (8) will reduce to corresponding Lyapunov function (7) ([31, 32]), so (7) can be seen as a special case of (8) which considers both the integration of state and the integration of state’s derivative. As is well known, when a system is considered, the character of the state’s derivative also plays an important role in the stability analysis. Besides, sometimes, it is difficult to get the same $Q_j$ in [31]. Fortunately, in our result, $Q_{ij}$ is allowed to be different. Therefore, the LMIs obtained in this paper is more flexible and general than those obtained by ADT method in [31]. Meanwhile, the method can also be used for the stability analysis of general switched delay system to derive less conservative conditions than those in [32] ($Q_{ij} = Q_j, R_{ij} = R_{1j}$).

Similarly, when $\varphi(t, z(t)) \in [K_1, K_2]$, we have the following result.

**Corollary 9.** For given $h > 0$, $\alpha_j > 0$, $\mu_j \geq 1$, $j \in M$, system (1) is absolutely exponentially stable in sector $[K_1, K_2]$ with MDADT $\tau_{aj} > \tau_{aj}^*$ if there exist a real number
\( \varepsilon > 0 \) and symmetric matrices \( P_i > 0, Q_{ij} > 0, R_{ij} > 0, i = 1, 2, 3, \ldots, n, j \in M \) such that the following LMIs hold

\[
\begin{bmatrix}
\tilde{Y}_{11} & \tilde{Y}_{12} & \tilde{Y}_{13} \\
0 & Y_{22} & 0 \\
0 & 0 & Y_{33}
\end{bmatrix} < 0 \tag{21}
\]

where

\[
\begin{align*}
\tilde{Y}_{11} &= P_j (B_j - C_j D_j) + (A_j - C_j K_1 D_j)^T + P_j + Q_{ij} \\
\tilde{Y}_{12} &= -\varepsilon E_j^T \begin{pmatrix} K_2 - K_1 \end{pmatrix}^T \\
\tilde{Y}_{13} &= P_j C_j - \varepsilon D_j^T \begin{pmatrix} K_2 - K_1 \end{pmatrix}^T
\end{align*}
\]

\[
\tilde{Y}_{11} = P_j (B_j - C_j K_1 D_j) + (A_j - C_j K_1 D_j)^T + P_j + Q_{ij} \\
\tilde{Y}_{12} = -\varepsilon E_j^T (K_2 - K_1)^T \\
\tilde{Y}_{13} = \tilde{P}_j C_j - \varepsilon \tilde{D}_j^T (K_2 - K_1)^T
\]

\[
\begin{align*}
\tilde{P}_j &= \begin{bmatrix} P_j \\ 0 \end{bmatrix}, \\
\tilde{E} &= \begin{bmatrix} E_a^T \\ E_b^T \end{bmatrix}
\end{align*}
\]

\[
\begin{bmatrix}
\tilde{Y}_{11} & \tilde{Y}_{12} & \tilde{Y}_{13} \\
0 & Y_{22} & 0 \\
0 & 0 & Y_{33}
\end{bmatrix} < 0 \tag{26}
\]

Corollary 10. For given \( h > 0, \alpha_j > 0, \mu_j \geq 1, j \in M, \) system (23) is absolutely exponentially stable in sector \([K_1, K_2]\) with MDADT \( r_{a_j} > r_{a_j}^* = (\ln \mu_j)/\alpha_j \), if there exist real numbers \( \varepsilon > 0, \lambda > 0 \) and symmetric matrices \( P_i > 0, Q_{ij} > 0, R_{ij} > 0, i = 1, 2, 3, \ldots, n, j \in M \) such that the following LMIs hold

\[
\begin{bmatrix}
\tilde{Y}_{11} & \tilde{Y}_{12} & \tilde{Y}_{13} & \tilde{P}_j L & \lambda \tilde{E} \\
0 & Y_{22} & 0 & 0 & 0 \\
0 & 0 & Y_{33} & h \sum_{i=1}^n R_{ij} L & 0 \\
0 & 0 & 0 & -\lambda I & 0 \\
0 & 0 & 0 & 0 & -\lambda I
\end{bmatrix} < 0
\]

4. Numerical Example

Consider uncertain system (23) with the following parameters:

\[
A_1 = \begin{pmatrix} -2 & 0 \\ 0.5 & -0.9 \end{pmatrix}, \\
B_1 = \begin{pmatrix} -1 & 0 \\ -1 & -1 \end{pmatrix},
\]

Corollary 9. It is easy to get Corollary 10.
$C_1 = \begin{pmatrix} -0.2 \\ -0.3 \end{pmatrix},$

$D_1 = \begin{pmatrix} 0.46 & 0.8 \end{pmatrix},$

$E_1 = \begin{pmatrix} 0 & 0 \end{pmatrix},$

$K_1 = 0.2,$

$K_2 = 0.5$

$A_2 = \begin{pmatrix} -3 & 0.5 \\ 0.8 & -0.8 \end{pmatrix},$

$B_2 = \begin{pmatrix} -1.5 & 0 \\ -1.2 & -1 \end{pmatrix},$

$C_1 = \begin{pmatrix} -0.4 \\ -0.8 \end{pmatrix},$

$D_2 = \begin{pmatrix} 0.5 & 0.3 \end{pmatrix},$

$E_2 = \begin{pmatrix} 0.2 & 0.5 \end{pmatrix},$

$L = \begin{pmatrix} \beta & 0 \\ 0 & \beta \end{pmatrix}, \quad \beta \geq 0,$

$E_a = E_b = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(28)

for $n = 2, \varepsilon = 0.3, \lambda = 0.5, \alpha_1 = 0.8, \alpha_2 = 0.5, \tau = 0.763, \mu_1 = 2, \mu_2 = 1.9, \beta = 0.5.$ Using the Matlab software, the LMI in Corollary 10 is solvable, and we can get the following.

$P_1 = \begin{pmatrix} 0.1021 & -0.0010 \\ -0.0010 & 0.0212 \end{pmatrix},$

$P_2 = \begin{pmatrix} 0.0107 & 0.0002 \\ 0.0002 & 0.0347 \end{pmatrix},$

$Q_{11} = \begin{pmatrix} 0.0035 & 0.0004 \\ 0.0004 & 0.0034 \end{pmatrix},$

$Q_{12} = \begin{pmatrix} 0.0040 & 0.0003 \\ 0.0003 & 0.0034 \end{pmatrix},$

$Q_{21} = \begin{pmatrix} 0.0025 & 0.0001 \\ 0.0001 & 0.0031 \end{pmatrix},$

$Q_{22} = \begin{pmatrix} 0.0020 & 0.0004 \\ 0.0004 & 0.0038 \end{pmatrix},$

$R_{11} = \begin{pmatrix} 1.6069 & -0.0107 \\ -0.0107 & 1.0008 \end{pmatrix},$

$R_{12} = \begin{pmatrix} 1.2019 & -0.0107 \\ -0.0107 & 1.1002 \end{pmatrix},$

$R_{21} = \begin{pmatrix} 1.2100 & -0.0120 \\ -0.0120 & 1.0008 \end{pmatrix},$

$R_{22} = \begin{pmatrix} 1.3019 & -0.0237 \\ -0.0237 & 1.1022 \end{pmatrix}$

(29)

The simulation is shown in Figures 1 and 2.

Table 1: Comparison with the result in [31].

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>Corollary 2 [31]</th>
<th>Corollary 10, $n=2$</th>
<th>Corollary 10, $n=3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta = 0.00$</td>
<td>2.2845</td>
<td>3.0079</td>
<td>3.1085</td>
</tr>
<tr>
<td>$\beta = 0.05$</td>
<td>2.2352</td>
<td>2.6738</td>
<td>2.7602</td>
</tr>
<tr>
<td>$\beta = 0.10$</td>
<td>2.0243</td>
<td>2.1654</td>
<td>2.4731</td>
</tr>
</tbody>
</table>

The simulation is shown in Figures 1 and 2.

Table 1 lists the maximum allowed time-delay bounds obtained by Corollary 10 for various $\beta$ in comparison with
those obtained by Corollary 2 in [31]. Clearly, the results in this paper are better than those in [31]. Besides, by setting $m = 1$, Corollary 10 reduces to Theorem 2 in [28].

5. Conclusions

In this article, we considered the stability of switched Lur’e systems. In order to derive less conservative criteria, we introduce an appropriate Lyapunov-Krasovskii functional by the method of delay length segmentation ([33]). Besides, the MDADT method is also used to gain more applicable results. In further research, nonlinear switched Lur’e systems with time-varying delays will be considered; in particular, we will consider if the concept ”average impulsive interval” [18] can be used in our further study.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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