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Research Article

Nonlinear Voltage Regulation Algorithm for DC-DC Boost Converter with Finite-Time Convergence

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For the DC-DC Boost converter system, this paper employs the finite-time control technique to design a new nonlinear fast voltage regulation control algorithm. Compared with the existing algorithm, the main advantage of the proposed algorithm lies in the fact that it can offer a fast convergent rate, i.e., finite-time convergence. Based on the average state space model of Boost converter system and finite-time control theory, rigorous stability analysis showed that the output voltage converges to the reference voltage in a finite time. Simulation results demonstrate the efficiency of the proposed method. Compared with PI control algorithm, it is shown that the proposed algorithm has a faster regulation performance and stronger robust performance on load-variation.

1. Introduction

As a kind of important power electronic devices, the main function of DC-DC converters is used to achieve energy conversion, which has been widely applied in many industrial occasions, such as switching power supply, direct current motor drives, and communication equipment. Boost type direct current-direct current (DC-DC) converter is a typical power converter which has many industrial applications such as direct current (DC) motor drives, computer systems, and communication equipment [1]. With the development of distributed power generations, it is required that the DC-DC converter has the high-quality, reliable, efficient power supplies, and other features. However, from the control viewpoint, how to improve the control system performance for DC-DC converter is challenging since DC-DC converters are usually time-varying systems due to their switching operation.

In the past decades, many researchers from automatic control and power electronic have investigated the control problem for this kind of devices. It is well-known that DC-DC power converters are typical switch systems and the Boost type DC-DC converter is non-minimum-phase system, which is a main obstacle for controller's design.

So far, many researchers have employed different nonlinear control methods to design control algorithms for Boost converter. In [2, 3], based on sliding mode control (SMC) method, some SMC algorithms were given and the analog control circuits were set up. In [4], combining SMC and feedback linearization technique, the corresponding SMC algorithm was also designed. Based on LMI technique and saturation control technique, the work [5] proposed a saturated nonlinear control algorithm. In [6], time-delayed compensation technique was employed to design a time-delayed control algorithm.

Actually, for a control system, the steady-state and dynamical performances (e.g., convergent rate) are two key indexes. Note that the most of existing voltage regulation algorithms for Boost converter system only guarantee that the convergence is at best exponential with infinite settling time. Clearly, in practice, it is more desirable if the output voltage can converges to the reference voltage in a finite time. Motivated by this, finite-time control theory has been introduced and developed in the literate [7–13], which guarantees that the system states converge to equilibrium in a finite time. Besides faster convergence rates, the closed-loop systems under finite-time control usually have some other nice features such as higher accuracies and better disturbance rejection properties [7, 14]. Because of the advantages of finite-time control, the work [15] employed the terminal sliding mode technique to design the finite-time voltage regulation algorithm for Buck converter. The works [16, 17] considered the finite-time control law for the DC-DC buck converter system. As for the Boost converter, the work [18] designed a class of finite-time voltage control algorithms, where the input voltage and load resistance are assumed to be known.

This paper will also employ the finite-time control technique to solve the voltage regulation problem for DC-DC Boost converter systems. Different from the existing work [18], this paper considers the design of fast finite-time control algorithm. The contribution/novelty of this paper is that a new nonlinear control algorithm is designed, i.e., the finite-time control algorithm. The main advantage of this algorithm is that the fast convergent rate of the closed-loop system can be guaranteed when the state is near the equilibrium. First, since the DC-DC Boost converter system has a nonlinear structure, a coordinate transform based on the total energy storage function is used to the average state space mode. Then the voltage control problem is equivalent to control the total storage energy. Based on the finite-time control theory, a second-order finite control algorithm is given. Finally, a rigorous proof is given to prove the global finite-time stability of the closed-loop system under the proposed controller. At the end, simulation results are provided to show the potentials of the proposed techniques.

2. Preliminaries and Problem Formulation

2.1. System Model. Figure 1 shows a typical boost type DC-DC converter. V_{in} is a DC input voltage source, S is a controlled switch, D is a diode, V_o is sensed output voltage, i_L denotes the inductance current, and L, C, R are the inductance, capacitance, and load resistance, respectively. If the switching frequency for S is sufficiently high, the dynamic of DC-DC converters can be described by an average state space model [3]. Based on the average state space model [3], the dynamic equation for the Boost converter is

$$\begin{aligned} \dot{i}_L &= -\frac{V_o}{L} + \frac{V_{in}}{L} + \frac{V_o}{L}\mu, \\ \dot{V}_o &= \frac{i_L}{C} - \frac{1}{RC}V_o - \frac{i_L}{C}\mu, \end{aligned} \tag{1}$$

where μ is the duty ratio function (called control input) and $\mu \in [0, 1]$. The Boost type DC-DC converters are used in applications where the required output voltage is larger than the input voltage. Let V_{ref} be the desired DC output reference voltage; then the reference current i_{Lref} can be described by

$$i_{Lref} = \frac{V_{ref}^2}{V_{in}R}. \tag{2}$$

Let $z_1 = (1/2)Li_L^2 + (1/2)CV_o^2$ be the total energy storage function for Boost circuit. Then it can be followed from system (1) that the state z_1 satisfies

$$\begin{aligned} \dot{z}_1 &= z_2 = -\frac{V_o^2}{R} + V_{in}i_L, \\ \dot{z}_2 &= -\frac{2}{RC}V_o i_L + \frac{2}{R^2C}V_o^2 - \frac{V_{in}}{L}V_o + \frac{V_{in}^2}{L} \\ &\quad + \left(\frac{2V_o i_L}{RC} + \frac{V_{in}V_o}{L}\right)\mu. \\ &:= u \\ y &= z_1. \end{aligned} \tag{3}$$

Based on this model, the reference values for system states z_1, z_2 are

$$\begin{aligned} z_{1ref} &= \frac{1}{2}Li_{Lref}^2 + \frac{1}{2}CV_{ref}^2, \\ z_{2ref} &= 0. \end{aligned} \tag{4}$$

Then, the control objective of this paper is to design a nonlinear control algorithm such that the system's output $y = z_1$ can track the reference signal z_{1ref} in *a finite time*. Actually, if there is a time T such that $z_1(t) \equiv z_{1ref}, \forall t \geq T$, then

$$\frac{1}{2}i_L^2 + \frac{1}{2}V_o^2 = \frac{1}{2}i_{Lref}^2 + \frac{1}{2}V_{ref}^2. \tag{5}$$

Meanwhile, note that $i_L = V_o^2/V_{in}R$ when the system state is kept steady. As a result, $z_1(t) \longrightarrow z_{1ref}$ is equivalent to $V_o \longrightarrow V_{ref}$ and $i_L \longrightarrow i_{Lref}$.

Define the tracking error as

$$\begin{aligned} e_1 &= z_1 - z_{1ref}, \\ e_2 &= z_2 - z_{2ref}; \end{aligned} \tag{6}$$

then the error dynamic equation is given as

$$\begin{aligned} \dot{e}_1 &= e_2, \\ \dot{e}_2 &= u. \end{aligned} \tag{7}$$

2.2. Some Useful Definitions and Lemmas. As that in [16, 17], in order to prove that the closed-loop system is finite-time stable, some essential definitions and lemmas are given.

Definition 1 (see [7, 16, 17]). Consider the nonlinear system

$$\dot{x} = f(x), \quad f(0) = 0, \ x \in \mathcal{R}^n, \tag{8}$$

where $f(\cdot) : \mathcal{R}^n \longrightarrow \mathcal{R}^n$ is a continuous vector function. The origin is finite-time stable equilibrium if it is Lyapunov stable and finite-time convergence. The finite-time convergence means that there is a function $T(x_0)$ such that $\lim_{t \longrightarrow T(x_0)} x(t, x_0) = 0$ and $x(t, x_0) \equiv 0, \forall t \geq T(x_0)$.

The homogeneous theory method will be used in this paper to construct the finite-time controller. The definition of homogeneity is given below.

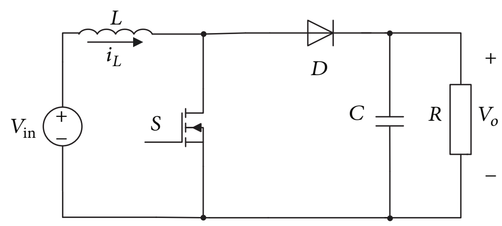

Figure 1: DC-DC Boost converter.

Definition 2 (see [16, 17, 19]). Consider system (8) and define the dilation $(r_1, \ldots, r_m) \in R^m$ with $r_i > 0, i = 1, \ldots, m$. Let

$$f(x) = [f_1(x), \ldots, f_m(x)]^T \tag{9}$$

be a continuous vector field. $f(x)$ is said to be homogeneous of degree $k \in R$ with respect to dilation $(r_1, \ldots, r_m)$ if, for any given $\varepsilon > 0, i = 1, \ldots, m$,

$$f_i(\varepsilon^{r_1} x_1, \ldots, \varepsilon^{r_m} x_m) = \varepsilon^{k+r_i} f_i(x), \quad \forall x \in R^m, \tag{10}$$

where $k > -\min\{r_i, i = 1, \ldots, m\}$. We can say that the system $\dot{x} = f(x)$ is homogeneous if $f(x)$ is homogeneous.

In addition, the following definition is given to make the controller design convenient.

Definition 3. Denote $\mathrm{sig}^{\alpha}(x) = \mathrm{sign}(x)|x|^{\alpha}$, where $\alpha \geq 0$, $x \in R$, and $\mathrm{sign}(\cdot)$ is the standard sign function.

Lemma 4 (see [20]). *For the following system,*

$$\dot{x} = f(x) + \hat{f}(x), \quad f(0) = 0, \ \hat{f}(0) = 0, \ x \in R^n, \tag{11}$$

where $f(x)$ is a continuous homogeneous vector space and is homogeneous of degree $k < 0$ with respect to the dilation $(r_1, \ldots, r_n)$. If $x = 0$ is the asymptotically stable equilibrium point of the system $\dot{x} = f(x)$, and $\forall x \neq 0$,

$$\frac{\hat{f}_i(\varepsilon^{r_1} x_1, \ldots, \varepsilon^{r_n} x_n)}{\varepsilon^{r_i+k}} = 0, \quad i = 1, 2, \ldots, n, \tag{12}$$

then $x = 0$ is a locally finite-time stable equilibrium of system (11). In addition, if system (11) is not only globally asymptotically stable, but also locally finite-time stable, then it is globally finite-time stable.

3. Main Results

In this section, we present the main results.

Theorem 5. *For system (7), if the controller is designed as*

$$u = -k_1 [\mathrm{sig}^{\alpha_1}(e_1) + e_1] - k_2 [\mathrm{sig}^{\alpha_2}(e_2) + e_2], \tag{13}$$

where $k_1 > 0, k_2 > 0, 0 < \alpha_1 < 1, \alpha_2 = 2\alpha_1/(1 - \alpha_1)$, then the states of system (7) will converge to zero in a finite time; i.e., $(e_1(t), e_2(t)) \longrightarrow 0$ in a finite time.

Proof. The proof can be divided into two steps, i.e., global asymptotic stability and local finite-time stability. The closed-loop system is

$$\begin{aligned} \dot{e}_1 &= e_2, \\ \dot{e}_2 &= -k_1 [\mathrm{sig}^{\alpha_1}(e_1) + e_1] - k_2 [\mathrm{sig}^{\alpha_2}(e_2) + e_2]. \end{aligned} \tag{14}$$

Step 1 (proof of global asymptotic stability). Choose Lyapunov function as follows:

$$\begin{aligned} V &= k_1 \int_0^{e_1} \mathrm{sig}^{\alpha_1}(\rho)\, d\rho + \frac{1}{2} e_2^2 + \frac{1}{2} k_1 e_1^2 \\ &= k_1 \frac{\alpha_1}{1 + \alpha_1} |e_1|^{1+\alpha_1} + \frac{1}{2} e_2^2 + \frac{1}{2} k_1 e_1^2, \end{aligned} \tag{15}$$

Clearly, the Lyapunov function V is positive definite and radially unbounded. Since

$$\frac{d \int_0^{e_1} \mathrm{sig}^{\alpha_1}(\rho)\, d\rho}{dt} = \mathrm{sig}^{\alpha_1}(e_1) e_2, \tag{16}$$

then

$$\begin{aligned} \dot{V} &= k_1 \mathrm{sig}^{\alpha_1}(e_1) \cdot e_2 + k_1 e_1 e_2 \\ &\quad + e_2 [-k_1 \mathrm{sig}^{\alpha_1}(e_1) - k_1 e_1 - k_2 \mathrm{sig}^{\alpha_2}(e_2) - k_2 e_2] \\ &= -k_2 \mathrm{sig}^{\alpha_2}(e_2) e_2 - k_2 e_2^2. \end{aligned} \tag{17}$$

Noticing that

$$e_2 \mathrm{sig}^{\alpha_2}(e_2) = |e_2|^{\alpha_2+1}, \tag{18}$$

then

$$\dot{V} = -k_2 \mathrm{sig}^{\alpha_2}(e_2) e_2 - k_2 e_2^2 \leq 0. \tag{19}$$

Define the set $\Psi = \{(e_1, e_2) \mid \dot{V}\} \equiv 0$. It follows from (19) that $\dot{V} \equiv 0$ means that $e_2 \equiv 0$, and further $\dot{e}_2 \equiv 0$. By (14), it can be obtained that $(e_2, \dot{e}_2) \equiv 0$ implies that $e_1 \equiv 0$. Thus, based on LaSalle invariant principle [21], it can be concluded that $(e_1(t), e_2(t)) \longrightarrow 0$ as $t \longrightarrow 0$. That is to say, system (14) is globally asymptotically stable.

Step 2 (proof of local finite-time stability). Rewrite system (14) as follows:

$$\begin{aligned} \dot{e}_1 &= e_2, \\ \dot{e}_2 &= -k_1 \mathrm{sig}^{\alpha_1}(e_1) - k_2 \mathrm{sig}^{\alpha_2}(e_2) + g(e_1, e_2), \end{aligned} \tag{20}$$

where $g(e_1, e_2) = -k_1 e_1 - k_2 e_2$.

Choose Lyapunov function as

$$W = k_1 \int_0^{e_1} \mathrm{sig}^{\alpha_1}(\rho)\, d\rho + \frac{1}{2} e_2^2. \tag{21}$$

According to (20), the derivative of (21) is

$$\dot{W} = -k_2 e_2 \mathrm{sig}^{\alpha_2}(e_2) = -k_2 |e_2|^{\alpha_2+1} \leq 0. \tag{22}$$

Similar to the proof in Step 1, it can be first proved that system (20) is asymptotically stable.

In addition, note that $0 < \alpha_1 < 1, \alpha_2 = 2\alpha_1/(1+\alpha_1)$; according to Definition 2, it can be verified that system (20) is homogeneous of degree $m = (\alpha_1 - 1)/2 < 0$ with respect to the dilation $(r_1, r_2) = (1, (\alpha_1+1)/2)$.

For any $(e_1, e_2) \neq (0,0)$, according to the definition of function $g(\cdot)$, we obtain

$$\lim_{\varepsilon \longrightarrow 0} \frac{g\left(\varepsilon^{r_1} e_1, \varepsilon^{r_2} e_2\right)}{\varepsilon^{r_2+m}} = 0. \tag{23}$$

Thus, according to Lemma 4, it can be concluded that system (20) is globally finite-time stable.

By the proposed results in Steps 1 and 2, it can be found that system (7) under the controller (13) is globally finite-time stable. The proof is completed. □

Based on this result, we now can design a finite-time voltage regulation algorithm.

Theorem 6. *For the Boost converter system (1), if the duty ratio function μ is designed as*

$$\begin{aligned}\mu = {} & \frac{1}{2V_o i_L/RC + V_{in}V_o/L} \cdot \left[\frac{2}{RC}V_o i_L - \frac{2}{R^2C}V_o^2\right. \\ & + \frac{V_{in}}{L}V_o - \frac{V_{in}^2}{L} - k_1 \operatorname{sig}^{\alpha_1}(e_1) - k_1 e_1 - k_2 \operatorname{sig}^{\alpha_2}(e_2) \\ & \left. - k_2 e_2\right],\end{aligned} \tag{24}$$

$$e_1 = \frac{1}{2}Li_L^2 + \frac{1}{2}CV_o^2 - \frac{1}{2}L\left(\frac{V_{ref}^2}{V_{in}R}\right)^2 - \frac{1}{2}CV_{ref}^2,$$

$$e_2 = -\frac{V_o^2}{R} + V_{in}i_L,$$

where $k_1 > 0, k_2 > 0, 0 < \alpha_1 < 1, \alpha_2 = 2\alpha_1/(1+\alpha_1)$, then the output voltage V_o will reach the reference voltage V_{ref} in a finite time.

Remark 7. Note that although the sign function is employed in the controller (24), the combination of the sign function and the term $|e_1|^{\alpha_1}$ and $|e_2|^{\alpha_2}$, i.e., $\operatorname{sign}(e_1)|e_1|^{\alpha_1}$ and $\operatorname{sign}(e_2)|e_2|^{\alpha_2}$), is continuous. In addition, it should be pointed out that there is no singularity problem in the proposed controller (24) since $0 < \alpha < 1$.

4. Numerical Simulations

All the simulation data is based on the PSIM (Power Simulation which is developed for power electronics and motor drive) software.

4.1. Simulation Parameters. The parameters of Boost converter are chosen as follows: input voltage $V_{in} = 15V$, inductance $L = 30mH$, capacitance $C = 50\mu F$, load resistance $R = 30\Omega$, and reference voltage $V_{ref} = 40V$.

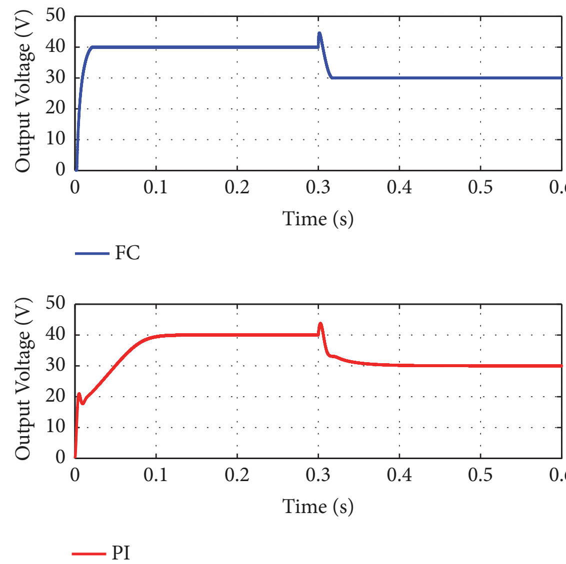

Figure 2: The response curves for output voltage under two control algorithms: finite-time control (FC) and PI control.

To have a comparison, two kinds of control algorithms are used. One is the proposed finite-time control (FC) algorithm (24) in this paper; the other one is PI control algorithm.

For the FC algorithm, the control parameters are chosen as follows: $\alpha_1 = 1/5, \alpha_2 = 1/3, k_1 = 2 \times 10^4, k_2 = 1 \times 10^3$.

For the PI control algorithm, the proportional gain is chosen as $K_p = 0.004$ and the integral gain $K_i = 0.007$.

Respectively, consider the system dynamical response performances under the conditions of boot process, reference voltage change, and load variation.

4.2. Dynamic Responses under Different Reference Voltages. The reference voltage is changed from $40V$ to $30V$ at 0.3 seconds, and the other parameters are kept unchanged. Under the two kinds of control algorithms, the response curves of output voltage are shown in Figure 2. By comparisons, the finite-time control algorithm can offer a faster convergent speed than that of PI control.

4.3. Dynamic Responses in the Presence of Load Variations. The load resistance is changed as follows:

$$R = \begin{cases} 30, & 0 \le t \le 0.2s; \\ 20, & 0.2s < t \le 0.4s; \\ 30, & t > 0.4s, \end{cases} \tag{25}$$

and the other parameters are kept unchanged. Under the finite-time control algorithm and PI control algorithm, the response curves are given in Figure 3. Clearly, the finite-time control algorithm also demonstrates faster regulation speed and stronger disturbance rejection ability.

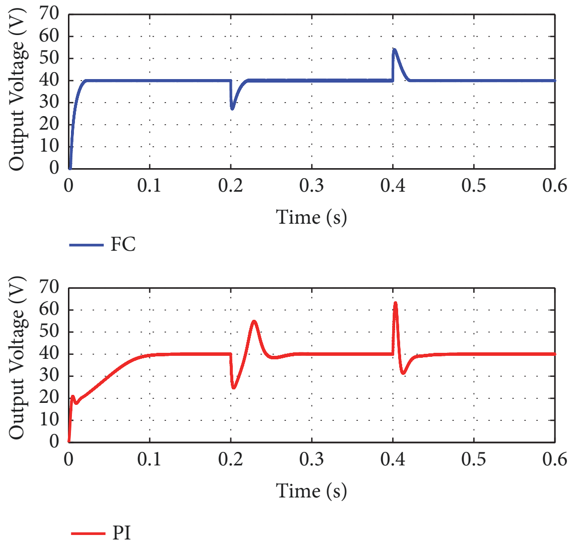

Figure 3: The response curves for output voltage under two control algorithms in the presence of load variations: finite-time control (FC) and PI control.

5. Conclusion

In this paper, a new finite-time control algorithm has been designed for boost converters. It has been proven that the output voltage can track the reference voltage in a finite time. The effectiveness of the proposed method has been verified through simulations.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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