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Research Article

Joint Quality Investment and Inventory Decisions for Perishable Items with Reference Quality Effect under the O2O Environment

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Reference quality, as a benchmark against which the purchase quality of a product is judged, has an important impact on consumers' purchase decision, especially perishable items. Thus, how to determine an appropriate quality investment and ordering strategies for perishable items to maximize retailers' profit is an essential task. A joint dynamic quality investment and inventory replenishment problem for perishable items with consideration of reference quality effect is investigated, where the retailer sells its products using a combination of the offline and online stores, and the sales function is sensitive to products' quality and consumers' reference quality. The optimal control model is established to maximize the retailer's total profit by making a joint quality and inventory strategy. The continuous time dynamic optimal quality investment and inventory replenishment strategies with reference quality effect are obtained by Pontryagin's maximum principle. Numerical experiments are accounted for the key system parameters on the optimal strategies.

1. Introduction

Along with the rapid development of Internet, many traditional retailers in catering and service industries are trying to lay out the offline to online (O2O) platform in order to expand market share and improve the competitiveness of the industry, such as McDonald's and KFC, the world's catering giants; JD.com, yhd.com, Beequick, and Missfresh, the well-known fresh goods retailers in China; and a large number of take-away retailers that are currently emerging. Owing to the fact that most of these O2O retailers mainly operate perishable goods, how to effectively control their inventory is a particularly critical problem. Besides, in recent years, competition is shifting from price to quality in specific segments of the market in some industries, particularly for catering and service industries [1]. Therefore, more retailers are adopting quality improvement as a power competitive tool in the market. For academic and practical perspective, it is necessary to investigate joint quality investment and inventory control decisions for perishable items. Nevertheless, recent studies on behavioral sciences [2] have recognized that consumers are subject to anchoring effects. They indicated that consumers often develop quality expectation for products in repeated transactions. This expectation, captured by the reference quality, acts as a benchmark against which consumers compare the quality of a product. In practice, the reference quality effect has an important impact on demand and therefore becomes an indispensable part of retailers' decision making.

Due to the significant effect of reference quality on consumers' purchasing behaviors, this paper considers a continuous time dynamic optimization model that determines the optimal quality investment and inventory replenishment strategies for perishable products under reference quality effect in the O2O environment.

Literature related to this paper focuses mainly on joint quality and inventory decisions for perishable products and reference quality effect. Many researchers have studied the joint decisions on quality and inventory for perishable products. Hsu et al. [3] develop a solution procedure to

determine the retailer's replenishment and preservation technology investment policy with a constant rate of deterioration and demand. Hsieh and Dye [4] investigate a production-inventory model with controllable deterioration rate and finite replenishment capacity. Liu et al. [5] study a joint pricing and preservation technology investment strategy while maximizing the total profit from selling a given initial inventory. Bardhan et al. [6] analyze an inventory model with stock-dependent demand and noninstantaneous deterioration; the optimal length of an inventory cycle and investment in preservation technology are obtained in two different inventory scenarios. In all the above quality technology investment papers, the amount of inventory is measured by physical quantity or by quality solely. However, the quality and physical quantity usually decay simultaneously during storage or on shelf life. Moreover, the demand for perishable goods is affected not only by its physical quantity but also by its quality. There are very limited studies on perishable products considering these two factors simultaneously. Qin et al. [7] formulate a pricing and inventory model with quality and physical quantity deteriorating simultaneously, where the demand rate is assumed to be deterministic and dependent on the quality of an item, the sales price per unit, and the on-display stock level. Rabbani et al. [8] analyze an optimal dynamic pricing and replenishment policy for perishable items with simultaneous deterioration of quality and physical quantity, where the demand rate is dependent on the quality of inventory and changes in price over time. They obtain the optimal replenishment cycle, initial price, and discount rate by maximizing the total profit of the system. Lin [9] studies an optimal replenishment model with dynamic pricing and quality investment for perishable products, where the quality and physical quantity deteriorate simultaneously and the demand rate is decreasing in sales price and increasing in quality level. A common characteristic to the aforementioned papers is that the consumers' behaviour is not considered.

The second stream of research related to our work is the reference quality effect in the field of operation and marketing. Hardie et al. [10] developed the notion of reference quality, a parallel concept to the reference price, and empirically demonstrated that the gap between observed quality and reference quality can significantly affect purchase probabilities. It is noteworthy that the reference price research has received widespread attention, such as pricing policies (see, e.g., [11–19]), cooperative pricing and inventory (see, e.g., [20–23]), cooperative pricing and preservation technology investment [24], cooperative advertising [25], and cooperative pricing and advertising [26]. These studies provide efficiency strategies to improve firms' profits by understanding the consumers' reference price effect. However, to our best knowledge, only a few studies consider the reference quality effect. Kopalle and Winer [27] extend Greenleaf's [11] results to incorporate the relationship between expected quality and reference price and consider a model of a monopolist who makes time-varying decisions regarding price and product quality. Gavious and Lowengart [28] examine the relationship between price and reference quality and their combined effect on profits. He et al. [29] develop a cooperative quality investment model that incorporates the effects of the reference quality and reference price on the demand function of a "single-supplier single-manufacturer" supply chain system. He et al. [30] consider a supply chain consisting of a supplier and a retailer and develop a joint quality, pricing, and advertising decisions model with the reference quality effect, where the retailer sells the product via the O2O mode. Xue et al. [31] study a problem of joint pricing and dynamic product quality investment with consumers' reference quality effect under the existence of quality inflation. Although the reference quality is considered by these papers, few of them delve into the discussion of inventory replenishment decision under the reference quality effect.

From the above literature review, there is no formal model presented to investigate the problem of joint quality investment and inventory replenishment decisions for perishable items while simultaneously considering the impact of consumers' reference quality. These motivate us to do the exploration in this aspect in our paper. This paper addresses this issue by establishing a joint quality investment and inventory replenishment model for perishable items with the reference quality effect. The differences between the existing literature and ours are as follows: (1) the existing literature is focused on the offline channel only, while we consider the O2O mode, i.e., combination of online and offline channel. (2) The reference quality effect is seldom considered in the existing literature, which we consider in our work. The model we studied incorporates the following features: (1) O2O business model; (2) perishable products; (3) products' quality and physical quantity deteriorating simultaneously; (4) a continuous time and dynamic environment; (5) quality and reference quality sensitivity and goodwill dependent sales function, where the goodwill is built up by the gap between the product quality and reference quality, as well as investing in product quality; (6) two controls at each time: product quality investment and inventory. The optimal quality investment and inventory replenishment policies are obtained by applying Pontryagin's maximum principle so that the retailer's total profit is maximized. In addition, numerical experiments are employed to assess the sensitivity analysis of the key system parameters on the optimal strategies.

The rest of this paper is organized as follows. We develop the finite planning horizon optimal control model for quality investment and inventory replenishment with reference quality effects in Section 2 and characterize the optimal policies in Section 3. Numerical examples are given in Section 4. Finally, a conclusion is made in Section 5.

2. Model Formulation

Consider a retailer that sells a single type of perishable products to the end consumers using a combination of the offline and online stores (the O2O business mode). The retailer, who faces the consumers' reference quality effect, controls the quality investment $u(t)$ and inventory $I(t)$ over an finite planning horizon $[0, T]$.

Let $G(t)$ denote the product's accumulated goodwill over time t. The quality investment effort $u(t)$ contributes to the accumulation of the goodwill $G(t)$, which evolves

according to the modified Nerlove-Arrow framework [32], i.e.,

$$\dot{G}(t) = \sigma\left(Q(t) - r_Q(t)\right) + \beta u(t) - \varphi G(t), \quad G(0) = G_0 \geq 0, \tag{1}$$

where $Q(t)$ is the quality of product at time t, G_0 is the initial goodwill at time 0, $r_Q(t)$ is the reference quality reflecting consumers' expectation on the product quality, σ and β are the efficiency of quality and quality effort to goodwill accumulation, respectively, and φ is the decay coefficient of goodwill.

As for the reference quality $r_Q(t)$, previous literature such as Weigelt and Camerer [33] and Hellofs and Jacobson [34] proved that consumers will expect a well-known company to provide high quality products, or a product with high goodwill should have a good quality. Following their results, the reference quality is supposed to be

$$r_Q(t) = \xi G(t), \tag{2}$$

with an initial reference quality $r_Q^0 = \xi G_0$, where ξ is a positive parameter.

As in Teng and Thompson (1996), Liu et al. [5], and Lin [9], the basic demand function is described as

$$D\left(p, Q(t)\right) = \gamma\left(a - bp\right)Q(t), \tag{3}$$

where $a, b, \gamma > 0$. The parameter a denotes the basic market size and b and γ represent the demand sensitivity to product sales price p and quality Q, respectively. The demand $D(p, Q(t))$ here is assumed to decrease in p and increase in Q.

It should be noted that the sales price p in (3) is assumed to be constant for the following reasons. First, the main purpose of this paper is to study the impact of the reference quality effect on the retailer's quality and inventory decision making. Second, in some industries, competition is shifting from price to quality in specific segments of the market [1]. In practice, retailers commonly adopt the same price policy but offer products with different qualities in the market. For example, McDonald's and KFC compete by providing products with different designs and taste. Similar examples are common in the sales of fashionable products such as mobile phones, bags, and accessories. Third, this assumption can be found in existing literature on reference effects, such as supply chain cooperative advertising decision [25] and cooperative quality decision [29]. Therefore, this assumption in our model is realistic and reasonable.

When the retailer sells the product via pure offline stores, the product quality will be exposed to the consumers. The quality gap between the expectation and reality will influence the sales significantly. Hence, the sales function can be represented as

$$S_{offline}(t) = \gamma\left(a - bp\right)Q(t) + \delta\left(Q(t) - r_Q(t)\right) + \eta G(t), \tag{4}$$

where $\delta(\delta > 0)$ reflects the offline consumers' reference quality effect. A higher δ implies that consumers are more sensitive to the gap between the actual quality and reference quality; $\delta = 0$ represents having no reference quality effect. The parameter $\eta(\eta > 0)$ is a coefficient representing the effectiveness of goodwill on the sales.

When the retailer sells the product via a pure online store, consumers cannot observe the product quality before the products arrive via an express company. The quality gap between the expectation and reality will not influence the sales under such a situation. Then the sales function can be specified as

$$S_{online}(t) = a - bp + \eta G(t). \tag{5}$$

When the retailer sells the products through both online and offline stores, i.e., the O2O mode, some consumers (called online consumers) purchase from the online store directly, while others will go to the offline stores (called offline consumers). Suppose that the ratios of the offline and online consumers are ϕ and $1-\phi$, respectively [30]. Then the retailer's sales quantity is

$$\begin{aligned} S_{O2O}(t) &= \phi S_{offline}(t) + \left(1 - \phi\right)S_{online}(t) \\ &= \left(a - bp\right)\left[\phi\gamma Q(t) + \left(1 - \phi\right)\right] \\ &\quad + \phi\delta\left(Q(t) - r_Q(t)\right) + \eta G(t). \end{aligned} \tag{6}$$

The sales functions (4) and (5) are two special cases of (6). So the following discussion will focus on the O2O situation.

We introduce the quality and inventory dynamics in this system. Due to the nature of perishable products, the quality and physical quantity deteriorate over time simultaneously. First, the dynamics that concerns the effects of deterioration and investment on quality is modeled as

$$\dot{Q}(t) = -\rho Q(t) + u(t), \tag{7}$$

with an initial quality $Q(0) = Q_0$, where $\rho(\rho > 0)$ is the decay rate of quality and $u(t) \geq 0$ is the quality investment rate at time t. Second, let $I(t)$ be the retailer's inventory level at time t; the inventory level variances can be expressed as

$$\dot{I}(t) = -\theta I(t) - S_{O2O}(t), \tag{8}$$

with an initial inventory level $I(0) = I_0$, where $\theta(\theta > 0)$ is the decay rate of quantity.

Following previous literature such as Xue et al. [31] and Lin [9], the quality expenditure is quadratic of its quality improvement effort $u(t)$, i.e.,

$$C_u = \frac{1}{2}ku^2(t), \tag{9}$$

where $k > 0$ implies an increasing marginal cost of quality investment. The inventory holding cost C_I is a linear function of the current inventory level as

$$C_I = hI(t), \tag{10}$$

where $h(h > 0)$ represents the unit holding cost of items. Furthermore, the purchasing cost is assumed to be c.

The retailer's objective is to make the quality investment and inventory strategies by maximizing its profit over the finite planning horizon $[0, T]$, specified as

$$\begin{aligned}
&\max_{u(\cdot),I(\cdot)} \quad J \\
&\quad = \int_0^T \Big[p\left[\gamma\left(a - bp\right)Q(t) + \phi\delta\left(Q(t) - r_Q(t)\right) + \eta G(t)\right] - hI(t) \\
&\quad - \frac{1}{2}ku^2(t)\Big]\,\mathrm{d}t - cI_0 \\
&\text{s.t.} \quad \dot{I}(t) = -\theta I(t) - S_{O2O}(t), \quad I(T) = 0, \\
&\quad \dot{Q}(t) = -\rho Q(t) + u(t), \quad Q(0) = Q_0, \\
&\quad \dot{G}(t) = \sigma\left(Q(t) - r_Q(t)\right) + \beta u(t) - \varphi G(t), \\
&\quad G(0) = G_0, \\
&\quad u(t) \geq 0,\ I(t) \geq 0,\ Q_0 \geq 0,\ G_0 \geq 0.
\end{aligned} \tag{11}$$

It should be pointed out that the replenishment quantity I_0 is also a decision variable in the optimization problem (11). However, if the decision variable $I(t)$ is determined, then the replenishment quantity I_0 can be determined by setting $I_0 = I(0)$. Thus, the replenishment quantity I_0 is not treated explicitly as a decision variable.

3. Optimal Quality and Inventory Strategies

In this section, we use Pontryagin's maximum principle proposed in Sethi and Thompson (2000) to solve the optimization problem (11) for the optimal quality investment and inventory strategies with reference quality effect.

For convenience, the following notations are introduced:

$$\begin{aligned}
\Delta_1 &= \frac{\eta - \phi\delta\xi}{\theta - \sigma\xi - \varphi}, \\
\Delta_2 &= \frac{\eta - \phi\delta\xi}{\sigma\xi + \varphi}\left(p + \frac{h}{\theta}\right), \\
\Delta_3 &= \frac{1}{\theta - \rho}\left[\frac{\phi h}{\theta}\left(\gamma\left(a - bp\right) + \delta\right) - \sigma\Delta_1\right], \\
\Delta_4 &= \frac{\sigma A_1}{\sigma\xi + \varphi - \rho}, \\
\Delta_5 &= \frac{1}{\rho}\left[\left(p + \frac{h}{\theta}\right)\phi\left(\gamma\left(a - bp\right) + \delta\right) + \sigma\Delta_2\right], \\
A_1 &= -\Delta_1 \mathrm{e}^{(\theta - \sigma\xi - \varphi)T} - \Delta_2 \mathrm{e}^{-(\sigma\xi + \varphi)T}, \\
A_2 &= -\Delta_3 \mathrm{e}^{(\theta - \rho)T} + \Delta_4 \mathrm{e}^{(\sigma\xi + \varphi - \rho)T} - \Delta_5 \mathrm{e}^{-\rho T}.
\end{aligned} \tag{12}$$

The retailer's optimal dynamic quality investment policy can be obtained by the following result.

Proposition 1. *The optimal dynamic quality investment policy $u^*(t)$ is characterized by*

$$\begin{aligned}
u^*(t) = \frac{1}{k}\Big[&A_2 \mathrm{e}^{\rho t} + \left(\beta\Delta_1 + \Delta_3\right)\mathrm{e}^{\theta t} \\
&+ \left(\beta A_1 - \Delta_4\right)\mathrm{e}^{(\sigma\xi + \varphi)t} + \beta\Delta_2 + \Delta_5\Big].
\end{aligned} \tag{13}$$

Proof. The current-value Hamiltonian function for the retailer over the sales period $[0, T]$ is given by

$$\begin{aligned}
H(u, I, Q, G) = {} & p\left\{\left(a - bp\right)\left[\phi\gamma Q(t) + \left(1 - \phi\right)\right]\right. \\
& \left. + \phi\delta\left(Q(t) - r_Q(t)\right) + \eta G(t)\right\} - hI(t) - \frac{1}{2}ku^2(t) \\
& - cI_0 + \lambda_1\left\{-\theta I(t) - \left(a - bp\right)\left[\phi\gamma Q(t) + \left(1 - \phi\right)\right]\right. \\
& \left. + \phi\delta\left(Q(t) - r_Q(t)\right) + \eta G(t)\right\} + \lambda_2\left(u(t)\right. \\
& \left. - \rho Q(t)\right) + \lambda_3\left[\sigma\left(Q(t) - r_Q(t)\right) + \beta u(t)\right. \\
& \left. - \varphi G(t)\right],
\end{aligned} \tag{14}$$

where λ_1, λ_2, and λ_3 are the adjoint variables associated with the state variables $I(t), Q(t)$, and $G(t)$, respectively.

Applying general control theory, we have the following optimization condition:

$$\frac{\partial H}{\partial u} = -ku(t) + \lambda_2 + \beta\lambda_3 = 0, \tag{15}$$

which yields the optimal quality investment policy as

$$u^*(t) = \frac{1}{k}\left(\lambda_2 + \beta\lambda_3\right) \tag{16}$$

Furthermore, the adjoint variables λ_1, λ_2, and λ_3 satisfy the following adjoint equations:

$$\dot{\lambda}_1 = -\frac{\partial H}{\partial I} = \theta\lambda_1 + h, \tag{17}$$

$$\begin{aligned}
\dot{\lambda}_2 &= -\frac{\partial H}{\partial Q} \\
&= \left(\lambda_1 - p\right)\left[\gamma\left(a - bp\right) + \phi\delta\right] + \rho\lambda_2 - \sigma\lambda_3,
\end{aligned} \tag{18}$$

$$\dot{\lambda}_3 = -\frac{\partial H}{\partial G} = \left(\phi\delta\xi - \eta\right)\left(p - \lambda_1\right) + \left(\sigma\xi + \varphi\right)\lambda_3. \tag{19}$$

Solving the adjoint equation (17) with the transversality condition $\lambda_1(0) = 0$ gives

$$\lambda_1 = \frac{h}{\theta}\left(\mathrm{e}^{\theta t} - 1\right). \tag{20}$$

Solving the adjoint equation (19) with the transversality condition $\lambda_3(T) = 0$ by substituting (19) yields

$$\lambda_3 = A_1 \mathrm{e}^{(\sigma\xi + \varphi)t} + \Delta_1 \mathrm{e}^{\theta t} + \Delta_2. \tag{21}$$

Substituting (20) and (21) into (18) and then solving (18) with the transversality condition $\lambda_2(T) = 0$, we get

$$\lambda_2 = A_2 \mathrm{e}^{\rho t} + \Delta_3 \mathrm{e}^{\theta t} - \Delta_4 \mathrm{e}^{(\sigma\xi + \varphi)t} + \Delta_5, \tag{22}$$

and this, together with (16), yields (13). The proof is complete. □

For convenience, the following notations are introduced in the following sequel:

$$\begin{aligned}
\Delta_6 &= \frac{A_2}{2\rho},\\
\Delta_7 &= \frac{\beta\Delta_1+\Delta_3}{\theta+\rho},\\
\Delta_8 &= \frac{\beta A_1-\Delta_4}{\sigma\xi+\varphi+\rho},\\
\Delta_9 &= \frac{\beta\Delta_2+\Delta_5}{\rho},\\
\Delta_{10} &= \frac{\sigma B_1}{\sigma\xi+\varphi-\rho},\\
\Delta_{11} &= \frac{1}{\sigma\xi+\varphi+\rho}\left(\frac{\beta A_2}{k}+\sigma\Delta_6\right),\\
\Delta_{12} &= \frac{1}{\sigma\xi+\varphi+\theta}\left[\frac{\beta(\beta\Delta_1+\Delta_3)}{k}+\sigma\Delta_7\right],\\
\Delta_{13} &= \frac{1}{2(\sigma\xi+\varphi)}\left[\frac{\beta(\beta A_1-\Delta_4)}{k}+\sigma\Delta_8\right],\\
\Delta_{14} &= \frac{\beta}{k(\sigma\xi+\varphi)}(\beta\Delta_2+\Delta_5),\\
\Delta_{15} &= \frac{1}{\theta-\rho}\left[\phi(\gamma(a-bp)+\delta)B_1+(\eta-\phi\delta\xi)\Delta_{10}\right],\\
\Delta_{16} &= \frac{1}{\theta+\rho}\left[\phi(\gamma(a-bp)+\delta)\Delta_6+(\eta-\phi\delta\xi)\Delta_{11}\right],\\
\Delta_{17} &= \frac{1}{2\theta}\left[\phi(\gamma(a-bp)+\delta)\Delta_7+(\eta-\phi\delta\xi)\Delta_{12}\right],\\
\Delta_{18} &= \frac{1}{\theta+\sigma\xi+\varphi}\left[\phi(\gamma(a-bp)+\delta)\Delta_8\right.\\
&\quad\left.+(\eta-\phi\delta\xi)\Delta_{13}\right],\\
\Delta_{19} &= \frac{\eta-\phi\delta\xi}{\theta-\sigma\xi-\varphi}B_2,\\
\Delta_{20} &= \frac{1}{\theta}\left[\phi(\gamma(a-bp)+\delta)\Delta_9+(\eta-\phi\delta\xi)\Delta_{14}\right],\\
B_1 &= Q_0-\Delta_6-\Delta_7-\Delta_8-\Delta_9,\\
B_2 &= G_0-\Delta_{10}-\Delta_{11}-\Delta_{12}-\Delta_{13}-\Delta_{14},\\
B_3 &= \Delta_{15}e^{(\theta-\rho)T}+\Delta_{16}e^{(\theta+\rho)T}+\Delta_{17}e^{2\theta T}\\
&\quad+\Delta_{18}e^{(\sigma\xi+\varphi+\theta)T}+\Delta_{19}e^{(\theta-\sigma\xi-\varphi)T}+\Delta_{20}e^{\theta T}.
\end{aligned}\tag{23}$$

As the dynamic optimal quality investment policy presented in Proposition 1, the retailer's optimal quality level $Q^*(t)$, optimal goodwill level $G^*(t)$, and optimal inventory level $I^*(t)$, the consumers' optimal reference quality path $r_Q^*(t)$ can be characterized by the following proposition.

Proposition 2. *The retailer's optimal quality level $Q^*(t)$, optimal goodwill level $G^*(t)$, and optimal inventory level $I^*(t)$ are characterized by*

$$Q^*(t) = B_1e^{-\rho t}+\Delta_6e^{\rho t}+\Delta_7e^{\theta t}+\Delta_8e^{(\sigma\xi+\varphi)t}+\Delta_9, \tag{24}$$

$$\begin{aligned}
G^*(t) &= B_2e^{-(\sigma\xi+\varphi)t}+\Delta_{10}e^{-\rho t}+\Delta_{11}e^{\rho t}+\Delta_{12}e^{\theta t}\\
&\quad+\Delta_{13}e^{(\sigma\xi+\varphi)t}+\Delta_{14},
\end{aligned}\tag{25}$$

$$\begin{aligned}
I^*(t) &= B_3e^{-\theta t}-\Delta_{15}e^{-\rho t}-\Delta_{16}e^{\rho t}-\Delta_{17}e^{\theta t}\\
&\quad-\Delta_{18}e^{(\sigma\xi+\varphi)t}-\Delta_{19}e^{-(\sigma\xi+\varphi)t}-\Delta_{20},
\end{aligned}\tag{26}$$

while the consumers' optimal reference quality path $r_Q^*(t)$ is

$$\begin{aligned}
r_Q^*(t) &= \xi G^*(t) = \xi\left[B_2e^{-(\sigma\xi+\varphi)t}+\Delta_{10}e^{-\rho t}+\Delta_{11}e^{\rho t}\right.\\
&\quad\left.+\Delta_{12}e^{\theta t}+\Delta_{13}e^{(\sigma\xi+\varphi)t}+\Delta_{14}\right].
\end{aligned}\tag{27}$$

Proof. First, substituting (13) into (7) and then solving (7) with the boundary condition $Q(0) = Q_0$, we can obtain (24). Second, substituting (13) and (24) into (1), the solution (25) to the differential equation (1) is obtained with the boundary condition $G(0) = G_0$. This, together with (2), yields (27). Third, substituting (24) and (25) into (8) and then solving (8) with the boundary condition $I(T) = 0$, we can thus get (26). The proof is complete. □

4. Numerical Analysis

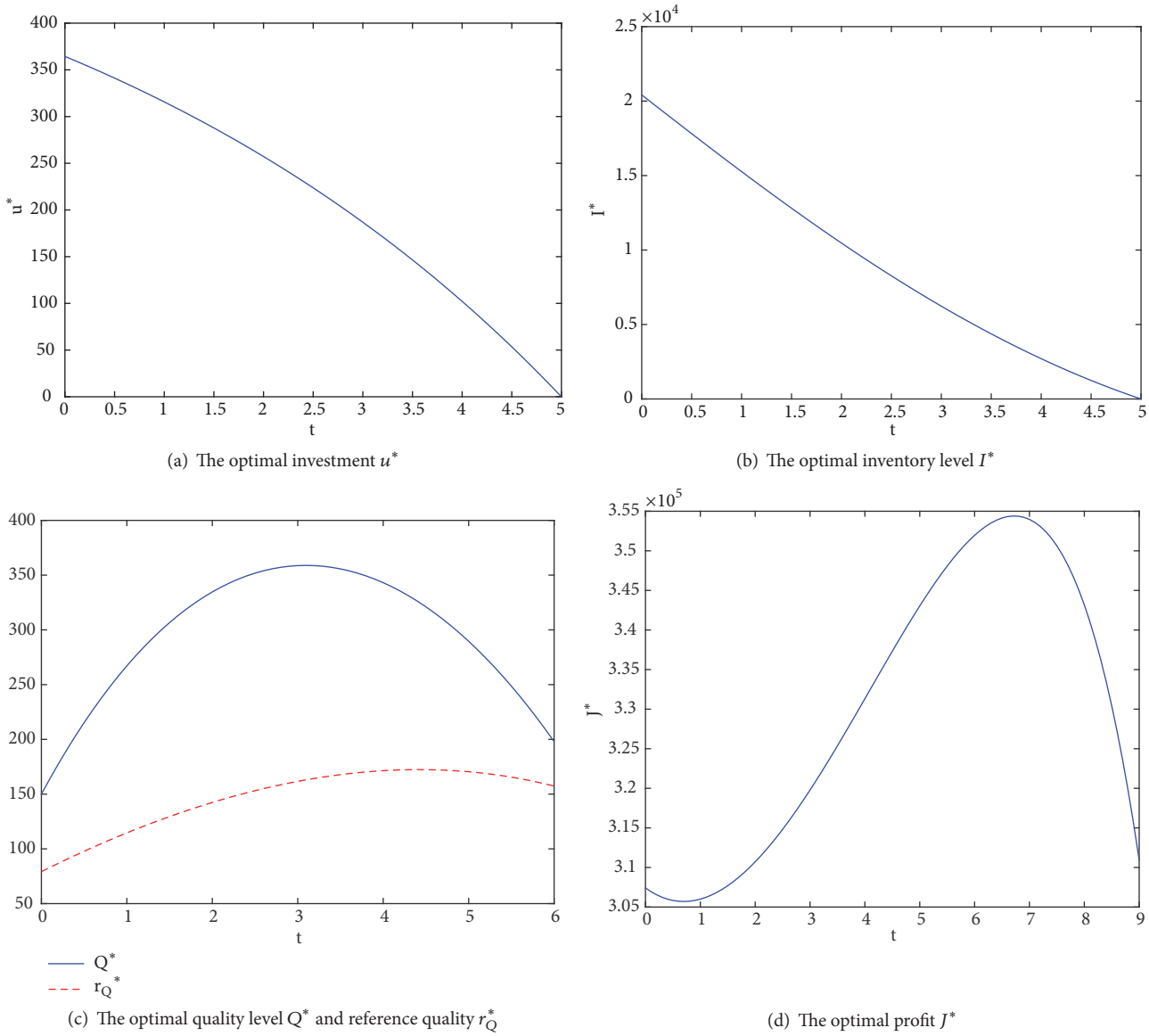
In this section, we present several numerical experiments to illustrate the above theoretical results and gain some managerial insights. All experiments below are performed in MATLAB R2014b on a laptop with an Intel(R) Core (TM) i5-7200U central processing unit CPU (2.50 GHz, 2.70GHz) and 8.0 GB of RAM running 64-bit Windows 10 Enterprise.

Consider the following initial parameters: $a = 30, b = 1, c = 0.5, \gamma = 0.3, \sigma = 0.02, \delta = 0.15, \beta = 0.2, \varphi = 0.1, \eta = 2, \phi = 0.5, \rho = 0.25, \theta = 0.2, \xi = 0.67, k = 0.5, h = 0.15, p = 10, T = 5, Q_0 = 150, G_0 = 120, r_Q^0 = \xi G_0 = 80$. Some of these parameters are chosen from the previous researches for a comprehensive illustration (e.g., [9, 17, 26, 31]). With the parameters above, we can get the graphs of the optimal quality investment u^*, the optimal inventory level I^*, the optimal quality level Q^*, the optimal reference quality level r_Q^*, and the optimal profit J^*, which are shown in Figure 1. Further, sensitivity analysis of the key system parameters, including the reference quality effect coefficient δ and the deterioration coefficients ρ and θ with respect to quality and quantity, as well as the ratio ϕ, is presented on the optimal quality investment and inventory strategies. We vary one of the four variables while keeping the other parameters fixed at the initial values. The results of sensitivity analysis on the optimal quality investment u^* and the optimal inventory level I^* are shown in Figures 2 and 3, respectively. Table 1 displays the impact of these system parameters on the optimal profit J^*.

As shown in Figure 1, the optimal quality investment u^* and inventory level I^* are decreasing in T. The optimal

Table 1: Sensitivity analysis of optimal profit J^* with respect to parameters δ, ρ, θ, and ϕ.

δ	J^*	ρ	J^*	θ	J^*	ϕ	J^*
0.15	35845	0.25	35845	0.1	37714	0.1	41145
0.25	35783	0.45	26530	0.2	34845	0.3	42302
0.35	35720	0.55	23476	0.3	27753	0.5	35845
0.45	35655	0.65	21102	0.4	16731	0.7	22076
0.55	35589	0.75	19226	0.5	1463.1	0.9	9930.6



(a) The optimal investment u^*

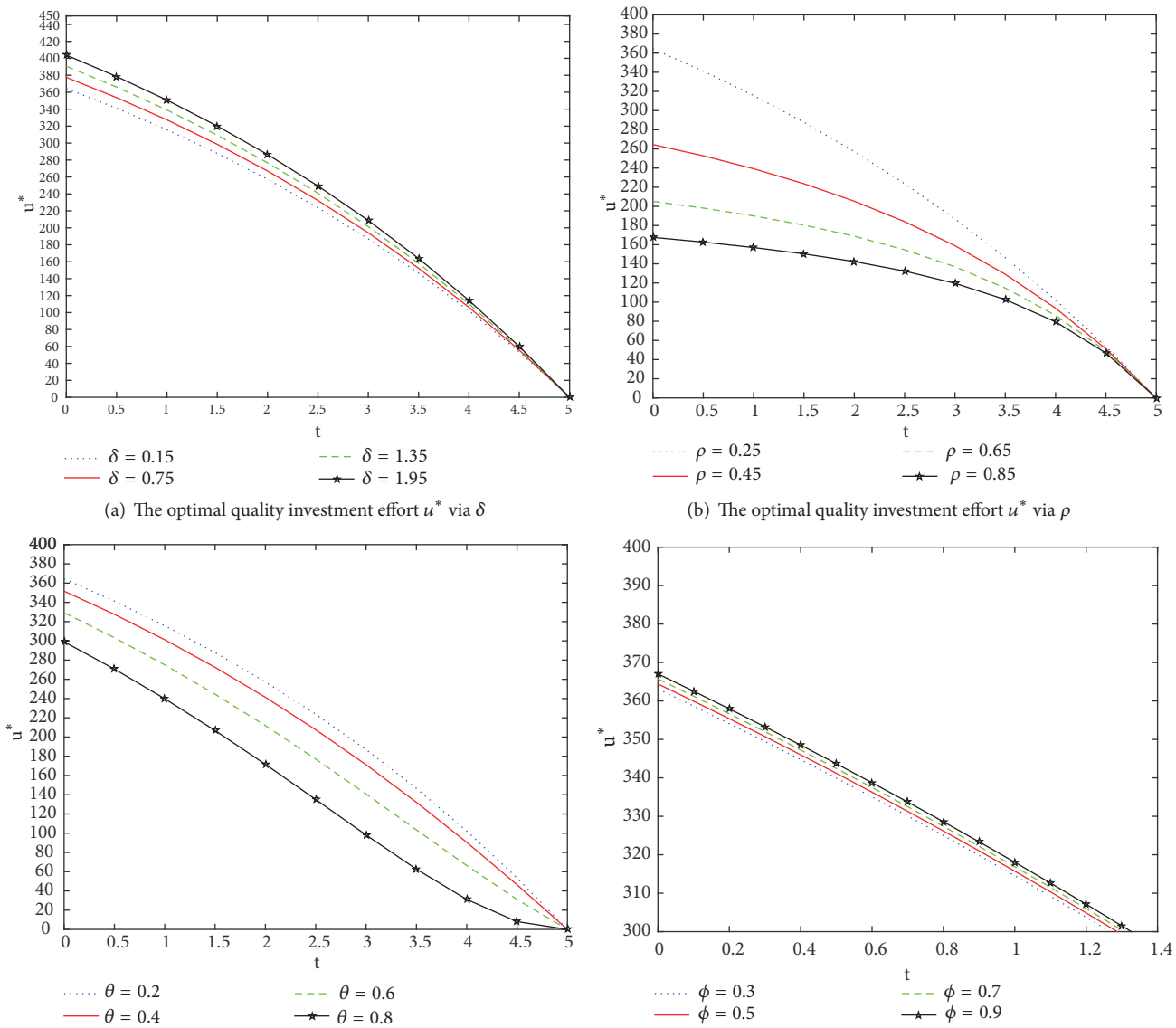
(b) The optimal inventory level I^*

(c) The optimal quality level Q^* and reference quality r_Q^*

(d) The optimal profit J^*

Figure 1: The optimal strategies versus T.

quality level Q^* and reference quality level r_Q^* are concave in T. The optimal profit J^* first decreases with T and then shows concavity. In addition, Figure 1(a) illustrates that, at the beginning of the planning horizon, a high quality investment is utilized to increase the products' quality level and stimulate demand. At the same time, it can further reduce the loss of deterioration. At the end of planning horizon, the quality investment effort is reduced gradually to zero to save quality investment cost. Figures 1(b) and 1(c) indicate that, at the beginning of the sales cycle, with the increase of quality level, the difference between the products' quality and reference quality is also increasing, so the inventory decreases quickly. As the quality level decreases, the gap between the products' quality and the reference quality



(a) The optimal quality investment effort u^* via δ

(b) The optimal quality investment effort u^* via ρ

(c) The optimal quality investment effort u^* via θ

(d) The optimal quality investment effort u^* via ϕ

Figure 2: The impacts of parameters δ, ρ, θ, and ϕ on the optimal quality investment effort u^*.

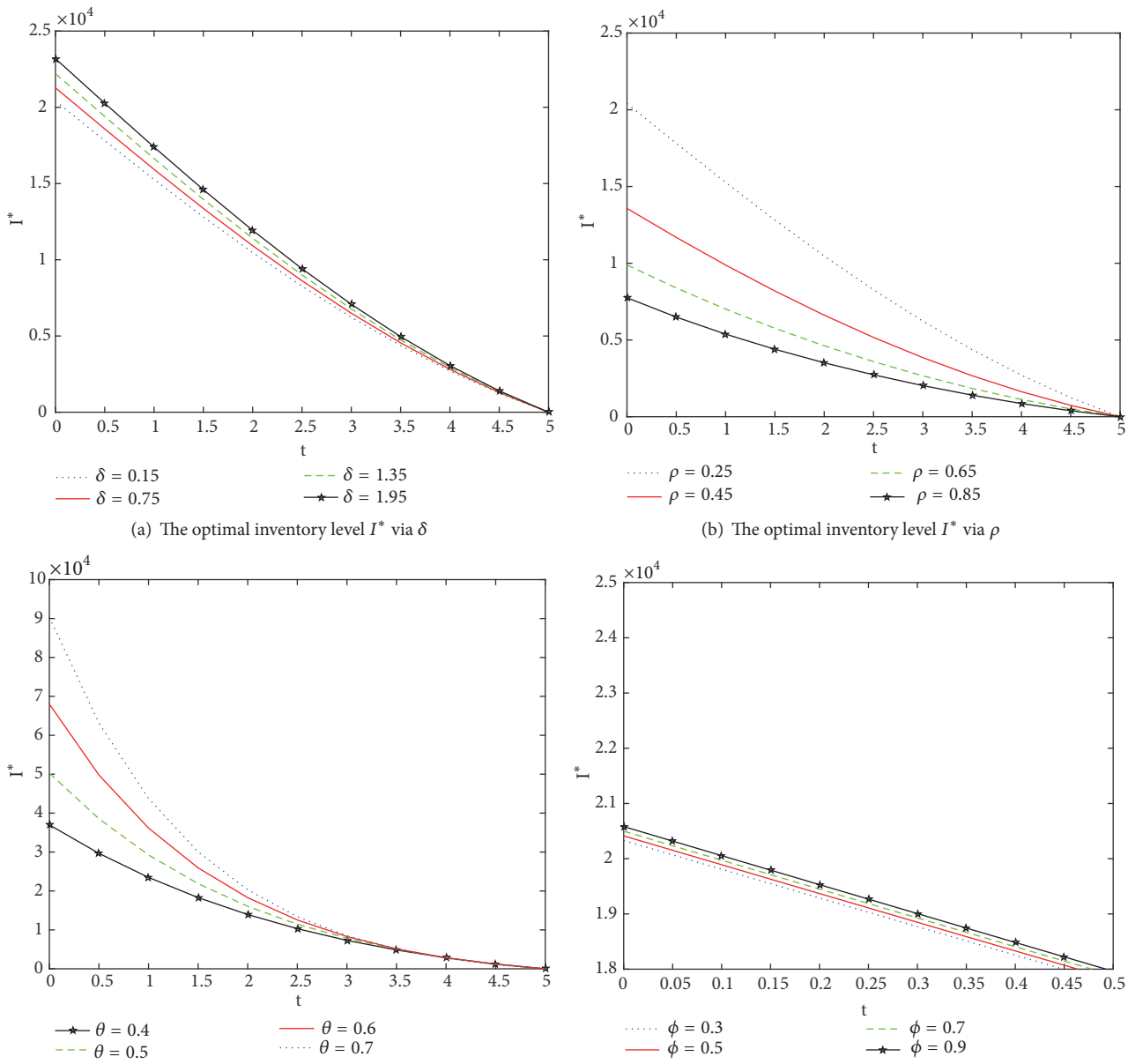
becomes small. Consequently, the inventory level decreases slowly. Figure 1(d) demonstrates that, at the beginning of the sales period, high quality investment and high inventory holding cost will reduce the retailer's profit. Thereafter, as the difference between the product quality and the reference quality increases, the demand of consumers will increase, which makes profit increase.

Figure 2 presents the effects of $\delta, \rho, \theta, \phi$ on the optimal quality investment effort u^*, where these parameters are taken separately from sets $\{0.15, 0.75, 1.35, 1.95\}$, $\{0.25, 0.45, 0.65, 0.85\}$, $\{0.2, 0.4, 0.6, 0.8\}$, and $\{0.3, 0.5, 0.7, 0.9\}$. From Figure 2, we can observe the following managerial insights.

(1) Figure 2(a) presents the impact of reference quality effect coefficient δ on the optimal quality investment effort u^*. From Figure 2(a), we can see that the optimal quality investment effort u^* increases with δ. When δ is large, which means the effect of the gap between the products' quality and consumer' reference quality is great, the retailer should invest more on products' quality. This result is consistent with that of Lin [9] on retailers in physical stores, whose research ignores the reference quality effect of consumers.

(2) Figures 2(b) and 2(c) provide the impact of the deterioration coefficients ρ and θ on the optimal quality investment effort u^*. Figures 2(b) and 2(c) show that the optimal quality investment effort u^* decreases as ρ and θ increase. The intuition is that items with high deterioration coefficients ρ and θ will reduce the reference quality of consumers, which means that the retailer should reduce its quality investment. This result is also consistent with that of Xue et al. [31] and Lin [9].

(3) Figure 2(d) reveals the impact of the ratio of offline consumers ϕ on the optimal quality investment effort u^*.



(a) The optimal inventory level I^* via δ

(b) The optimal inventory level I^* via ρ

(c) The optimal inventory level I^* via θ

(d) The optimal inventory level I^* via ϕ

Figure 3: The impacts of parameters δ, ρ, θ, and ϕ on the optimal inventory level I^*.

Figure 2(d) shows that the optimal quality investment effort u^* increases as ϕ increases. This illustrates that the more consumers patronize physical stores, the more retailers should invest in products' quality.

Figure 3 presents the effects of $\delta, \rho, \theta, \phi$ on optimal inventory level I^*, where these parameters are taken separately from sets $\{0.15, 0.75, 1.35, 1.95\}$, $\{0.25, 0.45, 0.65, 0.85\}$, $\{0.4, 0.5, 0.6, 0.7\}$, and $\{0.3, 0.5, 0.7, 0.9\}$. From Figure 3, we can observe the following managerial insights.

(1) Figure 3(a) presents the impact of reference quality effect coefficient δ on the optimal inventory level I^*. Figure 3(a) shows that the optimal inventory level I^* increases with δ. This is because when consumers are more sensitive to the gap between quality and reference quality, the retailer will invest more in quality, which not only increases the quality level of products, but also increases the goodwill of retailers. It follows from Figure 1(c) and (6) that the retailer's sales will increase, so the retailer will increase its inventory level.

(2) Figure 3(b) reveals the impact of the quality decay rate ρ on the optimal inventory level I^*. It is shown from Figure 3(b) that the optimal inventory level I^* decreases with ρ. The intuition is that products with high quality can not only improve the goodwill of the retailer, but also improve their sales, which makes the retailer increase its inventory level.

(3) Figure 3(c) provides the quantity decay rate θ on the optimal inventory level I^*. It is shown from Figure 3(c) that the optimal inventory level I^* decreases with θ. This suggests

that, for the products with high deteriorating rate θ, the retailer should reduce the replenishment quantity per cycle to reduce the loss caused by deterioration.

(4) Figure 3(d) reveals the impact of the ratio of offline consumers ϕ on the optimal inventory level I^*. Figure 3(d) shows that the optimal inventory level I^* increases as ϕ increases. This illustrates that more offline consumers' patronage will allow retailers to increase their inventory level.

Table 1 presents the effects of $\delta, \rho, \theta, \phi$ on optimal profit J^*. We can observe from Table 1 that the optimal profit J^* decreases with these parameters. Consequently, the reference quality effect coefficient δ and the deterioration coefficients ρ and θ have negative impacts on profit. The monotonicity of J^* with ϕ indicates that the high patronage rate of online consumers will increase the retailer's profit.

5. Conclusion

In this paper, we study a dynamic quality investment and inventory replenishment model of a retailer selling a single type of perishable products in the current O2O environment with reference quality effect in finite planning horizon. The purpose of this paper is to find the optimal dynamic quality investment and inventory replenishment strategies under reference quality effect so that the retailer's total profit is maximized. The continuous time dynamic quality investment and inventory replenishment strategies with reference quality effect are obtained by applying Pontryagins maximum principle. Furthermore, sensitivity analysis is employed to illustrate the impact of the reference quality effect coefficient δ and the deterioration coefficients ρ and θ with respect to quality and quantity, as well as the ratio ϕ on the optimal quality investment, inventory strategies, and optimal profit. Through sensitivity analysis, some management inspiration can be obtained, which can be summarized as follows.

First, due to the negative impact of reference quality effect coefficient δ on retailers' profitability, when consumers are more sensitive to the gap between products' quality and reference quality, the retailer should invest more in the quality of products, which not only increases the quality of products, but also improves the goodwill of the retailer, thus increasing the consumers' demand. This also promotes the retailer to increase its inventory level.

Second, larger deterioration coefficients ρ and θ will reduce the reference quality level of consumers; retailers should thus reduce quality investment and replenishment quantity per cycle to save the cost of quality investment and avoid the loss caused by deterioration. Moreover, this is precisely because of such impact of ρ and θ on consumers, which will also affect the increase in total profit.

Third, as shown in Table 1, in the current O2O environment, expanding the share of online consumers (i.e., a high ϕ) will help retailers to gain more profits. Therefore, for perishable goods, retailers should build an effective online platform in the current O2O environment to achieve high revenue.

Though this paper has identified the effects of reference quality on quality investment and ordering decisions, there are still some shortcomings that can be investigated in the future. First, this paper analyzes the quality and order decisions of a single retailer under reference quality effects and is unaware of the influence of reference quality effects on suppliers. An interesting future research topic is to examine the quality and inventory decisions for suppliers and to design an appropriate coordination mechanism so that a win-win outcome for both parties can be obtained. Second, this paper assumes that the price is constant. In reality, the retailer's quality investment strategy has an important impact on the price strategy. Hence, how to design an effective numerical algorithm to characterize the optimal price strategy is also one of the interesting future research topics. Third, in our study, the customers' reference quality can be observed by retailers. However, the information on customers' reference quality is difficult to get in reality. Thus, demand learning can be incorporated into formulating pricing, quality investment, and inventory strategies in the presence of the reference quality effects.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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