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*Research Article*

# Robust Output Feedback Passivity-Based Variable Structure Controller Design for Nonlinear Systems

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This paper examines the use of an output feedback variable structure controller with a nonlinear sliding surface for a class of SISO nonlinear systems in the presence of matched disturbances. With only the measurable system output, the discontinuous observer reconstructs the system states and ensures that the estimation errors exponentially approach zero. Using the estimation states, the proposed nonlinear sliding surface with variable damping ratio can simultaneously achieve low overshoot and short settling time. Then the passivity-based controller including a discontinuous term can guarantee that the closed-loop system asymptotically converges to the sliding surface. Compared with other sliding mode controllers, the proposed passivity-based control scheme has better transient performance and effectively reduces the control gain. Finally, simulation results demonstrate the validity of the proposed method.

## 1. Introduction

Variable structure control or sliding mode control [1, 2] utilizing a discontinuous control term to drive the plant onto a predesigned surface is a popular robust control method for nonlinear systems with unknown disturbances. The design approach of sliding mode controller is composed mainly of two parts, namely, the design of the sliding surface, which represents the desired behavior in the sliding mode, and the synthesis of control laws such that the closed-loop system can guarantee the reaching and sliding condition. Since the chosen sliding surface affects system performance and the dynamics once sliding mode occurs, the design of the desired sliding surface is very important. For linear systems, the sliding surface is generally designed by assigning the eigenvalues [3], minimizing a quadratic index [4] and Lyapunov function [5]. If the sliding surface is the linear combination of the system states, it becomes a linear sliding surface. However, linear sliding surfaces might not fit the global dynamic property of nonlinear plants [6]. Fulwani et al. [7] combined the composite nonlinear feedback technique to propose the nonlinear sliding surface with the variable damping ratio. These studies [8, 9] proposed the different designs of the nonlinear sliding surfaces to stabilize uncertain systems. Hence, nonlinear sliding surfaces offer a richer variety of design alternatives compared with linear sliding surfaces.

For SISO nonlinear systems with unknown uncertainties and disturbances, the main objective of the controller design is to achieve desired output performance. This problem becomes a great challenge when only output information is available and the system model is not exactly known. Observer-based controller design for nonlinear systems has been a long standing problem, in which high-gain observer [10, 11] is usually used to reconstruct the estimation states. In general, choosing the observer gain large enough (therefore the observer is called the high-gain observer), the estimation error can be made arbitrarily small. However, for high values, initial peaking phenomenon is generated in which large mismatched values between true and estimation values for the short initial period exist in its response. Due to peaking response, the observer-based controller generates high-gain control input, which usually creates an input saturation problem. In recent years, the discontinuous technique for designing observers has been intensively developed [12–17] due to their robustness property. Sliding mode observer [13–15] or robust exact differentiator [16–18] provides an alternative design for estimation of uncertain nonlinear systems.

This paper develops a robust passivity-based variable structure control method for a class of nonlinear systems with matched unknown disturbances based on the estimation information. This discontinuous observer can make the dynamics of estimation error satisfy a strictly positive real lemma. Although the nonlinear system has uncertain and disturbance terms, based on the proposed observer, the system states can be effectively estimated in which the estimation errors will be shown to exponentially approach zero. According to the estimation states, a nonlinear sliding surface with a variable damping ratio is developed in this paper. Hence, system response can obtain a quick response and low overshoot. The passivity-based control law is then proposed, which has a simple structured nonlinear part and a discontinuous control action, to guarantee that the system can asymptotically converge to the sliding surface. If the system is subjected to matched disturbances, it is shown that the closed-loop system can be asymptotically stabilized. The advantage of this approach is that it addresses the problem of designing a controller and the proposed control method overcomes the constraint of input saturation. Compared with the linear sliding surface, the nonlinear sliding surface proposed here has advantages which are examined in a numerical example.

In the next section, a class of SISO nonlinear systems with matched unknown perturbations is introduced. Section 3 first presents an estimation scheme to reconstruct the system states. According to the estimation information, the nonlinear sliding surface with the variable damping ratio is proposed and the passivity-based variable structure control algorithm is developed in this section. To demonstrate the proposed controller, a numerical example is given in Section 4. Finally, Section 5 gives concluding remarks.

## 2. Problem Formulation

Consider a single input single output (SISO) dynamical system with relative degree $n$ in the Brunovsky canonical form [10, 11, 15, 16, 19, 20] as

$$\begin{aligned}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= x_3(t) \\
&\vdots \\
\dot{x}_{n-1}(t) &= x_n(t) \\
\dot{x}_n(t) &= \rho(\boldsymbol{x}, t) + b_2(u(t) + d(t)) \\
y(t) &= x_1(t)
\end{aligned} \tag{1}$$

where $x_i \in \mathbb{R}$, $u \in \mathbb{R}$, $y \in \mathbb{R}$, and $d \in \mathbb{R}$ are the state variable, control input, system output, and unknown matched disturbance, respectively. Moreover, $\boldsymbol{x}^T = [x_1 \ x_2 \ \cdots \ x_n]$ are the system states, $b_2 \in \mathbb{R}$ is a nonzero gain, and $\rho(\boldsymbol{x}, t)$ is an uncertain smooth function. Many nonlinear systems can be transformed into this triangular form (1) by using the feedback linearization technique [19, 20]. If all the states are available, then a state feedback sliding mode controller is designed as

$$\begin{aligned}
u(t) = -\frac{1}{b_2}\Bigg(&\hat{\rho}(\boldsymbol{x}, t) + c_1 x_2(t) + \cdots + c_{n-1} x_n(t) \\
&+ \kappa \frac{\sigma(t)}{|\sigma(t)|}\Bigg)
\end{aligned} \tag{2}$$

where $\hat{\rho}(\boldsymbol{x}, t)$ is the nominal term of $\rho(\boldsymbol{x}, t)$ and $\kappa > 0$ is a gain, which is capable of stabilizing system (2). Moreover, the linear sliding surface $\sigma(t) = c_1 x_1(t) + c_2 x_2(t) + \cdots + x_n(t)$ is usually used in (2). Systems with a relative degree equal to the order of the system have good stabilization qualities [20]. In reality, in most engineering systems only the output of the system is measurable. It follows from (1) that the zero dynamics is constant and equal to zero. In this case, the observer designs including the high-gain observer [10, 11, 19] and sliding mode observer [12–18] can be usually used to estimate the system states. In this paper, the observer structure, which is similar to the sliding mode observer but uses the different analysis method to design the parameters, can precisely estimate the real states.

For transient performance, settling time and overshoot are two important parameters to be selected. To obtain a quick response without any overshoot is the desired goal of the controller design. Fast response and small overshoot cannot be simultaneously obtained using linear sliding surfaces because there is always a tradeoff between these two parameters in linear cases. Based on the estimation information, the nonlinear sliding surface proposed here can guarantee a fast response and low overshoot characteristic. Then a passivity-based variable structure controller design that effectively decreases the control gain and obtains high performance is presented. The observer-based controller gives globally asymptotical stability for the overall closed-loop system.

## 3. Robust Output Feedback Variable Structure Controller Design

In practical control systems, not all state variables are available. There might be a part or only one state measureable. Therefore, it is convenient to develop output feedback robust controllers. One possible solution of this problem is to reconstruct the system states based on measureable system outputs. In this study, the passivity-based variable structure controller including the design of the nonlinear sliding surface is developed by integrating the observer. The proposed nonlinear sliding surface with the variable damping ratio has relatively smaller damping ratio to accelerate the output response during the initial phase. On the other hand, it has relatively bigger damping ratio to avoid system output overshoot during the steady state phase. Since passivity uses energy concepts that are normally used for practical problems, the proposed control method asymptotically stabilizes the closed-loop system and avoids the need for high-gain control.

Let $\hat{x}_i$ for $1 \leq i \leq n$ denote the estimation states of system (1) and $\hat{\boldsymbol{x}} = [\hat{x}_1 \ \cdots \ \hat{x}_n]^T$. Also the function $\hat{\rho}(\hat{\boldsymbol{x}}, t)$ is known a priori. In order to estimate the system states, the discontinuous observer is designed as

$$\begin{aligned}\dot{\hat{x}}_1(t) &= \hat{x}_2(t) + k_1\tilde{x}_1(t) - l_1 v(t)\\ \dot{\hat{x}}_2(t) &= \hat{x}_3(t) + k_2\tilde{x}_1(t) - l_2 v(t)\\ &\vdots\\ \dot{\hat{x}}_n(t) &= \hat{\rho}(\hat{\boldsymbol{x}}, t) + b_2 u(t) + k_n\tilde{x}_1(t) - l_n v(t)\end{aligned} \tag{3}$$

where $\tilde{x}_1 = x_1 - \hat{x}_1$ is the estimation error of the system output, the parameters $k_i > 0$ and $l_i > 0$ designed in the latter are all positive constants and $v \in \mathbb{R}$ is a discontinuous switching term. Slotine et al. [13] have first proposed the sliding observer structure in (3). They applied the concept of equivalent output injection to analyze the estimation performance. The evaluation of equivalent control is not straightforward in usual practice. The different design method of (3) will be presented in the following. First, it follows from (1) and (3) that the estimation error dynamic equations are given by

$$\begin{aligned}\dot{\tilde{x}}_1(t) &= \tilde{x}_2(t) - k_1\tilde{x}_1(t) + l_1 v(t)\\ \dot{\tilde{x}}_2(t) &= \tilde{x}_3(t) - k_1\tilde{x}_1(t) + l_2 v(t)\\ &\vdots\\ \dot{\tilde{x}}_n(t) &= -k_n\tilde{x}_1(t) + l_n v(t) + \tilde{\rho}(t) + b_2 d(t) = -k_n\tilde{x}_1(t)\\ &\quad + l_n v(t) + \omega(t)\end{aligned} \tag{4}$$

where $\tilde{x}_i = x_i - \hat{x}_i$ for $1 \leq i \leq n$ are estimation errors, $\tilde{\rho} = \rho(\boldsymbol{x}, t) - \hat{\rho}(\hat{\boldsymbol{x}}, t)$, and $\omega = \tilde{\rho} + b_2 d$. Let $\tilde{\boldsymbol{x}} = \boldsymbol{x} - \hat{\boldsymbol{x}}$ and write (4) as a matrix form

$$\begin{aligned}\dot{\tilde{\boldsymbol{x}}}(t) &= \boldsymbol{A}\tilde{\boldsymbol{x}}(t) + \boldsymbol{b}v(t) + \boldsymbol{h}\omega(t)\\ \tilde{x}_1(t) &= [1 \ 0 \ \cdots \ 0]\,\tilde{\boldsymbol{x}}(t) = \boldsymbol{c}\tilde{\boldsymbol{x}}(t)\end{aligned} \tag{5}$$

where

$$\begin{aligned}\boldsymbol{A} &= \begin{bmatrix} -k_1 & 1 & 0 & \cdots & 0\\ -k_2 & 0 & 1 & & \\ \vdots & & & \ddots & \\ & & & & 1\\ -k_n & 0 & & \cdots & 0\end{bmatrix},\\ \tilde{\boldsymbol{x}} &= \begin{bmatrix}\tilde{x}_1\\ \tilde{x}_2\\ \vdots\\ \tilde{x}_n\end{bmatrix},\\ \boldsymbol{h} &= \begin{bmatrix}0\\ \vdots\\ 0\\ 1\end{bmatrix},\\ \boldsymbol{b}^T &= [l_1 \ l_2 \ \cdots \ l_n],\\ \text{and } \boldsymbol{c} &= [1 \ 0 \ \cdots \ 0].\end{aligned} \tag{6}$$

Taking the Laplace transformation of the above equation yields

$$\begin{aligned}\tilde{X}_1(s) &= \frac{1}{s^n + k_1 s^{n-1} + k_2 s^{n-2} + \cdots + k_n} T(s)\\ &\quad + \frac{l_1 s^{n-1} + l_2 s^{n-2} + \cdots + l_n}{s^n + k_1 s^{n-1} + k_2 s^{n-2} + \cdots + k_n} V(s)\end{aligned} \tag{7}$$

where $\tilde{X}_1(s) = \mathfrak{I}(\tilde{x}_1(t))$, $T(s) = \mathfrak{I}(\tau(t))$, and $V(s) = \mathfrak{I}(v(t))$ are the Laplace transformations of $\tilde{x}_1(t)$, $\tau(t)$, and $v(t)$, respectively. The parameters $l_i$ for $1 \leq i \leq n$ are chosen such that

$$l_1 s^{n-1} + l_2 s^{n-2} + \cdots + l_n = l_1 (s + \lambda_2)^{n-1} \tag{8}$$

where $l_1 > 0$ is a gain and $\lambda_2 > 0$ is a real value. Besides, the parameters $k_i$ for $1 \leq i \leq n$ are designed as

$$s^n + k_1 s^{n-1} + k_2 s^{n-2} + \cdots + k_n = (s + \lambda_1)^n \tag{9}$$

where $\lambda_1 > 0$ is a real value, so (7) can be rewritten as follows.

$$\tilde{X}_1(s) = \frac{1}{(s + \lambda_1)^n} T(s) + \frac{l_1 (s + \lambda_2)^{n-1}}{(s + \lambda_1)^n} V(s) \tag{10}$$

**Lemma 1** (see [19]). *Let $\overline{\boldsymbol{c}}$ is of $\mathbb{R}^{1\times n}$, $\overline{\boldsymbol{A}}$ is of $\mathbb{R}^{n\times n}$, and $\overline{\boldsymbol{b}}$ is of $\mathbb{R}^{n\times 1}$. Define $H(s) = \overline{\boldsymbol{c}}(s\boldsymbol{I} - \overline{\boldsymbol{A}})^{-1}\overline{\boldsymbol{b}}$ where $(\overline{\boldsymbol{A}}, \overline{\boldsymbol{b}})$ is controllable and $(\overline{\boldsymbol{c}}, \overline{\boldsymbol{A}})$ is observable. The transfer function $H(s)$ is strictly positive real if and only if there exist matrices $\boldsymbol{P} = \boldsymbol{P}^T > 0$, $\boldsymbol{L}$, and a positive constant $\eta > 0$ such that*

$$\begin{aligned}\overline{\boldsymbol{A}}^T\boldsymbol{P} + \boldsymbol{P}\overline{\boldsymbol{A}} &= -\boldsymbol{L}^T\boldsymbol{L} - \eta\boldsymbol{P}\\ \boldsymbol{P}\overline{\boldsymbol{b}} &= \overline{\boldsymbol{c}}^T.\end{aligned} \tag{11}$$

**Lemma 2.** *Consider a transfer function as follows.*

$$\begin{aligned}H(s) &= \frac{l_1 s^{n-1} + l_2 s^{n-2} + \cdots + l_n}{s^n + k_1 s^{n-1} + k_2 s^{n-2} + \cdots + k_n}\\ &= \frac{l_1 (s + \lambda_2)^{n-1}}{(s + \lambda_1)^n}\end{aligned} \tag{12}$$

*The transfer function $H(s)$ is strictly positive real if and only if $n\lambda_1 > (n-1)\lambda_2$.*

*Proof.* According to the definition of strictly positive real function [19], it is known that the transfer function $H(s)$ is strictly positive real if and only if (i) $H(s)$ is Hurwitz, (ii) $\mathrm{Re}(H(j\omega)) > 0$, and (iii) $\lim_{\omega\longrightarrow\infty}\omega^2\mathrm{Re}(H(j\omega)) > 0$. We consider the three cases (1) $n = 2$, (2) $n = 3$, and (3) $n = 4$.

*Case 1* ($n = 2$). First the transfer function is written as $H(s) = l_1(s+\lambda_2)/(s+\lambda_1)^2$ and its real part of $H(j\omega)$ is as follows.

$$\mathrm{Re}\left(H\left(j\omega\right)\right) = \frac{l_1\left(\lambda_2\lambda_1^2 + \left(2\lambda_1 - \lambda_2\right)\omega^2\right)}{\left(\lambda_1^2 - \omega^2\right)^2 + \left(2\lambda_1\omega\right)^2} \tag{13}$$

Since the definition of strictly positive real function requires $\lim_{\omega\longrightarrow\infty}\omega^2\mathrm{Re}(H(j\omega)) > 0$, one can obtain that the condition $2\lambda_1 > \lambda_2$ holds.

*Case 2* ($n = 3$). Similar to the work of Case 1, the real part of $H(j\omega)$ is given by the following.

$$\begin{aligned}\mathrm{Re}\left(H\left(j\omega\right)\right) &= \mathrm{Re}\left(\frac{l_1\left(\left(\lambda_2^2-\omega^2\right)+j2\lambda_2\omega\right)}{\left(\lambda_1^3-3\lambda_1\omega^2\right)+j\left(3\lambda_1^2\omega-\omega^3\right)}\right)\\ &= \frac{l_1\left(\left(\lambda_1^3+3\lambda_1\omega^2\right)\lambda_2^2+\left(3\lambda_1-2\lambda_2\right)\omega^4\right)}{\left(\lambda_1^3-3\lambda_1\omega^2\right)^2+\left(3\lambda_1^2\omega-\omega^3\right)^2}\end{aligned} \tag{14}$$

To satisfy the requirement that the transfer function $H(s)$ is strictly positive real, the condition $3\lambda_1 > 2\lambda_2$ holds.

*Case 3* ($n = 4$). The real part of $H(j\omega)$ is written as follows.

$$\begin{aligned}\mathrm{Re}\left(H\left(j\omega\right)\right) &= \frac{l_1\left(\left(\lambda_2^3-3\lambda_2\omega^2\right)\left(\lambda_1^4-6\lambda_1^2\omega^2\right)+\lambda_2^3\omega^4\right)}{\left(\lambda_1^4-6\lambda_1^2\omega^2+\omega^4\right)^2+16\left(\lambda_1^3\omega-\lambda_1\omega^3\right)^2}\\ &+\frac{l_1\left(4\left(\lambda_1^3\omega-\lambda_1\omega^3\right)3\lambda_2^2\omega-4\lambda_1^3\omega^4+\left(4\lambda_1-3\lambda_2\right)\omega^6\right)}{\left(\lambda_1^4-6\lambda_1^2\omega^2+\omega^4\right)^2+16\left(\lambda_1^3\omega-\lambda_1\omega^3\right)^2}\end{aligned} \tag{15}$$

Hence, the condition $4\lambda_1 > 3\lambda_2$ must hold to satisfy the definition of strictly positive real function. According to the above three cases and the definition of strictly positive real function, the transfer function $H(s) = l_1(s+\lambda_2)/(s+\lambda_1)^2$ is strictly positive real if and only if the condition $n\lambda_1 > (n-1)\lambda_2$ holds. The proof of this lemma is completed. □

**Theorem 3.** *For system (1), the discontinuous observer is designed as*

$$\begin{aligned}\dot{\hat{x}}_1(t) &= \hat{x}_2(t) + k_1\tilde{x}_1(t) - l_1 v(t)\\ \dot{\hat{x}}_2(t) &= \hat{x}_3(t) + k_2\tilde{x}_1(t) - l_2 v(t)\\ &\vdots\\ \dot{\hat{x}}_n(t) &= \hat{\rho}(\hat{\boldsymbol{x}},t) + b_2 u(t) + k_n\tilde{x}_1(t) - l_n v(t)\end{aligned} \tag{16}$$

*where $v(t) = -\rho(\tilde{x}_1(t)/|\tilde{x}_1(t)|^2)$ and $\rho > 0$, chosen to satisfy the certain condition, is a positive constant. If the parameters $k_i$ and $l_i$ are chosen such that the transfer function $H(s)$ is strictly positive real and the uncertain term and unknown disturbance satisfy the following bounded condition:*

$$\left|\tilde{\rho} + b_2 d\right| \le \overline{d} \tag{17}$$

*where $\overline{d} > 0$ is a known constant, then the estimation error exponentially approaches zero. It follows that $\hat{\boldsymbol{x}}(t) \longrightarrow \boldsymbol{x}(t)$ as $t \longrightarrow \infty$.*

*Proof.* First, according to (5) and (6) the error dynamics can be written as follows.

$$\begin{aligned}\dot{\tilde{\boldsymbol{x}}}(t) &= \boldsymbol{A}\tilde{\boldsymbol{x}}(t) - \boldsymbol{b}\rho\frac{\tilde{x}_1(t)}{\left|\tilde{x}_1(t)\right|^2} + \boldsymbol{h}\omega(t)\\ \tilde{x}_1(t) &= \boldsymbol{c}\tilde{\boldsymbol{x}}(t)\end{aligned} \tag{18}$$

From Lemma 2, we apply the condition $n\lambda_1 > (n-1)\lambda_2$ to choose the parameters $k_i$ and $l_i$ such that the transfer function $H(s)$ is strictly positive real. It follows from Lemma 1 that these exists a positive definite matrix $\boldsymbol{P} \in \mathbb{R}^{n\times n}$ such that

$$\begin{aligned}\boldsymbol{A}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A} &= -\boldsymbol{L}^T\boldsymbol{L} - \eta\boldsymbol{P}\\ \boldsymbol{P}\boldsymbol{b} &= \boldsymbol{c}^T.\end{aligned} \tag{19}$$

Let $V(t) = (1/2)\tilde{\boldsymbol{x}}(t)^T\boldsymbol{P}\tilde{\boldsymbol{x}}(t)$ be the Lyapunov function. Taking its time derivative and applying the above relation into it give

$$\begin{aligned}\dot{V} &= \frac{\partial V}{\partial\tilde{\boldsymbol{x}}}\left(\boldsymbol{A}\tilde{\boldsymbol{x}} + \boldsymbol{b}v + \boldsymbol{h}\omega\right)\\ &= \tilde{\boldsymbol{x}}^T\boldsymbol{P}\left(\boldsymbol{A}\tilde{\boldsymbol{x}} - \boldsymbol{b}\rho\frac{\tilde{x}_1(t)}{\left|\tilde{x}_1(t)\right|^2} + \boldsymbol{h}\omega\right)\\ &= \frac{1}{2}\tilde{\boldsymbol{x}}^T\left(\boldsymbol{P}\boldsymbol{A} + \boldsymbol{A}^T\boldsymbol{P}\right)\tilde{\boldsymbol{x}} - \rho + \tilde{\boldsymbol{x}}^T\boldsymbol{P}\boldsymbol{h}\omega\\ &\le -\frac{1}{2}\eta\lambda_{\min}(\boldsymbol{P})\left\|\tilde{\boldsymbol{x}}\right\|^2 - \rho + g_1\left\|\tilde{\boldsymbol{x}}\right\|\end{aligned} \tag{20}$$

where $\lambda_{\min}(\bullet)$ denotes the minimum eigenvalue of $\bullet$ and $g_1 = \|\boldsymbol{Ph}\|\overline{d} > 0$. Hence, if the gain $\rho > 0$ is chosen such that the condition $\rho > g_1^2/2\eta\lambda_{\min}(\boldsymbol{P})$ holds, then the above equation becomes as follows.

$$\begin{aligned}\dot{V} &= -\frac{1}{2}\eta\lambda_{\min}(\boldsymbol{P})\left(\left\|\tilde{\boldsymbol{x}}\right\| - \frac{g_1}{\eta\lambda_{\min}(\boldsymbol{P})}\right)^2 - \rho\\ &+ \frac{g_1^2}{2\eta\lambda_{\min}(\boldsymbol{P})}\\ &\le -\frac{1}{2}\eta\lambda_{\min}(\boldsymbol{P})\left(\left\|\tilde{\boldsymbol{x}}\right\| - \frac{g_1}{\eta\lambda_{\min}(\boldsymbol{P})}\right)^2\end{aligned} \tag{21}$$

According to the definition of Lyapunov stability [19] and the above inequalities, it can be concluded that the estimation error states exponentially approach zero. It follows that $\tilde{\boldsymbol{x}}(t) \longrightarrow 0$ as $t \longrightarrow \infty$ and $\hat{\boldsymbol{x}}(t) \longrightarrow \boldsymbol{x}(t)$ as $t \longrightarrow \infty$. The proof of this theorem is finished. □

Although the system has unknown input and has the relative degree $n$, the proposed discontinuous observer (16) can precisely estimate the system states. This observer can be separately designed from a controller; therefore, the separation principle is satisfied and the total closed-loop system stability is guaranteed.

*Remark 4.* In this paper, inspired by but different from the sliding mode observer, the modified discontinuous observer can overcome some of the typical problems that may be posed to a sliding mode observer. The main contribution of the proposed observer is that we do not apply the concept of equivalent control but use the strictly positive real condition to analyze the estimation performance.

According to the estimation states, conventional sliding mode controllers [1, 2, 15, 16] for system (1) design the linear sliding surface as

$$\sigma(t) = c_1\hat{x}_1(t) + c_2\hat{x}_2(t) + \cdots + \hat{x}_n(t) \tag{22}$$

with the linear sliding surface poles being fixed, and there is always a tradeoff between settling time and overshoot. When large mismatched values between the real and estimation states exist, peaking phenomenon is generated by the observer. Since the dynamic behavior of the system is determined by the nature of the sliding surface, when using the linear sliding surface, it follows that a large control input is required. However, the need for high-gain control usually creates an input saturation problem. To eliminate the peaking response induced by the observer, the control input is saturated outside a compact set of interest. The calculation of bounded region might not be straightforward [11] and the closed-loop system stability becomes complex. In the following, the design methods of the nonlinear sliding surface and the passivity-based controller using the estimation states are addressed to avoid the high-gain control.

Let $\boldsymbol{x}_1^T = [x_1 \ \ x_2 \ \ \cdots \ \ x_{n-2}] \in \mathbb{R}^{n-2}$ and rewrite system (1) as

$$\begin{aligned} \dot{\boldsymbol{x}}_1(t) &= \boldsymbol{A}_{11}\boldsymbol{x}_1(t) + \boldsymbol{A}_{12}x_{n-1}(t) \\ \dot{x}_{n-1}(t) &= x_n(t) \\ \dot{x}_n(t) &= \rho(\boldsymbol{x}, t) + b_2(u(t) + d(t)) \end{aligned} \tag{23}$$

where $\boldsymbol{A}_{11} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & & \\ & & \ddots & & \\ & & & 0 & 1 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \in \mathbb{R}^{(n-2)\times(n-2)}$ and $\boldsymbol{A}_{12} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \in \mathbb{R}^{(n-2)}$. Since the pair $(\boldsymbol{A}_{11}, \boldsymbol{A}_{12})$ is controllable, there exists a gain matrix $\boldsymbol{g} \in \mathbb{R}^{1\times(n-2)}$ such that the matrix $\boldsymbol{A}_{11} - \boldsymbol{A}_{12}\boldsymbol{g}$ is Hurwitz. Define $z_{n-1} = x_{n-1} + \boldsymbol{g}\boldsymbol{x}_1$ and its dynamics can be obtained as

$$\dot{z}_{n-1}(t) = \dot{x}_{n-1}(t) + \boldsymbol{g}\dot{\boldsymbol{x}}_1(t) = x_n(t) + \boldsymbol{g}\boldsymbol{x}_2(t) \tag{24}$$

where $\boldsymbol{x}_2^T = [x_2 \ \ x_3 \ \ \cdots \ \ x_{n-1}] \in \mathbb{R}^{n-2}$. Let $z_n = x_n + \boldsymbol{g}\boldsymbol{x}_2$ and obtain

$$\begin{aligned} \dot{z}_n(t) &= \dot{x}_n(t) + \boldsymbol{g}\dot{\boldsymbol{x}}_2(t) \\ &= \rho(\boldsymbol{x}, t) + b_2(u(t) + d(t)) + \boldsymbol{g}\boldsymbol{x}_3(t) \end{aligned} \tag{25}$$

where $\boldsymbol{x}_3^T = [x_3 \ \ x_4 \ \ \cdots \ \ x_n] \in \mathbb{R}^{n-2}$. Hence, system (23) becomes as follows.

$$\begin{aligned} \dot{\boldsymbol{x}}_1(t) &= (\boldsymbol{A}_{11} - \boldsymbol{A}_{12}\boldsymbol{g})\boldsymbol{x}_1(t) + z_{n-1}(t) \\ \dot{z}_{n-1}(t) &= z_n(t) \\ \dot{z}_n(t) &= \rho(\boldsymbol{x}, t) + \boldsymbol{g}\boldsymbol{x}_3 + b_2(u(t) + d(t)) \end{aligned} \tag{26}$$

Let $\hat{z}_{n-1} = \hat{x}_{n-1} + \boldsymbol{g}\hat{\boldsymbol{x}}_1$ and $\hat{z}_n = \hat{x}_n + \boldsymbol{g}\hat{\boldsymbol{x}}_2$. To avoid peaking phenomena and address the input saturation, based on estimation states we design the nonlinear sliding surface for system (26) as

$$\sigma(t) = \hat{z}_n(t) + \frac{c}{\beta + |\hat{z}_{n-1}(t)|}\hat{z}_{n-1}(t) \tag{27}$$

where $c \in \mathbb{R}$ and $\beta \in \mathbb{R}$ are the positive parameters designed by the user.

**Lemma 5.** *Consider system (26) and use the estimation states to design the sliding surface (27). When $\sigma(t) = 0$, the system performance satisfies $\boldsymbol{x}(t) \longrightarrow \boldsymbol{0}$ as $t \longrightarrow \infty$.*

*Proof.* First, there exist two vectors $\boldsymbol{h}_1 \in \mathbb{R}^{1\times n}$ and $\boldsymbol{h}_2 \in \mathbb{R}^{1\times n}$ such that $z_n = \hat{z}_n + \boldsymbol{h}_1\tilde{\boldsymbol{x}}$ and $z_{n-1} = \hat{z}_{n-1} + \boldsymbol{h}_2\tilde{\boldsymbol{x}}$ where $\boldsymbol{h}_1 = [0 \ \ \boldsymbol{g} \ \ 1]$ and $\boldsymbol{h}_2 = [\boldsymbol{g} \ \ 1 \ \ 0]$. Now the nonlinear sliding surface is written as follows.

$$\sigma(t) = z_n(t) + \frac{c}{\beta + |\hat{z}_{n-1}(t)|}\hat{z}_{n-1}(t) - \boldsymbol{h}_1\tilde{\boldsymbol{x}}(t) \tag{28}$$

With $\sigma(t) = 0$, one can obtain the following.

$$z_n(t) = -\frac{c}{\beta + |\hat{z}_{n-1}(t)|}\hat{z}_{n-1}(t) + \boldsymbol{h}_1\tilde{\boldsymbol{x}}(t) \tag{29}$$

Substituting this term into the dynamics of $z_{n-1}$ in system (26) yields

$$\begin{aligned} \dot{z}_{n-1}(t) &= -\frac{c}{\beta + |\hat{z}_{n-1}(t)|}\hat{z}_{n-1}(t) + \boldsymbol{h}_1\tilde{\boldsymbol{x}} \\ &= -\frac{c}{\beta + |\hat{z}_{n-1}(t)|}z_{n-1}(t) + f_1(\tilde{\boldsymbol{x}}) \end{aligned} \tag{30}$$

where $f_1(\tilde{\boldsymbol{x}}) = (\boldsymbol{h}_1 + (c/(\beta + |\hat{z}_{n-1}(t)|))\boldsymbol{h}_2)\tilde{\boldsymbol{x}}$. According to Theorem 3 and the definition of exponential stability, there exist a vector function $\boldsymbol{f}_o \in \mathbb{R}^n$ and a scalar $\mu > 0$ such that

$$\begin{gathered} \dot{\tilde{\boldsymbol{x}}} = \boldsymbol{f}_o(\tilde{\boldsymbol{x}}) \\ \text{and } \ \|\tilde{\boldsymbol{x}}(t)\| \le p e^{-\mu t}\|\tilde{\boldsymbol{x}}(0)\| \end{gathered} \tag{31}$$

where $\boldsymbol{f}_o(\boldsymbol{0}) = \boldsymbol{0}$ and $p > 0$. Let $a_1 = c/(\beta + |\hat{z}_{n-1}(t)|) > 0$ and $a_2 = \|\boldsymbol{h}_1 + (c/(\beta + |\hat{z}_{n-1}(t)|))\boldsymbol{h}_2\| > 0$ where $\|\bullet\|$ denotes the 2-norm of the matrix $\bullet$. It follows from (30) and (31) that the bound of $z_{n-1}$ satisfies

$$\begin{aligned} |z_{n-1}(t)| &\le |z_{n-1}(0)| e^{-a_1 t} \\ &\quad + a_2 p\|\tilde{\boldsymbol{x}}(0)\| \int_0^t \left| e^{-a_1(t-\tau)} e^{-\mu\tau} \right| d\tau \\ &\le p_1 e^{-a_1 t} + p_2 e^{-\mu t} \end{aligned} \tag{32}$$

where $p_1 > 0$ and $p_2 > 0$ are two positive constants. Hence, it can be concluded from the above equation that $z_{n-1}(t) \longrightarrow 0$ as $t \longrightarrow \infty$. Since $\sigma(t) = 0$ and $z_{n-1}(t) \longrightarrow 0$ as $t \longrightarrow \infty$, the property $z_n(t) \longrightarrow 0$ as $t \longrightarrow \infty$ can be obtained. From $z_{n-1} = x_{n-1} + \boldsymbol{g}\boldsymbol{x}_1$ and $z_n = x_n + \boldsymbol{g}\boldsymbol{x}_2$, it can be concluded that $x_i(t) \longrightarrow 0$ for $i = 1, 2, \cdots, n$ as $t \longrightarrow \infty$ because of $\boldsymbol{A}_{11} - \boldsymbol{A}_{12}\boldsymbol{g}$ being Hurwitz. It follows from (30) that $\boldsymbol{x}(t) \longrightarrow \mathbf{0}$ as $t \longrightarrow \infty$. The proof of this lemma is completed. □

Although the estimation states are used in the sliding surface, it follows from Lemma 5 that the closed-loop stability is guaranteed when $\sigma(t) = 0$. However, the main implementation problem of the nonlinear sliding surface is that the time derivative of the term $(c/(\beta + |\widehat{z}_{n-1}(t)|))\widehat{z}_{n-1}(t)$ is difficult to obtain. In the following, the passivity concept is used to design the controller. Passivity and its application to control of nonlinear systems have been widely studied [21–24] and there have been continuous improvements in recent years in many different areas [24]. A system is passive if and only if the rate of increase of the storage function is not bigger than the supply rate [19], so passivity-based control is an energy-shaping approach. Let us first consider the well-known definitions of passivity.

*Definition 6* (see [19]). A nonlinear system of the form

$$\begin{aligned} \dot{x}(t) &= f(x) + g(x)u \\ y(t) &= h(x) \end{aligned} \tag{33}$$

is *passivity* from input $u$ to the output $y$ if there exists a nonnegative function $V(t)$, with $V(0) = 0$ such that for all $u \in \mathbb{R}$ and all solutions

$$V(x(t)) - V(x(0)) \leq \int_0^t y(\tau)u(\tau)\,d\tau \tag{34}$$

is satisfied for all possible inputs and initial conditions. Moreover, if system (1) satisfies the following more demanding condition

$$\begin{aligned} V(x(t)) - V(x(0)) \leq{}& \int_0^t y(\tau)u(\tau)\,d\tau \\ &- \int_0^t S(x(\tau))\,d\tau \end{aligned} \tag{35}$$

where $S(x)$ is a positive definite function, then the system is said to be *strictly passive*.

For system (1), if $f(0) = 0$, $h(0) = 0$, and, in addition, $V(x)$ is positive definite and proper, it is easy to conclude that a strictly passive system with $u(t) = 0$ has $x = 0$ as a globally asymptotically stable equilibrium point.

*Definition 7* (see [19]). The dynamic system (1) is said to be *feedback passive* if there exists a feedback law

$$u = \boldsymbol{\Omega}(x, r) \tag{36}$$

such that the system with the new input $r \in \mathbb{R}$ is passive.

**Theorem 8.** *For system (26), the state observer is designed as (16) and the sliding surface is chosen as (27). If the uncertain term and unknown disturbance satisfy the bounded condition in (17) and the passivity-based variable structure controller is designed as*

$$\begin{aligned} u(t) = -\frac{1}{b_2}\Bigg(&\widehat{\rho}(\widehat{\boldsymbol{x}}, t) + \left(\overline{d} + \gamma_1\right)\frac{\widehat{z}_n}{|\widehat{z}_n|} \\ &+ \frac{c}{\beta + |\widehat{z}_{n-1}(t)|}\widehat{z}_{n-1} + \boldsymbol{g}\widehat{\boldsymbol{x}}_3 \\ &- (\sigma(t)r(t) - \gamma_2)\frac{\widehat{z}_n}{|\widehat{z}_n|^2}\Bigg) \end{aligned} \tag{37}$$

*where $\gamma_1 > 0$ and $\gamma_2 > 0$ are two positive constants and $r$ is a dummy input, then the system states $\boldsymbol{x}(t)$ are globally convergent to zero; i.e., $\boldsymbol{x}(t) \longrightarrow 0$ as $t \longrightarrow \infty$.*

*Proof.* First let

$$\begin{aligned} &S(\widehat{z}_{n-1}, c, \beta) \\ &\quad = \begin{cases} c(\widehat{z}_{n-1} - \beta\ln(\beta + \widehat{z}_{n-1})), & \widehat{z}_{n-1} > 0 \\ -c(\widehat{z}_{n-1} + \beta\ln(\beta - \widehat{z}_{n-1})), & \widehat{z}_{n-1} < 0. \end{cases} \end{aligned} \tag{38}$$

It follows that $S(\widehat{z}_{n-1}, c, \beta) \geq 0$ and $\partial S(\widehat{z}_{n-1}, c, \beta)/\partial\widehat{z}_{n-1} = c\widehat{z}_{n-1}(t)/(\beta + |\widehat{z}_{n-1}(t)|)$. From the proof of Lemma 5, one can yield $\dot{\widehat{z}}_{n-1} = \widehat{z}_n - \boldsymbol{h}_2\boldsymbol{f}_o(\widetilde{\boldsymbol{x}}) + \boldsymbol{h}_1\widetilde{\boldsymbol{x}}$ and $\dot{\widehat{z}}_n = \dot{z}_n - \boldsymbol{h}_1\boldsymbol{f}_o(\widetilde{\boldsymbol{x}})$. According to the definition of passivity, the control input (37) is designed where, in this part, $r$ and $\sigma$ are taken as the new input and the new output, respectively. To make the sliding surface (27) strictly passive via feedback passivation, a storage function $V(t) = (1/2)\widehat{z}_n^2 + S(\widehat{z}_{n-1}, c, \beta)$ is chosen. Taking its time derivative and substituting the control input (37) into the above equation can yield

$$\begin{aligned} \dot{V} &= \widehat{z}_n\left(\dot{z}_n - \boldsymbol{h}_1\boldsymbol{f}_o(\widetilde{\boldsymbol{x}})\right) + \frac{\partial S(\widehat{z}_{n-1}, c, \beta)}{\partial\widehat{z}_{n-1}}\dot{\widehat{z}}_{n-1} \\ &= \widehat{z}_n\left(\rho(\boldsymbol{x}, t) + \boldsymbol{g}\boldsymbol{x}_3 + b_2(u(t) + d(t)) - \boldsymbol{h}_1\boldsymbol{f}_o(\widetilde{\boldsymbol{x}})\right) \\ &\quad + \frac{c\widehat{z}_{n-1}}{\beta + |\widehat{z}_{n-1}|}\dot{\widehat{z}}_{n-1} \\ &= \widehat{z}_n\left(\omega + \boldsymbol{g}\widetilde{\boldsymbol{x}}_3 - \boldsymbol{h}_1\boldsymbol{f}_o(\widetilde{\boldsymbol{x}})\right) - \left(\overline{d} + \gamma_1\right)|\widehat{z}_n| \\ &\quad + (\sigma r - \gamma_2) + \frac{c\widehat{z}_{n-1}}{\beta + |\widehat{z}_{n-1}|}\left(\boldsymbol{h}_1\widetilde{\boldsymbol{x}} - \boldsymbol{h}_2\boldsymbol{f}_o(\widetilde{\boldsymbol{x}})\right) \end{aligned} \tag{39}$$

where $\widetilde{\boldsymbol{x}}_3^T = [\widetilde{x}_3\ \widetilde{x}_4\ \cdots\ \widetilde{x}_n] \in \mathbb{R}^{n-2}$. Since the estimation error exponentially approaches zero, there exist two domains $\boldsymbol{\Omega}_1 = \{\widetilde{\boldsymbol{x}} \in \mathbb{R}^n : |\boldsymbol{g}\widetilde{\boldsymbol{x}}_3 - \boldsymbol{h}_1 f_o(\widetilde{\boldsymbol{x}})| < \gamma_1 - \alpha_1\}$ and $\boldsymbol{\Omega}_2 = \{\widetilde{\boldsymbol{x}} \in \mathbb{R}^n : |(c\widehat{z}_{n-1}(t)/(\beta + |\widehat{z}_{n-1}(t)|))(\boldsymbol{h}_1\widetilde{\boldsymbol{x}} - \boldsymbol{h}_2 f_o(\widetilde{\boldsymbol{x}}))| < \gamma_2 - \alpha_2\}$ where $\alpha_1$ and $\alpha_2$ are small scalars satisfying $0 < \alpha_1 < \gamma_1$ and

$0 < \alpha_2 < \gamma_2$ such that the trajectory of $\tilde{\boldsymbol{x}}(t)$ will enter each domain in finite time. Then it follows that

$$\begin{aligned}\dot{V} &\leq -\gamma_1 \left|\widehat{z}_n\right| + \sigma r - \gamma_2 + \widehat{z}_n \left(\boldsymbol{g}\tilde{\boldsymbol{x}}_3 - \boldsymbol{h}_1 f_o\left(\tilde{\boldsymbol{x}}\right)\right) \\ &\quad + \frac{c\widehat{z}_{n-1}}{\beta + \left|\widehat{z}_{n-1}\right|}\left(\boldsymbol{h}_1\tilde{\boldsymbol{x}} - \boldsymbol{h}_2 f_o\left(\tilde{\boldsymbol{x}}\right)\right) \\ &\leq -\alpha_1 \left|\widehat{z}_n\right| + \sigma r - \alpha_2.\end{aligned} \tag{40}$$

Taking the integral of the above equation over $\tau \in [0, t]$ yields

$$\int_0^t \sigma(\tau) r(\tau)\, d\tau - \int_0^t S_1(\tau)\, d\tau \geq V(t) - V(0) \tag{41}$$

where $S_1 = \alpha_1|\widehat{z}_n| + \alpha_2 > 0$, which implies the strict passivity of the sliding surface $\sigma(t)$ with the new input $r$. When $\sigma(t) = 0$, it follows from Lemma 5 that the system states satisfy $\boldsymbol{x}(t) \longrightarrow \boldsymbol{0}$ as $t \longrightarrow \infty$. Hence, the proof of this theorem is finished. □

*Remark 9.* For the nonlinear sliding surface (27), the damping ratio term is $c/(\beta + |\widehat{z}_{n-1}|)$. It is known that the damping ratio is low at the initial time and increases when the system states approach the origin. With the variable damping ratio, the quick response of system with a small damping ratio and the small overshoot of system with a large damping ratio can be simultaneously obtained. Hence, the nonlinear sliding surface (27) not only has the advantage of a conventional linear sliding surface but also has monotonously increasing damping ratio characteristics.

*Remark 10.* The passivity concept and a method of converting a nonlinear system into a passive system with new inputs and outputs are established using Theorem 8. Since the system is strictly passive for input (37), the system is asymptotically stable even if $r = 0$ and then the controller becomes as follows.

$$\begin{aligned}u = -\frac{1}{b_2}\Bigg(&\widehat{\rho}\left(\widehat{\boldsymbol{x}}, t\right) + \left(\overline{d} + \gamma_1\right)\frac{\widehat{z}_n}{\left|\widehat{z}_n\right|} + \boldsymbol{g}\widehat{\boldsymbol{x}}_3 + \gamma_2\frac{\widehat{z}_n}{\left|\widehat{z}_n\right|^2} \\ &+ \frac{c}{\beta + \left|\widehat{z}_{n-1}\right|}\widehat{z}_{n-1}\Bigg)\end{aligned} \tag{42}$$

Note that the control input in (42) cannot guarantee the finite-time convergence to the sliding surface but can obtain asymptotic convergence to the surface.

*Remark 11.* In this paper, the discontinuous techniques are used in the observer and controller designs. To practically implement the proposed method, the discontinuous term $\kappa(\delta(t)/|\delta(t)|^2)$ is smoothened by $\kappa(\delta(t)/(|\delta(t)|^2 + \varepsilon))$ where $\varepsilon > 0$ is a small positive constant. As a result, the system performance will not asymptotically converge to the sliding surface but be constrained in a bounded region.

## 4. Numerical Example

To demonstrate the proposed design techniques, the ship roll stabilization problem proposed by Fulwani's paper [7] is taken as

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -2 & -2.7 & -1.7\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix}(u+d) \tag{43}$$

where the unknown disturbance is set as $d(t) = \cos(\pi t/3) + \sin(\pi t/4)$ and its upper bound is given as $|d(t)| < 1.4$. Since $n = 3$, we choose $\lambda_1 = 6$ and $\lambda_2 = 8$ to satisfy the condition in Lemma 2. The discontinuous observer is designed as

$$\begin{aligned}\dot{\widehat{x}}_1 &= \widehat{x}_2 + 18\tilde{x}_1 - 0.1v \\ \dot{\widehat{x}}_2 &= \widehat{x}_3 + 108\tilde{x}_1 - 1.6v \\ \dot{\widehat{x}}_3 &= -2\widehat{x}_1 - 2.7\widehat{x}_2 - 1.7\widehat{x}_3 + u + 216\tilde{x}_1 - 6.4v\end{aligned} \tag{44}$$

where the discontinuous term $v$ is smoothened by $v(t) = 0.1(\tilde{x}_1(t)/(|\tilde{x}_1(t)|^2 + 0.001))$. Note that the switching gain is smaller than the upper bound of disturbance and, hence, the sliding observer method proposed by Slotine's paper [13] cannot be implemented. Figures 1–3 show the estimation performance under $u = 0$. As can be seen, the proposed discontinuous observer can precisely estimate the real states. To demonstrate the advantages of the proposed method, two cases are also simulated as follows.

*Case 1.* By using the conventional sliding mode control design method, the linear sliding surface and the corresponding controller are described, respectively, by the following.

$$\begin{aligned}s_1 &= 10x_1 + \widehat{x}_2 + \widehat{x}_3 \\ u &= 2x_1 - 7.3\widehat{x}_2 + 0.7\widehat{x}_3 - 2s_1 - 1.5\frac{s_1}{\left|s_1\right| + 0.01}\end{aligned} \tag{45}$$

*Case 2.* First let $z_2 = x_2 + gx_1$ and $z_3 = x_3 + gx_2$ be two new states. The above system (43) can be transformed into the following form.

$$\begin{aligned}\dot{x}_1 &= -gx_1 + z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= -2x_1 - 2.7x_2 - \left(1.7 - g\right)x_3 + u + d\end{aligned} \tag{46}$$

Based on Theorem 8, the nonlinear sliding surface and its passivity-based variable structure controller are designed, respectively, as

$$\begin{aligned}s_2 &= \widehat{z}_3 + \frac{40}{8 + \left|\widehat{z}_2\right|}\widehat{z}_2 \\ u &= 2x_1 + 2.7\widehat{x}_2 + (1.7 - k)\,\widehat{x}_3 - 1.5\frac{\widehat{z}_3}{\left|\widehat{z}_3\right| + 0.01} \\ &\quad - \frac{40}{8 + \left|\widehat{z}_2\right|}\widehat{z}_2 - 0.1\frac{\widehat{z}_3}{\left|\widehat{z}_3\right|^2 + 0.01}\end{aligned} \tag{47}$$

where $k = 5$, $g = 8$, $\widehat{z}_2 = \widehat{x}_2 + kx_1$, and $\widehat{z}_3 = \widehat{x}_3 + k\widehat{x}_2$.

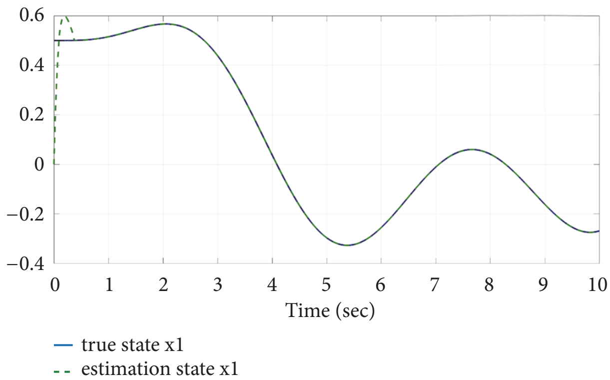


Figure 1: Real and estimation states $\boldsymbol{x}_1$.

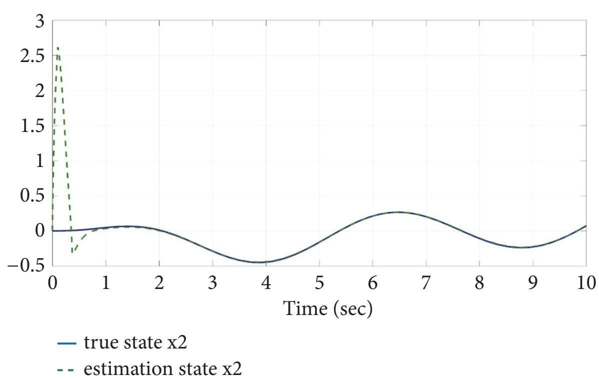


Figure 2: Real and estimation states $\boldsymbol{x}_2$.

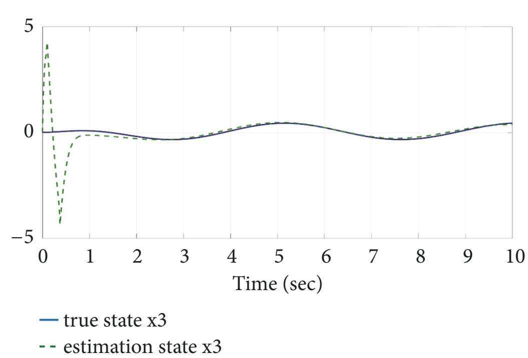


Figure 3: Real and estimation states $\boldsymbol{x}_3$.

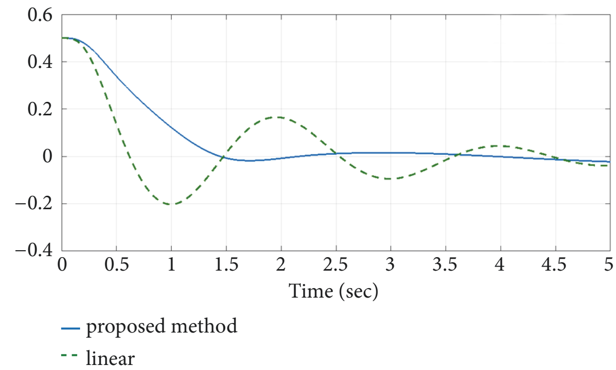


Figure 4: System states $\boldsymbol{x}_1$ of two cases.

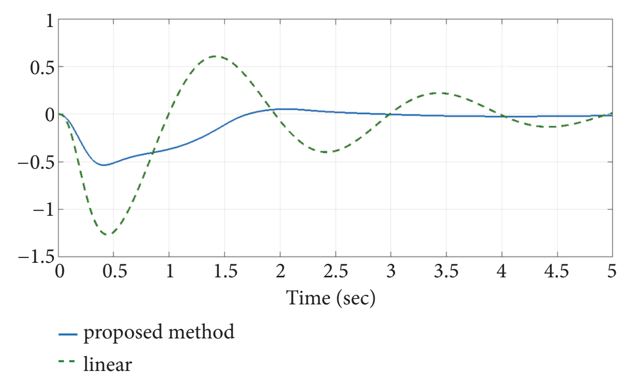


Figure 5: System states $\boldsymbol{x}_2$ of two cases.

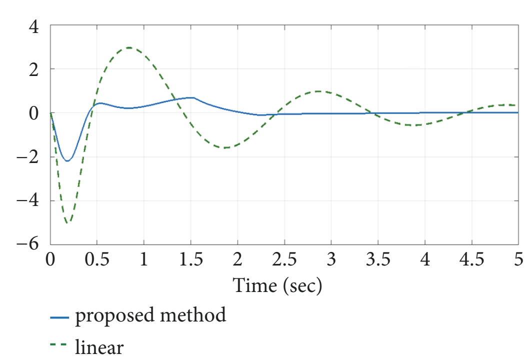


Figure 6: System states $\boldsymbol{x}_3$ of two cases.

Two cases are simultaneously simulated where the initial states are set $\boldsymbol{x}(0) = [0.5 \ \ 0 \ \ 0]^T$ and $\widehat{\boldsymbol{x}}(0) = \mathbf{0}$. Figures 4–6 show the responses of the system states for the two methods. As can be seen, the nonlinear sliding surface proposed in this paper obtains a quick response with low overshoot, indicating that the nonlinear sliding surface performs better if the nonlinear part is chosen correctly. Figures 7 and 8 display, respectively, the control inputs and the sliding surfaces for the two cases. From these figures, the proposed method can obtain better transient performance without increasing in the magnitude of the control input. Compared to the conventional linear sliding surface, the proposed control scheme also guarantees robust stability for the closed-loop system and addresses the problem of high-gain control.

## 5. Conclusion

For SISO nonlinear systems with matched disturbances, an output feedback variable structure controller with the nonlinear sliding surface has been proposed in this paper. A robust discontinuous observer is first used to estimate the system states in which exponential stability of the estimation performance is proved. Based on the estimation states, the proposed nonlinear sliding surface is capable of achieving low overshoot and quick response simultaneously. By using

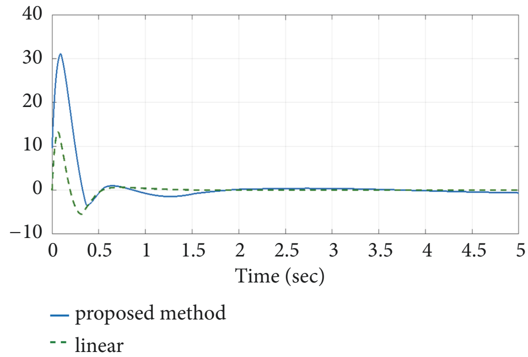


Figure 7: Sliding surfaces of two cases.

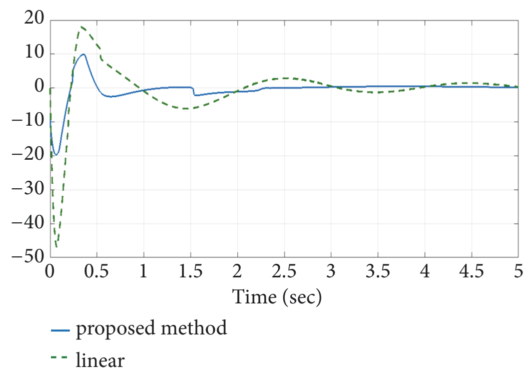


Figure 8: Control inputs of two cases.

the links between passivity and discontinuous control design, the control law is developed to guarantee asymptotic stability of the closed-loop system. Simulation results verify the effectiveness of the proposed design method and demonstrate the advantages of nonlinear sliding surface compared with the linear sliding surface. The future work is to extend our results for MIMO cases.

## Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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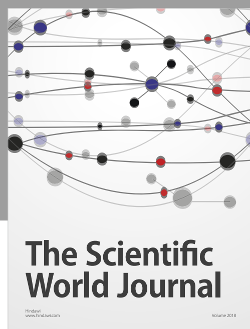
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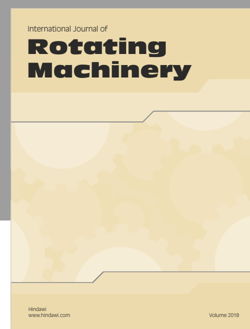
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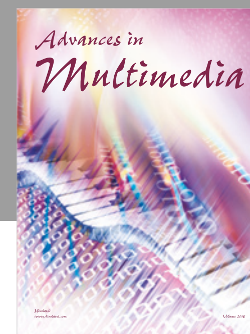
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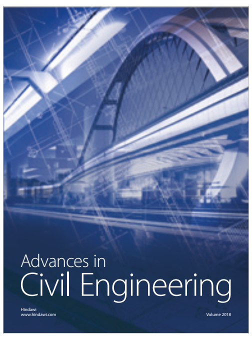
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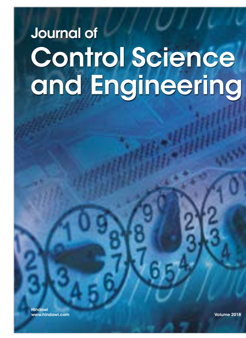
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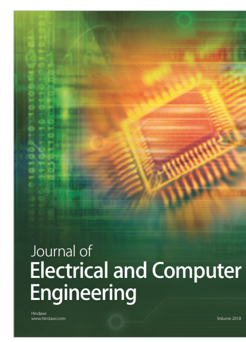
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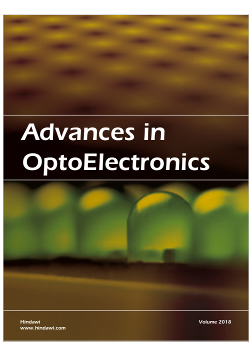
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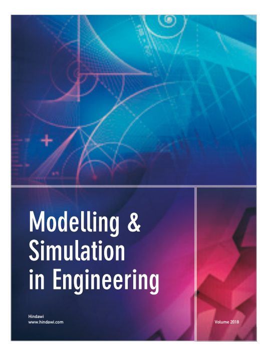
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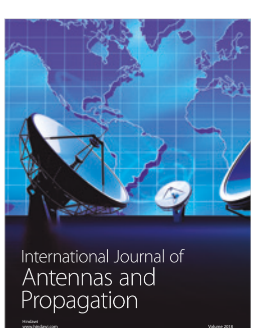
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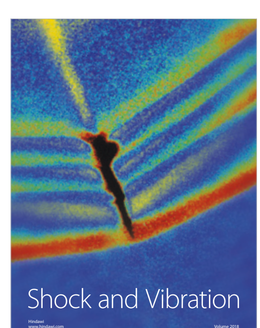
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