Research Article

Hybrid Robust H_2/H_∞ Fault-Tolerant Control with Random Delay NCS

Jun Wang ^{1,2} and Miaomiao Ling ^{1,2}

¹College of Electrical and Information Engineering, Lanzhou University of Technology, Lanzhou 730050, China

²Key Laboratory of Gansu Advanced Control for Industrial Processes, Lanzhou University of Technology, Lanzhou 730050, China

Correspondence should be addressed to Miaomiao Ling; 18893825891@163.com

Received 6 September 2019; Revised 20 December 2019; Accepted 27 December 2019; Published 21 January 2020

Academic Editor: Yongji Wang

Copyright © 2020 Jun Wang and Miaomiao Ling. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

In this paper, a hybrid fault-tolerant control method with off-line design and online scheduling is proposed for NCS with actuator faults, random delay, and external finite energy disturbance. The problem of less conservatism of robust generalized H_2/H_∞ hybrid fault-tolerant control is studied. Firstly, a closed-loop fault model of the system with random delay parameters was established according to the Bernoulli 0-1 distribution; all possible prior faults are divided into a few intervals according to certain rules, and then an interval fault-tolerant controller is designed off-line according to the prior faults of each interval. Secondly, when the fault is estimated online, the corresponding interval fault-tolerant controller is called through the scheduling mechanism to achieve rapid fault tolerance of prior faults within the interval and mitigate the impact of other faults within the interval, which provides a guarantee for subsequent safe reconstruction control. Finally, the effectiveness of the proposed method is verified by Matlab simulation.

1. Introduction

With the development of science and technology, a networked control system (NCS) has been widely used in aerospace, chemical industry, and other fields due to its convenient operation and strong integrated control. As one of the important components of an NCS, the actuator is susceptible to the influence of the external environment and long-term operation, which will inevitably affect the normal operation of the system, resulting in poor product quality and even system collapse. Therefore, fault-tolerant control for NCS actuator faults is a high concern in industry.

Passive fault-tolerant control (PFTC) [1–3] has been studied by a large number of scholars due to its advantages of simple design. In addition, the active fault-tolerant control (AFTC) [4–6] method, which can design the controller online for each fault, is also discussed. From a large number of studies, it is not difficult to find that the above two classical control methods have their own

shortcomings. Because PFTC contains a large number of fault modes, the solutions are often conservative, which cannot meet some performance requirements. AFTC requires a lot of computation time to estimate faults online and design controllers, so its fault-tolerant reliability is questionable. In order to further improve the reliability of fault-tolerant control, in recent years a few scholars have proposed a hybrid fault-tolerant control method combining their respective advantages [7–11]. Jiang and Yu [7] compare the advantages and limitations of active and passive fault-tolerant control methods and verifies them in the UAV fault calculation example. Su et al. [8] first combined AFTC and passive decentralized unconditional stability control technology to deduce the stability conditions of the FTC system in the case of sensor failure. Yu and Jiang [9] make use of the advantages of active and passive control to establish a hybrid control strategy for a UAV with control failure. The robustness of PFTC not only guarantees the stability of the system, but also provides favorable conditions for fault detection and diagnosis

(FDD) to obtain accurate fault information and reconstruct faults. The above research on hybrid control methods is still in the preliminary exploration stage and has not been involved in the field of the NCS. However, network congestion, limited bandwidth, and other conditions widely exist in an NCS. Therefore, a discrete event-triggered communication scheme (DETCS) was established in reference [10] to alleviate the blocking of data transmission by screening effective data. Meanwhile, an active-passive hybrid fault-tolerant controller was integrated and designed for arbitrary actuator faults. In reference [11], under the action of DETCS, the hybrid robust fault-tolerant design problem with external interference and a parameter uncertainty system was discussed based on T-S fuzzy models. It is worth mentioning that, due to the existence of passive robustness, the above control methods are still some conservatism. How to use hybrid control to effectively reduce the conservatism of results and ensure the satisfactory performance of the system needs to be further studied.

At the same time, the NCS is affected by the difference of transmission codes and topological structures, which causes the network-induced delay to be random. In practical systems, randomness usually manifests as random uncertainty, which will cause the delay to occur with a certain probability during the propagation process, resulting in partial loss of time-delay information. Therefore, based on the traditional delay rate change method, the system cannot be guaranteed to be stable. To overcome this difficulty, scholars have recently studied the stability problem under probabilistic delay. In reference [12], the Bernoulli 0-1 distribution is introduced to construct stochastic time-delay functions, and the robust stability conditions of stochastic systems are discussed. On the basis of [12], Li and Wu [13] use the state observer as the state feedback to establish the closed-loop augmented system and design the controller with exponential stability of mean square. In reference [14], the stability analysis of nonlinear disturbance and random time delay in the sampling process was studied. Obviously, none of the above random delay stability design problems take system failure into consideration. However, complex situations such as system failure under random delay cannot be ignored in the field of stability, and only a few scholars discuss this problem. For example, Wang et al. [15] study the synchronous design problem of network node coupling faults. Zhang et al. [16] discuss the design of the controller under actuator failure by considering relatively comprehensive random delay information. Therefore, the stability problem of NCS actuator faults with probabilistic delay has certain research value.

In this paper, based on the Bernoulli 0-1 distribution, the random delay is applied to two different probability intervals, which effectively compensates for the shortcomings of the delay variation rate in the probabilistic delay problem. At the same time, the event-triggering conditions and scheduling mechanism are established. Aiming at network congestion and actuator failure, a hybrid robust fault-tolerant control method of “off-line design, online scheduling” with less conservative intervals is proposed based on DETCS.

Under the action of FDD, when the interval prior fault occurs, the off-line designed interval fault-tolerant controller is quickly invoked for control. When other faults occur within the interval, FDD compensates the interval fault-tolerant controller for the purpose of fast fault tolerance and makes the system have H_2/H_∞ performance.

2. Problem Description

2.1. Event-Triggered Communication Scheme. DETCSs are introduced to conserve network resources when network bandwidth is limited. Its system structure diagram is shown in Figure 1.

For the convenience of presentation, the following assumptions are made.

Hypothesis 1. The sampling period of the sampler is h , the set of sampling times $i_k h$ is $\{i_k h | i_k \in N\}$, and the set of transmission times $t_k h$ is $\{t_k h | t_k \in N\}$.

Hypothesis 2 (see [17]). The system state is completely measurable.

It can be seen from Figure 1 that the actuator, the controlled plant, the sensor, the controller, and the network jointly constitute the NCS, where the time delay exists in the sensor-to-controller τ_{sc} , controller-to-actuator τ_{ca} , and calculation process τ_c . The data sampled by the sampler will be sent to the network through an event-triggering scheme to save network resources. According to system requirements, the following trigger conditions are created:

$$t_{k+1}h = t_k h + \min \left\{ lh \mid e_x^T(i_k h) \Gamma e_x(i_k h) \geq \sigma x^T(i_k h) \Gamma x(i_k h) \right\}, \quad (1)$$

where Γ is the positive definite symmetric matrix, $\sigma \in [0, 1]$ is the trigger parameter, and the state error is

$$e_x(i_k h) = x(i_k h) - x(t_k h). \quad (2)$$

2.2. The System Model. Establish linear NCS fault model under the DETCS:

$$\begin{cases} \dot{x} = Ax(t) + Bu(t) + Ef(t) + Dw(t), \\ y(t) = C_{11}x(t), \\ z(t) = C_{12}x(t), \end{cases} \quad (3)$$

where $x(t) \in R^n$ and $u(t) \in R^m$ are system states and control inputs; $y(t) \in R^p$ is the measurement output, and $z(t) \in R^p$ is the adjusted output; $w(t) \in L_2$ is the external finite energy disturbance; the system matrices A , B , C_{11} , C_{12} , E , and D are constant matrices with appropriate dimensions; $f(t) \in R^r$ is the additive fault of the actuator, and $\|f(t)\| \leq f_0$. Suppose the system has m actuators, then

$$f(t) = [f_1 \ f_2 \ \cdots \ f_m], \quad q = 1, 2, \dots, m. \quad (4)$$

When $f_q = 0$, it means that the q^{th} actuator works normally. When $f_q \neq 0$, the fault of the q^{th} actuator is indicated.

FIGURE 1: NCS structure diagram under event-triggering scheme.

For the convenience of subsequent proof, let the fault distribution matrix $E = -B$, and define the actuator fault $f(t)$ as

$$\begin{aligned} f(t) &= \hat{L}u(t), \\ \hat{L} &= I - L, \end{aligned} \quad (5)$$

where $L = \text{diag}\{l_1, l_2, \dots, l_m\}$ is the fault degree matrix and $l_q \in [0, 1], q = 1, 2, \dots, m$. When $l_q = 1$, the q^{th} actuator is normal. When $l_q = 0$, the q^{th} actuator completely fails. When $l_q \in (1, 0)$, the q^{th} actuator partially fails.

According to equations (3)–(5), $Lu(t) = u(t) - f(t)$ can be obtained; that is, the fault model (3) can be transformed into

$$\begin{cases} \dot{x} = Ax(t) + BLu(t) + Dw(t), \\ y(t) = C_{11}x(t), \\ z(t) = C_{12}x(t). \end{cases} \quad (6)$$

It can be known from the above that (3) and (6) are equivalent, so the state feedback control law is $u(t) = K(x(t - \tau(t)) - e_x(i_k h))$, $t \in [t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}}]$, where τ_i is the delay corresponding to the transmission time of i . In the actual system, $\tau(t)$ has randomness, so there is a constant d_1 ($0 \leq d_1 \leq d_2$) which makes the delay $\tau(t)$ have a certain probability when it is distributed on $[0, d_1]$ and $(d_1, d_2]$. Therefore, Bernoulli sequence $\delta(t)$ is defined to satisfy the following formula:

$$\delta(t) \sim \begin{cases} 1, & \tau_1 \in [0, d_1], \\ 0, & \tau_2 \in (d_1, d_2], \end{cases} \quad (7)$$

and then the probability distribution of random delay $\tau(t)$ is assumed to be

$$\begin{aligned} P\{\delta(t) = 1\} &= P\{0 \leq \tau_1(t) \leq d_1\} = \delta_0, \\ P\{\delta(t) = 0\} &= P\{d_1 < \tau_2(t) \leq d_2\} = 1 - \delta_0, \\ \delta_0 &\in [0, 1], \dot{\tau}_1(t) < \mu_1 < \infty, \dot{\tau}_2(t) < \mu_2 < \infty. \end{aligned} \quad (8)$$

The closed-loop fault system model with random delay is as follows:

$$\begin{cases} \dot{x} = Ax + \delta_0 BLKx_{\tau_1} + (1 - \delta_0) BLKx_{\tau_2} \\ \quad + (\delta - \delta_0) BLK(x_{\tau_1} - x_{\tau_2}) - BLK e_x(i_k h) + Dw, \\ y(t) = C_{11}x(t), \\ z(t) = C_{12}x(t). \end{cases} \quad (9)$$

To facilitate further derivation and proof, the following lemma is introduced.

Lemma 1 (see [18]). Any matrix $W > 0$, for a scalar $b > a$ and a function $w: [a, b] \rightarrow \mathbb{R}^n$, has the following integral inequality:

$$\left(\int_a^b \bar{w}(s) ds \right)^T W \left(\int_a^b \bar{w}(s) ds \right) \leq (b - a) \int_a^b \bar{w}^T(s) W \bar{w}(s) ds. \quad (10)$$

Lemma 2 (see [19] (Schur)). Given the symmetric matrix

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}, \text{ following conditions are equivalent:}$$

- (1) $Z < 0$
- (2) $Z_{11} < 0, Z_{22} - Z_{12}^T Z_{11}^{-1} Z_{12} < 0$
- (3) $Z_{22} < 0, Z_{11} - Z_{12}^T Z_{22}^{-1} Z_{12} < 0$

Lemma 3 (see [20]). For any matrix $M = \begin{bmatrix} R & U \\ * & R \end{bmatrix} \geq 0$, if its scalar satisfies $0 \leq d(t) \leq d$ for the vector $\dot{x}: [-d, 0] \rightarrow R^n$, then the inequality is set up as $-d \int_{t-d}^t \dot{x}^T(a) R \dot{x}(a) da \leq S^T(t) Z s(t)$, and

$$\begin{aligned} S^T(t) &= [x^T(t) \quad x^T(t-d(t)) \quad x^T(t-d)], \\ Z &= \begin{bmatrix} -R & R-U & U \\ * & -2R+U+U^T & R-U \\ * & * & -R \end{bmatrix}, \end{aligned} \quad (11)$$

where $*$ is the corresponding matrix block obtained by the symmetric matrix block.

3. Main Results

3.1. Failure Detection Observer Design. Considering the system model (3) for the NCS, a fault detection observer model with random delay is established:

$$\begin{cases} \dot{\hat{x}} = A\hat{x}(t) + Bu(t) + \delta_0 G(y(t-\tau_1) - \hat{y}(t-\tau_1)) \\ \quad + (1-\delta_0)G(y(t-\tau_2) - \hat{y}(t-\tau_2)) + (\delta-\delta_0) \\ \quad G((y(t-\tau_1) - \hat{y}(t-\tau_1)) - (y(t-\tau_2) - \hat{y}(t-\tau_2))), \\ \hat{y}(t) = C_{11}\hat{x}(t), \end{cases} \quad (12)$$

where G is the observer gain matrix.

The system output residual $r(t)$, state error $e(t)$, and fault estimation error $r_e(t)$ are defined as follows:

$$r(t) = W(y(t) - \hat{y}(t)), \quad (13)$$

$$e(t) = x(t) - \hat{x}(t), \quad (14)$$

$$r_e(t) = r(t) - f(t), \quad (15)$$

where W is the system output residual weight matrix.

By substituting equations (13)–(15) into equation (12), the observer error system of fault detection can be written as

$$\begin{cases} \dot{e}(t) = Ae(t) - \delta_0 GC_{11}e(t-\tau_1) \\ \quad - (1-\delta_0)GC_{11}e(t-\tau_2) - (\delta-\delta_0)GC_{11}(e(t-\tau_1) \\ \quad - e(t-\tau_2)) - Bf(t) + Dw(t), \\ r_e(t) = WC_{11}e(t) - f(t). \end{cases} \quad (16)$$

Therefore, the observer design problem is equivalent to solving G and W , making the error system (16) asymptotically stable and satisfying the following performance:

$$\|r_e(t)\|_2 \leq \gamma_1^2 \|f(t)\|_2 + \gamma_2^2 \|w(t)\|_2. \quad (17)$$

Note: it is easy to write and remember

$$\begin{aligned} e(t) &= e, \\ e(t-\tau(t)) &= e_\tau, \\ e(t-\tau_1(t)) &= e_{\tau_1}, \\ e(t-\tau_2(t)) &= e_{\tau_2}, \\ x(t) &= x, \\ x(t-\tau(t)) &= x_\tau, \\ x(t-\tau_1(t)) &= x_{\tau_1}, \\ x(t-\tau_2(t)) &= x_{\tau_2}, \\ w(t) &= w, \\ f(t) &= f, \\ r_e(t) &= r_e. \end{aligned} \quad (18)$$

Theorem 1. For the error system 16 and performance index (17), given positive numbers $d_1, d_2, \mu_1, \mu_2, \delta_0, \gamma_1, \gamma_2, a$, and b , if symmetric positive definite matrices P, V, W , and $Q_i (i = 1, 2, 3, 4)$ exist, the following matrix inequality (19) is feasible:

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & 0 & \Phi_{14} & 0 & \Phi_{16} & \Phi_{17} & \Phi_{18} & \Phi_{19} & \Phi_{110} \\ * & \Phi_{22} & \Phi_{23} & 0 & 0 & 0 & 0 & 0 & \Phi_{29} & \Phi_{210} \\ * & * & \Phi_{33} & \Phi_{34} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Phi_{44} & \Phi_{45} & 0 & 0 & 0 & \Phi_{49} & \Phi_{410} \\ * & * & * & * & \Phi_{55} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Phi_{66} & 0 & \Phi_{68} & \Phi_{69} & \Phi_{610} \\ * & * & * & * & * & * & \Phi_{77} & 0 & \Phi_{79} & \Phi_{710} \\ * & * & * & * & * & * & * & \Phi_{88} & 0 & 0 \\ * & * & * & * & * & * & * & * & \Phi_{99} & 0 \\ * & * & * & * & * & * & * & * & * & \Phi_{1010} \end{bmatrix} < 0, \quad (19)$$

where

$$\begin{aligned}
\Phi_{11} &= A^T P + PA + Q_1 + Q_2 - Q_3, \\
\Phi_{12} &= -\delta_0 V C_{11} + Q_3, \\
\Phi_{14} &= -(1 - \delta_0) V C_{11}, \\
\Phi_{16} &= -PB, \\
\Phi_{17} &= PD, \\
\Phi_{18} &= C_{11}^T W^T, \\
\Phi_{19} &= d_1 A^T P, \\
\Phi_{110} &= (d_2 - d_1) A^T P, \\
\Phi_{22} &= -(1 - \mu_1) Q_1 - 2Q_3, \\
\Phi_{23} &= Q_3, \\
\Phi_{29} &= -d_1 \delta_0 C_{11}^T V^T, \\
\Phi_{210} &= -(d_2 - d_1) \delta_0 C_{11}^T V^T, \\
\Phi_{33} &= -Q_3 - Q_4, \\
\Phi_{34} &= Q_4, \\
\Phi_{44} &= -(1 - \mu_2) Q_2 - 2Q_4, \\
\Phi_{45} &= Q_4, \\
\Phi_{49} &= -d_1 (1 - \delta_0) C_{11}^T V^T, \\
\Phi_{410} &= -(d_2 - d_1) (1 - \delta_0) C_{11}^T V^T, \\
\Phi_{55} &= -Q_4, \\
\Phi_{66} &= -\gamma_1^2 I, \\
\Phi_{68} &= -I, \\
\Phi_{69} &= -d_1 B^T P, \\
\Phi_{610} &= -(d_2 - d_1) B^T P, \\
\Phi_{77} &= -\gamma_2^2 I, \\
\Phi_{79} &= d_1 D^T P, \\
\Phi_{710} &= (d_2 - d_1) D^T P, \\
\Phi_{88} &= -I, \\
\Phi_{99} &= -2aP + a^2 Q_3, \\
\Phi_{1010} &= -2bP + b^2 Q_4.
\end{aligned} \tag{20}$$

The designed fault detection observer can effectively detect faults and estimate the size of faults, $G = P^{-1}V$.

Proof. construct the Lyapunov–Krasovskii functional as follows:

$$\begin{aligned}
V(t) &= V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t), \\
V_1(t) &= e^T P e, \\
V_2(t) &= \int_{t-\tau_1}^t e^T(s) Q_1 e(s) ds, \\
V_3(t) &= \int_{t-\tau_2}^t e^T(s) Q_2 e(s) ds, \\
V_4(t) &= \int_{-d_1}^0 \int_{t+\theta}^t d_1 \dot{e}^T(s) Q_3 \dot{e}(s) d\theta ds, \\
V_5(t) &= \int_{-d_2}^{-d_1} \int_{t+\theta}^t (d_2 - d_1) \dot{e}^T(s) Q_4 \dot{e}(s) d\theta ds.
\end{aligned} \tag{21}$$

If we define

$$\begin{cases} E\{\delta - \delta_0\} = 0, \\ E\{(\delta - \delta_0)^2\} = \delta_0(1 - \delta_0), \end{cases} \tag{22}$$

then taking the derivative of $V(t)$, we can get

$$\begin{aligned}
E\{\dot{V}_1(t)\} &= E\{2e^T(t)P\dot{e}(t)\}, \\
E\{\dot{V}_2(t)\} &\leq E\{e^T(t)Q_1e(t) - (1 - \mu_1)e_{\tau_1}^T Q_1 e_{\tau_1}\}, \\
E\{\dot{V}_3(t)\} &\leq E\{e^T(t)Q_2e(t) - (1 - \mu_2)e_{\tau_2}^T Q_2 e_{\tau_2}\}, \\
E\{\dot{V}_4(t)\} &\leq E\left\{d_1^2 \dot{e}^T(t)Q_3 \dot{e}(t) - d_1 \int_{t-d_1}^t \dot{e}^T(s)Q_3 \dot{e}(s) ds\right\}, \\
E\{\dot{V}_5(t)\} &\leq E\left\{(d_2 - d_1)^2 \dot{e}^T(t)Q_4 \dot{e}(t) - (d_2 - d_1) \cdot \int_{t-d_2}^{t-d_1} \dot{e}^T(s)Q_4 \dot{e}(s) ds\right\}.
\end{aligned} \tag{23}$$

According to Lemma 1,

$$\begin{aligned}
E\left\{-d_1 \int_{t-d_1}^t \dot{e}^T(s)Q_3 \dot{e}(s) ds\right\} &\leq -[e(t) - e(t - \tau_1(t))]^T \\
&\quad \cdot Q_3[e(t) - e(t - \tau_1(t))] \\
&\quad - [e(t - \tau_1(t)) - e(t - d_1(t))]^T \\
&\quad \cdot Q_3[e(t - \tau_1(t)) - e(t - d_1(t))].
\end{aligned} \tag{24}$$

In the same way,

$$\begin{aligned}
E\left\{-(d_2 - d_1) \int_{t-d_2}^{t-d_1} \dot{e}^T(s)Q_4 \dot{e}(s) ds\right\} &\leq -[e(t - d_1) - e(t - \tau_2(t))]^T \\
&\quad \cdot Q_4[e(t - d_1) - e(t - \tau_2(t))] \\
&\quad - [e(t - \tau_2(t)) - e(t - d_2)]^T \\
&\quad \cdot Q_4[e(t - \tau_2(t)) - e(t - d_2)].
\end{aligned} \tag{25}$$

Let $\varepsilon = \begin{bmatrix} A & -\delta_0 G C_{11} & 0 & -(1 - \delta_0) G C_{11} & 0 & -B & D \end{bmatrix}$ and $\xi^T(t) = \begin{bmatrix} e^T & e_{\tau_1}^T & e_{d_1}^T & e_{\tau_2}^T & e_{d_2}^T & f^T & w^T \end{bmatrix}$.
Then,

$$E\{\dot{V}(t)\} - \gamma_1^2 f^T(t)f(t) - \gamma_2^2 w^T(t)w(t) \leq \xi^T(t) (\Phi' + d_1^2 \varepsilon^T Q_3 \varepsilon + (d_2 - d_1)^2 \varepsilon^T Q_4 \varepsilon) \xi(t),$$

$$\Phi' = \begin{bmatrix} \Phi'_{11} & \Phi'_{12} & 0 & \Phi'_{14} & 0 & \Phi'_{16} & \Phi'_{17} \\ * & \Phi'_{22} & \Phi'_{23} & 0 & 0 & 0 & 0 \\ * & * & \Phi'_{33} & \Phi'_{34} & 0 & 0 & 0 \\ * & * & * & \Phi'_{44} & \Phi'_{45} & 0 & 0 \\ * & * & * & * & \Phi'_{55} & 0 & 0 \\ * & * & * & * & * & \Phi'_{66} & 0 \\ * & * & * & * & * & * & \Phi'_{77} \end{bmatrix} < 0, \quad (26)$$

where

$$\begin{aligned} \Phi'_{11} &= A^T P + PA + Q_1 + Q_2 - Q_3, \\ \Phi'_{12} &= -\delta_0 PGC_{11} + Q_3, \\ \Phi'_{14} &= -(1 - \delta_0)PGC_{11}, \\ \Phi'_{16} &= -PB, \\ \Phi'_{17} &= PD, \\ \Phi'_{22} &= -(1 - \mu_1)Q_1 - 2Q_3, \\ \Phi'_{23} &= Q_3, \\ \Phi'_{33} &= -Q_3 - Q_4, \\ \Phi'_{34} &= Q_4, \\ \Phi'_{44} &= -(1 - \mu_2)Q_2 - 2Q_4, \\ \Phi'_{45} &= Q_4, \\ \Phi'_{55} &= -Q_4, \\ \Phi'_{66} &= -\gamma_1^2 I, \\ \Phi'_{77} &= -\gamma_2^2 I. \end{aligned} \quad (27)$$

According to equation (16), it is easy to obtain

$$\begin{aligned} r_e^T(t)r_e(t) &= \xi^T(t)Y^T Y \xi(t), \\ Y &= [WC_{11} \ 0 \ 0 \ 0 \ 0 \ -I \ 0]. \end{aligned} \quad (28)$$

Then,

$$\begin{aligned} E\{\dot{V}(t)\} + r_e^T(t)r_e(t) - \gamma_1^2 f^T(t)f(t) - \gamma_2^2 w^T(t)w(t) \\ \leq \xi^T(t) (\Phi' + d_1^2 \varepsilon^T Q_3 \varepsilon + (d_2 - d_1)^2 \varepsilon^T Q_4 \varepsilon + Y^T Y) \xi(t), \\ \Phi'' = \Phi' + d_1^2 \varepsilon^T Q_3 \varepsilon + (d_2 - d_1)^2 \varepsilon^T Q_4 \varepsilon + Y^T Y. \end{aligned} \quad (29)$$

According to the performance index of H_∞ ,

$$\begin{aligned} J_1 &= \int_0^t (r_e^T(t)r_e(t) - \gamma_1^2 f^T(t)f(t) - \gamma_2^2 w^T(t)w(t)) dt \\ &\leq E \left\{ \int_0^t (\dot{V}(t) + r_e^T(t)r_e(t) - \gamma_1^2 f^T(t)f(t) - \gamma_2^2 w^T(t)w(t)) dt \right\} \leq 0. \end{aligned} \quad (30)$$

When $f(t) = 0$ and $w(t) = 0$, if $\Phi'' < 0$ exists, the system (16) is stable in mean square. When $f(t) \neq 0$ and $w(t) \neq 0$, the following integral is obtained for J_1 :

$$\begin{aligned} E\{V(t) - V(t_0)\} &\leq - \int_{t_0}^t (r_e^T(t)r_e(t) - \gamma_1^2 f^T(t)f(t) \\ &\quad - \gamma_2^2 w^T(t)w(t)) dt. \end{aligned} \quad (31)$$

Under zero initial conditions, $t_0 = 0$; when $t \rightarrow \infty$, $\int_0^\infty r_e^T(t)r_e(t) dt \leq \int_0^\infty (\gamma_1^2 f^T(t)f(t) + \gamma_2^2 w^T(t)w(t)) dt$ is reasonable; that is, the H_∞ performance index is satisfied.

Furthermore, according to Lemma 2, we can get

$$\begin{aligned} \Phi'' &= \begin{bmatrix} \Phi' & \Pi \\ * & \Theta \end{bmatrix} < 0, \\ \Pi &= \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} \\ 0 & \Pi_{22} & \Pi_{23} \\ 0 & 0 & 0 \\ 0 & \Pi_{42} & \Pi_{43} \\ 0 & 0 & 0 \\ \Pi_{61} & \Pi_{62} & \Pi_{63} \\ 0 & \Pi_{72} & \Pi_{73} \end{bmatrix}, \\ \Theta &= \begin{bmatrix} \Theta_{11} & 0 & 0 \\ * & \Theta_{22} & 0 \\ * & * & \Theta_{33} \end{bmatrix}, \end{aligned} \quad (32)$$

where

$$\begin{aligned}
\Pi_{11} &= C_{11}^T W^T, \\
\Pi_{12} &= d_1 A^T Q_3, \\
\Pi_{13} &= (d_2 - d_1) A^T Q_4, \\
\Pi_{22} &= -d_1 \delta_0 C_{11}^T G^T Q_3, \\
\Pi_{23} &= -(d_2 - d_1) \delta_0 C_{11}^T G^T Q_4, \\
\Pi_{42} &= -d_1 (1 - \delta_0) C_{11}^T G^T Q_3, \\
\Pi_{43} &= -(d_2 - d_1) (1 - \delta_0) C_{11}^T G^T Q_4, \\
\Pi_{61} &= -I, \\
\Pi_{62} &= -d_1 B^T Q_3, \\
\Pi_{63} &= -(d_2 - d_1) B^T Q_4, \\
\Pi_{72} &= d_1 D^T Q_3, \\
\Pi_{73} &= (d_2 - d_1) D^T Q_4, \\
\Theta_{11} &= -I, \\
\Theta_{22} &= -Q_3, \\
\Theta_{33} &= -Q_4.
\end{aligned} \tag{33}$$

In addition, when the left and right sides of Φ'' are multiplied by the diagonal matrix $\{I, I, I, I, I, I, I, PQ_3^{-1}, PQ_4^{-1}\}$ at the same time and then by the following inequality:

$$-\alpha^2 Z_1 + 2\alpha P - PZ_1^{-1}P = -(\alpha Z_1 - P)Z_1^{-1}(\alpha Z_1 - P) \leq 0. \tag{34}$$

We can get

$$\begin{cases} -PQ_3^{-1}P = -2aP + a^2Q_3, \\ -PQ_4^{-1}P = -2bP + b^2Q_4. \end{cases} \tag{35}$$

So, the fault detection observer gain is $G = P^{-1}V$. The theorem is proved. $\square$

3.2. Design of Interval Fault-Tolerant Controller. An interval fault-tolerant controller is designed for an NCS closed-loop fault system (9) with random time delay. The state feedback control law with Bernoulli 0-1 distribution parameters is

$$\begin{aligned}
u_i &= K_i(\delta x(t - \tau_1(t)) + (1 - \delta)(x(t - \tau_2(t))) - e_x(i_k h)), \\
i &= 1, 2, 3, 4,
\end{aligned} \tag{36}$$

which guarantees the stability of the system's mean-square index and H_2/H_∞ performance.

Theorem 2. For the closed-loop fault system (9) with the DETCS, given positive numbers $d_1, d_2, \mu_1, \mu_2, \gamma_1', \gamma_2', \delta_0$, and $\sigma \in [0, 1)$, if there are symmetric positive definite matrices $X, R_j (j = 1, \dots, 6)$ and matrices V, Y_i , then the following matrix inequality (37), (38) are valid

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & 0 & \Omega_{16} & \Omega_{17} & \Omega_{18} & \Omega_{19} & \Omega_{110} \\ * & \Omega_{22} & \Omega_{23} & 0 & 0 & 0 & 0 & \Omega_{28} & \Omega_{29} & 0 \\ * & * & \Omega_{33} & \Omega_{34} & \Omega_{35} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Omega_{44} & \Omega_{45} & 0 & 0 & \Omega_{48} & \Omega_{49} & 0 \\ * & * & * & * & \Omega_{55} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Omega_{66} & 0 & \Omega_{68} & \Omega_{69} & 0 \\ * & * & * & * & * & * & \Omega_{77} & \Omega_{78} & \Omega_{79} & 0 \\ * & * & * & * & * & * & * & \Omega_{88} & 0 & 0 \\ * & * & * & * & * & * & * & * & \Omega_{99} & 0 \\ * & * & * & * & * & * & * & * & * & \Omega_{1010} \end{bmatrix} < 0, \tag{37}$$

$$\begin{bmatrix} X & XC_{12}^T \\ * & \gamma_2'^2 I \end{bmatrix} > 0, \tag{38}$$

where

$$\begin{aligned}
\Omega_{11} &= AX + XA^T + 2X - R_1 - R_2 + R_3, \\
\Omega_{12} &= \delta_0 BLY_i - R_3 + R_5, \\
\Omega_{13} &= 2X - R_5, \\
\Omega_{14} &= (1 - \delta_0)BLY_i, \\
\Omega_{16} &= -BLY_i, \\
\Omega_{17} &= D, \\
\Omega_{18} &= d_1 XA^T, \\
\Omega_{19} &= (d_2 - d_1)XA^T, \\
\Omega_{110} &= XC_{11}^T, \\
\Omega_{22} &= \delta_0 \sigma [2X - V] - (1 - \mu_1)[2X - R_1] + 2R_3 - R_5 - R_5^T, \\
\Omega_{23} &= -R_3 + R_5, \\
\Omega_{28} &= \delta_0 d_1 Y_i^T L^T B^T, \\
\Omega_{29} &= \delta_0 (d_2 - d_1) Y_i^T L^T B^T, \\
\Omega_{33} &= -4X + R_3 + R_4, \\
\Omega_{34} &= -R_4 + R_6, \\
\Omega_{35} &= 2X - R_6, \\
\Omega_{44} &= \sigma(1 - \delta_0)[2X - V] - (1 - \mu_2)[2X - R_2] + 2R_4 - R_6 - R_6^T, \\
\Omega_{45} &= -R_4 + R_6, \\
\Omega_{48} &= (1 - \delta_0)d_1 Y_i^T L^T B^T, \\
\Omega_{49} &= (1 - \delta_0)(d_2 - d_1) Y_i^T L^T B^T, \\
\Omega_{55} &= -2X + R_4, \\
\Omega_{66} &= -2X + V, \\
\Omega_{68} &= -d_1 Y_i^T L^T B^T, \\
\Omega_{69} &= -(d_2 - d_1) Y_i^T L^T B^T, \\
\Omega_{77} &= -\gamma_1'^2 I, \\
\Omega_{78} &= d_1 D^T, \\
\Omega_{79} &= (d_2 - d_1)D^T, \\
\Omega_{88} &= -R_3, \\
\Omega_{99} &= -R_4, \\
\Omega_{1010} &= -I.
\end{aligned} \tag{39}$$

Then, there is the state feedback control law (36), which makes the closed-loop NCS fault system (9) mean-square exponential stable and have performance of γ'_1 and γ'_2 , where the controller gain matrix $K_i = YX^{-1}$, and event-triggered weight matrix $\Gamma = V^{-1}$.

Proof. Construct a Lyapunov–Krasovskii functional:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t), \quad (40)$$

where

$$\begin{aligned} V_1(t) &= x^T P x, \\ V_2(t) &= \int_{t-\tau_1}^t x^T(s) Q_1 x(s) ds, \\ V_3(t) &= \int_{t-\tau_2}^t x^T(s) Q_2 x(s) ds, \\ V_4(t) &= \int_{-d_1}^0 \int_{t+\theta}^t d_1 \dot{x}^T(s) Q_3 \dot{x}(s) d\theta ds, \\ V_5(t) &= \int_{-d_2}^{-d_1} \int_{t+\theta}^t (d_2 - d_1) \dot{x}^T(s) Q_4 \dot{x}(s) d\theta ds. \end{aligned} \quad (41)$$

Take the derivative of $V(t)$ and get the following inequalities from equation (22):

$$\begin{aligned} E\{\dot{V}_1(t)\} &= E\{2x^T(t)P\dot{x}(t)\}, \\ E\{\dot{V}_2(t)\} &\leq E\{x^T(t)Q_1x(t) - (1-\mu_1)x_{\tau_1}^T Q_1 x_{\tau_1}\}, \\ E\{\dot{V}_3(t)\} &\leq E\{x^T(t)Q_2x(t) - (1-\mu_2)x_{\tau_2}^T Q_2 x_{\tau_2}\}, \\ E\{\dot{V}_4(t)\} &\leq E\left\{d_1^2 \dot{x}^T(t)Q_3 \dot{x}(t) - d_1 \int_{t-d_1}^t \dot{x}^T(s)Q_3 \dot{x}(s) ds\right\}, \\ E\{\dot{V}_5(t)\} &\leq E\left\{(d_2 - d_1)^2 \dot{x}^T(t)Q_4 \dot{x}(t) - (d_2 - d_1) \cdot \int_{t-d_2}^{t-d_1} \dot{x}^T(s)Q_4 \dot{x}(s) ds\right\}, \\ E\{\dot{V}(t)\} &\leq E\{\dot{V}_1(t)\} + E\{\dot{V}_2(t)\} + E\{\dot{V}_3(t)\} + E\{\dot{V}_4(t)\} + E\{\dot{V}_5(t)\} \\ &\quad + e_x^T(i_k h) \Gamma e_x(i_k h) - e_x^T(i_k h) \Gamma e_x(i_k h) + y^T(t) y(t) \\ &\quad - \gamma_1'^2 w^T(t) w(t) + \gamma_1'^2 w^T(t) w(t) - y^T(t) y(t). \end{aligned} \quad (42)$$

According to Lemma 3, we have

$$\begin{aligned} E\left\{-d_1 \int_{t-d_1}^t \dot{x}^T(s) Q_3 \dot{x}(s) ds\right\} &\leq \begin{bmatrix} x \\ x_{\tau_1} \\ x_{d_1} \end{bmatrix}^T \begin{bmatrix} -Q_3 & Q_3 - U_1 & U_1 \\ * & -2Q_3 + U_1 + U_1^T & Q_3 - U_1 \\ * & * & -Q_3 \end{bmatrix} \begin{bmatrix} x \\ x_{\tau_1} \\ x_{d_1} \end{bmatrix}, \\ E\left\{-(d_2 - d_1) \int_{t-d_2}^{t-d_1} \dot{x}^T(s) Q_4 \dot{x}(s) ds\right\} &\leq \begin{bmatrix} x_{d_1} \\ x_{\tau_2} \\ x_{d_2} \end{bmatrix}^T \begin{bmatrix} -Q_4 & Q_4 - U_2 & U_2 \\ * & -2Q_4 + U_2 + U_2^T & Q_4 - U_2 \\ * & * & -Q_4 \end{bmatrix} \begin{bmatrix} x_{d_1} \\ x_{\tau_2} \\ x_{d_2} \end{bmatrix}. \end{aligned} \quad (43)$$

When

$$\begin{aligned} \xi'^T &= [x^T \ x_{\tau_1}^T \ x_{d_1}^T \ x_{\tau_2}^T \ x_{d_2}^T \ e_x^T(i_k h) \ w^T], \\ \varepsilon' &= [A \ \delta_0 B L_i K_i \ 0 \ (1 - \delta_0) B L_i K_i \ 0 \ -B L_i K_i \ D], \end{aligned} \quad (44)$$

we can rewrite $E\{\dot{V}(t)\}$ as follows:

$$E\{\dot{V}(t)\} \leq \xi'^T \left(\Omega'_1 + d_1^2 \varepsilon'^T Q_3 \varepsilon' + (d_2 - d_1)^2 \varepsilon'^T Q_4 \varepsilon' \right) \xi',$$

$$\Omega' = \begin{bmatrix} \Omega'_{11} & \Omega'_{12} & \Omega'_{13} & \Omega'_{14} & 0 & \Omega'_{16} & \Omega'_{17} \\ * & \Omega'_{22} & \Omega'_{23} & 0 & 0 & 0 & 0 \\ * & * & \Omega'_{33} & \Omega'_{34} & \Omega'_{35} & 0 & 0 \\ * & * & * & \Omega'_{44} & \Omega'_{45} & 0 & 0 \\ * & * & * & * & \Omega'_{55} & 0 & 0 \\ * & * & * & * & * & \Omega'_{66} & 0 \\ * & * & * & * & * & * & \Omega'_{77} \end{bmatrix} < 0, \quad (45)$$

where

$$\begin{aligned} \Omega'_{11} &= A^T P + P A + Q_1 + Q_2 - Q_3, \\ \Omega'_{12} &= \delta_0 P B L_i K_i + Q_3 - U_1, \\ \Omega'_{13} &= U_1, \\ \Omega'_{14} &= (1 - \delta_0) P B L_i K_i, \\ \Omega'_{16} &= -P B L_i K_i, \\ \Omega'_{17} &= P D, \\ \Omega'_{22} &= \sigma \delta_0 \Gamma - (1 - \mu_1) Q_1 - 2Q_3 + U_1 + U_1^T, \\ \Omega'_{23} &= Q_3 - U_1, \\ \Omega'_{33} &= -Q_3 - Q_4, \\ \Omega'_{34} &= Q_4 - U_2, \\ \Omega'_{35} &= U_2, \\ \Omega'_{44} &= \sigma (1 - \delta_0) \Gamma - (1 - \mu_2) Q_2 - 2Q_4 + U_2 + U_2^T, \\ \Omega'_{45} &= Q_4 - U_2, \\ \Omega'_{55} &= -Q_4, \\ \Omega'_{66} &= -\Gamma, \\ \Omega'_{77} &= -\gamma_1'^2 I. \end{aligned} \quad (46)$$

The arrangement of equation (45) can be obtained as

$$E\{\dot{V}(t)\} + y^T(t)y(t) - \gamma_1'^2 w^T(t)w(t) \leq \xi'^T(t) \cdot (t) \left(\Omega' + d_1^2 \varepsilon'^T Q_3 \varepsilon' + (d_2 - d_1)^2 \varepsilon'^T Q_4 \varepsilon' + \phi^T \phi \right) \xi'(t), \quad (47)$$

where $\phi = [C_{11}^T \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T$.

According to Lemma 2, Ω'' can be rewritten as

$$\Omega'' = \Omega' + d_1^2 \varepsilon'^T Q_3 \varepsilon' + (d_2 - d_1)^2 \varepsilon'^T Q_4 \varepsilon' + \phi^T \phi. \quad (48)$$

Before and after multiplying Ω'' with $\text{diag}\{P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, I, Q_3^{-1}, Q_4^{-1}, I\}$, we obtain Ω .

Consider the H_2/H_∞ performance indicators:

$$J_2 = \int_0^t \left(y^T(t)y(t) - \gamma_1'^2 w^T(t)w(t) \right) dt \leq E \left\{ \int_0^t \left[\dot{V}(t) + y^T(t)y(t) - \gamma_1'^2 w^T(t)w(t) \right] dt \right\} \leq 0. \quad (49)$$

When $w(t) = 0$, if $\Omega \leq 0$, then $E = \{\dot{V}(t)\} \leq 0$, and the system (9) is asymptotically stable in mean square. If $w(t) \in [0, \infty)$, the integral of $t_0 \rightarrow t$ on both sides of J_2 is expressed as

$$E\{V(t)\} - E\{V(t_0)\} \leq - \int_{t_0}^t \left(y^T(t)y(t) - \gamma_1'^2 w^T(t)w(t) \right) dt. \quad (50)$$

Under the zero initial condition, $\int_0^\infty y^T(t)y(t)dt \leq \int_0^\infty \gamma_1'^2 w^T(t)w(t)dt$ can be obtained; that is, the performance index $\|y(t)\|_2 \leq \gamma_1' \|w(t)\|_2$ is valid.

In addition,

$$E\{x^T(t)Px(t)\} \leq E\{V(t)\} < E \left\{ \int_0^t w^T(s)w(s)ds \right\}, \quad (51)$$

$$C_{12}^T C_{12} < \gamma_2'^2 P,$$

can be obtained from equation (40) and Lemma 2, so

$$E\{z^T(t)z(t)\} \leq \gamma_2'^2 E\{x^T(t)Px(t)\} < \gamma_2'^2 \int_0^\infty w^T(s)w(s)ds. \quad (52)$$

Namely, the generalized H_2/H_∞ index is supportable. The theorem is proved. $\square$

3.3. Fault-Tolerant Controller Scheduling. Based on prior knowledge, all possible actuator faults L are divided into four intervals $[1 \ 0.8]$, $[0.8 \ 0.5]$, $[0.5 \ 0.2]$, and $[0.2 \ 0]$

according to certain rules, and the interval controller set is designed off-line.

Firstly, model set $\Lambda_i = [\Lambda_j \ \Lambda_j]^T$ of actuator fault f is constructed, where $\Lambda_j \in [0, 1]$, $i \in (1, 2, 3, 4)$ is the scheduling table of the interval controller, and $j \in (1, 2, \dots, m)$ is the number of actuators. When $\Lambda_j \cap \Lambda_j < [0.2 \ 0.2]^T$, the default strategy is adopted, and when $\Lambda_j \cup \Lambda_j \geq [0.2 \ 0.2]^T$, the switching strategy is started. Secondly, an online scheduling algorithm is established. When FDD detects the fault $f_i = [f_p \ f_q]^T$, $\min_{1,2}|L_j - f_p|$, and $\min_{1,2}|L_j - f_q|$ are calculated, respectively, and then $\max(\min_{1,2}, \min_{1,2})$ is solved to classify the current fault interval. Finally, the corresponding interval controller is called to satisfy the fault tolerance rapidly. The scheduling principle is shown in Figure 2.

3.4. Controller Compensation Design. It can be seen from Figure 2 that, when the random delay with Bernoulli distribution exists in the system, the interval fault-tolerant controller designed by Theorem 2 is fault-tolerant for the interval prior faults, so as to ensure the stability of the mean-square exponential of the system and have certain dynamic performance. In case of other early faults in the interval, it can buffer the rate of system performance decline. Based on Theorem 1, the interval controller is reconstructed to compensate for the impact of other faults on the system.

According to the delay closed-loop fault system (9), the compensation method for the interval controller $u_p(t)$ is

$$u(t) = u_p(t) - F^+ E \hat{f}(t), \quad (53)$$

where F^+ is the right pseudo-inverse of B and satisfies the conditions of $(I - FF^+)E = 0$ and $E = -B$. Substitute equation (53) into equation (3) to get

$$\begin{cases} \dot{x}(t) = Ax(t) + \delta_0 BKx_{\tau_1} + (1 - \delta_0)BKx_{\tau_2} + (\delta - \delta_0) \\ \quad \cdot BK(x_{\tau_1} - x_{\tau_2}) - BKe_x(i_k h) + \widehat{D}\widehat{w}(t), \\ y(t) = C_{11}x(t), \\ z(t) = C_{12}x(t). \end{cases} \quad (54)$$

Let $e_f = f - \hat{f}$. Then, Theorem 1 can get e_f asymptotic convergence, so there are $\widehat{D} = [D \ -B]$ and $\widehat{w} = [w^T \ e_f^T]^T$.

Theorem 3. For the closed-loop system (54), given positive numbers $d_1, d_2, \mu_1, \mu_2, \gamma_1', \gamma_2', \delta_0$, and $\sigma \in [0, 1]$, if symmetric positive definite matrices $X, V, Y_i (i = 1, 2, 3, 4)$, and $R_j (j = 1, \dots, 6)$ exist to satisfy inequalities (55) and (56), then the system has both γ_1' and γ_2' performance and mean square stability.

$$H = \begin{bmatrix} H_{11} & H_{12} & H_{13} & H_{14} & 0 & H_{16} & H_{17} & H_{18} & H_{19} & H_{110} & H_{111} \\ * & H_{22} & H_{23} & 0 & 0 & 0 & 0 & 0 & H_{29} & H_{210} & 0 \\ * & * & H_{33} & H_{34} & H_{35} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & H_{44} & H_{45} & 0 & 0 & 0 & H_{49} & H_{410} & 0 \\ * & * & * & * & H_{55} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & H_{66} & 0 & 0 & H_{69} & H_{610} & 0 \\ * & * & * & * & * & * & H_{77} & 0 & H_{79} & H_{710} & 0 \\ * & * & * & * & * & * & * & H_{88} & H_{89} & H_{810} & 0 \\ * & * & * & * & * & * & * & * & H_{99} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & H_{1010} & 0 \\ * & * & * & * & * & * & * & * & * & * & H_{1111} \end{bmatrix} < 0, \quad (55)$$

$$\begin{bmatrix} X & XC_{12}^T \\ * & \gamma_2'^2 I \end{bmatrix} > 0, \quad (56)$$

where

$$\begin{aligned} H_{11} &= AX + XA^T + 2X - R_1 - R_2 + R_3, \\ H_{12} &= \delta_0 BY_i - R_3 + R_5, \\ H_{13} &= 2X - R_5, \\ H_{14} &= (1 - \delta_0)BY_i, \\ H_{16} &= -BY_i, \\ H_{17} &= D, \\ H_{18} &= -B, \\ H_{19} &= d_1 XA^T, \\ H_{110} &= (d_2 - d_1)XA^T, \\ H_{111} &= XC_{11}^T, \\ H_{22} &= \delta_0 \sigma [2X - V] - (1 - \mu_1)[2X - R_1] + 2R_3 - R_5 - R_5^T, \\ H_{23} &= -R_3 + R_5, \\ H_{29} &= \delta_0 d_1 Y_i^T B^T, \\ H_{210} &= \delta_0 (d_2 - d_1)Y_i^T B^T, \\ H_{33} &= -4X + R_3 + R_4, \\ H_{34} &= -R_4 + R_6, \\ H_{35} &= 2X - R_6, \\ H_{44} &= \sigma (1 - \delta_0)[2X - V] - (1 - \mu_2)[2X - R_2] + 2R_4 - R_6 - R_6^T, \\ H_{45} &= -R_4 + R_6, \\ H_{49} &= (1 - \delta_0)d_1 Y_i^T B^T, \\ H_{410} &= (1 - \delta_0)(d_2 - d_1)Y_i^T B^T, \\ H_{55} &= -2X + R_4, \\ H_{66} &= -2X + V, \\ H_{69} &= -d_1 Y_i^T B^T, \\ H_{610} &= -(d_2 - d_1)Y_i^T B^T, \\ H_{77} &= -\gamma_1'^2 I, \\ H_{79} &= d_1 D^T, \\ H_{710} &= (d_2 - d_1)D^T, \\ H_{88} &= -\gamma_1'^2 I, \\ H_{89} &= -d_1 D^T, \\ H_{810} &= -(d_2 - d_1)D^T, \\ H_{99} &= -R_3, \\ H_{1010} &= -R_4, \\ H_{1111} &= -I. \end{aligned}$$

The proof process is similar to Theorem 2, and this section will not go into details.

4. Simulation Examples and Results

In order to verify the effectiveness of the control method described in this paper, the numerical model of reference [21] is cited, and the correlation matrix is as follows:

$$\begin{aligned} A &= \begin{bmatrix} -1.3 & -0.5 \\ 0.7 & -1.8 \end{bmatrix}, \\ B &= \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}, \\ C_{11} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \\ C_{12} &= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \\ D &= \begin{bmatrix} 0.4 \\ 0 \end{bmatrix}, \\ w(t) &= \begin{cases} \cos(2 * \pi i * t) * \exp(-0.2 * t), & 6 \leq t \leq 12, \\ 0, & \text{else.} \end{cases} \end{aligned} \quad (58)$$

Assuming the initial state of the system is $x(0) = [2 \ -2]^T$, the scalar parameters selected in the theorem are $d_1 = 0.5$, $d_2 = 0.8$, $\mu_1 = 0.1$, $\mu_2 = 0.6$, $\delta_0 = 0.75$, $a = 1$, $b = 0.5$, $\gamma_1 = 1.1$, $\gamma_2 = 0.6$, $\gamma_1' = 1.9$, and $\gamma_2' = 0.6$.

Considering the minor fault $L_0 = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.9 \end{bmatrix}$ and unknown fault $f_0 = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}$, the correlation matrix of the fault detection observer can be obtained as

(57)

FIGURE 2: Schematic diagram of scheduling mechanism.

$$G = \begin{bmatrix} 0.1941 & -0.0307 \\ -0.0128 & 0.0199 \end{bmatrix}, \quad (59)$$

$$W = \begin{bmatrix} -0.1141 & -0.1163 \\ -0.1163 & -0.0589 \end{bmatrix},$$

according to Theorem 1.

Similarly, select interval prior faults

$$L_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$L_2 = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}, \quad (60)$$

$$L_3 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix},$$

$$L_4 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},$$

to obtain the event-triggered weight matrix Γ and interval fault-tolerant controller K_i ($i = 1, 2, 3, 4$) according to Theorem 2 as follows:

$$\Gamma = \begin{bmatrix} 5.1368 & 0.1529 \\ 0.1529 & 4.0107 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} -0.5648 & -0.1028 \\ -0.1046 & 0.0113 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -0.7060 & -0.1284 \\ -0.1308 & 0.0141 \end{bmatrix}, \quad (61)$$

$$K_3 = \begin{bmatrix} -1.1297 & -0.2055 \\ -0.2093 & 0.0226 \end{bmatrix},$$

$$K_4 = \begin{bmatrix} -2.8242 & -0.5138 \\ -0.5232 & 0.0566 \end{bmatrix}.$$

FIGURE 3: Fault estimation diagram.

FIGURE 4: Response curve of state x_1 .

The following is the simulation diagram of the observer estimation fault and the effect of different control strategies for the prior faults and other faults within the interval.

If the system runs to $t = 6$ s, a minor fault L_0 occurs; when it runs to $t = 12$ s, an unknown fault f_0 occurs. It can be seen from Figure 3 that the observer designed in this paper can effectively detect and estimate the fault size, laying a foundation for the subsequent start-up scheduling mechanism and controller reconstruction. Figures 4 and 5, respectively, show the fault-tolerant effects of state x_1 and state x_2 in three control modes (T1 is a traditional passive fault-tolerant controller, T2 is an interval fault-tolerant controller, and T3 is an interval hybrid fault-tolerant controller). When $t = 6$ s, a prior fault occurs within the interval, and all three control methods have good robustness to the fault. However, since T2 requires the online dispatching controller and T3 to compensate for the fault, these two control methods will consume part of the time, so the stable trend is slowed down. When $t = 12$ s, other faults occur in the interval. Since T1 cannot effectively tolerate faults for unknown faults, it is finally in a state of instability. By contrast, T2 reduces conservatism and slows down the rate

FIGURE 5: Response curve of state x_2 .

FIGURE 6: Transmission time and transmission interval diagram of NCS under DETCS.

of system deterioration, providing a reliable guarantee for T3's compensation. Based on the above analysis, interval hybrid fault-tolerant control T3 enables the system to have certain dynamic performance and meet the performance index H_2/H_∞ .

Figure 6 is the partial interception diagram of data transmission time and transmission interval under the sampling period $h = 0.05$ s and $\sigma = 0.36$ of the NCS. The system sampled data for a total of 400 times under the periodic time-triggered communication scheme (PTTCS), and the data in the figure were transmitted for only 245 times. It follows that the DETCS acting on the NCS can effectively save network space.

5. Conclusion

In this paper, an interval fault-tolerant control method with less conservatism is proposed based on the hybrid control idea. Firstly, a closed-loop fault model with Bernoulli 0-1 distribution is constructed according to the delay information. On this basis, a fault-tolerant controller set with little conservatism is designed off-line by using prior interval faults. Secondly, the fault size is estimated by the fault detection observer so as to dispatch the corresponding interval fault-tolerant controller rapidly. Finally, the interval

compensation algorithm is designed to satisfy the system's mean square stability and H_2/H_∞ performance index in the case of failure. The simulation example shows that if the prior fault occurs within the interval, the pre-established interval fault-tolerant controller can be quickly invoked, and if other faults occur within the interval, the control method of compensating the interval fault-tolerant controller is effective. In addition, compared to the PTTCS, the DETCS can better meet the requirements of efficient data transmission when network resources are limited to improve the utilization of network space.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This research was sponsored by the National Natural Science Foundation of China (Grant nos. 61463030 and 61563031).

References

- [1] H.-Q. Xiao, Y. He, M. Wu, and S.-P. Xiao, " H_∞ output tracking control for sampled-data networked control systems in T-S fuzzy model," *Acta Automatica Sinica*, vol. 41, no. 3, pp. 661–668, 2015.
- [2] J. D. Stefanovski, "Fault tolerant control of descriptor systems with disturbances," *IEEE Transactions on Automatic Control*, vol. 63, no. 3, pp. 976–988, 2018.
- [3] J. H. Fan, Z. Q. Zheng, and Y. M. Zhang, "Robust fault-tolerant control against time-varying actuator faults and saturation," *IET Control Theory & Applications*, vol. 6, no. 14, pp. 2198–2208, 2012.
- [4] Y. Li, Z. Ma, and S. Tong, "Adaptive fuzzy output-constrained fault-tolerant control of nonlinear stochastic large-scale systems with actuator faults," *IEEE Transactions on Cybernetics*, vol. 47, no. 9, pp. 2362–2376, 2017.
- [5] C. Liu, K. Qian, Y.-H. Li, and Q. Ding, "Integrated robust active fault tolerance controller design based on linear matrix inequality," *Control and Decision*, vol. 33, no. 1, pp. 53–59, 2018.
- [6] Z.-H. Wang and Y. Shen, "Fault estimator design for descriptor systems in finite frequency domain," *Acta Automatica Sinica*, vol. 44, no. 3, pp. 545–554, 2018.
- [7] J. Jiang and X. Yu, "Fault-tolerant control systems: a comparative study between active and passive approaches," *Annual Reviews in Control*, vol. 36, no. 1, pp. 60–72, 2012.
- [8] S. W. Su, B. Jie, and P. L. Lee, "A hybrid active-passive fault-tolerant control approach," *Asia-Pacific Journal of Chemical Engineering*, vol. 1, no. 1-2, pp. 54–62, 2010.
- [9] X. Yu and J. Jiang, "Hybrid fault-tolerant flight control system design against partial actuator failures," *IEEE Transactions on Control Systems Technology*, vol. 20, no. 4, pp. 871–886, 2012.
- [10] J. Wang, X. Yao, and W. Li, "Hybrid active-passive robust fault-tolerant control of event-triggered nonlinear NCS," *The Open Electrical & Electronic Engineering Journal*, vol. 11, no. 1, pp. 68–86, 2017.

- [11] J. Wang, X.-W. Yao, and W. Li, "Hybrid fault-tolerant control method research of NNCS," *Control and Decision*, vol. 33, no. 2, pp. 316–329, 2018.
- [12] Y.-J. Li, F.-Q. Deng, and Y.-J. Peng, "Robust delay probability distribution stability of linear stochastic systems with time-varying delay," *Control Theory & Applications*, vol. 28, no. 7, pp. 1015–1020, 2011.
- [13] Y.-H. Li and D. Wu, "Robust L_2 - L_∞ control for nonlinear networked control systems with random time delays," *Control Theory & Applications*, vol. 34, no. 7, pp. 931–937, 2017.
- [14] K. Subramanian, P. Muthukumar, and Q. Zhu, "Exponential stability of impulsive systems with random delays under sampled-data control," *IET Control Theory & Applications*, vol. 11, no. 16, pp. 2834–2847, 2017.
- [15] C. Wang, C.-X. Fan, and L.-S. Gong, "Fault tolerant synchronization for a general complex dynamical network with random delays," *Journal of Harbin Institute of Technology (New Series)*, vol. 24, no. 1, pp. 51–56, 2017.
- [16] H. Zhang, J. Hu, and X. Yu, "Adaptive sliding mode fault-tolerant control for a class of uncertain systems with probabilistic random delays," *IEEE Access*, vol. 7, pp. 64234–64246, 2019.
- [17] W. Li and Y.-J. Li, "Collaborative design of fault regulation and communication satisfaction of non-uniform transmission networked control system based on discrete event-triggered communication mechanism," *Journal of Jilin University (Engineering Edition)*, vol. 48, no. 1, pp. 245–258, 2018.
- [18] B. Wu, C.-L. Wang, J.-F. Xu, and Y.-J. Hu, "Stability criteria for time-varying delay systems with nonlinear perturbations," *Control Theory & Applications*, vol. 34, no. 5, pp. 692–700, 2017.
- [19] Y.-C. Liu, B. Liu, Y. Zhang, and W.-G. Li, "Stability analysis for networked control systems with bilateral random delay and packet dropout," *Control and Decision*, vol. 32, no. 9, pp. 1565–1573, 2017.
- [20] F. Yang, H. Zhang, and Y. Wang, "An enhanced input-delay approach to sampled-data stabilization of T-S fuzzy systems via mixed convex combination," *Nonlinear Dynamics*, vol. 75, no. 3, pp. 501–512, 2014.
- [21] J. Wang, S.-Z. Li, and W. Li, "Hybrid active-passive robust fault-tolerant control for a networked control system based on an event-triggered scheme," *Information and Control*, vol. 46, no. 2, pp. 144–152, 2017.