

Research Article

Parameter Optimization of Ultrafine Comminution Based on Analytic Hierarchy Process: Fuzzy Comprehensive Evaluation

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This paper proposes a fuzzy comprehensive evaluation of ultrafine powders, namely, yield and quality value-based feature selection. Three indicators reflecting product yield and quality were selected to construct a simple and practical fuzzy comprehensive evaluation protocol. The weight set of the indices and the fuzzy evaluation set were calculated based on the analytic hierarchy process (AHP) method. The fuzzy comprehensive evaluation value was worked out as the only comprehensive index for the evaluation of product. The best ultrafine comminution condition will be established through the comparison of the fuzzy comprehensive evaluation values. Single-factor experiments and orthogonal experiments of the main influencing factors of ultrafine comminution were conducted. It was concluded that the importance of each factor is sequentially the concentration, specific surface area (SSA) of the media, and percentage of critical speed (PCS). Moreover, the concentration and SSA of the media were equally important. Ultrafine comminution by ball mill had the best overall performance under the PCS of 85%, the SSA of the media of $0.24 \text{ m}^2/\text{kg}$, and the concentration of 75%.

1. Introduction

Grinding has been utilized in manufacturing fine and ultrafine powders for the development of new materials and for improving product quality [1, 2]. The grinding technology can significantly affect the particle characteristics [3], but not a single-objective process [4]. It is difficult to pulverize the particles directly to required particle size, which generally includes pulverization and classification or prepulverization and ultrafine pulverization [5]. The grinding optimization has been studied in the fields of cement production, chemical industry, metallurgical fine grinding, mineral grinding, and other industries [6, 7]. It can be seen that the proportion of the grinding cost is a large part [8, 9]. Therefore, the study on grinding optimization is valuable and significant, so as to make the equipment have a better grinding response.

In order to solve the problem of multiobjective optimization, many scholars have established comprehensive

evaluation criteria or methods, such as power coefficient measurement method, constraint method, and failure mode and effect analysis (FMEA) method [10, 11]. The optimization of the mineral ultrafine grinding process is a multiobjective optimization problem [4, 12]. In many cases, the unit of measurement of each index is different. Therefore, it is difficult to objectively evaluate whether the optimized multiobjective problem is good or not.

Fuzzy set theory, introduced by Zadeh [13], resembles human reasoning in its use of approximate information and uncertainty to generate decisions. It was specifically designed to provide formalized tools for dealing with the imprecision intrinsic to many problems [14]. The fuzzy comprehensive evaluation method has been well applied in many fields [15, 16]; however, it is less involved in the field of mineral crushing. In this study, the fuzzy comprehensive evaluation system, which is based on the analytic hierarchy process, was introduced into the field of ultrafine grinding;

comprehensive indicators were used to optimize the ultrafine grinding process of mineral powders. The study aimed at optimizing the three main factors (the mass of powders smaller than $10\text{ }\mu\text{m}$, the fractal dimension of particle size distribution and d_{97}) affecting the ultrafine grinding of mineral powders.

2. Materials and Methods

2.1. Materials. Potassium feldspar powders were used in this study, which were collected in Jiangsu Province. The d_{97} of sample was $63.68\text{ }\mu\text{m}$, and the particle size of smaller than $10\text{ }\mu\text{m}$ was about 35.32%, which was measured by a laser particle size analyzer (Model: BT-9300H). Sodium polyacrylate dispersant was purchased from the Chengxin Chemical Material Supply Station in Zhejiang Province.

2.2. Experiments. The samples were ground in a steel tank ball mill using the wet ultrafine pulverization method, by the ball grinder, with the rated power of 1000 W and the critical speed of 85 rpm. The ceramic tank with the inner diameter of 250 mm, the inner height of 275 mm, and the volume of 13.5 L were used. Two kinds of steel balls with the sizes of 12 mm and 17 mm were used in this experiment. Refer to previous research experiments [17]; the mass of ore powders was fixed at 3.66 kg at a slurry's mass concentration of 55% and the amount of dispersant was of 0.2% of the mineral powder weight. The specific surface area of the grinding media that consists of 6.6 kg of 12 mm steel balls and 28 kg of 17 mm steel balls was $0.20\text{ m}^2/\text{kg}$. The operational percentage of critical speed was 95%. The relevant data of the product ultrafine pulverized after 3 h were analyzed. A single-factor experiment was designed to analyze the main influence factors of the ball mill (rotation rate, SSA of the media, and concentration) in turn, and then the orthogonal experiments were designed to find the optimal process conditions.

2.3. Calculation of the Weight Based on AHP. Three indicators reflecting product quantity and quality were selected to construct a simple and practical fuzzy comprehensive evaluation scheme for ultrafine powders. The mass of powders smaller than $10\text{ }\mu\text{m}$ (m_{-10}) was used to measure the yield. The fractal dimension of particle size distribution (PSD) and d_{97} were chosen to measure the quality of the production.

The analytic hierarchy process established by Saaty [18] is a decision-making method combined qualitatively and quantitatively [19, 20]. The function $f(x, y)$ indicates how important the indicator x is compared to the indicator y ; the function $f(y, x)$ indicates how important the indicator y is compared to the indicator x . The values of $f(x, y)$ and $f(y, x)$ can be defined according to subjective judgment. If x is as important as y , $f(x, y) = 1$; if x is especially more important than y , $f(x, y) = 9$, and so on. $f(x, y) \in [1, 9]$; $f(y, x) \in [1/9, 1]$.

Three indicators were compared pairwise, and the description matrix of importance is given as follows:

$$\begin{bmatrix} \text{Equal} & \text{Evident} & \text{Between equal and evident} \\ \text{Equal} & \text{Equal} & \text{Equal} \\ \text{Equal} & \text{Equal} & \text{Equal} \end{bmatrix}. \quad (1)$$

This description matrix is converted into a judgment matrix

$$A = (a_{ij})_{3 \times 3} = \begin{bmatrix} 1 & 5 & 4 \\ 0.2 & 1 & 1 \\ 0.25 & 1 & 1 \end{bmatrix}. \quad (2)$$

The judgment matrix is usually not a consistent matrix, and the consistency ratio should be calculated. The first eigenvalue of A can be calculated according to the following equation:

$$\lambda_1 = \frac{1}{n} \sum_{i=1}^n \sum_{j=1}^n a_{ij} w_j, \quad (3)$$

where $\lambda_1 = 3.006$.

The consistency index CI of A can be calculated according to the following equation:

$$CI = \frac{-1}{n-1} \sum_{i=2}^n \lambda_i = \frac{\lambda_1 - n}{n-1}, \quad n = 3, \quad (4)$$

where $CI = 0.003$.

The consistency ratio CR can be calculated according to

$$CR = \frac{CI}{RI}. \quad (5)$$

If $CR \leq 0.1$, it can be acceptable. When potential conflict emerges in evaluation [21], the CR is unacceptable and the decision-maker is encouraged to repeat the pairwise comparisons, or some approaches should be adopted to process highly conflicting data [22, 23].

From the relevant chart [24, 25], the index RI of the 3-dimensional judgment matrix is 0.58:

$$CR = \frac{0.003}{0.58} = 0.005 < 0.1. \quad (6)$$

Therefore, the judgment matrix A is reliable and consistent.

According to the AHP theory, the weight vector is calculated using the power method, sum method, root method, and the characteristic roots method, among which the sum method is the simplest. In the case where the judgment matrix is consistent, each calculation method can obtain an accurate solution. The sum method was selected to calculate the weight vector in this study. The weight of each indicator is obtained through related calculation formulas. The weight of m_{-10} (w_1) is 0.69, the weight of D (w_2) is 0.149, and the weight of d_{97} (w_3) is 0.161, so the weight vector can be represented as $W = (0.69, 0.149, 0.161)^T$.

2.4. Comprehensive Evaluation. The degree of membership μ_1 of index m_{-10} value was calculated by membership function $f(m_{-10})$, degree of membership μ_2 of index D value

calculated by membership function $f(D)$, and degree of membership μ_3 of index d_{97} value calculated by membership function $f(d_{97})$, respectively. The membership functions $f(m_{-10})$, $f(D)$, and $f(d_{97})$ are constructed in Section 3.1.1. The weight of m_{-10} (w_1), the weight of D (w_2), and the weight of d_{97} (w_3) was calculated according to the AHP theory.

The construction process of the fuzzy comprehensive evaluation is shown in Figure 1.

3. Results and Discussion

3.1. Application of Comprehensive Evaluation

3.1.1. Evaluation Set and Membership Functions. The initial experiment conditions were set first. The ore powder slurry was prepared at 55% solid concentration by mass. SSA of the grinding media was $0.20 \text{ m}^2/\text{kg}$. The additive amount of dispersant was 0.2% of mineral powder. The PCS of the tank was 95%. The result shows that $m_{-10} = 2.656 \text{ kg}$, $D = 2.225$. According to PSD curve calculation, it can be calculated that $d_{97} = 25.36 \mu\text{m}$. Therefore, the evaluation set $U = (m_{-10}, D, d_{97}) = (2.656, 2.225, 25.36)$.

In the raw material, m_{-10} was 1.293 kg . And, the total mass of the raw material was 3.66 kg , so the value range of the mass $m_{-10} \in [1.293, 3.66]$. From geometric knowledge [26], the range of $D \in [1, 3]$.

Ideally, when all raw mineral powders are pulverized to $-10 \mu\text{m}$, d_{100} was $10 \mu\text{m}$ and d_{97} was less than $10 \mu\text{m}$. In this

study, d_{97} could be approximated as d_{100} , and thus $d_{97} \approx 10 \mu\text{m}$. In the worst case, there were no fresh $-10 \mu\text{m}$ powders formed in the product, so the upper limit of d_{97} was $63.68 \mu\text{m}$. Therefore, $d_{97} \in [10, 63.68]$.

The kinetic equation [27], which has been revised by Aliavden [28], used to describe the grinding process of materials is obtained as the following equation:

$$y(t) = y_0 \exp(-kt^n), \quad (7)$$

where y_0 is the initial sieve residue of ground materials with a certain particle size; k is the grinding rate constant; and n is the time index and determined by the property of ground material and its grinding conditions.

The comminution kinetics equation is as follows:

$$y_{+10}(t) = 84.76e^{-0.64t^{0.66}}. \quad (8)$$

With the propelling of the ultrafine pulverization, the difficulty became greater. The second-degree parabolic membership function, which corresponded to pulverization kinetic curve [29], was selected. Because the indicator m_{-10} was a benefit-type indicator, the larger the value, the better the product. While D and d_{97} were the cost-type indicators, the smaller the value, the better the product. Therefore, the membership functions used for the three indicators were different. The following formulas (equation (6)) are the membership functions of the indicators m_{-10} , D , and d_{97} :

$$f(m_{-10}; 1.293, 3.66, 2) = \begin{cases} 0, & m_{-10} \leq 1.293, \\ \left(\frac{m_{-10} - 1.293}{2.367}\right)^2, & 1.293 < m_{-10} < 3.66, \\ 1, & m_{-10} \geq 3.66, \end{cases}$$

$$f(D; 1, 3, 2) = \begin{cases} 1, & D \leq 1, \\ \left(\frac{3 - D}{2}\right)^2, & 1 < D < 3, \\ 0, & D \geq 3, \end{cases}$$

$$f(d_{97}; 10, 63.68, 2) = \begin{cases} 1, & d_{97} \leq 10, \\ \left(\frac{(63.68 - d_{97})}{53.68}\right)_2, & 10 < d_{97} < 63.68, \\ 0, & d_{97} \geq 63.68. \end{cases}$$
(9)

3.1.2. Fuzzification and Comprehensive Evaluation. The values of indicators were changed into membership degrees μ , $\mu \in [0, 1]$. Take one indicator of them, m_{-10} , as an

example. In the product, $m_{-10} = 2.656 \text{ kg}$, and $1.293 < 2.656 < 3.66$, so the membership degree of m_{-10} could be obtained by equation (6).

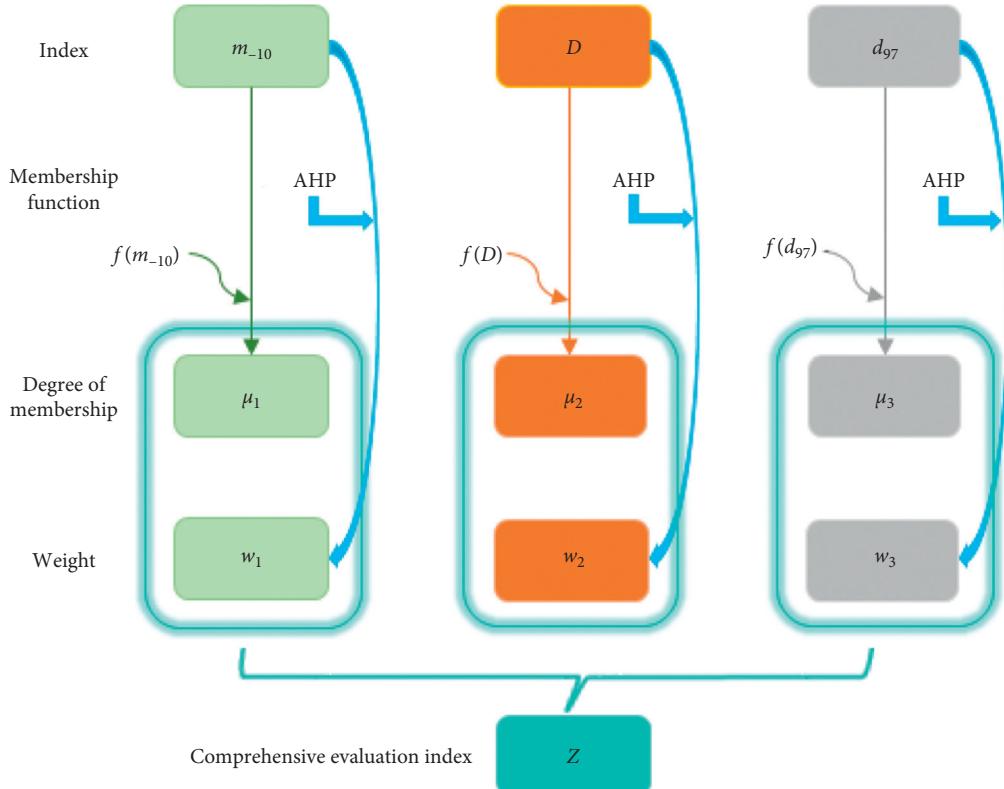


FIGURE 1: Flow chart of fuzzy comprehensive evaluation.

$$\mu_1 = f(2.656; 1.293, 3.66, 2) = \left(\frac{(2.656 - 1.293)^2}{2.367} \right) = 0.332. \quad (10)$$

In turn, the membership degrees μ_2 and μ_3 of D and d_{97} could be calculated as 0.15 and 0.51, respectively. The fuzzy evaluation set of each indicator is $U = (0.332, 0.15, 0.51)$:

$$Z = U_{1 \times 3} W_{3 \times 1}. \quad (11)$$

According to equation (7), the multi-indicator fuzzy comprehensive evaluation value Z can be calculated from the fuzzy evaluation set U and weight set of each indicator W , where $U = (0.332, 0.15, 0.51)$ and $W = (0.69, 0.149, 0.161)^T$; therefore, $Z = 0.334$.

3.2. Effect of Factors on the Ultrafine Powders. The single-factor experiment was designed to find the influence of single factor. Then, the orthogonal experiments were followed to analyze the best process conditions. Taguchi L₉(3⁴) orthogonal arrays were generated by the IBM SPSS Statistics 25.0.

3.2.1. Effect of PCS on the Ultrafine Powders. The best parameters of PCS were explored under the conditions where the operational concentration was 55% and SSA of the media was 0.24 m²/kg. The results are shown in Table 1.

When the experimental PCS was set to 85%, the effect of ultrafine pulverization of mineral powders was best. With a

gradual increase in the PCS, the fuzzy comprehensive evaluation value of the experiment results increased rapidly. After the curve reached the highest point, it gradually decreased, but the decrease of Z value was not as rapid as before. When the PCS exceeded 100%, the fuzzy comprehensive evaluation value of the results also dropped rapidly.

With the increase of the PCS, the movement of balls was changed from the principal tumbling form to the combinative impacting form and the tumbling form. The continuous increase of the PCS could cause the media to adhere to the wall, and the force on the particles would reduce. Changes of PCS changed the force distribution (impact force, shear force, frictional force, etc.) [30] between media and powders. The synthetic effect of several forces according to a certain distribution ratio made the possibility of particle refinement increase; thus, the effect was better.

3.2.2. Effect of SSA of the Media on the Ultrafine Powders. The best parameters of SSA of the media were explored under the conditions where the operational concentration was 55% and the PCS was 85%. The different SSAs of the media were achieved by adjusting the ratio of the two different-sized media. The results are shown in Table 2.

When the SSA of the media was 0.22 m²/kg, ultrafine pulverization of mineral powders performs best. Small SSA of the media would greatly reduce the number of point contact among the balls, and the chance of the contact of slurry with grinding balls would be dramatically reduced [31]. During the movement of the media, although the

TABLE 1: Variation of product feature at different speeds.

Product feature PCS (%)	m_{-10} (kg)	D (-)	d_{97} (μm)	Z (-)
65	1.867	2.291	35.12	0.105
75	2.154	2.263	32.68	0.165
85	2.433	2.158	25.57	0.267
95	2.447	2.325	29.14	0.248
105	2.079	2.259	33.71	0.147

TABLE 2: Variation of product features at different SSAs of the media.

Product feature SSA of the media (m^2/kg)	m_{-10} (kg)	D (-)	d_{97} (μm)	Z (-)
0.20	2.190	2.262	33.07	0.172
0.22	2.501	2.257	28.65	0.269
0.24	2.433	2.158	25.57	0.267
0.26	2.410	2.201	29.41	0.243

impact force of a single ball on the material was increased, the frictional force and shear force were greatly reduced. The SSA of the media continued to increase, and the fuzzy comprehensive evaluation value of the product did not decrease greatly, which indicates that the impact force was not the main force. The friction force and shear force were the main forces in the pulverization process. In the late stage of ultrafine pulverization, the particles became round and the edges and corners basically disappeared.

3.2.3. Effect of Concentration on the Ultrafine Powders. The optimal concentration was explored under the conditions where the operational PCS was 85% and SSA of the media was $0.22 \text{ m}^2/\text{kg}$. The results are shown in Table 3.

When the concentration was 65%, the ultrafine pulverization of mineral powders works best. When the concentration was low, the PSD of product was narrow; however, the yield of product was low. In reverse, the PSD of product was extremely wide; although the yield of product had advantages, there were many large particles in the product, which affected the total quality. If the concentration was low, there were more opportunities for the occurrence of contact between the balls and the particles, but the contact between the particles was insufficient. When the concentration was too high, the liquidity of the slurry reduced sharply and the motion of balls and particles were obstructed, leaving some large particles uncrushed. According to “Bed comminution” [32], when there were more particles, there were more “particle-particle contact”; smaller particles would definitely be produced under the same pulverization condition. Therefore, an appropriate increase in the concentration would increase the fuzzy comprehensive evaluation value of the product.

3.2.4. Optimization Parameters. As shown in Table 4, the fuzzy comprehensive evaluation values of products were calculated. Due to the change in the concentration, the range of m_{-10} changed. Thus, $m_{-10} \in [1.293, 5.49]$.

The sequence of the factors influencing the product feature was listed by polar value analysis. With the indicator m_{-10} as the appraising indicator, maximum differences (R) of each factor were calculated orderly. R (concentration) = 0.994, R (PCS) = 0.227, R (SSA of the media) = 0.330; the factors were ranked in order of importance, and concentration > SSA of the media > PCS. With the indicator D as the appraising indicator, R (concentration) = 0.181, R (PCS) = 0.130, R (SSA of the media) = 0.163, and concentration > SSA of the media > PCS. With the indicator d_{97} as the appraising indicator, R (concentration) = 17.817, R (PCS) = 6.343, R (SSA of the media) = 5.233, and concentration > PCS > SSA of the media. Obviously, the significance ordering of the factors was contradictory when choosing different appraising indicators. With the indicator Z as the appraising indicator, R (concentration) = 0.054, R (PCS) = 0.039, and R (SSA of the media) = 0.053. Concentration > SSA of the media > PCS; moreover, the concentration and SSA of the media were equally important.

Using the fuzzy comprehensive evaluation value as an indicator, combined with polar value analysis, the optimal process conditions were obtained. The optimal PCS should be at 85%; the optimal SSA of the media should be at $0.24 \text{ m}^2/\text{kg}$ and the optimal concentration should be at 75%.

Under the optimal conditions, a verification experiment was conducted to analyze the relevant parameters of the product, and the results are shown as follows: $m_{-10} = 3.415 \text{ kg}$, $D = 2.483$, and $d_{97} = 47.96 \mu\text{m}$. Based on the membership function used in the orthogonal experiments, the membership degrees of each indicator were calculated,

TABLE 3: Variation of product feature at different concentrations.

Product feature Concentration (%)	m_{-10} (kg)	D (-)	d_{97} (μm)	Z (-)
45	1.913	2.154	23.98	0.145
55	2.433	2.158	25.57	0.181
65	2.674	2.258	30.14	0.182
75	2.601	2.317	46.79	0.124

TABLE 4: Results of the orthogonal experiments.

No.	Concentration (%)	PCS (%)	SSA of the media (m^2/kg)	m_{-10} (kg)	D (-)	d_{97} (μm)	Z (-)
1	55	75	0.20	1.791	2.472	36.57	0.061
2	55	85	0.22	2.501	2.257	28.65	0.146
3	55	95	0.24	2.447	2.325	29.14	0.136
5	65	75	0.22	2.583	2.464	39.21	0.109
4	65	85	0.24	2.674	2.258	30.14	0.158
6	65	95	0.20	2.464	2.532	40.45	0.092
7	75	75	0.24	3.267	2.472	51.33	0.172
8	75	85	0.20	3.148	2.54	49.29	0.154
9	75	95	0.22	3.308	2.587	47.19	0.181

$\mu_1 = 0.256$, $\mu_2 = 0.067$, and $\mu_3 = 0.086$. Finally, the maximum fuzzy comprehensive evaluation value under the optimal process parameters was obtained, $Z = 0.200$. Therefore, the optimal operational factors obtained by polar value analysis were correct.

4. Conclusions

- (1) Three indicators reflecting quantity and quality have been selected, and a comprehensive index has been constructed using a fuzzy comprehensive evaluation method based on hierarchical analysis, and this has been used to successfully find the optimal comminution operating conditions.
- (2) The effect of different indicators on the ultrafine powders was analyzed with the AHP. The weight set of three indicators of m_{-10} , D , and d_{97} was $W = (0.69, 0.149, 0.161)^T$.
- (3) The significance ordering of the factors of ultrafine pulverization was concentration, SSA of the media, and PCS. Moreover, the concentration and SSA of the media were equally important.
- (4) When the optimal operating conditions were at the PCS of 85%, the SSA of the media of $0.24 \text{ m}^2/\text{kg}$, and the concentration of 75%, the comprehensive performance of the product was the best and $Z = 0.200$.

Data Availability

The partial data used to support the findings of this study are included within the article, and other partial data are available from the first author upon request.

Additional Points

- (1) The significance of three indicators of the ultrafine powders is analyzed by the analytic hierarchy process

method. (2) According to the three indicators, the fuzzy comprehensive evaluation based on analytic hierarchy process is used to obtain the comprehensive indicator. (3) The three main factors influencing ultrafine comminution, concentration, specific surface area of the media, and percentage of critical speed are significantly ranked by the orthogonal test. The optimal operating conditions have been obtained subsequently.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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